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QUARK FLAVOR MIXING, CP VIOLATION, AND ALL THAT*

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ABSTRACT

We review the present state of knowledge of the mixing of quark flavors under weak interactions and the associated explanation of CP violation inherent in the single nontrivial phase present in the three-generation mixing matrix. In this context we present the phenomenological basis for the increasing possibility that large CP violation asymmetries can be experimentally observed in the B meson system.

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Introduction

First and foremost, we study weak decays, flavor mixing and CP violation from the perspective of finding evidence for physics which lies outside the standard model. Thus, for example, we search for:

- Processes forbidden in the standard model, such as would be induced by lepton-flavor changing neutral currents
- Indications that CP violating phenomena have an origin other than from the nontrivial phase in the quark flavor mixing matrix
- Deviations from expected rates, especially for rare processes such as those forbidden at tree level in the electroweak interactions. These can be sensitive to heavy virtual particles (from a fourth generation, supersymmetry, left-right electroweak gauge symmetry, etc.) This is especially true of CP violating amplitudes, which, when they involve one loop amplitudes, arise first at momentum scales due to second and third generation quarks rather than those characteristic of Λ_{QCD} or light quarks.
- Theoretical relations between masses and mixing angles. These are both put into the standard model by hand, and therefore originate from outside of it.

From a less revolutionary perspective, we look at these phenomena from inside the standard model to study:

- The interplay of strong and electroweak interactions in weak decays of hadrons
- The parameters of the standard model (masses and mixing angles). Eventually we will pin down these parameters, permitting us to calculate the standard model contributions to these processes unambiguously.

These two attitudes are interrelated. As each *a priori* free parameter of the standard model is measured, we use the then updated predictions of the standard model and return to the former perspective of looking for physics beyond the standard model by examining the consistency of all previous data and by pointing to further experimental measurements with which to compare standard model predictions.

In what follows we examine a few of the aspects from both perspectives for which there have been recent improvements in our knowledge and understanding. These are: the Kobayashi–Maskawa matrix, CP violation, and CP violation in B decay.

The Kobayashi–Maskawa Matrix

In the standard model with SU(2) $\times$ U(1) as the gauge group of electroweak interactions, both the quarks and leptons are assigned to be left-handed doublets and right-handed singlets. The quark mass eigenstates are not the same as the weak eigenstates, and the matrix connecting them has become known as the Kobayashi–Maskawa[1] (K–M) matrix since an explicit parametrization in the six-quark case was first given by them in 1973. It generalizes the four-quark case, where the matrix is parametrized by a single angle, the Cabibbo angle.[2]

By convention, the three charge 2/3 quarks (u, c, and t) are unmixed, and all the mixing is expressed in terms of a 3×3 unitary matrix V operating on the charge $-1/3$ quarks (d, s, b):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{1}$$

There are several parametrizations of the K–M matrix. In the 1988 edition of the Review of Particle Properties a "standard" form is advocated:[3]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} . \tag{2}$$

This is the notation of Harari and Leurer[4] for a form generalizable to an arbitrary number of "generations" and also proposed by Fritzsch and Plankl.[5] The choice of rotation angles follows that of Maiani,[6] and the placement of the phase follows that of Wolfenstein.[7] The three "generation" form was proposed earlier by Chau and Keung.[8] Here $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, with i and j being "generation" labels, $\{i, j = 1, 2, 3\}$. In the limit $\theta_{23} = \theta_{13} = 0$ the third generation decouples, and the situation reduces to the usual Cabibbo mixing of the first two generations with θ_{12} identified with the Cabibbo angle.[2] The real angles θ_{12}, θ_{23}, θ_{13} can all be made to lie in the first quadrant by an appropriate redefinition of quark field phases. Then all s_{ij} and c_{ij} are positive, and $|V_{us}| = s_{12}c_{13}$, $|V_{ub}| = s_{13}$, and $|V_{cb}| = s_{23}c_{13}$. ~~As~~ c_{13} deviates from unity only in the fifth decimal place (from experimental measurement of s_{13}), $|V_{us}| = s_{12}$, $|V_{ub}| = s_{13}$, and $|V_{cb}| = s_{23}$ to an excellent approximation. The phase δ_{13} lies in the range $0 \leq \delta_{13} < 2\pi$, with non-zero values generally breaking CP invariance for the weak interactions.

