SLAC – PUB – 4632
May 1988
(T/E)

THE TAU DECAY MODE PROBLEM*

MARTIN L. PERL

Stanford Linear Accelerator Center,
Stanford University, Stanford, California 94309

ABSTRACT

The problem of understanding the branching fractions of the 1-charged particle decay modes of the τ lepton is reviewed. The emphasis is on a recent study by K. G. Hayes and M. L. Perl of the statistical validity of the branching fraction measurements. Unconventional explanations of the problem, none of them satisfactory, are also discussed.

Invited talk given at Les Recontres de Physique, de La Valle D'Aoste,
La Thuile, Aoste Valley, Italy, February 28—March 5, 1988

*Work supported by the Department of Energy, contract DE–AC03–76SF00515.

Table of Contents

I. INTRODUCTION AND BASIC CONCEPTS OF TAU LEPTON DECAYS

A. Introduction

At present we do not completely understand the decay modes of the tau lepton. About 87% of τ decays have 1 charged particle, Sec. II. But when we add up the branching fractions of the individual modes which contain 1 charged particle, Sec. III, we can only account for about 81%. This is the tau decay mode problem. We do not know if it is caused by experimental error, by the misuse of conventional theory, or by an unknown phenomena in τ decay.

My colleague Kenneth G. Hayes suggested and led the statistical studies of τ branching ratio measurements which I recount in Sec. IV and V. I am indebted to him for this work.

This paper concludes with Sec. VI, describing some failed attempts to find an unconventional explanation for the τ decay mode problem. Final remarks are in Sec. VII.

To begin, I remind you of the conventional theory of τ decay.

B. Conventional Theory of τ Decay

The conventional theory of τ decays assumes strict lepton conservation

$$\tau^- \rightarrow \nu_\tau + \text{ other particles } \quad ; \tag{1}$$

the numbers and types of other particles being constrained by conventional conservation laws and[1,2]

$$\begin{aligned} \tau \text{ mass } &= 1784.2 \pm 3 \text{ MeV/c}^2 \\ \nu_\tau \text{ mass } &\leq 35 \text{ MeV/c}^2 \end{aligned} \tag{2}$$

The branching fraction B_α for the α decay mode is

$$B_\alpha = \Gamma_\alpha/\Gamma \tag{3}$$

where Γ_α is the decay width for that mode and Γ is the total decay width. I will usually give B_α in per cent.

The widths for the purely leptonic decays

$$\Gamma_e : \quad \tau^- \to \nu_\tau + e^- + \bar{\nu}_e \tag{4a}$$

$$\Gamma_\mu : \quad \tau^- \to \nu_\tau + \mu^- + \bar{\nu}_\mu \tag{4b}$$

are given exactly by weak interaction theory[3] with the constraint from the $e-\mu$ mass difference

$$B_e/B_\mu = \Gamma_e/\Gamma_\mu = 0.973 \tag{5}$$

The calculation[3] of the widths for the simplest hadronic decays

$$\Gamma_\pi : \quad \tau^- \to \nu_\tau + \pi^- \tag{6a}$$

$$\Gamma_K : \quad \tau^- \to \nu_\tau + K^- \tag{6b}$$

requires in addition to weak interaction theory, knowledge of the decay rates for $\pi^- \to \mu^- + \bar{\nu}_\mu$ and $K^- \to \mu^- + \bar{\nu}_\mu$. These rates have been carefully measured, so there are strict predictions for Γ_π and Γ_K. In particular

$$B_\pi/B_e = \Gamma_\pi/\Gamma_e = 0.607 \tag{7}$$

Once the decay mode contains several hadrons there is no presently known exact method to calculate the width in all cases. There is a special method available when the weak decay of the τ can be related to an electron-positron electromagnetic annihilation process through the conserved vector current (CVC) rule. [3,4,5] Thus

$$\Gamma_{\pi^-\pi^0} : \quad \tau^- \to \nu_\tau + \pi^- + \pi^0 \tag{8a}$$

is related to the cross section for

$$e^+ + e^- \to \pi^+ + \pi^- \quad ; \tag{8b}$$

$$\Gamma_{\pi^-3\pi^0} : \quad \tau^- \to \nu_\tau + \pi^- + \pi^0 + \pi^0 + \pi^0 \tag{9a}$$

is reliable to the cross section for

$$e^+ + e^- \to \pi^+ + \pi^- + \pi^+ + \pi^- \quad ; \tag{9b}$$

$$\Gamma_{2\pi^-\pi^+\pi^0} : \quad \tau^- \rightarrow \nu_\tau + \pi^- + \pi^- + \pi^+ + \pi^0 \tag{10a}$$

is related to the cross sections for

$$e^+ + e^- \rightarrow \pi^+ + \pi^- + \pi^+ + \pi^-, \; \pi^+ + \pi^- + \pi^0 + \pi^0 \quad ; \tag{10b}$$

and

$$\Gamma_{\pi^-\pi^0\eta} : \quad \tau^- \rightarrow \nu_\tau + \pi^- + \pi^0 + \eta \tag{11a}$$

is related to the cross section for

$$e^+ + e^- \rightarrow \pi^+ + \pi^- + \eta \tag{11b}$$

Two comments on the previous paragraph: (1) The use of CVC gives Γ_α with an error dependent on the errors in the measurement of the e^+e^- cross section. (2) The decay in Eq. 8a takes place almost completely through

$$\Gamma_\rho : \quad \tau^- \rightarrow \nu_\tau + \rho \rightarrow \nu_\tau + \pi^- + \pi^0 \tag{12}$$

In the literature the entire $\Gamma_{\pi^-\pi^0}$ is replaced by Γ_ρ, and $\sigma(e^+ + e^- \rightarrow \pi^+ + \pi^-)$ is replaced by $\sigma(e^+ + e^- \rightarrow \rho)$ as a definition.[6]

There remain decay modes for which there is no general method for calculating the widths. Examples are:

$$\Gamma_{\pi^-2\pi^0} : \quad \tau^- \rightarrow \nu_\tau + \pi^- + 2\pi^0 \tag{13a}$$

$$\Gamma_{2\pi^-\pi^+} : \quad \tau^- \rightarrow \nu_\tau + \pi^- + \pi^+ + \pi^- \tag{13b}$$

$$\Gamma_{\pi^-2\pi^0\eta} : \quad \tau^- \rightarrow \nu_\tau + \pi^- + 2\pi^0 + \eta \tag{13c}$$

C. Calculation of Γ_{hadron}

Is there a way to exactly calculate the width for all hadronic modes, Γ_{hadron}? There is a crude approximation method[7] and a recent better approximation[8] using QCD. In the crude method the final state interactions of the hadrons are ignored and Γ_{hadron} is calculated by counting quark pairs.[7] This gives

$$\Gamma_{hadrons,quarks}/\Gamma_e = 3 \quad . \tag{14a}$$

yielding

$$B_e \approx 20\%, \; B_\mu \approx 20\%, \; B_{hadron} \approx 60\% \tag{14b}$$

Braaten[8] recently made a QCD calculation. Perturbative QCD replaces Eq. 14a by

$$\Gamma_{hadron,QCD}/\Gamma_e = 3.29 \pm 0.04 \tag{15a}$$

yielding

$$B_e = 19.0\%,\ B_\mu = 18.5\%, B_{hadron} = 62.5\% \tag{15b}$$

Braaten says the non-perturbative contribution is small compared to 3.29 and negative. The errors on the branching fraction predictions in Eq. 15b depend in part upon how well the non-perturbative term can be calculated.

