SLAC–PUB–4608
May 1988
(T/E)

GRAND UNIFICATION AT SLC/LEP

D. C. KENNEDY* AND B. W. LYNN†

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

and

Department of Physics
Stanford University, Stanford, California 94309

ABSTRACT

We present a general relationship between grand unification parameters and observables at the SLC and LEP e^+e^- colliders. These include the Z and W vector boson masses and, in particular, the polarization asymmetry A_{LR}, a highly sensitive measure of the electroweak mixing $\sin^2\theta_W$. We show that A_{LR} provides a considerably more accurate test of grand unification than heretofore possible. Predictions of A_{LR}, M_Z and M_W are provided for the minimal SU(5) and supersymmetric SU(5) and E_6 models.

Submitted to *Nuclear Physics B*

* Work supported by the Department of Energy, contract DE–AC03–76SF00515.
† Work supported by the National Science Foundation, contract NSF–PHY–86–12280.

1. Introduction

The unification of the forces of Nature is one of the great dreams of physics. Since the invention of modern field theories and quantum mechanics, the drive towards unification has not produced an unambiguously successful "theory of everything." Yet progress in that direction in the last fifty years has been remarkable. Nuclear and particle physicists have, through painstaking effort, produced an apparently complete and satisfactory gauge theory of the basic interactions (excluding gravity): SU(3) $\times$ SU(2) $\times$ U(1), the standard model.

The desire to simplify this picture has prompted theorists to hypothesize the principle of grand unification and to construct more fundamental theories that unify these three forces into a single structure. Such theories include conventional grand unified theories (GUTs) and supersymmetric GUTs; as well as more ambitious models that include gravity, such as supergravity and superstrings. The line between speculation and established result, however, has not shifted for a decade, and the backlog of such theories has grown. With the arrival of the SLC and LEP e^+e^- colliders will come the first new precise tests of the standard model that can place important constraints on this cornucopia of proposed new physics, including theories with grand unification.

The electroweak mixing $\sin^2\theta_W$, along with proton decay, is a classic prediction of grand unification and provides a direct test of this principle. Although many measurements of $\sin^2\theta_W$ have been performed and analyzed, the use of polarized e^- beams at SLC and LEP will allow its determination to much greater accuracy than previously possible [1]. Along with M_Z (SLC/LEP) and M_W (LEP II), as well as already well-known electroweak parameters (α_{em} and G_μ, Fermi's constant), $\sin^2\theta_W$ will be one of the few standard model parameters known to sufficient accuracy to probe radiative corrections and grand unification. A new radiative correction technique, developed in a previous paper, allows for a simple and complete calculation of electroweak measurables; in particular, the polarization asymmetry $A_{LR}(Z)$ at the Z resonance, a direct measure of $\sin^2\theta_W$. In section 2, this technique is applied to embedding the electroweak SU(2) × U(1) groups into an arbitrary simple group and to providing a general theoretical relationship between $A_{LR}(Z)$ and the grand unification parameters. In section 3, we illustrate this relationship with the minimal SU(5), SUSY SU(5) and SUSY E_6 models, showing the dependence of measurables (A_{LR}, M_Z, M_W) on the unknown mass scales of new physics, including the SUSY and grand unification masses. We will see that the accuracy of an SLC/LEP A_{LR} measurement is such that different models can be easily distinguished.

2. SLC/LEP Observables and Grand Unification

To predict $\sin^2\theta_W$ requires, at a minimum, the embedding of the two electroweak groups, SU(2) $\times$ U(1), into a single simple Lie group that we will call G. The couplings of weak isospin SU(2) and hypercharge U(1), g_2 and g', respectively, are in principle unrelated in the standard model but become related in a GUT by being related to the common coupling of the group G. Recall in the standard model: $\tan^2\theta_W = g'^2/g_2^2$. Let the coupling of G be g. Then, since G and SU(2) are both non-Abelian and have fixed normalizations of their generators, $g_2 = g$. On the other hand, g' is Abelian and its normalization is arbitrary. Embedded in G, it acquires a fixed normalization, being related to one of the generators of G. This fixes $\sin^2\theta_W$. To use a more compact notation, let $\sin^2\theta_W = s_\theta^2$. Then:

$$s_\theta^2 = \frac{Tr(I_3^2)}{Tr(Q^2)} \quad , \tag{2.1}$$

where I_3 and Q are the weak isospin and electric charge generators for a single representation of G [2].