The values of individual K–M matrix elements can in principle all be determined from weak decays of the relevant quarks, or, in some cases, from deep inelastic neutrino scattering. Our present knowledge of the matrix elements comes from the following sources:

(1) Nuclear beta decay, when compared to muon decay, gives[9,10]

$$|V_{ud}| = 0.9747 \pm 0.0011 . \tag{3}$$

(2) Analysis of K_{e3} decays yields[11] $|V_{us}| = 0.2196 \pm 0.0023$. The analysis of hyperon decay data has larger theoretical uncertainties because of first order SU(3) symmetry breaking

effects in the axial-vector couplings, but due account of symmetry breaking gives a consistent value[12] of $0.220 \pm 0.001 \pm 0.003$. The average of these two results is[3]

$$|V_{us}| = 0.2197 \pm 0.0019 \ . \tag{4}$$

(3) The magnitude of $|V_{cd}|$ may be deduced from neutrino and antineutrino production of charm off valence d quarks. When the dimuon production cross sections of the CDHS group[13] are supplemented by more recent measurements of the semileptonic branching fractions and the production cross sections in neutrino reactions of various charmed hadron species, the value[14]

$$|V_{cd}| = 0.21 \pm 0.03 \ . \tag{5}$$

is extracted.

(4) Values of $|V_{cs}|$ from neutrino production of charm are dependent on assumptions about the strange quark density in the parton-sea. The most conservative assumption, that the strange-quark sea does not exceed the value corresponding to an SU(3) symmetric sea, leads to a lower bound,[13] $|V_{cs}| > 0.59$. It is more advantageous to proceed analogously to the method used for extracting $|V_{us}|$ from K_{e3} decay; namely, we compare the experimental value for the width of D_{e3} decay with the expression[15] that follows from the standard weak interaction amplitude. This gives:[3]

$$|f_+^D(0)|^2 |V_{cs}|^2 = 0.51 \pm 0.07 \quad .$$

With sufficient confidence in a theoretical calculation of $|f_+^D(0)|$ a value of $|V_{cs}|$ follows,[17] but even with the very conservative assumption that $|f_+(0)| < 1$ it follows that

$$|V_{cs}| > 0.66 \quad . \tag{6}$$

The constraint of unitarity when there are only three-generations gives a much tighter bound (see below).

(5) The ratio $|V_{ub}/V_{cb}|$ can be obtained from the semileptonic decay of B mesons by fitting to the lepton energy spectrum as a sum of contributions involving $b \to u$ and $b \to c$. The relative overall phase space factor between the two processes is calculated from the usual four-fermion interaction with one massive fermion (c quark or u quark) in the final state. The value of this factor depends on the quark masses, but is roughly one-half. The lack of observation of the higher momentum leptons characteristic of $b \to u\ell\bar{\nu}_\ell$ as compared to $b \to c\ell\bar{\nu}_\ell$ has resulted thus far only in upper limits which depend on the lepton energy spectrum assumed for each decay.[17,18,19] Using the lepton momentum region near the

endpoint for $b \to c\ell\bar{\nu}_\ell$ and taking the calculation[19] of the lepton spectrum that gives the least restrictive limit results in[20]

$$|V_{ub}/V_{cb}| < 0.20 \; . \tag{7}$$

A lower bound on $|V_{ub}|$ can be established from the observation[21] of exclusive baryonic B decays into $p\bar{p}\pi$ and $p\bar{p}\pi\pi$ which involve $b \to u + d\bar{u}$ at the quark level. A chain of assumptions on the relative phase space, the fraction of the quark level process which hadronizes into baryonic channels, and the fraction of those that occur in the observed modes is required. No other channels that reflect $b \to u$ at the quark level have been observed.[22] Given the branching fractions of the two observed modes, a reasonable lower limit is[21]

$$|V_{ub}/V_{cb}| > 0.07 \; . \tag{8}$$

(6) The magnitude of V_{cb} itself can be determined if the measured semileptonic bottom hadron partial width is assumed to be that of a b quark decaying through the usual $V - A$ interaction:[3]

$$|V_{cb}| = 0.046 \pm 0.010 \; . \tag{9}$$

Most of the error quoted in eq. (9) is not from the experimental uncertainty in the value of the b lifetime, but in the theoretical uncertainties in choosing a value of m_b and in the use of the quark model to represent inclusively semileptonic decays which, at least for the B meson, are dominated by a few exclusive channels. We have made the error bars larger than they are sometimes stated to reflect these uncertainties. They include the central values obtained for $|V_{cb}|$ by using a model for the exclusive final states in semileptonic B decay and extracting $|V_{cb}|$ from the absolute width for one or more of them.[17,19,23]

From eqs. (3) through (9), plus unitarity (assuming only three-generations), the 90% confidence limits on the magnitude of the elements of the complete matrix are:[3]

$$\begin{pmatrix} 0.9748 \text{ to } 0.9761 & 0.217 \text{ to } 0.223 & 0.003 \text{ to } 0.010 \\ 0.217 \text{ to } 0.223 & 0.9733 \text{ to } 0.9754 & 0.030 \text{ to } 0.062 \\ 0.001 \text{ to } 0.023 & 0.029 \text{ to } 0.062 & 0.9980 \text{ to } 0.9995 \end{pmatrix} \; . \tag{10}$$

The ranges shown are for the individual matrix elements. The constraints of unitarity connect different elements, so choosing a specific value for one element restricts the range of the others. The ranges given in eq. (10) are consistent with the one standard deyiation errors on the input matrix elements.