II. DIRECT MEASUREMENTS OF BRANCHING FRACTIONS

A. Topological Branching Fractions and Averaging Method

All measurements of the τ decay modes come from studies of the processes

$$\begin{aligned} &e^+ + e^- \to \tau^+ + \tau^- \\ &\tau^+ \to \bar{\nu}_\tau + \text{ other particles} \\ &\tau^- \to \nu_\tau + \text{ other particles} \end{aligned} \tag{16}$$

The average measured values of the inclusive or topological, branching fractions into 1, 3, 5, or 7 charged particles are[7,9–11]

$$\begin{aligned} B_1 &= (86.6 \pm 0.3)\% \\ B_3 &= (13.3 \pm 0.3)\% \\ B_5 &= (0.10 \pm 0.03)\% \\ B_7 &\le 0.019\% \quad , \quad 90\% \text{ CL} \end{aligned} \tag{17}$$

Before proceeding I will use B_1 to illustrate how the average value and error is obtained for a branching fraction measured many times.[1,9] Table 1 from Ref. 9 lists all measurements of B_1 and B_3. Consider N measurements $y_1,\ y_2 \dots y_N$ with $\sigma_{stat,n}$ and $\sigma_{sys,n}$ the statistical and systematic errors assigned by the experimenters to y_n. Then define the combined error on y_n

$$\sigma_n = \left[\sigma^2_{stat,n} + \sigma^2_{sys,n}\right]^{1/2} \quad ; \tag{18a}$$

and the weight

$$w_n = 1/\sigma_n^2 \tag{18b}$$

The average of the y_n's is

$$y = \sum_{n=1}^{N} w_n y_n \Big/ \sum_{n=1}^{N} w_n \tag{19}$$

Table 1. τ topological branching fractions in per cent. The statistical error is given first, the systematic error second. We list all measurements provided the measurement is described in a preprint, journal article, or Ph.D. thesis authored by the experimenters, and the authors have not stated the measurement is superseded by a more recent measurement.

B_1			B_3		Energy (GeV)	Experimental Group	Reference
Measurement	Combined Error	Weight	Measurement	Combined Error			
70.*	±10.	—	30.†*	±10.	3.6 to 5.0	PLUTO	J.Burmester *et al.* Phys.Lett.**68B**, 297 (1977)
68.†*	±5.	—	32.*	±5.	3.1 to 7.4	DELCO	W.Bacino *et al.* Phys.Rev.Lett.**41**, 13 (1978)
65.†*	±11.	—	35.*	±11.	3.9 to 5.2	DASP	R.Brandelik *et al.* Phys.Lett.**73B**, 109 (1978)
82.†*	±6.5	—	18.*	±6.5	6 to 7.4	MARK I	J.Jaros *et al.* Phys.Rev.Lett.**40**, 1120 (1978)
76.*	±6.	—	24.*	±6.	12 to 31.6	TASSO	R.Brandelik *et al.* Phys.Lett.**92B**, 199 (1980)
84.0	±2.0	.019	15.0	±2.0	32.0 to 36.8	CELLO	H.J.Behrend *et al.* Phys.Lett.**114B**, 282 (1982)
86.0±2.0±1.0*	±2.2	—	14.0±2.0±1.0*	±2.2	29.0	MARK II	C.A.Blocker *et al.* Phys.Rev.Lett.**49**, 1369 (1982)
85.2±0.9±1.5*	±1.7	—	14.8±0.9±1.5*	±1.7	29.0	TPC	H.Aihara *et al.* Phys.Rev.**D30**, 2436 (1984)
85.2±2.6±1.3	±2.9	.009	14.8±2.0±1.3	±2.4	14.0	CELLO	H.J.Behrend *et al.* Z.Phys.**C23**, 103 (1984)
85.1±2.8±1.3	±3.1	.008	14.5±2.2±1.3	±2.6	22.0	CELLO	H.J.Behrend *et al.* Z.Phys.**C23**, 103 (1984)
87.8±1.3±3.9	±4.1	.005	12.2±1.3±3.9	±4.1	34.6 average	PLUTO	Ch.Berger *et al.* Z.Phys.**C28**, 1 (1985)
$84.7\pm1.1^{+1.6}_{-1.3}$	$^{+1.9}_{-1.7}$	.024	$15.3\pm1.1^{+1.3}_{-1.6}$	$^{+1.7}_{-1.9}$	13.9 to 43.1	TASSO	M.Althoff *et al.* Z.Phys.**C26**, 521 (1985)
86.7±0.3±0.6	±0.7	.157	13.3±0.3±0.6	±0.7	29.0	MAC	E.Fernandez *et al.* Phys.Rev.Lett.**54**, 1624 (1985)
86.9±0.2±0.3	±0.4	.482	13.0±0.2±0.3	±0.4	29.0	HRS	C.Akerlof *et al.* Phys.Rev.Lett.**55**, 570 (1985)
86.1±0.5±0.9	±1.0	.077	13.6±0.5±0.8	±0.9	30.0 to 46.8	JADE	W.Bartel *et al.* Phys.Lett.**161B**, 188 (1985)
87.9±0.5±1.2	±1.3	.046	12.1±0.5±1.2	±1.3	29.0	DELCO	W.Ruckstuhl *et al.* Phys.Rev.Lett.**56**, 2132 (1986)
87.2±0.5±0.8	±0.9	.095	12.8±0.5±0.8	±0.9	29.0	MARK II	W.B.Schmidke *et al.* Phys.Rev.Lett.**57**, 527 (1986)
87.1±1.0±0.7†*	±1.2	—	12.8±1.0±0.7*	±1.2	29.0	MARK II	P.R.Burchat *et al.* Phys.Rev.**D35**, 27 (1987)
84.7±0.8±0.6	±1.0	.077	15.1±0.8±0.6	±1.0	29.0	TPC	H.Aihara *et al.* Phys.Rev.**D35**, 1553 (1987)

†Calculated from B_1 or B_3 measurement using $B_1 + B_3 + B_5 = 1.$ with $B_5 = 0.1\%$.