The relationship eq. (2.1) holds at tree level but is rendered ambiguous by radiative corrections. Counterterm renormalization and the renormalization group (RNG) are traditionally used to handle radiative corrections and were first used to compute s_θ^2 at the weak scale by Georgi, Quinn and Weinberg [3]. This defines s_θ^2 in some renormalization scheme (RS), but the RS is arbitrary and s_θ^2 has only an indirect relationship to an observable such as A_{LR}. A new approach to radiative corrections, developed in ref. [4] for electroweak physics, dispenses altogether with RS's and expresses RS-invariant physical measurables in terms of RS-independent bare parameters (denoted by a "0" subscript) and radiative corrections. In particular, the set of vector boson self-energies ("oblique" corrections) and certain parts of vertex corrections can be collected to define a universal set of running functions (denoted by a "$*$" subscript) that replace the bare, or tree-level, parameters of the theory. These running parameters are analogous to the renormalization group (RNG) running couplings, but are defined in terms of the complete proper self-energies (not just leading logarithms, but all the one-loop subleading terms as well), which can be computed exactly and in closed form using the loop functions of Passarino and Veltman. There is no arbitrary separation into finite and infinite parts, and physical observables are related directly to other physical observables [5].

In the electroweak theory by itself, each starred function is defined by a single experimental input at some energy (the analogue of a renormalization point) and the corresponding bare parameter, which is unknowable, can be eliminated [4]. (The infinite parts of the radiative corrections cancel in the process.) The function $s_*^2(q^2)$ is not computable in SU(2) $\times$ U(1), but must be defined by an experimental input. Once embedded into a simple group G, however, $s_*^2(q^2)$ becomes a computable quantity requiring no experimental measurement. Given the group G, s_θ^2 in eq. (2.1) becomes the *bare* s_0^2 :

$$s_0^2 = \frac{Tr(I_3^2)}{Tr(Q^2)} \quad , \tag{2.1a}$$

the ratio of bare couplings. In terms of the weak isospin and electric charge currents, J_3 and J_Q, we can define the running functions $e_*^2(q^2)$ and g_{2*}^2 (replacing e^2 and g_2^2, respectively):

$$\begin{aligned} \frac{1}{e_*^2(q^2)} &= \frac{1}{e_0^2} - [\Pi'_{QQ}(q^2) + 2\Gamma'(q^2)] \quad ; \\ \frac{1}{g_{2*}^2(q^2)} &= \frac{1}{g_{2_0}^2} - [\Pi'_{3Q}(q^2) + 2\Gamma'(q^2)] \quad , \end{aligned} \tag{2.2}$$

where the Π's are the self-energy functions from the currents and Γ' is a proper vertex function necessary in a non-Abelian theory. Then $s_*^2(q^2) = e_*^2(q^2)/g_{2*}^2(q^2)$. Since $s_0^2 = e_0^2/g_{2_0}^2$,

$$\begin{aligned} s_*^2(Z) - s_0^2 &= -e_*^2(Z)P(Z) \quad ; \\ P(Z) &= \Pi'_{3Q}(Z) + \Gamma'(Z) - s_0^2[\Pi'_{QQ}(Z) + \Gamma'(Z)] \quad , \end{aligned} \tag{2.3}$$

taken at the Z resonance, $q^2 = -M_Z^2$. The same equation [eq. (2.1a)] that defines s_0^2 guarantees that the function $P(Z)$ is finite in a GUT, although divergent in the electroweak theory alone.

After the divergences cancel in $P(Z)$, we are left with finite parts of various types. Their significance depends on the masses of the particles in the loops relative to the scale M_Z at which s_*^2 will be measured.

• *Light particles* ($m \ll M_Z$): These will have self-energies $\propto \ln M_Z^2 + O(m^2/M_Z^2)$. Since the divergences cancel over a complete multiplet of G, the logs will also cancel, leaving the terms $O(m^2/M_Z^2)$. Light particles thus do not contribute much to $P(Z)$ if they do not belong to multiplets with heavy partners. This applies to the fermions of the standard model, with the exception of the top-bottom quark doublet.

• *Electroweak particles* ($m \sim M_Z$): These will have self-energies $\propto \ln M_Z^2 + O(M_Z^2/m^2)$. The gauge bosons themselves do not belong here, as they fall into a complete multiplet under G (see below), but the top quark is split from the bottom quark, and $P(Z)$ will receive a contribution $\propto \ln(m_t^2/M_Z^2) + O(M_Z^2/m_t^2)$.