The data do not preclude there being more than three-generations. Of course, the constraints deduced from unitarity are loosened when the K–M matrix is expanded to accommodate more generations. Still, the known entries restrict the possible values of additional elements if the matrix is expanded to account for additional generations. For example, unitarity and the known elements of the first row require that any additional element in the first row have a magnitude $|V_{ub'}| < 0.07$, and the known elements of the first column require that $|V_{t'd}| < 0.15$

Further information on the angles requires theoretical assumptions. For example, $B_d - \bar{B}_d$ mixing, if it originates from short-distance contributions to ΔM_B dominated by box diagrams involving virtual t quarks, gives information on $V_{tb}\, V_{td}^*$ once hadronic matrix elements and the t quark mass are known.[24] A similar comment holds for $V_{tb}\, V_{ts}^*$ and $B_s - \bar{B}_s$ mixing. Even at the present stage of knowledge, we may use the published data claiming the observation of $B - \bar{B}$ mixing[25] to obtain a significant lower bound on $|V_{td}|$ within the three-generation standard model. This is because the magnitude of the mixing depends on m_t, an hadronic matrix element, and $|V_{td}|$. Taking $m_t < 180$ GeV,[26] and the relevant matrix element parametrized as $|B_B f_B^2|$ to be less than $(200\ \mathrm{MeV})^2$, we obtain

$$|V_{td}| > 0.006 \quad . \tag{11}$$

This is a considerable improvement over the constraint provided by unitarity and the measured values of other matrix elements in eq. (10).

Up to this point we have discussed only information on magnitudes of K–M matrix elements. In principle, such measurements of magnitudes could tell us about the phase, δ_{13}, as well as the "rotation angles" θ_{12}, θ_{23}, and θ_{13} in eq. (2). This is most easily seen for the case at hand, where the "rotation angles" are small, by using the unitarity of the K–M matrix applied to the first and third columns to derive that (c_{ij} have been set to unity):

$$1 \cdot V_{ub}^* - s_{12} \cdot V_{cb}^* + V_{td} \cdot 1 \approx 0 \quad . \tag{12}$$

This equation is represented graphically in figure 1 in terms of a triangle in the complex plane, the length of whose sides is $|V_{ub}^*|$, $|s_{12} \cdot V_{cb}^*|$, and $|V_{td}|$. This triangle has been implicit, and even occasionally explicit, in many people's work on the constraints on the K–M matrix implied by various data involving mixing or CP violation, but has been particularly emphasized by Bjorken.[27,28]

With this representation of the unitarity of the K–M matrix, it is possible to see more directly the interplay of various pieces of experimental information. For example, an increase in the magnitude of the $b \to u$ transition obviously increases the side whose length is $|V_{ub}|$. The present upper bound on $|V_{ub}/V_{cb}|$ means that this side at most is as long as the side whose length is $|s_{12} V_{cb}^*|$. On the other hand, an increased magnitude for $B_d - \bar{B}_d$ mixing implies

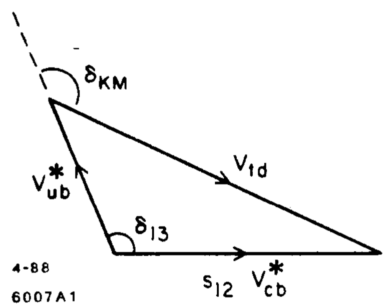

Fig. 1. Representation in the complex plane of the triangle formed by the Kobayashi–Maskawa matrix elements V_{ub}^*, $s_{12} \cdot V_{cb}^*$, and V_{td}.

(keeping m_t and the appropriate hadronic matrix element, $B_B f_B^2$, fixed) stretching the side whose length is $|V_{td}|$. With the other sides set by independent measurements, the triangle gets flatter and flatter and eventually "breaks." At that point $B - \bar{B}$ mixing has become incompatible with other data plus assumed values of m_t and the hadronic matrix element. Hence the derivation of a lower bound on m_t from $B - \bar{B}$ mixing.[24][29]

In principle, accurate measurement of the lengths of all three sides could show that the triangle can not exist (and we must go beyond the three-generation standard model), or cause the triangle to collapse to a line (and we must go beyond the standard model for an explanation of CP violation), or demand the existence of a nontrivial triangle with δ_{13} not equal to 0° or 180°. Unfortunately, given our present experimental knowledge and our limited theoretical ability to compute hadronic matrix elements, the three sides are not known with sufficient accuracy to discriminate between these situations, let alone determine the value of δ_{13}. To do this we are forced to consider a CP violating quantity and assume it can be understood within the three-generation standard model.