*Not included in average.

I call this the formal average because the definition of σ_n, Eq. 18a, is a formal procedure in the addition by quadrature of $\sigma_{stat,n}$ and $\sigma_{sys,n}$. The formal error is

$$\sigma = \left[\sum_{n=1}^{N} \sigma_n^{-2}\right]^{1/2} = \left[\sum_{n=1}^{N} w_n\right]^{1/2} \tag{20}$$

In computing the formal average of B_1 some measurements are omitted[9] because the same data was used in two publications. Those omitted were MARK II 1982, TPC 1984, and MARK II 1987. Five other measurements were not used, all pre-1981 publications, for reasons given in Ref. 9. This leaves 11 measurements.

It is important to notice that the computed average value of B_1 is somewhat independent of the measurements and weighting method used. The full set of the 11 selected measurements give

$$\text{Full set}: \quad B_1 = (86.6 \pm 0.3)\% \tag{21}$$

We can also select a smaller set of the greatest weight measurements. We use the smallest set whose total weight is larger than $.9^2 = .81$. This will give a formal error no larger than 1.11 of the full set error. The small set contains MAC 1985, HRS 1985, JADE 1985, MARK II 1986 and TPC 1987. We find

$$\text{Small set}: \quad B_1 = (86.6 \pm 0.3)\% \tag{21b}$$

Another example is provided by removing the HRS 1985 measurement[12] which contributes almost half the weight. We find

$$\text{Full set less HRS 1985}: \quad B_1 = (86.3 \pm 0.4)\% \tag{21c}$$

We can also calculate a formal average using just the statistical errors, obtaining:

$$\text{Full set, statistical weights}: \quad B_1 = (86.6 \pm 0.14)\% \tag{21d}$$

$$\text{Small set, statistical weights}: \quad B_1 = (86.7 \pm 0.15)\% \tag{21e}$$

The formal errors are wrong here since they exclude systematic errors.

As a final example I include the five pre-1981 measurements with the full set. Four of the five give smaller B_1 values. This gives

$$\text{Full set plus pre-1981}: \quad B_1 = (86.5 \pm 0.3)\% \tag{21f}$$

There is little change in B_1 because the pre-1981 measurements have large errors.

Thus the formal average of B_1 is not dependent on any single measurement or on $\sigma_{sys,n}$. Looking ahead, the formal averages of B_e, B_μ, B_π, and B_ρ show a similar independence.[9]

B. 1-charged Particle Branching Fractions With Multiple Measurements

As listed in Ref. 9 there are 14 measurements of B_e and 19 measurements of B_μ, some of them constrained by Eq. 5; there are 7 measurements of B_π; and there are 7 measurements of B_ρ. Using the method of Eq. 19 the following formal averages are obtained.[9]

Using only unconstrained measurements of B_e and B_μ

$$B_e = (17.6 \pm 0.4)\% \tag{22a}$$

$$B_\mu = (17.7 \pm 0.4)\% \tag{22b}$$

Using all B_e and B_μ measurements, and requiring from Eq. 5 that $B_\mu = 0.973 B_e$

$$B_e = (18.0 \pm 0.3)\% \tag{23a}$$

$$B_\mu = (17.5 \pm 0.3)\% \tag{23b}$$

Notice that the sum $B_e + B_\mu$ does not change significantly. The other two repeatedly measured branching fractions are

$$B_\pi = (10.8 \pm 0.6)\% \tag{24}$$

$$B_\rho = (22.5 \pm 0.9)\% \tag{25}$$

C. Other Directly Measured 1-charged Particle Branching Fractions

There are three recent measurements[13–15] of

$$B_{\pi 2\pi^0}: \quad \tau^- \to \nu_\tau + \pi^- + 2\pi^0 \quad ; \tag{26}$$

Ref. 15 uses the most persuasive method giving

$$B_{\pi 2\pi^0} = (7.4 \pm 1.4)\% \tag{27a}$$

In this paper I use the formal average

$$B_{\pi 2\pi^0} = (7.6 \pm 0.8)\% \tag{27b}$$

The 1-charged particle decay modes containing one or more K mesons are complicated, App. A, fortunately their total contribution has been measured[16]

$$B_{Kn\pi^0, n\geq 0} = (1.7 \pm 0.3)\% \tag{28}$$

D. 1-Charged Particle Branching Fractions with Upper Limits or Unmeasured

There remain 1-charged particle modes whose branching fractions have only a measured upper limit or are unmeasured. For example the best direct measurement[15] of

$$B_{\pi 3\pi^0}: \quad \tau^- \to \nu_\tau + \pi^- + 3\pi^0 \tag{29}$$

is

$$B_{\pi 3\pi^0} = (0.54 \pm 1.10)\% \quad ;$$

better given as an upper limit

$$B_{\pi 3\pi^0} < 2.5\% \quad , \quad 95\% \text{ CL} \tag{30}$$

The modes

$$\begin{aligned} B_{\pi 4\pi^0}: &\quad \tau^- \to \nu_\tau + \pi^- + 4\pi^0 \\ B_{\pi 5\pi^0}: &\quad \tau^- \to \nu_\tau + \pi^- + 5\pi^0 \end{aligned} \tag{31}$$

with 8 and 10 final photons have not been separated from other modes with large numbers of photons such as

$$\begin{aligned} \tau^- \to \nu_\tau + \pi^- + \eta + n\pi^0 \quad &, \quad n \geq 1 \\ \tau^- \to \nu_\tau + \pi^- + 2\eta + n\pi^0 \quad &, \quad n \geq 0 \end{aligned}$$

where $\eta \to 2\gamma$ or $\eta \to 3\pi^0 \to 6\gamma$. There is no published measurement of an upper limit on $B_{\pi 4\pi^0} + B_{\pi 5\pi^0}$. From studies[13,17–19] of decays with 1-charged particle and many photons I estimate

$$B_{\pi 4\pi^0} + B_{\pi 5\pi^0} < \text{ about } 4\% \quad . \tag{32}$$

There are measured upper limits[15,17–19] on modes containing η mesons obtained using $\eta \to 2\gamma$ or $\eta \to \pi^+ + \pi^- + \pi^0$.

E. Comparison of Directly Measured Branching Fractions

The directly measured branching fractions, measured upper limits and guessed upper limits are in Table 2. Denoting an individual 1-charged particle branching fraction by $B_{1\alpha}$ for the α mode; we expect

$$\sum_\alpha B_{1\alpha} = B_1 \tag{33}$$

Indeed, this is the definition of B_1. Within the errors and upper limits, the measurements in Table 2 are consistent with that definition, that is, Rows N and 0 agree. At present there is no tau decay mode problem if considerations are limited to the information in Table 2.

