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ALGORITHMS FOR THE SELF-CONSISTENT SIMULATION OF HIGH POWER KLYSTRONS*

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ABSTRACT

We discuss an improvement to the algorithm developed by Yu[1] for modelling rf cavities in klystrons using the port approximation. In this method, the cavity is simulated by imposing an rf voltage as a boundary condition across the outer wall. The voltage and phase are chosen to be consistent with the cavity impedence and with the rf current induced by the electron beam. In the original method, each cavity was calculated successively using either linear theory or an iterative method to achieve a self-consistent voltage. The new method relaxes the voltage and phase of several cavities simultaneously during the simulation. The time dependence of the voltages are calculated from a relaxation equation. The new algorithm reduces the total computation time by about a factor of five for a complete klystron.

INTRODUCTION

In the port approximation[1] to the modelling of rf cavities using an electromagnetic particle in cell code such as MASK[2], the cavities are simulated by imposing an rf voltage as a boundary condition across an opening or "port" in the outer wall of the drift tube (Figure 1). This method ignores the transient and looks only for the steady-state solution at a single operating frequency. In a real cavity, electromagnetic energy flows across the gap between the cavity and the electron beam in the drift tube. By writing the equations for energy flow across the gap one splits the problem into two much simpler pieces:

$$V \cdot I_{ind} = \int^{*} E \cdot J dV \quad . \tag{1}$$

(Dot product involves the integration over rf cycle of the complex phase; for vector quantities it also subsumes the spatial dot product.)

Looking from the cavity side, in steady state the voltage across the gap and the current flowing in the walls uniquely determine the state of the cavity - thus they can be expressed in terms of the cold cavity impedence. From the drift tube side the energy flow into or out of the beam is completely determined in steady state by the voltage and phase across the gap. The key trick is noticing

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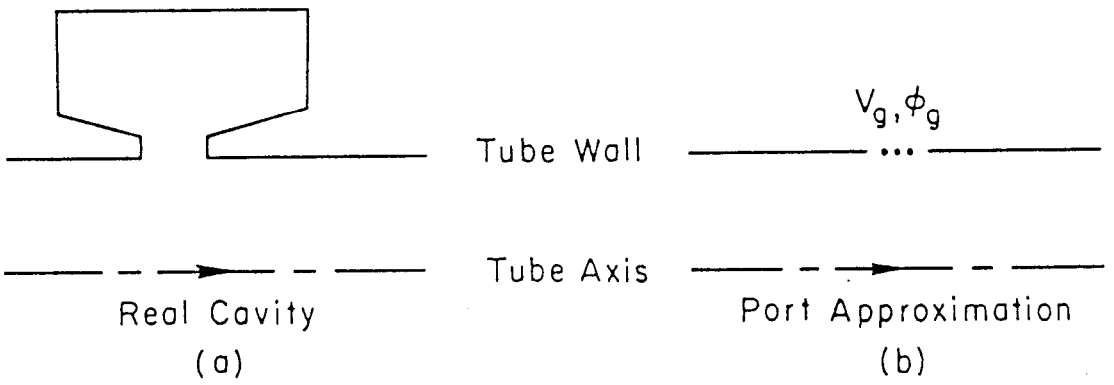
that the current flowing in the walls (the induced current) can be calculated simply in terms of the transform of the volume integral of $E \cdot J$. Of course, the current distribution is changed by the presence of the cavity voltage.

Figure 1. Simulation of a real cavity by a port boundary condition.



The voltage and phase must be chosen (by some means) to be consistent with the cavity impedence and with the rf current induced by the electron beam. Note that the induced current is not identical to the rf current flowing through the drift tube. The currents can be related through the transit angle factor, since the energy transfer to the beam is

$$P = V \cdot I_{rf} T \quad . \tag{2}$$

Thus

$$I_{ind} = I_{rf} T \quad . \tag{3}$$

The relation to cold cavity parameters comes through the relation

$$V = I_{ind} Z \quad . \tag{4}$$

It is straightforward to relate Z to cavity Q, ω, and R/Q (taking care to be consistent if voltages are measured on axis or across the gap), using the relationship:

$$Z = e^{j\psi}/\alpha \quad . \tag{5}$$

where

$$\alpha = I_{ind}/V = [Q_0^{-2} + 4(\Delta\omega/\omega)^2]^{1/2} \quad . \tag{6}$$

and

$$\psi = \phi_I - \phi_V = tan^{-1}[-2Q_0\Delta\omega/\omega] \quad . \tag{7}$$

The alternative to the port approximation would be to model the cavity boundaries in MASK. The advantage of using the port approximation is that it becomes possible, using current computational resources, to fully model the cavities of a klystron and their interaction with an electron beam. There are two reasons for preferring the port approximation:

1. To obtain the correct voltages, if the entire cavity were modelled, the frequencies of the gain cavities would need to be accurate to better than 0.1 percent, which would require prohibitively fine zoning.
2. The cavities typically have loaded Q's of order several hundred, which would necessitate excessive computation to reach steady state.

NEW PORT ALGORITHM

In Simon Yu's original method, each cavity was calculated successively using either linear theory or an iterative method to achieve a self-consistent voltage. We have devised a faster method which relaxes the voltage and phase of several cavities simultaneously. In steady state, the gap voltage should satisfy the condition

$$V_{ss} = I_{ss} \cdot Z \quad . \tag{8}$$

Here Z is the complex cavity impedence and V_{ss} and I_{ss} are the Fourier components in steady state of the gap voltage and the induced current at the operating frequency. Now we assume a time dependence of the form:

$$V_t = V(t)e^{-j\omega t} \quad . \tag{9}$$

Here V_t is the instantaneous voltage across the port and V(t) is an envelope which varies slowly in an rf cycle. Asymptotically we want $V(t)$ to converge to V_{ss}. We can achieve this by making $V(t)$ satisfy a relaxation equation, i.e.,

$$dV(t)/dt = -k \cdot (V(t) - I(t) \cdot Z) \quad . \tag{10}$$

Thus $V(t)$ will adjust itself until the impedence relation is satisfied self-consistently. We compute the induced current at the operating frequency by

keeping a running table of the volume integral of $E \cdot J$. The equation converges faster if one takes into account the beam loading, assuming that the change in induced current is a linear function of the change in voltage, i.e.

$$\Delta I = \alpha_l \cdot \Delta V \quad . \tag{11}$$

Then

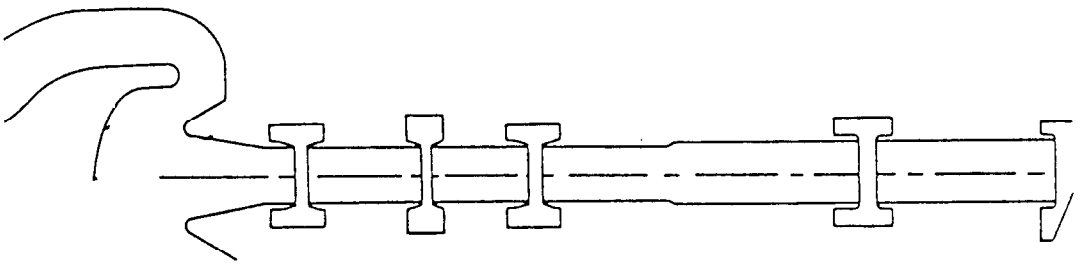
$$\Delta V = -k\Delta t(V - IZ) \div (1 - \alpha_l Z) \quad . \tag{12}$$

The constant α_l depends on dc current, frequency, and drift-tube size but is insensitive to gap width and beam profile.