In this connection, note that the law of sines applied to the triangle gives:

$$\frac{\sin \delta_{KM}}{|s_{12} V_{cb}^*|} = \frac{\sin \delta_{13}}{|V_{td}|} \quad .$$

Setting cosines of small angles to unity and expressing V_{cb} as s_{23}, but V_{td} as $s_1 s_2$ in the original notation of Kobayashi and Maskawa,[1] allows this equation to be converted to ($s_{12} \approx s_1$):

$$s_1^2 s_2 s_3 \sin \delta_{KM} = s_{12} s_{23} s_{13} \sin \delta_{13} \quad . \tag{13}$$

This is twice the area of the triangle and, aside from cosines of small angles having been set to unity, is just proportional to the measure of CP violation in the three-generation standard model proposed by Jarlskog.[30]

CP Violation

As noted in the previous section, the standard model allows for CP violation in the form of phases originating in the quark mixing matrix, and when there are three-generations of quarks and leptons, there is just one nontrivial CP violating phase. The computation of any difference of rates between a given process and its CP conjugate process always has the form

$$\Gamma - \bar{\Gamma} \;\propto\; \text{coef.} \times\; s_1^2 s_2 s_3 \sin\delta_{KM} c_1 c_2 c_3 = \text{coef.} \times s_{12} s_{23} s_{13} \sin\delta_{13} c_{12} c_{23} c_{13}^2 \tag{14}$$

where we express things first in the original parametrization of the quark mixing matrix[1] and then in the "new" parametrization used in the previous section. Our present experimental knowledge means that the approximation of setting the cosines to unity induces errors of at most a few percent. In what follows we will usually write only the factor involving sines of angles. Then eq. (13) of the last section would have already permitted us to relate the appropriate factor in the two parametrizations. For old times sake, we henceforth revert to the K–M parametrization.

The combination of sines and cosines of K–M angles that occurs in eq. (14) is mandatory for a CP violating effect with three generations. It is precisely this combination of factors that occurs in the determinant of the commutator of mass matrices introduced by Jarlskog[30] to formulate a general condition for CP violation, if her basis-independent condition is restated in the K–M parametrization. We see explicitly from eq. (14) that the presence of non-zero mixing for all three-generations is required in order to have a CP violating effect. This is not surprising; we know that with only two generations there is no CP violation from the quark mixing matrix (all the potential phases can be absorbed into the quark fields) and this is exactly the situation we would be in if we set one of the mixing angles to 0 or $\pi/2$ and decoupled one of the generations from the other two.

When we form a CP violating asymmetry we divide a difference in rates by their sum:

$$\text{Asymmetry} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \quad . \tag{15}$$

If we do this for K decay, the decay rates for the dominant hadronic and leptonic modes all involve a factor of s_1^2, *i.e.*, essentially the Cabibbo angle squared. A CP violating asymmetry will then have the general dependence on K–M factors:

$$\text{Asymmetry}_{K\ Decay} \propto\; s_2 s_3 s_\delta \quad . \tag{16}$$

The right-hand side is of order 10^{-3} (see the discussion below). This is both a theoretical plus and an experimental minus. The theoretical good news is that CP violating asymmetries in the neutral K system are naturally at the 10^{-3} level, in agreement with the measured value of $|\epsilon|$. The experimental bad news is that, no matter what the K decay process, it is always going to be at this level, and therefore difficult to get at experimentally with the precision necessary to sort out the standard model explanation of its origin from other explanations.

Note also that because CP violation must involve all three-generations while the K has only first and second generation quarks in it (and its decay products only involve first generation quarks), CP violating effects must come about through heavy quarks in loops. There is no CP violation arising from tree graphs alone.