Table 2. Summary of direct measurements of branching fractions of 1-charged particle modes using only 1-charged particle decays.

Type of Information	Row	Decay Mode	Branching Fraction (%)	Reference
Measured in 1-charged particle decays	A	$\nu_\tau e^- \bar{\nu}_e$	17.6 ± 0.4	9
	B	$\nu_\tau \mu^- \bar{\nu}_\mu$	17.7 ± 0.4	9
	C	$\nu_\tau \pi^-$	10.8 ± 0.6	9
	D	$\nu_\tau \rho^-$	22.5 ± 0.9	9
	E	$\nu_\tau K^- n\pi^0 m K^0_L$ $n \geq 0, m \geq 0$	1.7 ± 0.4	16
	F	$\nu_\tau \pi^- 2\pi^0$	7.6 ± 0.8	Eq. 27b
Sum of rows A–F	G		77.9 ± 1.5	
Upper limit deduced or estimated in 1-charged particle decays	H	$\nu_\tau \pi^- 3\pi^0$	< 2.5	15
	I	$\nu_\tau \pi^- 4\pi^0 + \nu_\tau \pi^- 5\pi^0$	$\lesssim 4.$	Eq. 32
	J	$\nu_\tau \eta$	< 0.3	15
	K	$\nu_\tau \eta n \pi^0$	< 2.1	19
	L	$\nu_\tau 2\eta$	< 1.4	15
Sum of rows H–L	M		$\lesssim 10.3$	
Sum of rows A–F and H–L	N		$\lesssim 88.2 \pm 1.5$	
1-charged particle topological B_1	O		86.6 ± 0.3	9

III. USE OF OTHER DATA AND CONVENTIONAL THEORY

A. Other Data and Theory

The decay mode problem appears when other measurements and conventional theoretical concepts are used to evaluate or set smaller upper limits on the branching fractions in Rows H–L of Table 2. This is the seminal observation of Gilman[4,5] and of Truong.[20] Four methods are used.

In method (a), a directly measured 3-charged particle or 5-charged particle branching fraction is used to set an upper limit on a 1-charged particle branching fraction by invoking strong isospin conservation. For example, direct measurement[10,11] gives

$$B(3\pi^- 2\pi^+ \nu_\tau) = (0.051 \pm 0.020)\%$$

and strong isospin conservation requires

$$B(\pi^- 4\pi^0 \nu_\tau) \leq \frac{3}{4} B(3\pi^- 2\pi^+ \nu_\tau) \quad ,$$

hence

$$B(\pi^- 4\pi^0 \nu_\tau) \leq 0.06\% \quad , \quad 95\% \text{ CL}$$

(My usual notation for $B(\pi^- 4\pi^0 \nu_\tau)$ is $B_{\pi 4\pi^0}$.)

In method (b) the η decay mode

$$\eta \rightarrow \pi^+ + \pi^- + \pi^0$$

is used in the direct measurement of an η containing mode. Then the 1-charged particle branching fraction is deduced. For example the modes

$$\tau^- \rightarrow \nu_\tau + \pi^- + 2\eta + n\pi^0 \quad , \quad n \geq 0$$

can result in 1, 3, or 5 charged particles. The 5-charged particle final states[10,11] are limited by $B_5 = (0.10 \pm 0.03)\%$. This gives the upper limit.[18]

$$B(\pi^- 2\eta n\pi^0 \nu_\tau \rightarrow \text{ 1-charged particle}) \leq 0.6 \quad , \quad 95\% \text{ CL}$$

In method (c) we calculate a 1-charged particle branching fraction using the conserved vector current rule[4,20] and a corresponding e^+e^- cross section; Eqs. 8–11 are examples.

In method (d) the rule against a second class current forbids the decay mode

$$\tau^- \rightarrow \pi^- + \eta + \nu_\tau \tag{34}$$

Table 3 gives the results of these methods.

Table 3. Values and upper limits of branching fractions for 1-charged particle modes deduced from theory and other measurements. The methods are described in Sec. III.

Mode	Method	Value (%)	Upper Limit (%) 95% CL
$\nu_\tau \pi^- 3\pi^0$	c	1.0 ± 0.15	1.25
$\nu_\tau \pi^- 4\pi^0$	a		0.06
$\nu_\tau \pi^- 5\pi^0$	a		0.11
$\nu_\tau \pi^- \eta$	d		0.00
$\nu_\tau \pi^- \eta \pi^0$	c		0.24[a]
$\nu_\tau \pi^- \eta 2\pi^0$	a		0.40
$\nu_\tau \pi^- \eta\eta n\pi^0, n \geq 0$	b		0.60
sum			2.7[b]

[a] Reference 23 gives this upper limit, and says 0.66% is an absolute maximum if one of the e^+e- cross section measurements is wrong.

[b] Sum does not include modes with $\nu_\tau \pi\eta n\pi^0, n > 2$.

Table 4. Comparison of B_1 with $\sum B_1\alpha$.

Source of Information	Branching Fraction (%)
Sum of direct measurements from Row G of Table 2	77.9 ± 1.5
Sum of 95% CL upper limits from Table 3	≤ 2.7
Sum of above	$\leq 80.6 \pm 1.5$
Topological branching fraction B_1	86.6 ± 0.3

B. The τ Decay Mode Problem

The problem appears when the directly measured upper limits in Rows H–L of Table 2 are replaced by the limits in Table 3; this is done in Table 4. Thus

$$\sum_{\alpha} B_{1\alpha} \leq (80.6 \pm 1.5)\% \quad , \tag{35a}$$

but

$$B_1 = (86.6 \pm 0.3)\% \tag{35b}$$

A puzzling contradiction.

If method (c) is applied to $B_{\pi 2\pi^0}$

$$B(\pi^- 2\pi^0 \nu_\tau) \leq B(\pi^-\pi^+\pi^-\nu_\tau) = (6.7 \pm 0.4)\% \quad , \tag{36}$$

the latter being a direct measurement. Replacing the directly measured $B_{\pi 2\pi^0}$, Eq. 27b, by the deduced value in Eq. 36, increases the contradiction in Eqs. 35 by an additional 0.9%.