• *$M_X - M_W$ splittings* ($m \sim M_Z, M_X$): For particles belonging to multiplets with light (M_W) and superheavy (M_X) partners, we will get contributions to $P(Z)$ of the classic RNG type, $\propto \ln(M_X^2/M_W^2) + O(M_Z^2/M_X^2)$. The gauge bosons of the standard model fall into this class, as will other multiplets with light and superheavy partners.

• *Heavy and Superheavy Particles* ($m \gg M_Z$): Full multiplets under G none of whose members are light enough to see at the weak scale will contribute to $P(Z)$ if they contain mass splittings.

The function $P(Z)$ represents the knowledge of grand unification obtainable at SLC/LEP: a sum over the mass splittings of all the multiplets of G carrying electric charge and weak isospin, including the large $M_X - M_W$ splitting. $P(Z)$ probes energy scales up to 10^{15} GeV and more, and early universe times of 10^{-35} seconds, and thus is truly a 'remembrance of things past' [6].

The quantity A_{LR} is defined by the polarization states of the e^- beam in e^+e^- colliders [4,7,8]:

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \quad . \tag{2.4}$$

At the Z resonance:

$$A_{LR}(Z) \simeq \frac{2[1 - 4s_*^2(Z)]}{1 + [1 - 4s_*^2(Z)]^2} \quad , \tag{2.5}$$

for all light final-state fermions (excluding e^-), apart from other electroweak corrections. These include photon channel parts, weak vertices and bremsstrahlung, all of which have been computed to high accuracy [9]. The photon channel induces a slight dependence on the final state in $A_{LR}(Z)$; we use the standard $A_{LR}^{\mu^+\mu^-}(Z)$ ($e^+e^- \to \mu^+\mu^-$) [4,7]. Given s_0^2 and $e_*^2(Z)$, $A_{LR}(Z)$ directly measures $P(Z)$.

•For a given model, $P(Z)$ is computable. This requires M_X, obtained by running g_{2*}^2 and g_{3*}^2 (QCD coupling) together at high energy [3]. Strong interaction uncertainties then creep into M_X. In this paper, we will take M_X as an independent input. The standard model contributions to $P(Z)$ are known, as is $e_*^2(Z)$.

•More interesting is that, for any model, *independent of the other details of the model* and knowing s_0^2 alone, $P(Z)$ can be extracted from $A_{LR}(Z)$. This provides a *direct measurement* of M_X and a number for model-builders to chew on. $P(Z)$ also places an indirect constraint on g_{3*}^2 (Λ_{QCD}).

Combining the experimental errors in $A_{LR}(Z)$ ($\delta A_{LR}^{exp}(Z) \simeq 0.004$) [10,11] and the error in $e_*^2(Z)$ [from hadronic contributions [12] to the vacuum polarization: $\delta e_*^2(Z)/e_*^2(Z) \simeq 0.002$], we obtain the experimental limits on $P(Z)$:

$$\delta P(Z) \simeq 0.005 \quad . \tag{2.6}$$

For the minimal SU(5) model, this will measure M_X to an accuracy of $10^{\pm 0.04}$ or about ±9% , improving the current accuracy (from Λ_{QCD}) of 50–100% [13].

3. Some Examples

In this section, we present results from three different GUTs for the SLC/LEP observables A_{LR}, M_Z and M_W. The minimal SU(5) model is used first as a baseline, since it is the simplest possible GUT; although current low-energy data already appear to conflict with it [13]. The addition of supersymmetry, which has strong theoretical interest in its own right, eliminates this conflict, leading to the second example, SUSY SU(5). SU(5) is by no means the end of the story, as it can be embedded in more elaborate and symmetrical theories, starting with SO(10), leading to E_6. E_6 has a number of attractive properties and will serve as our final example [14,15]. SUSY E_6 also appears as a natural candidate GUT from the $E_8 \times E_8$ superstring theory. The supersymmetric models add new particles in the 100–1000 GeV region, but all contain a "desert" between 1 TeV and the unification scale. $s_0^2 = 3/8$ in all cases.