This is not the case in B decay (or B mixing and decay). First, the decay rate for the leading decays is very roughly proportional to s_2^2, which happens to be much smaller than the corresponding quantity (s_1^2) in K decay. But, more importantly, we can look at decays which have rates that are K–M suppressed by factors of $(s_1 s_2)^2$ or $(s_1 s_3)^2$, just to choose two examples. By choosing particular decay modes, it is then possible to have asymmetries which behave like

$$\text{Asymmetry}_{B\ Decay} \propto s_\delta \quad . \tag{17}$$

With luck, this could be of order unity! Note, though, that we have to pay the price of CP violation somewhere. That price, the product $s_1^2 s_2 s_3 s_\delta$, is given in the CP violating difference of rates in eq. (14). The K–M factors either are found in the basic decay rate, resulting in a very small branching ratio, or they enter the asymmetry, which is then correspondingly small. This is a typical pattern: the rarer the decay, the bigger the potential asymmetry. The only escape from this pattern comes from outside of K–M factors: to find a decay mode where the coefficient of the right-hand side of eq. (14) is large. A good example of this is provided by $B-\bar{B}$ mixing, which can be large because of a combination of the values of a hadronic matrix element and m_t, as well as a K–M matrix element.

The fact that asymmetries in K and B decay can be different by orders of magnitude is part and parcel of the origin of CP violation in the standard model. It "knows" about the quark mass matrices and can tell the difference between a b quark and an s quark. This is entirely different from what we expect in general from explanations of CP violation that come from very high mass scales, as in the superweak model or in left-right symmetric gauge theories. Then, all quark masses are negligible compared to the new, very high mass scale. Barring special provisions, there is no reason why such theories would distinguish one quark from another; we expect all CP violating effects to be roughly of the same order, namely that already observed in the neutral K system.

As the last year has unfolded, the standard model "explanation" of CP violation has looked better and better. In particular, there have been two important new experimental results for ϵ'/ϵ. First came the result from a test run of the Fermilab experiment[31] which has been updated to:

$$\epsilon'/\epsilon = 3.2 \pm 2.8 \pm 1.2 \times 10^{-3} \quad ,$$

and then the result from the CERN experiment[32]

$$\epsilon'/\epsilon = 3.3 \pm 1.1 \times 10^{-3} \quad .$$

It appears that CP is violated not only in the neutral K mass matrix (ϵ), but in the $K \to \pi\pi$ decay amplitude itself (ϵ'). If $\epsilon'/\epsilon \sim 3.3 \times 10^{-3}$, then CP violating effects from heavy quark loops is a likely interpretation and, especially if m_t is large, the result is within the ballpark of standard model expectations. It would seem that the wind is blowing in the direction of the standard model and the explanation of CP violation in terms of the K–M phase.

CP Violation in B Decay

The possibilities for observation of CP violation in B decays are much richer than for the neutral K system. The situation is even reversed, in that for the B system the variety and size of CP violating asymmetries in decay amplitudes far overshadows that in the mass matrix.[33]

To start with the familiar, however, it is useful to consider the phenomenon of CP violation in the mass matrix of the neutral B system. Here, in analogy with the neutral K system, one defines a parameter ϵ_B. It is related to p and q, the coefficients of the B° and $\bar{B}^\circ$, respectively, in the combination which is a mass matrix eigenstate by

$$\frac{q}{p} = \frac{1-\epsilon_B}{1+\epsilon_B} \quad .$$

The charge asymmetry in $B^\circ\bar{B}^\circ \to \ell^\pm\ell^\pm + X$ is given by[34]

$$\frac{\sigma(B^\circ\bar{B}^\circ \to \ell^+\ell^+ + X) - \sigma(B^\circ\bar{B}^\circ \to \ell^-\ell^- + X)}{\sigma(B^\circ\bar{B}^\circ \to \ell^+\ell^+ + X) + \sigma(B^\circ\bar{B}^\circ \to \ell^-\ell^- + X)} = \frac{|\frac{p}{q}|^2 - |\frac{q}{p}|^2}{|\frac{p}{q}|^2 + |\frac{q}{p}|^2} \tag{18}$$

$$= \frac{\mathrm{Im}(\Gamma_{12}/M_{12})}{1 + \frac{1}{4}|\Gamma_{12}/M_{12}|^2} \tag{19}$$

where we define $< B^\circ|H|\bar{B}^\circ >= M_{12} - \frac{i}{2}\Gamma_{12}$. The quantity $|M_{12}|$ is measured in $B - \bar{B}$ mixing and we may estimate Γ_{12} by noting that it gets contributions from B° decay channels which are common to both B° and $\bar{B}^\circ$, *i.e.*, K–M suppressed decay modes. This causes the charge asymmetry for dileptons most likely to be in the ballpark of a few times 10^{-3}, and at best 10^{-2}. For the foreseeable future, we might as well forget it experimentally.