APPLICATIONS

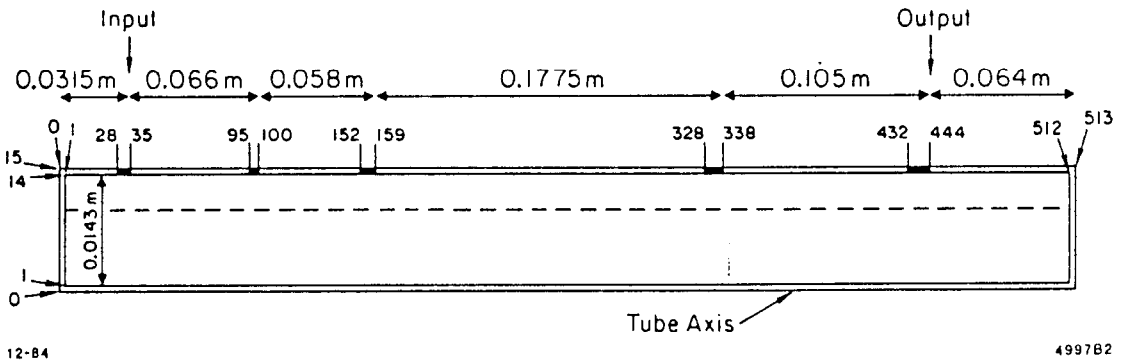
Applications of the original method have been described previously[1,3]. We now give an example of the application of the new algorithm to a similar problem. Figures 2 and 3 show a schematic of a SLAC XK-5 klystron, first with cavities drawn, then as it would be modelled using the port approximation in a MASK simulation.

Figure 2. Schematic of the SLAC XK-5 klystron.



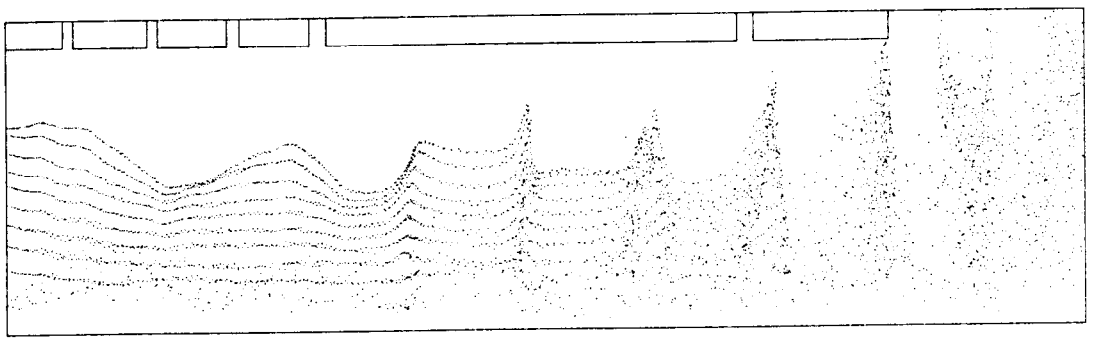
Here the electron gun is on the left, with the output cavity on the right hand side. Notice that there is an increase in tube radius between cavities three and four which was not modelled in the early simulation shown in Figure 3.

Figure 3. XK-5 klystron modelled in MASK using ports.



Figures 4 and 5 show position space and momentum space plots from a MASK simulation of a SLAC 5045 klystron. In Figure 4 the ports are located at the gaps in the blocks, with the output port just after the final block. The horizontal axis is Z, with the beam injected on the left, and the vertical axis is R, with the lower boundary being the symmetry axis R=0. The beam energy was 315 kV, and current was 393 amps. The magnetic field increased from zero at the cathode to a peak of 1200 gauss. The drift tube radius was 1.59 cm at the input and 1.75 cm at the output. The distance between input and output cavities was 56 cm.