The standard model contributions to $P(Z)$ appear as:

$$P(Z) = \frac{1}{192\pi^2}\left[-55\ln M_W^2 + \frac{1}{2}\ln M_W^2 - 2\ln\left(\frac{m_{top}^2}{M_Z^2}\right) + O\left(\frac{M_Z^2}{M_W^2}\right)\right] \quad , \quad (3.1)$$

where the first term is due to the gauge bosons, the second due to the would-be Goldstone bosons eaten by the W and the Z, the third from the top-bottom quark splitting, and the fourth from electroweak threshold effects at M_Z. The last two turn out to be small in comparison to the large RNG logarithms, for reasonable values of the top quark mass ($<$ 200 GeV, the current limit from ρ-parameter measurements) [13], but are included in the complete self-energies in $P(Z)$. The top quark mass *can* have a strong effect on M_Z and M_W, the result of global isospin breaking [4,16]. (The Higgs boson mass has an analogous but smaller effect.) We will assume, as suggested by recent $B - \bar{B}$ mixing data, that $m_{top} = 60$ GeV [17]. In the SUSY theories, another contribution to M_Z and M_W will come from heavy degenerate chiral fermions (such as gauginos and Higgsinos) carrying weak isospin; the effect is independent of their masses if they are well above the weak scale. Such fermions were discussed in ref. [4] and their effects are included in the SUSY SU(5) and E_6 predictions for M_W and M_Z. We assume that all the superheavy particles have common mass M_X, although in general these will have splittings that contribute to $P(Z)$. The logarithms of dimensionful quantities will combine with the contributions of the superheavy particles to form logarithms of dimensionless ratios.

In Table I we show the results for observables from the minimal SU(5) model [18]. The leading logarithms in $P(Z)$ are:

$$P(Z) = \frac{1}{192\pi^2}\left[55\ln M_X^2 - \frac{1}{2}\ln M_X^2\right] \quad . \tag{3.2}$$

The general effect of raising M_X is to lower $s_*^2(Z)$and raise $A_{LR}(Z)$. The currently predicted M_X (from Λ_{QCD}) is $\sim 2.0 \times 10^{14}$ GeV [13]. A general result: gauge bosons lower $s_*^2(Z)$, raising $A_{LR}(Z)$; while fermions and scalars raise $s_*^2(Z)$.

For the SUSY theories, we introduce a new mass scale in addition to the weak (M_W) and grand unification (M_X) scales, called μ. The new heavy (100–1000 GeV) particles required by these theories, including SUSY partners of the standard model, new neutral gauge bosons (Z'), new fermions, as well as the Higgs bosons, are all assumed to have common mass μ. From current ρ-parameter measurements, none of these new multiplets can have dramatic mass splittings in any case. Table II shows the predictions of SUSY SU(5) as a function of M_X and μ. Λ_{QCD} predicts an M_X of $\sim 5 \times 10^{15}$ GeV [13]. The leading logarithms of SUSY SU(5) in $P(Z)$ are:

$$P(Z) = \frac{1}{192\pi^2}\left[55\ln M_X^2 - \frac{1}{2}\ln M_X^2 + \frac{25}{2}\ln\left(\frac{\mu^2}{M_X^2}\right)\right] \quad . \tag{3.3}$$

The simplest SUSY E_6 model includes an additional low-energy U(1) or Z' [14]. The Z' does not contribute to $P(Z)$, as it is neutral; we assume that the Z' is heavy enough to decouple from M_Z and M_W. The superstring-inspired Wilson lines are assumed to break the E_6 symmetry in place of superHiggses. Table III shows the SUSY E_6 predictions for such a model. The leading logarithms of E_6 are:

$$P(Z) = \frac{1}{192\pi^2}\left[55\ln(M_X^2) - \frac{1}{2}\ln(\mu^2) + 10\ln\left(\frac{\mu^2}{M_X^2}\right)\right] \quad . \tag{3.4}$$

M_X has been estimated as $\sim 5 \times 10^{17}$ GeV [15]. Note that raising the scale μ of new heavy physics (while holding M_X^2 fixed) tends to raise the predictions for $A_{LR}(Z)$, M_W and M_Z. SUSY theories, because of their new representations of scalars and fermions, also have higher unification scales, bringing them closer to the Planck scale.

Figure 1 summarizes the predictions for $A_{LR}(Z)$ from these three models, including two different values of μ (100 and 1000 GeV). The current lower bound from proton decay on M_X [in minimal SU(5)] is approximately 1.8×10^{15} GeV (2×10^{32} years) [19] and is obviously in conflict with the prediction based on Λ_{QCD}. Figures 2 and 3 show the corresponding predictions for M_Z and M_W. Note that the experimental accuracy of $A_{LR}(Z)$ is clearly sufficient to distinguish among the sample models.