Turning now to CP violation in decay amplitudes; in principle, this can occur whenever there is more than one path to a common final state. For example, let us consider decay to a CP eigenstate, f, like ψK_s°. Since there is substantial $B^\circ - \bar{B}^\circ$ mixing, one can consider two decay chains of an initial B° meson:

$$\begin{array}{lcl} B^\circ \to B^\circ & \searrow & \\ & & f \quad , \\ B^\circ \to \bar{B}^\circ & \nearrow & \end{array}$$

where f is a CP eigenstate. The second path differs in its phase because of the mixing of $B^\circ \to \bar{B}^\circ$, and because the decay of a $\bar{B}$ involves the complex conjugate of the K–M factors involved in B decay. The strong interactions, being CP invariant, give the same phases for the two paths. The amplitudes for these decay chains can interfere and generate non-zero asymmetries between $\Gamma(B^\circ(t) \to f)$ and $\Gamma(\bar{B}^\circ(t) \to f)$. Specifically,

$$\Gamma(\bar{B}^\circ(t) \to f) \sim e^{-\Gamma t} \left(1 - \sin[\Delta m\ t] Im\left(\frac{p}{q}\rho\right)\right) \tag{20a}$$

and

$$\Gamma(B^\circ(t) \to f) \sim e^{-\Gamma t} \left(1 + \sin[\Delta m\ t] Im\left(\frac{p}{q}\rho\right)\right) \quad . \tag{20b}$$

Here we have neglected any lifetime difference between the mass matrix eigenstates (thought to be very small) and set $\Delta m = m_1 - m_2$, the difference of the eigenstate masses, and $\rho = A(B \to f)/A(\bar{B} \to f)$, the ratio of the amplitudes, and we have used the fact that $|\rho| = 1$ when f is a CP eigenstate in writing eqs. (20). From this we can form the asymmetry:

$$A_{\text{CP Violation}} = \frac{\Gamma(B) - \Gamma(\bar{B})}{\Gamma(B) + \Gamma(\bar{B})} = \sin[\Delta m\ t] Im\left(\frac{p}{q}\rho\right) \quad . \tag{21}$$

In the particular case of decay to a CP eigenstate, the quantity $Im\left(\frac{p}{q}\rho\right)$ is given entirely by the K–M matrix and is independent of hadronic amplitudes. However, to measure the asymmetry experimentally, one must know if one starts with an initial B° or $\bar{B}^\circ$, *i.e.*, one must "tag."

We can also form asymmetries where the final state f is not a CP eigenstate. Examples are $B_d \to D\pi$ compared to $\bar{B}_d \to \bar{D}\bar{\pi}$; $B_d \to \bar{D}\pi$ compared to $\bar{B}_d \to D\bar{\pi}$; or $B_s \to D_s^+ K^-$ compared to $\bar{B}_s \to \bar{D}_s^- K^+$. These is a decided disadvantage here in theoretical interpretation, in that the quantity $Im\left(\frac{p}{q}\rho\right)$ is now dependent on hadron dynamics.

It is instructive to look not just at the time-integrated asymmetry between rates for a given decay process and its CP conjugate, but to follow the time dependence,[35] as given in eqs. (20a) and (20b). As a first example, figures 2, 3, and 4 show[36] the time dependence for the process $\bar{b} \to \bar{c}u\bar{d}$ (solid curve) in comparison to that for $b \to c\bar{u}d$ (dashed curve). At the hadron level this could be, for example, $B_d \to \bar{D}^- \pi^+$ in comparison to $\bar{B}_d \to D^+\pi^-$. The direct process is very much Kobayashi–Maskawa favored over that which is introduced through mixing, and hence the magnitude of the ratio of amplitudes, $|\rho|$, is very much greater than unity. Figures 2, 3, and 4 show[37] the situation for $\Delta m/\Gamma = 0.2$ (at the high end of theoretical prejudice before the ARGUS result, Ref. 25, for B_d mixing), $\Delta m/\Gamma = \pi/4$ (near the central value from ARGUS), and $\Delta m/\Gamma = 5$ (roughly the minimum value expected for the B_s in the three-generation standard model, given the central value of ARGUS for B_d). In none of these cases are the dashed and solid curves distinguishable within "experimental errors" in drawing the graphs. This is simply because $|\rho|$ is so large that even with "big"

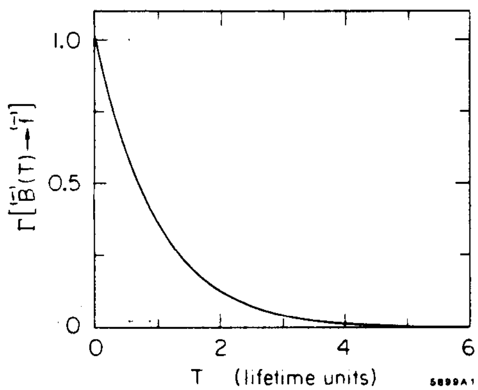

Fig. 2. The time dependence for the quark level process $\bar{b} \to \bar{c}u\bar{d}$ (solid curve) in comparison to that for $b \to c\bar{u}d$ (dashed curve). At the hadron level this could be, for example, $B_d \to \bar{D}^- \pi^+$ in comparison to $\bar{B}_d \to D^+\pi^-$. $\Delta m/\Gamma = 0.2$.