IV. A CLOSER LOOK AT THE DIRECT MEASUREMENTS: ERRORS AND BIASES

A. Introduction

One's first thought in facing the contradiction displayed in Table 4 is that unreasonably small errors have been assigned to B_1 and the major individual decay modes, B_e, B_μ B_π, B_ρ, and $B_{\pi 2\pi^0}$. As discussed at the end of the last section, there is evidence that we have $B_{\pi 2\pi^0}$ about right; there is no further information on this branching fraction. The other major branching fractions have been repeatedly measured and we can examine their errors. Looking back to Sec. II.A, the formal error σ is obtained from the individual statistical errors $\sigma_{stat,n}$, and the individual systematic errors, $\sigma_{sys,n}$. It is the latter which can be difficult to estimate, and may be wrongly estimated by the experimenters. Using the method described in the next section, Hayes and I[9] have shown that this first thought — that the errors are unreasonably small — is wrong. On the whole the formal errors associated with B_1, B_e, B_μ, B_π, and B_ρ are either slightly too small, Sec. V.A, reasonable, or in some cases too large!

B. The σ_{scat} Test of Errors

Consider a set of N measurements $y_1, \ y_2 \ldots y_N$ with formal average y and formal error σ. The various y_n are scattered about the average y; the weighted root mass square scatter distance is

$$\sigma_{scat} = \left[\frac{\sum_{m=1}^{N} w_n (y - y_n)^2}{N - 1} \right]^{1/2} \tag{37}$$

The individual errors enter through the weights $w_n = 1/\sigma_n^2$ but not directly in the calculation of σ_{scat}. This σ_{scat} should be about the same size as σ if the $\sigma_{sys,n}$ have been correctly estimated and the individual y_n's are not biased.

To make this quantitative, we defined in Ref. 9

$$r = \frac{\sigma_{scat}}{\sigma} \tag{38}$$

Figure 1 shows the distribution of r for N measurements; the larger N, the narrower the distribution about $r = 1$. The 11 measurements in Table 1 used to calculate $B_1 = 86.6\%$ provide an example. From Ref. 9

$$\begin{aligned} \sigma &= 0.28\% \\ \sigma_{scat} &= 0.27\% \\ r &= \frac{\sigma_{scat}}{\sigma} = 0.96 \end{aligned} \tag{39}$$

Thus the formal error σ assigned to B_1 is reasonable; there is no evidence from σ_{scat} that σ is too small. For example, if one wanted to set σ of B_1 at 1.% to help explain away the $B_1 - \sum B_{1\alpha}$ discrepancy, then $r = 0.3$, which has a very small probability of being a fluctuation from $r = 1$.

The σ_{scat} test applied to B_e, B_μ, B_π, and B_ρ gives the following evaluations of the σ's.

B_e not constrained by $B_e = 0.973\ B_\mu$, 10 measurements, $B_e = 17.6\%$

$$\begin{aligned} &\sigma = 0.44\%, \quad \sigma_{scat} = 0.37\% \\ &r = \sigma_{scat}/\sigma = 0.83 \\ &\text{probability of } r < 0.83 \text{ is } 30\% \end{aligned} \tag{40a}$$

B_μ not constrained by $B_e = 0.973\ B_\mu$, 16 measurements, $B_\mu = 17.7\%$

$$\begin{aligned} &\sigma = 0.41\%, \quad \sigma_{scat} = 0.37\% \\ &r = \sigma_{scat}/\sigma = 0.91 \\ &\text{probability of } r < 0.91 = 35\% \end{aligned} \tag{40b}$$

B_e constrained to $B_e = 0.973\ B_\mu$, 21 measurements, $B_e = 18.0\%$

$$\begin{aligned} &\sigma = 0.26\% \quad , \quad \sigma_{scat} = 0.19\% \\ &r = \sigma_{scat}/\sigma = 0.73 \\ &\text{probability of } r < 0.73 \text{ is } 4.7\% \end{aligned} \tag{40c}$$

B_π, 7 measurements, $B_\pi = 10.8\%$

$$\begin{aligned} &\sigma = 0.60\% \quad , \quad \sigma_{scat} = 0.35 \\ &r = \sigma_{scat}/\sigma = 0.59 \\ &\text{probability of } r < 0.59 \text{ is } 8.3\% \end{aligned} \tag{40d}$$

B_ρ, 6 measurements, $B_\rho = 22.5\%$

$$\begin{aligned} &\sigma = 0.85\% \quad , \quad \sigma_{scat} = 0.18 \\ &r = \sigma_{scat}/\sigma = 0.21 \\ &\text{probability of } r < 0.21 \text{ is } 0.1\% \end{aligned} \tag{40e}$$

In these cases $r \lesssim 1$, hence on the basis of the σ_{scat} test there is no evidence that the formal errors are too small.

C. Interpretations of the σ_{scat} Tests

The primary conclusion from these σ_{scat} tests has just been given, the formal error σ is either reasonable or too large for B_1, B_e, B_μ, B_π, and B_ρ. On the basis of these tests we cannot explain away the decay mode problem by proposing yet larger σ's for some or all of these B's.

In some cases in Eqs. 40, $r = \sigma_{scat}/\sigma$ has values less than 1 which have small probabilities of having occurred by chance. There are three interpretations of the significance of σ being larger than σ_{scat}.

(i) σ Overestimated:

The experimenters may have overestimated the $\sigma_{sys,n}$ they assign to their y_n. Cautious experimenters may do this.

(ii) Bias Towards an Accepted Value:

The formal error σ might be right but σ_{scat} might be too small because some of the individual measurements may be biased toward an accepted or central value. This could happen because in the course of obtaining a final value of B_1 or B_α from a raw measurement, the experimenter must use many properties of the detector and data sample: detector solid angle acceptance, trigger efficiency and so forth. Some of these parameters may be difficult to calculate or measure correctly. Other correction parameters may have small effects on the final value of B and it may be a judgement question as to how these corrections are used. An unconscious bias towards a particular expected final value of B might influence the evaluation of the acceptance, efficiency, and correction parameters. The small values of r for constrained B_e, B_π, and B_ρ could be caused by unconscious biases leading to a clumping together of individual measurements. It is not possible to trace such effects directly. Nor can I tell what sets the central value about which the measurements clump, if clumping has occurred. Such central values are certainly not the ones which solve the decay mode problem.

Reference 9 investigates in more detail the possibility of bias toward an accepted or central value. One interesting test considers the B_ρ measurements and sets each $\sigma_{sys,n}$ to 0. Then $\sigma_n = \sigma_{scat,n}$, the minimum value of σ_n. We find [9]

$$\begin{aligned} &B_\rho = 22.5\% \quad \sigma = 0.35\% \quad \sigma_{scat} = 0.14\% \\ &r = \sigma_{scat}/\sigma = 0.39 \\ &\text{probability of } r < 0.39 \text{ is } 2.0\% \end{aligned} \tag{41}$$

Thus there is evidence for clumping of the B_ρ measurements even when the σ_n's are reduced to their smallest values given by the $\sigma_{stat,n}$'s.