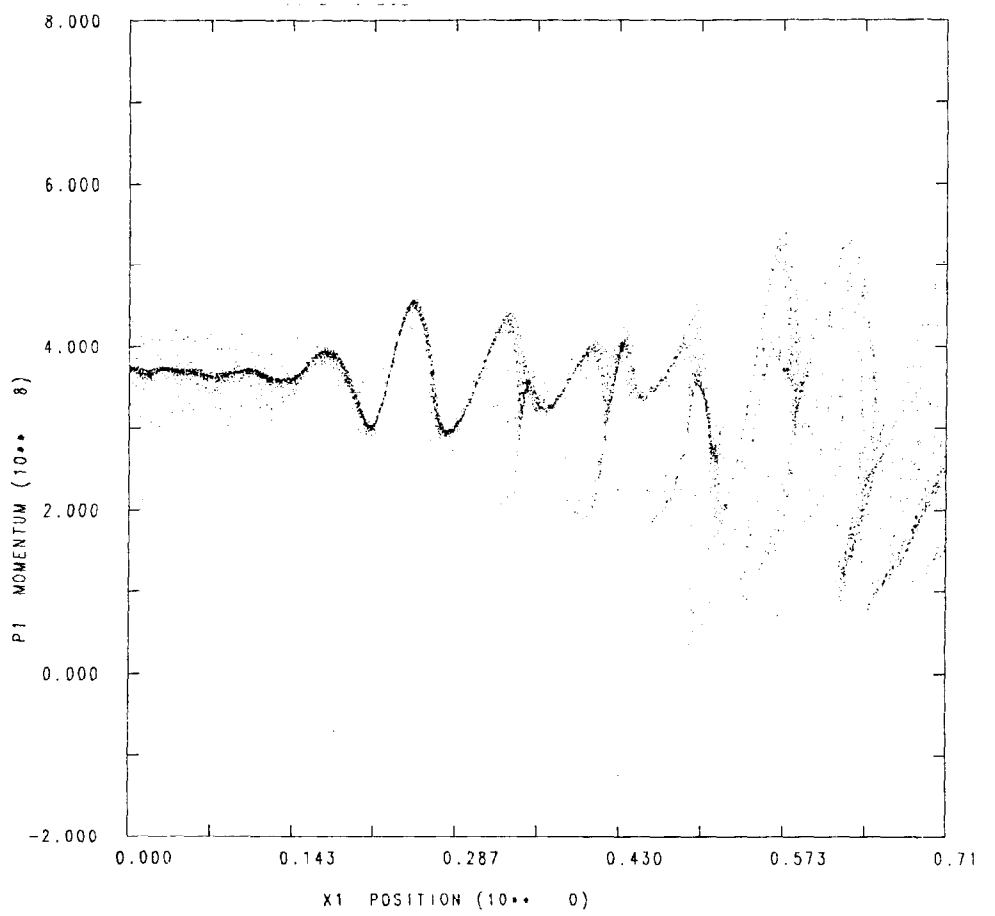
Figure 4. Position space distribution for the 5045.



In this tube the radius increased at the downstream nose of the output cavity. The varying radius was modelled with the conducting blocks shown in the figure. The input beam distribution was calculated using the EGUN code on the gun geometry, and the magnetic field with POISSON from the electromagnet design. The scalloping of the beam at the left is due to the magnetic field, while the longitudinal bunching downstream results from the rf modulation. Note that

the bunching is nearly as much a tranverse effect as it is longitudinal.

Figure 5. Longitudinal momentum (γv_z) for the 5045.



The initial modulation is sinusoidal. The trailing "fingers" are the result of space charge effects. The momentum profile "stands up" to produce maximum bunching at the output cavity. Because some current was left in the "antibunch," i.e., the portion of the rf cycle with the wrong phase to be deaccelerated by the output cavity, some electrons gain rather than lose energy there, limiting the efficiency to under 50%.

Figures 6 and 7 show the time history of the voltages and phases on each cavity using the algorithm described above as they relax to convergence. Cavities three through six were not turned on until cavity two had converged. (See discussion below.)

Figure 6. Port voltage vs. time in relaxation of the 5045.