4. Conclusion

The prediction of the electroweak mixing from grand unification has not been subject to exacting tests, but has produced results in the neighborhood of current experimental values for a wide class of models. The measurement of the polarization asymmetry at SLC/LEP will mark a leap in the accuracy of electroweak measurements, for the first time allowing easy discrimination among different grand unification schemes. Besides testing the predictions of specific models, $A_{LR}(Z)$ can directly measure the unification scale of any model, circumventing the need to predict M_X from Λ_{QCD}. The electroweak gauge boson masses M_W and M_Z can then be predicted from knowledge of $A_{LR}(Z)$, although they require additional assumptions about the particle content of the theory. Tests of grand unification will thus be raised to a new level of precision by the use of polarized beams at SLC/LEP, underscoring the importance of polarization in exploiting the full potential of these accelerators.

Acknowledgements

We would like to acknowledge our debt to the Mark II collaboration at SLAC for their support and encouragement. One of us (DCK) would also like to thank Michael Peskin of SLAC for valuable discussions.

References

[1] A. Blondel, in 22^{nd} *Rencontre de Moriond,* J. Tran Thanh Van, editor (1987), Vol. 1, pp. 3–32.

[2] R. N. Mohapatra, *Unification and SUSY* (Springer Verlag, New York, 1986), Chapter 5.

[3] H. Georgi, H. R. Quinn and S. Weinberg, *Phys. Rev. Lett.* **33** (1974) 451.

[4] D. C. Kennedy and B. W. Lynn, SLAC–PUB–4039 (1988), submitted to *Nucl. Phys. B.*

[5] A somewhat similar attempt in this direction with "effective field theories" may be found in S. Weinberg, *Phys. Lett.* **91B** (1980) 51.

[6] F. J. Dyson, *Infinite in All Directions* (Harper & Row, New York, 1988), p. 44.

[7] B. W. Lynn and R. G. Stuart, *Nucl. Phys.* **B253** (1985) 216.

[8] B. W. Lynn, M. E. Peskin and R. G. Stuart, SLAC–PUB–3725 (1985); *LEP Yellow Book,* CERN 86–02 (1986), Vol. I, p. 90.

[9] D. C. Kennedy, B. W. Lynn, C. J.-C. Im and R. G. Stuart, SLAC–PUB–4128 (1988), submitted to *Nucl. Phys. B.*

[10] B. W. Lynn and C. Verzegnassi, *Phys. Rev.* **D35** (1987) 3326.

[11] SLC Polarization Group, SLAC/Mark II, SLC Polarization Proposal.

[12] B. W. Lynn, G. Penso and C. Verzegnassi, *Phys. Rev.* **D35** (1987) 42.

[13] U. Amaldi, A. Böhm et al., U. of Penn. preprint, UPR–0331T (1987).

[14] M. E. Peskin, *SLAC Summer School Lectures,* 1987; SLAC–PUB–4601 (1988).

[15] M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory,* Vol. 2: *Loop Amplitudes, Anomalies and Phenomenology* (Cambridge University Press, Cambridge, England, 1987), Chapter 16.

[16] M. Veltman, *Nucl. Phys.* **B123** (1977) 89.

[17] K. Wachs, DESY 87/084 (1987); I. I. Y. Bigi, SLAC–PUB–4439 (1987).

[18] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **33** (1974) 451. The predictions of Table I agree with those published in W. Marciano, *Phys. Rev.* **D20** (1979) 274.

[19] R. N. Mohapatra, in *Gauge Theories of Fundamental Interaction,* R. N. Mohapatra and C. H. Lai, editors (World Scientific, Singapore, 1981); P. Langacher, U. of Penn. preprint UPR–0263T (1984).

Table Captions

Predictions of $s_*^2(Z)$, $A_{LR}^{\mu^+\mu^-}(Z)$, M_Z and M_W.

I. Minimal SU(5).

II. Supersymmetric SU(5); $\mu = 100$ and 1000 GeV.

III. Supersymmetric E_6; $\mu = 100$ and 1000 Gev.

Table I. Minimal SU(5) predictions.