(iii) Common Systematic Error:

A third interpretation of small values of r is that most or all of the measurements y_1, $y_2 \ldots y_N$ have a common systematic error, $\sigma_{sys,common}$. Equation 18a is replaced by

$$\sigma_n = \left[\sigma^2_{stat,n} + \sigma^2_{sys,n} - \sigma^2_{sys,common}\right]^{1/2} \tag{42a}$$

or

$$\sigma_n = \left[\sigma^2_{stat,n} + \left(\sigma_{sys,n} - \sigma^2_{sys,common}\right)\right]^{1/2} \tag{42b}$$

depending on how $\sigma_{sys,n}$ was evaluated. Then σ_{scat} should be compared with

$$\sigma_{partial} = \left[\sum_{n=1}^{N} \frac{1}{\sigma_n^2}\right]^{1/2} \tag{43}$$

and the formal error is

$$\sigma = \left[\sigma_{partial}^2 + \sigma_{sys,common}^2\right]^{1/2} \tag{44}$$

with this procedure σ_{scat} is compared with a smaller part of the formal error, and the formal error is larger than that calculated by setting $\sigma_{sys,common} = 0$. In the next section I describe searches for a common systematic error, in particular for a common asymmetric error whose discovery might change the formal average of some of the major branching fractions.

V. THE SEARCH FOR COMMON SYSTEMATIC ERRORS

A. Introduction

When the 1-charged particle decay mode problem became more apparent in the past year, searches began for one or more common systematic errors which might have occurred in measurements of one or more of the branching fractions: B_1, B_e, B_μ, B_π, B_ρ. Such common errors might increase the formal error, Eq. 44, or shift the formal average. In the latter case the discrepancy, Eq. 35, would disappear if B_1 were smaller by 4% or more, or if the sum

$$B_{e\mu\pi\rho} = B_e + B_\mu + B_\pi + B_\rho \tag{45}$$

were larger by 4% or more.

A search for a common systematic error faces three kinds of difficulties. First several different methods are used to measure branching fractions. One method depends on the total luminosity, another uses the ratio B_α/B_1 and does not depend on luminosity. One method may tag an $e^+ + e^- \rightarrow \tau^+ + \tau^-$ event by the 3-charged particle decay mode of one of the τ's, another method uses a restricted sample of 1-charged particle decay modes. A common systematic error must occur in most of the methods used to obtain a measurement set.

The second kind of difficulty is that a set of measurement comes from several, sometimes, many different experiments studying electron-positron annihilation at energies from 4 GeV to 44 GeV. A common systematic error must persist through at least part of this variety of experimental apparatus and energies.

The third difficulty is the constraints from conventional theory; Sec. I.B:

$$B_\mu/B_e = 0.973 \quad (46a)$$

$$B_\pi/B_e = 0.607 \quad (46b)$$

$$B_\rho/B_e = 1.23 \pm 0.008 \quad (46c)$$

I don't know what we would do with the discovery of a common error in say B_π measurements which when corrected increased B_π from 10.8% to 15%, but didn't change B_e or B_μ. We would then face a conflict with the conventional theory of τ decay. Therefore experimenters have tended to look for a common systematic error which when corrected would increase B_e, B_μ, B_π and B_ρ. Perhaps there is too much timidity.

B. Common Systematic Error in B_1 Measurements

I have not heard of any quantitative proposal of a common systematic error which when corrected would reduce B_1 to 83% or less. Table 1 shows that the first and lowest energy measurements of B_1 gave 65% to 82%. I believe these low energy measurements are contaminated by hadronic events. It will be useful to measure B_1 at the Z^0 energy

$$e^+ + e^- \rightarrow Z^0 \rightarrow \tau^+ + \tau^- \quad (47)$$

where the contamination from hadronic events would be even less than it was in the PETRA and PEP energy range.

C. Radiative Corrections to $e^+ + e^- \rightarrow \tau^+ + \tau^-$

Radiative corrections to

$$e^+ + e^- \rightarrow \tau^+ + \tau^- \quad (48a)$$

including

$$e^+ + e^- \rightarrow \tau^+ + \tau^- + \gamma, \tau^+ + \tau^- + 2\gamma \quad (48b)$$

can affect branching fraction measurements in several ways. First the τ pairs are produced by an E_{total} energy spectrum broader than the combined widths of the e^+ and e^- beams; the spectrum has a low energy radiative tail. This affects the momentum spectrum of the charged particle in the 1-charged particle decay modes. Second, a related effect is that the τ pair production cross section is larger than the cross section at $E_{total} = 2E_{beam}$, about 30% larger at 30 GeV total energy.

All the experiments which measured τ branching fractions used similar equations and computer codes to correct for these radiative effects. This is where a common error could occur. But no one has reported the discovery of a significant error. Burchat and Smith[21] have considered radiative effects by comparing the equations and methods used in the Mark II experiment and in the MAC experiment. Within the area they examined they find no significant common error.

D. Radiative τ Decays

A few months ago I found a common error in about all τ decay mode studies. The radiative decays of the τ:

$$\tau^- \to e^- + \bar{\nu}_e + \nu_\tau + \gamma \tag{49a}$$

$$\tau^- \to \mu^- + \bar{\nu}_\mu + \nu_\tau + \gamma \tag{49b}$$

$$\tau^- \to \pi^- + \nu_\tau + \gamma \tag{49c}$$

$$\tau^- \to \rho^- + \nu_\tau + \gamma \tag{49d}$$

had not been considered. My idea was that such 1-charged particle decays would be counted in a B_1 measurement because the number of photons is ignored in such measurements. But a B_e, B_μ, B_π, or B_ρ measurement might exclude these radiative modes, only counting the non-radiative modes. The decay width of a radiative weak decay, Γ_{rad} is mostly subtracted from the decay width calculated ignoring radiation, $\Gamma_{ignore\ rad}$.

$$\Gamma_{no\ rad} \approx \Gamma_{ignore\ rad} - \Gamma_{rad} \tag{50}$$

Hence a strict exclusion of Γ_{rad} would result in a measured B_e, B_μ, B_π, or B_ρ which is smaller than correct. This could produce a too small $\sum B_{1\alpha}$, Eq. 35a.