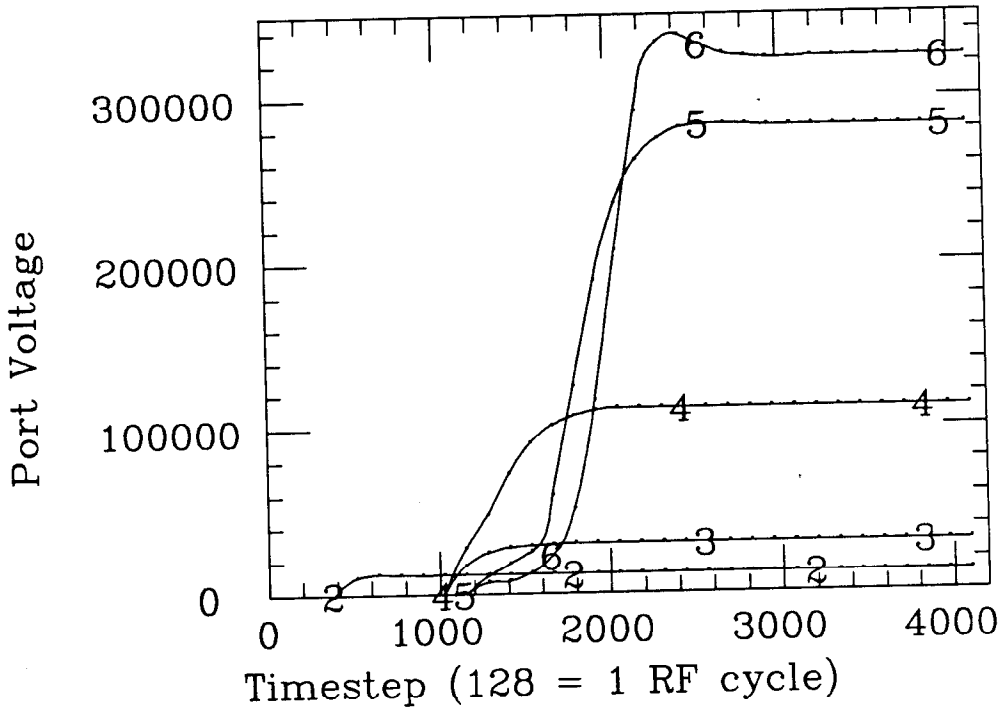
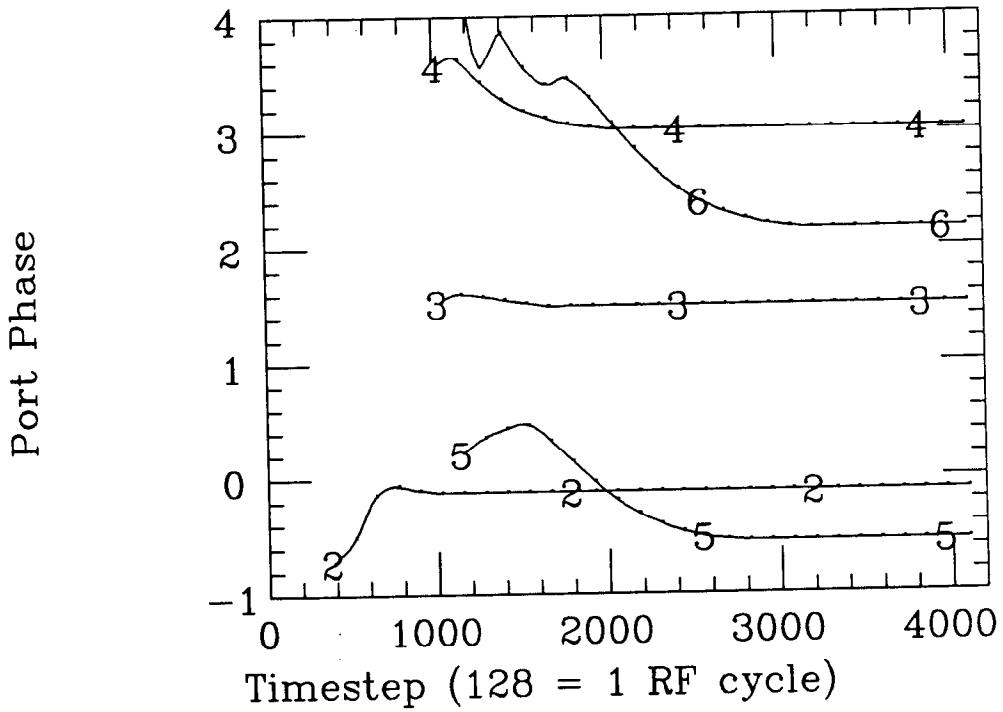


Figure 7. Port phase vs. time.