$m_{top} = 60$ GeV, $m_{Higgs} = 100$ GeV
All masses in GeV

M_X	$s_*^2(Z)$	$A_{LR}^{\mu^+\mu^-}(Z)$	M_Z	M_W
10^{14}	0.2186	0.238	93.36	82.44
10^{15}	0.2055	0.336	95.50	84.99
10^{16}	0.1927	0.427	97.84	87.76
10^{17}	0.1799	0.511	100.48	90.83
10^{18}	0.1671	0.589	103.46	94.25

Table II. Supersymmetric SU(5) predictions.

$m_{top} = 60$ GeV, $\mu = 100$ GeV
All masses in GeV

M_X	$s_*^2(Z)$	$A_{LR}^{\mu^+\mu^-}(Z)$	M_Z	M_W
10^{14}	0.2541	− 0.042	89.24	76.88
10^{15}	0.2438	0.041	90.50	78.45
10^{16}	0.2339	0.119	91.80	80.09
10^{17}	0.2240	0.197	93.20	81.83
10^{18}	0.2141	0.272	94.73	83.64

$m_{top} = 60$ GeV, $\mu = 1000$ GeV
All masses in GeV

M_X	$s_*^2(Z)$	$A_{LR}^{\mu^+\mu^-}(Z)$	M_Z	M_W
10^{14}	0.2511	− 0.018	89.67	77.34
10^{15}	0.2408	0.064	90.95	78.83
10^{16}	0.2309	0.143	92.28	80.51
10^{17}	0.2211	0.219	93.72	82.29
10^{18}	0.2112	0.294	95.29	84.19

Table III. Supersymmetric E_6 predictions.

$m_{top} = 60$ GeV, $\mu = 100$ GeV

All masses in GeV

M_X	$s_*^2(Z)$	$A_{LR}^{\mu^+\mu^-}(Z)$	M_Z	M_W
10^{14}	0.2456	0.026	90.18	78.19
10^{15}	0.2346	0.114	91.61	79.97
10^{16}	0.2240	0.197	93.11	81.83
10^{17}	0.2134	0.277	94.75	83.83
10^{18}	0.2028	0.355	96.55	85.98

$m_{top} = 60$ GeV, $\mu = 1000$ GeV

All masses in GeV

M_X	$s_*^2(Z)$	$A_{LR}^{\mu^+\mu^-}(Z)$	M_Z	M_W
10^{14}	0.2433	0.044	90.54	78.56
10^{15}	0.2323	0.131	91.99	80.26
10^{16}	0.2218	0.214	93.53	82.15
10^{17}	0.2112	0.294	95.20	84.19
10^{18}	0.2006	0.372	97.04	86.38