But when I looked at various measurements, I found that the selection criteria for B_e were sufficiently loose to include the radiative decay mode, Eq. 49a. This is because the γ in Eq. 49a is emitted close to the direction of motion of the e, the γ's energy is measured close to where the e showers in an electromagnetic calorimeter, and the γ is usually not separately detected. Similarly the selection criteria for measuring B_π and B_ρ usually accept a γ close to the π^-, allowing for π^- interactions. The B_μ selection criteria might eliminate the radiative decay mode, Eq. 49b. I calculate[22] for this mode

$$\Gamma_{rad}/\Gamma_{ignore\ rad} = 0.013 \tag{51}$$

when the τ energy is about 15 GeV and the photon is detected when its energy is above 0.2 GeV. This by itself is insufficient to explain the decay mode problem.

I have explained this idea, although it doesn't work, because it illustrates how a common systematic error might cause the decay mode problem. The stricter criteria used to select events for the measurement of B_e, B_μ, B_π, or B_ρ could be much more affected by the common error than the looser criteria used to select events for the B_1 measurement. But I have not heard of the discovery of such an error or errors.

E. The τ Lifetime

Conventional theory relates the τ lifetime T_τ to the μ lifetime T_μ through

$$T_\tau = T_\mu \left(\frac{m_\mu}{m_\tau}\right)^5 B_e \tag{52a}$$

$$T_\tau = T_\mu \left(\frac{m_\mu}{m_\tau}\right)^5 \frac{B_\mu}{.973} \tag{52b}$$

Equation 52b has rarely been used although B_μ is as well measured as B_e, Sec. II.B. In Ref. 9 we constrain all B_e and B_μ measurements to $B_\mu/B_e = 0.973$, hence using both parts of Eq. 52. We find using $B_e = 17.96\%$

$$T_\tau \text{ (predicted) } = (2.87 \pm 0.04) \times 10^{-13} \text{ s} \tag{53a}$$

compared to

$$T_\tau \text{ (measured) } = (3.02 \pm 0.09) \times 10^{-13} \text{ s} \tag{53b}$$

where the formal average and error of T_τ comes from the measurements in Table 5. The difference is

$$T_\tau \text{ (measured) } - T_\tau \text{ (predicted) } = (0.15 \pm 0.10) \times 10^{-13} \text{ s} \tag{53c}$$

Earlier comparisons using less data and just B_e sometimes calculated a larger difference.

The 1.5 standard deviation difference in Eq. 53c does not have enough significance to require that B_e should be larger than 17.96%.

VI. THE SEARCH FOR AN UNCONVENTIONAL EXPLANATION

A. Introduction

Since the search for experimental errors has not been rewarded with an explanation of the decay mode problem, physicists in the past few years have searched for an explanation using unconventional concepts. I give two examples.

B. Large Branching Fractions Containing η's?

About two years ago there was preliminary evidence that the branching fractions for modes containing η's might sum to from 5 to 8%. Later measurements[23] have negated this result. Some measured 95% CL upper limits are[15,18]

$$B(\tau^- \to \pi^- \eta \nu_\tau) < 0.3\% \tag{54a}$$

$$B(\tau^- \to \pi^- \pi^0 \eta \nu_\tau) < 0.9\% \tag{54b}$$

$$B(\tau^- \to \pi^- \eta \eta n \pi^0 \nu_\tau) < 0.6\% \tag{54c}$$

C. A second τ Neutrino?

Suppose there is an unconventional physics effect in τ decay processes which distorts the momentum spectra of the charged particle in the 1-charged particle decay modes. Echoing the ideas in Sec. V, the measurements of B_e, B_μ, B_π, or B_ρ might be more affected than B_1. I recently tried a simple model[24] for this concept; it doesn't explain the decay mode problem, but it illustrates the ideas.

Consider that in addition to ν_τ, there is a second τ neutrino, N_τ, with

$$m_\tau > m_{N_\tau} \gtrsim 1.0 \text{ GeV/c}^2 \qquad (55)$$

Here m_{N_τ} is the N_τ mass. Let the $\tau - \nu_\tau$ and $\tau - N_\tau$ coupling be the same, and conventional in form and strength. The major decay modes with N_τ are

$$\tau^- \to N_\tau + e^- + \bar{\nu}_e \qquad (56a)$$

$$\tau^- \to N_\tau + \mu^- + \bar{\nu}_\mu \qquad (56b)$$

$$\tau^- \to N_\tau + \pi^- \qquad (56c)$$

Figure 2 shows the e and π spectra. As m_{N_τ} increases, the spectra shrink to smaller energies.

Most branching fraction measurements of B_e, B_μ, B_π and sometimes B_ρ required the charged particle energy to be above a threshold in the 1 to 2 GeV range; this is not required for B_1 measurement. When $m_{N_\tau} \gtrsim 1$ GeV/c^2 the N_τ events would not be correctly counted in the exclusive measurements, but would be correctly counted in the B_1 measurement. However, as shown in Ref. 24 the effect is too small to explain the tau decay mode problem.

VII. NEXT STEPS

A. Present

The problem in the 1-charged particle decay modes of the τ remains unsolved. Are there undetected common systematic errors? Is the problem caused by an unlikely confluence of small errors, each undetected by itself, but adding up to the discrepancy? Has there been a clumping of measurements toward expected central values but not the right values? Is there something unconventional in the τ decay process? What are the next steps?

Some of my colleagues and I are still looking at τ data from the Mark II at PEP. We cannot expect to shift the formal averages in Table 2 with one more measurement based on modest statistics. We do hope to gain insight into error and bias problems, and perhaps get new ideas. If one doesn't work on a problem one can't progress. Unfortunately work does not guarantee progress.

I hope for more results and insight from τ studies with the ARGUS and Crystal Ball detectors. With their better photon detection they have already contributed tremendously to the elucidation of modes containing η's and π^0's.

B. Future

In the past in physics, the way out of a dilemma such as this decay mode problem has been new experiments with improved apparatus and much more data.

The rebuilt CLEO detector at the CESR electron-positron collider is such an experiment. A very large sample of τ decays will accumulate as CESR operates at a luminosity of 10^{32} cm^{-2} s^{-1} and higher

More studies will come from experiments at the SLC and LEP on

$$e^+ + e^- \rightarrow Z^0 \rightarrow \tau^+ + \tau^-$$

The high energy of the τ will make photon dependent measurements more difficult because the τ decay products lie in a narrower cone. But there is less hadronic event contamination and sometimes a new energy range brings new insights to an old problem.

Perhaps the experiments at TRISTAN are in the best energy range: 40 to 65 GeV. A balance can be found between hadronic event contamination and the size of the τ decay products cone.

C. Hopes

There are several proposals[25] for building very high luminosity electron-positron colliders in energy ranges where experimenters can study very large numbers of charm mesons, tau leptons, and B mesons. The combination of very large numbers of observed τ decays and new detectors may be needed to solve the τ decay mode problem.

ACKNOWLEDGEMENT

I am indebted to K. G. Hayes for his suggesting and leading our statistical studies of tau branching fractions. I appreciate many conversations and much help from K. K. Gan.