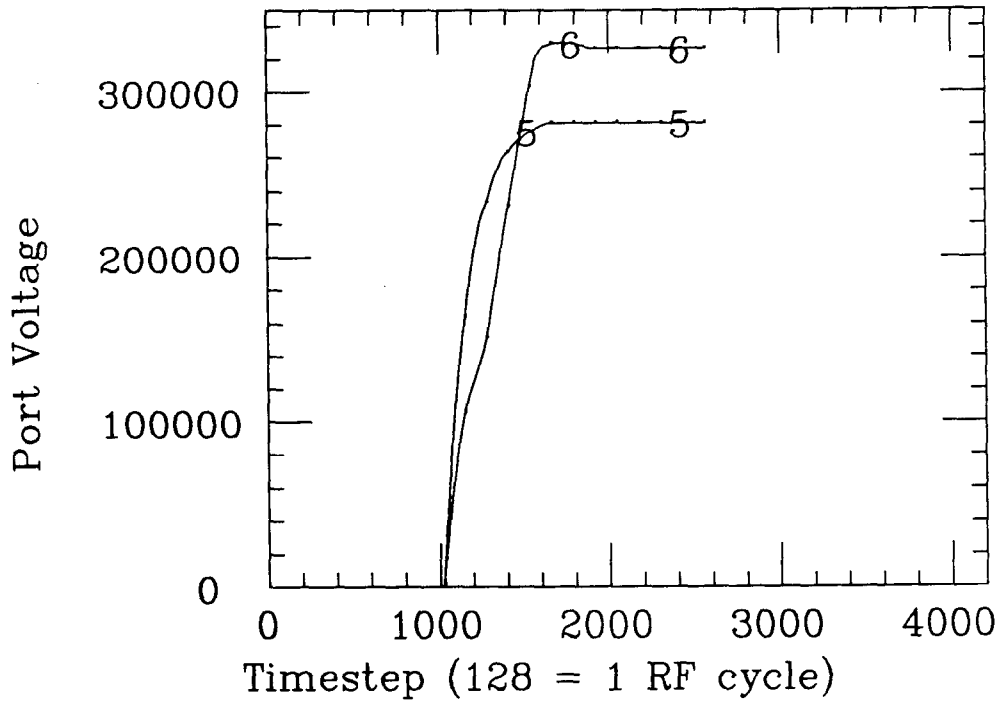


In this simulation cavity 2 was relaxed alone until it converged. Then cavities

3-6 were relaxed simultaneously. The reason this needed to be done is that in this particular klystron the detuning of the second cavity is very small. As a result the voltage on the second cavity is very sensitive to noise, and the high intensity of the second harmonic component produced downstream can distort the calculation of the induced current for the second cavity. With a larger detuning on this cavity, it would be possible to relax all cavities simultaneously.

In the first run (relaxing all cavities) the solution converged after about 24 RF cycles. The solution for the low power cavities is linear. Thus once one has a solution for a particular input power, one can find values for other power levels by multiplying all gain cavity voltages by a constant, and relaxing only the high-power cavities (5 and 6). This is shown in Figures 8 and 9. Here the cavities converged after about 16 RF cycles.

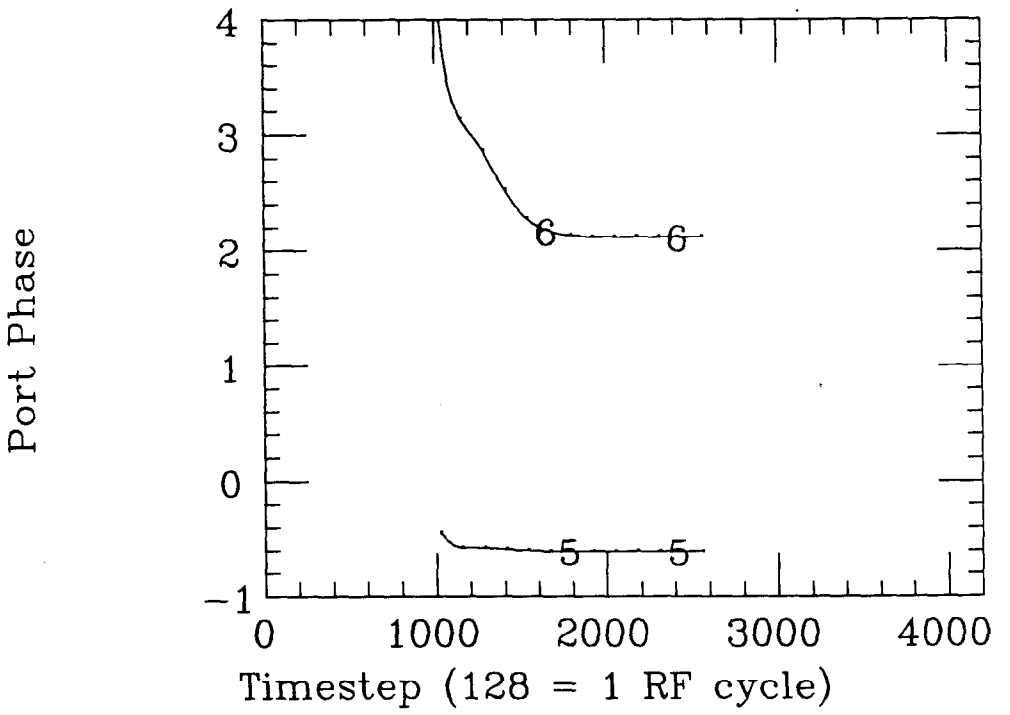
Figure 8. Port voltage vs. time for last two cavities, all others fixed.