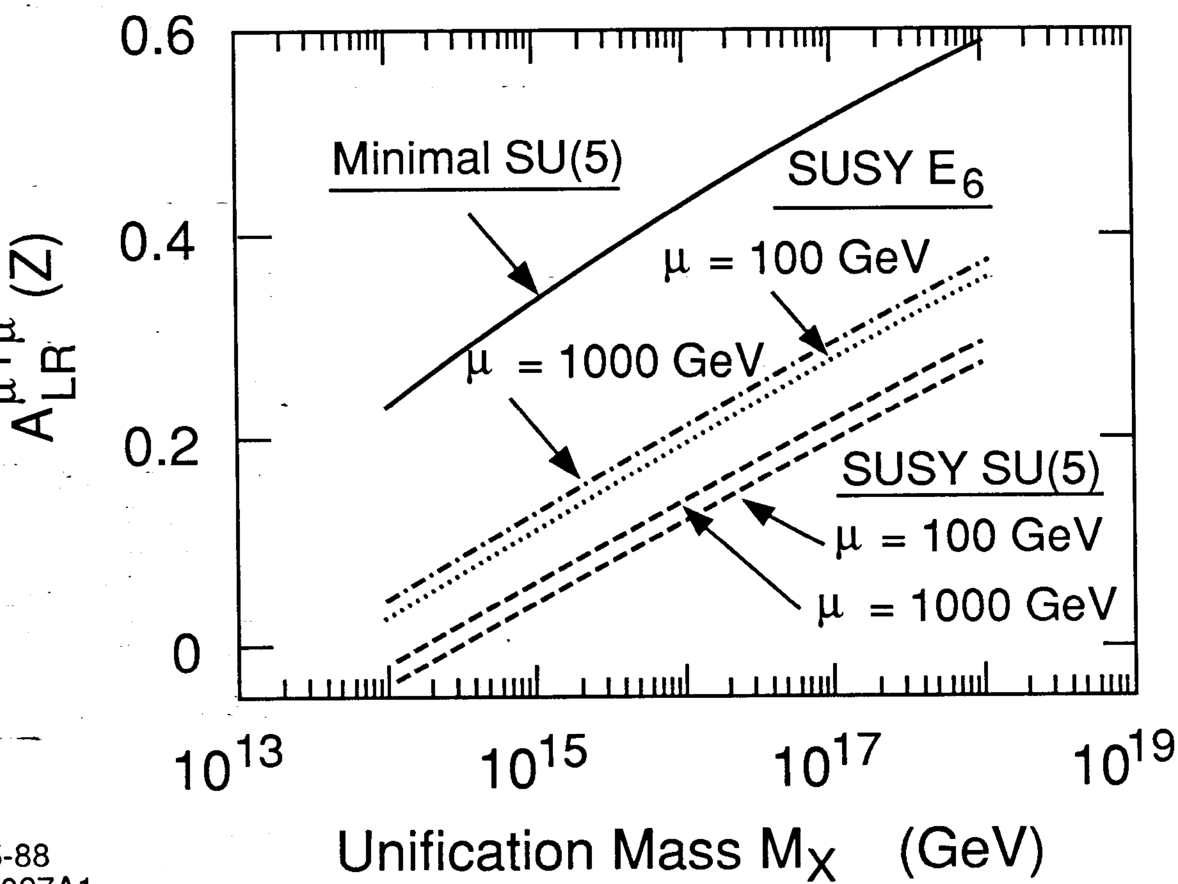
Figure Captions

Predictions for minimal SU(5), supersymmetric SU(5) and E_6.

I. Polarization asymmetry at Z resonance $A_{LR}^{\mu^+\mu^-}$ (Z).

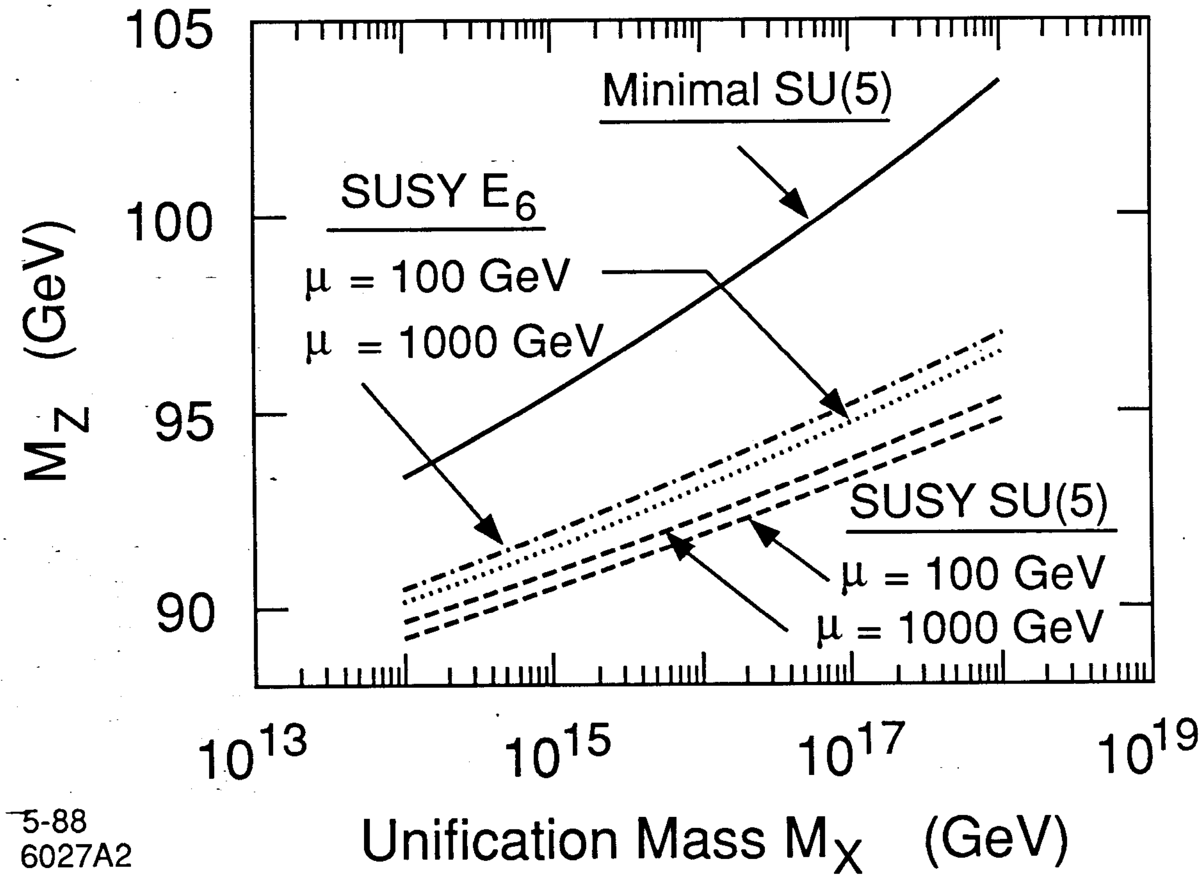
II. Z vector boson mass M_Z.