REFERENCES

1. M. Aguilar-Benitez *et al.*, Rev. Mod. Phys. **170B** (1986) 1.
2. H. Albrecht *et al.*, DESY preprint 87/148 (1987).
3. Y. S. Tsai, Phys. Rev. **D4** (1971) 2821.
4. F. J. Gilman and S. H. Rhie, Phys. Rev. **D31** (1985) 1066.
5. F. J. Gilman, Phys. Rev. **D35** (1987) 3541.

6. The question of non-resonant or non-ρ resonant contributions to $\tau^- \rightarrow \nu_\tau + \pi^- + \pi^0$ is complicated but should not offset the use of CVC. See for example Ref. 4 and K. K. Gan, SLAC–PUB–4512 (1987).

7. K. K. Gan and M. L. Perl, Int. J. Mod. Phys. A. **3** (1988) 531, Fig. 2.

8. E. Braaten, Phys. Rev. Lett. **60** (1988) 1606.

9. K. G. Hayes and M. L. Perl, SLAC–PUB–4471 (1988), submitted to Physical Review.

10. B. G. Bylsma, Ph.D. Thesis, Purdue Univ. PU–87–696 (1987).

11. B. G. Bylsma *et al.*, Phys. Rev. **D35** (1987) 2269.

12. C. Akerlof *et al.*, Phys. Rev. Lett. **55** (1985) 570.

13. K. K. Gan *et al.*, Phys. Rev. Lett. **59** (1987) 411.

14. H. R. Band *et al.*, Phys. Lett. **198B** (1987) 297.

15. S. T. Lowe, SLAC–PUB–4449, to be published in Proc. Int. Symp. on Production and Decay of Heavy Flavors (Stanford, 1987).

16. G. B. Mills *et al.*, Phys. Rev. Lett. **52** (1984) 1944.

17. T. Skwarnicki, DESY 87–166 (1987).

18. S. Abachi *et al.*, Phys. Lett. **197B** (1987) 291.

19. P. Baringer *et al.*, Phys. Rev. Lett. **59** (1987) 1993.

20. T. N. Truong, Phys. Rev. **D30** (1984) 1509.

21. P. R. Burchat and J. Smith, private communication.

22. T. Kinoshita and A. Sirlin, Phys. Rev. Lett. **2** (1959) 177.

23. K. K. Gan, SLAC–PUB–4468 (1987), to be published in Proc. SLAC Summer Inst. Particle Physics (Stanford, 1987).

24. M. L. Perl, SLAC–PUB–4533 (1987), to be published in Phys. Rev.

25. P. R. Burchat, Univ. of California, Santa Cruz preprint SCIPP 88/12 (1988).

Appendix A

K Meson Contributions to 1-Charged Particle Modes

We are interested in the branching fraction $B_{Kn\pi^0,n\geq 0}$ for

$$\tau^- \rightarrow \nu_\tau + K^- + n\pi^0 + mK_L^0,\ n \geq 0,\ m \geq 0$$

The following modes contribute:

$$\tau^- \rightarrow \nu_\tau + K^- + n\pi^0, \quad n \geq 0$$

$$\tau^- \rightarrow \nu_\tau + K^- + K^0 + n\pi^0, \quad n \geq 0$$
$$\tau^- \rightarrow \nu_\tau + K^- + 2K^0 + n\pi^0, \quad n \geq 0$$

These decay processes are complicated by the subsequent decays of the K^0. Half the time the K^0 is a K_L^0, escaping the detector before it decays. The other half the time it is a K_s^0, decaying 68.6% to $\pi^+\pi^-$ and 31.4% to $\pi^0\pi^0$. Thus the decay mode

$$\tau^- \rightarrow \nu_\tau + K^- + K^0$$

may appear as

$$\tau^- \rightarrow \nu_\tau + K^- + K_L^0$$

with only the K^- detected; as

$$\tau^- \rightarrow \nu_\tau + K^- + 2\pi^0 \quad ;$$

or as

$$\tau^- \rightarrow \nu_\tau + K^- + \pi^- + \pi^- \quad ,$$

a 3-charged particle decay. Fortunately Refs. 16 and 17 have directly measured

$$B_{Kn\pi^0, n\geq 0} = (1.7 \pm 0.3\%) \quad .$$

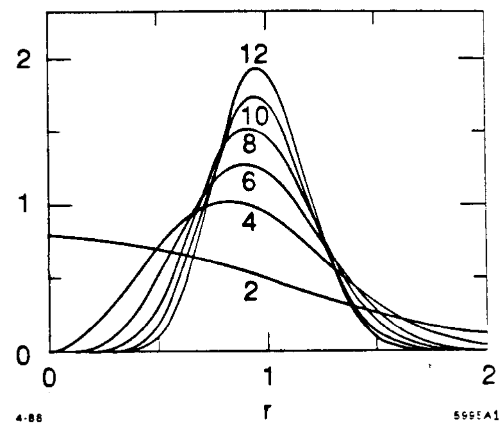

Fig. 1. r distribution for different values of N.

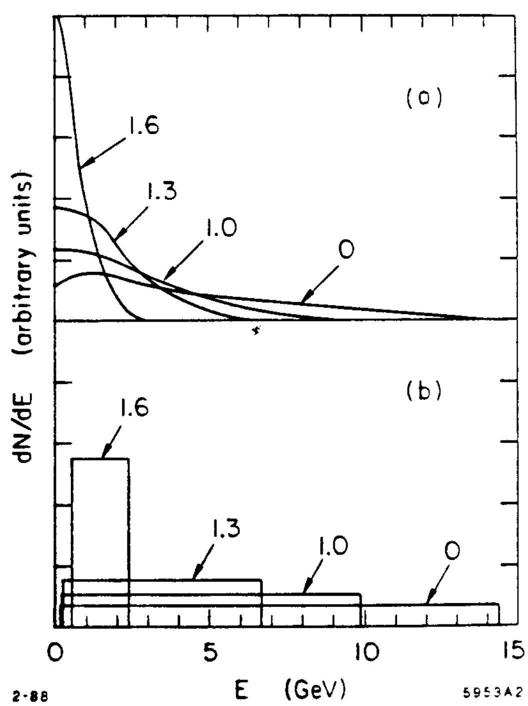

Fig. 2. The energy spectra for $m_{N_\tau} = 0.0$, 1.0, 1.3 and 1.6 GeV/c^2 for (a) the e in $\tau^- \rightarrow N_\tau + e^- + \bar{\nu}_e$ and (b) the π in $\tau^- \rightarrow N_\tau + \pi^-$. The energy of the τ is 14.5 GeV.