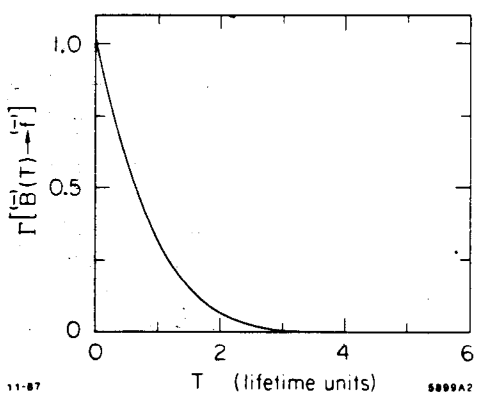

Fig. 3. Same as figure 2, but with $\Delta m/\Gamma = \pi/4$.

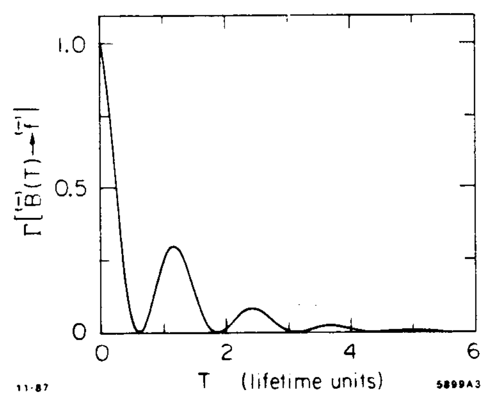

Fig. 4. Same as figure 2, but with $\Delta m/\Gamma = 5$.

mixing the second path to the same final state has a very small amplitude, and hence not much of an interference effect.

A much more interesting case is shown in figures 5, 6, and 7 for the time dependence at the quark level for the process $\bar{b} \to \bar{c}c\bar{s}$ (solid curve) in comparison to that for $b \to c\bar{c}s$ (dashed curve). At the hadron level this could be, for example, B_d in comparison to $\bar{B}_d$ decaying to the same, (CP self-conjugate) final state, ψK_s°. As discussed before, $|\rho| = 1$ in this case. The advantages of having $\Delta m/\Gamma$ for the B_d° system as suggested by ARGUS (figure 6) rather than previous theoretical estimates (figure 5) are very apparent. When we go to mixing parameters expected for the B_s° system (figure 7), the effects are truly spectacular.

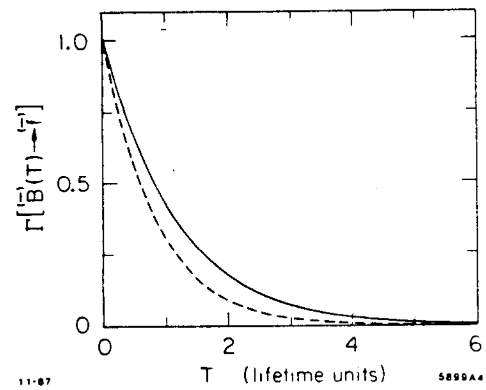

Fig. 5. The time dependence for the quark level process $\bar{b} \to \bar{c}c\bar{s}$ (solid curve) in comparison to that for $b \to c\bar{c}s$ (dashed curve). At the hadron level this could be, for example, $B_d \to \psi K_s^\circ$ (dashed curve) in comparison to $\bar{B}_d \to \psi K_s^\circ$ (solid curve). (The curves are interchanged for the ψK_s° final state because it is odd under CP.) $\Delta m/\Gamma = 0.2$.

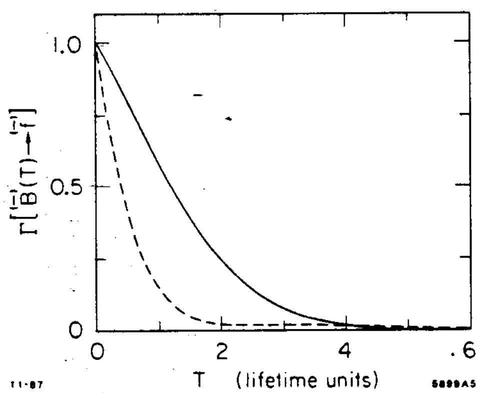

Fig. 6. Same as figure 5, but with $\Delta m/\Gamma = \pi/4$.