III. W vector boson mass M_W.



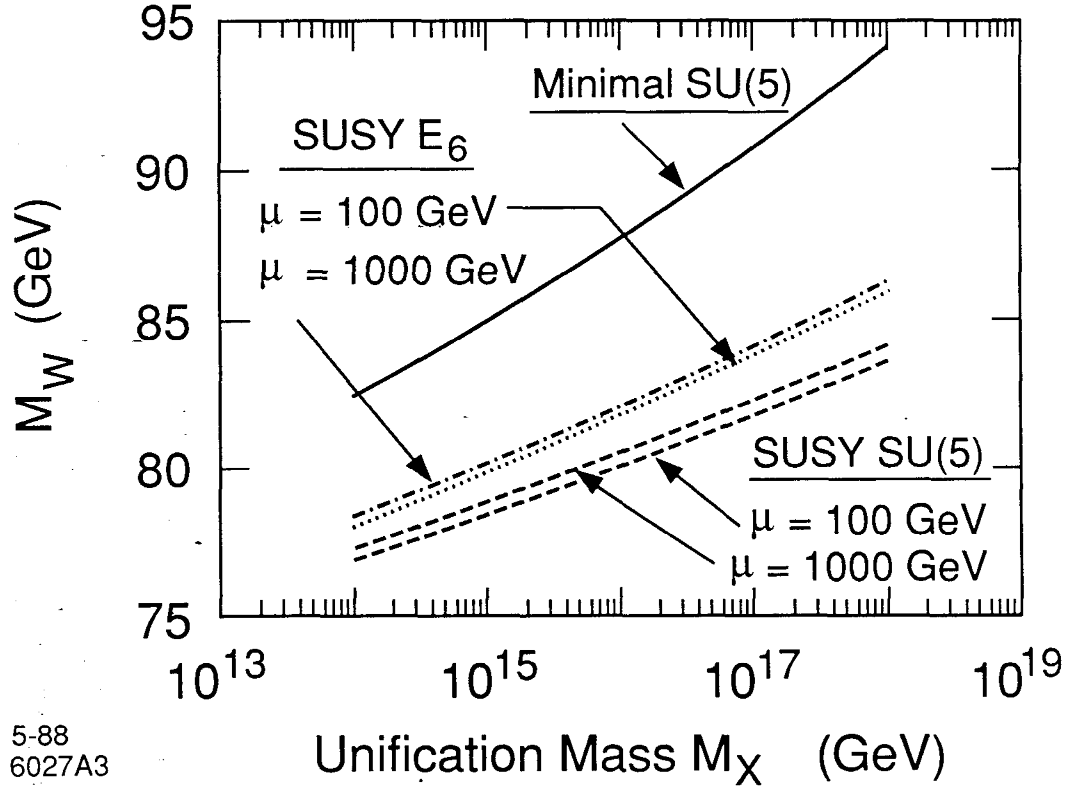
5-88
6027A1

Fig. 1



105
100
95
90
M_Z (GeV)
Minimal SU(5)
SUSY E_6
μ = 100 GeV
μ = 1000 GeV
SUSY SU(5)
μ = 100 GeV
μ = 1000 GeV
10^{13}
10^{15}
10^{17}
10^{19}
Unification Mass M_X (GeV)
5-88
6027A2

Fig. 2



95
90
85
80
75
Minimal SU(5)
SUSY E6
μ = 100 GeV
μ = 1000 GeV
SUSY SU(5)
μ = 100 GeV
μ = 1000 GeV
M_W (GeV)
10^13
10^15
10^17
10^19
Unification Mass M_X (GeV)
5-88
6027A3

Fig. 3