This new algorithm reduces the total computation time (for a gain curve with three or four points) by a factor of about four to five for the complete klystron, compared to the original method. The voltages and phases agree to within a few percent of those obtained with the old algorithm. The new method actually is more accurate for the low power cavities, and will converge where the older method would sometimes fail. In a simulation of the SLAC 5045 S-band klystron, the old method predicted saturation at about 65 watts at a beam voltage of 315 kV, while the new algorithm gives saturation at about 120 watts,

which is in better agreement to the data.

Figure 9. Port phase vs. time.



In some situations it is desirable to apply a fixed voltage and relax only the phase of the cavity. For example, for most non-externally loaded cavities the phase difference between voltage and induced current will be close to $-\pi/2$ (unless the detuning is very small), while for output cavities the phase difference is usually close to zero. If one is designing a klystron it is often simpler to specify the voltages than the impedences. A modification of the algorithm described above relaxes the phase only, in a simple way:

$$d\phi_V/dt = k \cdot (\psi - \psi_0) \quad . \tag{13}$$

Here ϕ_V is the gap phase, ψ is the phase difference $\phi_I - \phi_V$, and ψ_0 is the desired phase difference.

CHAINING ALGORITHM

These new methods have been applied to the simulation of the "hybrid relativistic klystron" using magnetic induction technology, now being studied by SLAC and LLNL. The length of this device would have made the simulation prohibitively expensive with the older method. The length also required a method of chaining several simulations together, feeding the output from one segment

into the input of a second. This required modification of the Neumann boundary conditions to prevent excessive transverse deflection at the junction between segments.

The chaining algorithm itself was straightforward. It assumes a beam injected from the left which does not reverse direction. All particles removed from the right boundary are put into a buffer, which is dumped at specified time intervals. The fields are calculated by assuming Neumann boundary conditions, so that the field quantities do not have to be dumped. A history of the number of particles removed on each cycle is stored in a separate array which is also dumped. This algorithm is useful either for a single pulse of finite length (in which case the entire pulse is written out) or for a steady-state problem. For a periodic problem an integral number of cycles can be dumped after steady state is reached, and then recycled periodically when they are read in. For a dc problem, the same procedure applies, with the "cycle" time being any interval long enough to produce a reasonable distribution.

The original Neumann boundary solver used in MASK was found to produce large transverse momentum disruptions in the reinjected beam whenever the space charge in the beam was significant. The algorithm solved for the correction potential whose gradient, when added to the fields, would produce a solution of Poisson's equation. However, the solution on the boundary is not the same that one gets by solving for the full potential. This is because the radial derivative of the radial fields on the left and right boundaries does not enter into the calculation of the Neumann boundary condition, and thus is not corrected by the solution. Another way of looking at this is that the solution is made to solve Poisson's equation in the interior, but not on the boundary itself.

To correct the problem, we solved Poisson's equation using the full charge distribution to get the total potential. Then we took the one dimensional (radial) gradient on the left and right boundaries to get the radial fields there. This technique greatly reduced the radial disruption across the boundaries.

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