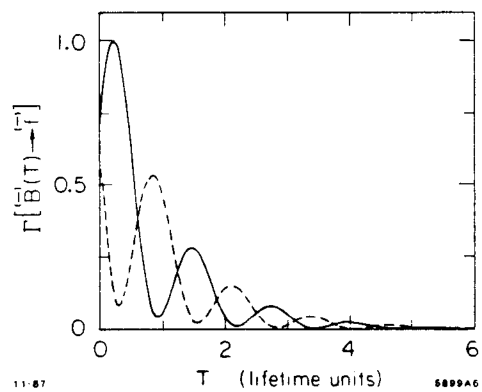

Fig. 7. Same as figure 5, but with $\Delta m/\Gamma = 5$.

Figures 8, 9, and 10 illustrate the opposite situation to that in figures 2–4; mixing into a big amplitude from a small one. We are explicitly comparing the quark level process $\bar{b} \to \bar{u}c\bar{d}$ (solid curve) to $b \to u\bar{c}d$ (dashed curve). At the hadron level this could be, for example, $B_d \to D^+\pi^-$ in comparison to $\bar{B}_d \to \bar{D}^-\pi^+$. The direct process is very much Kobayashi-Maskawa suppressed compared to that which occurs through mixing and hence the magnitude of the ratio of amplitudes, $|\rho|$, is very much less than unity. Here we have an example where too much mixing can be bad for you! As the mixing is increased (going from figure 8 to 10), the admixed amplitude comes to completely dominate over the original amplitude, and their interference (leading to an asymmetry) becomes less important in comparison to the dominant term.

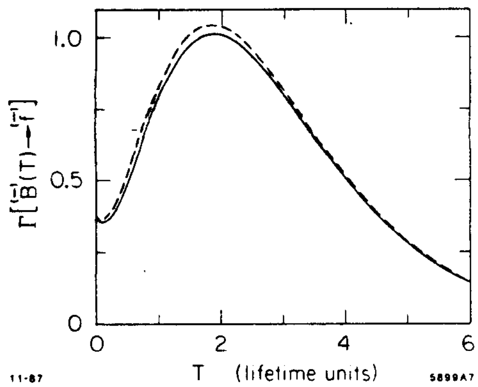

Fig. 8. The time dependence for the quark level process $\bar{b} \to \bar{u}c\bar{d}$ (solid curve) in comparison to that for $b \to u\bar{c}d$ (dashed curve). At the hadron level this could be, for example, $B_d \to D^+\pi^-$ in comparison to $\bar{B}_d \to \bar{D}^-\pi^+$. $\Delta m/\Gamma = 0.2$.

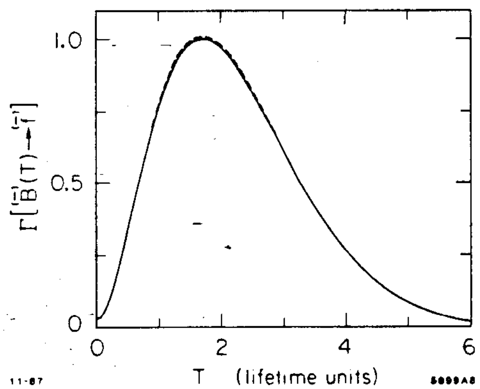

Fig. 9. Same as figure 8, but with $\Delta m/\Gamma = \pi/4$.

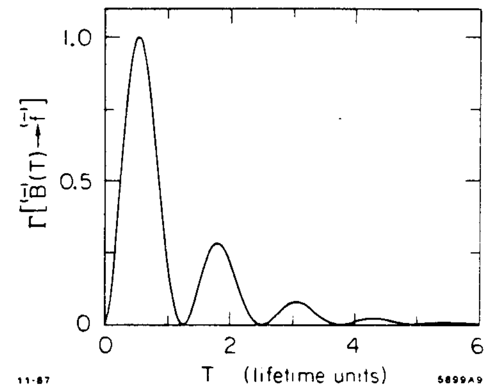

Fig. 10. Same as figure 8, but with $\Delta m/\Gamma = 5$.

A more likely example of the situation for B_s mixing is shown[38] in figure 11(c). The oscillations are so rapid that even with a very favorable difference in the time dependence for an initial B_s versus an initial $\bar{B}_s$, the time-integrated asymmetry is quite small. Measurement of the time dependence becomes a necessity for CP violation studies.

A second path to the same final state could arise in several other ways besides through mixing. For example, one could have two cascade decays that end up with the same final state, such as:

$$B_u^- \to D^\circ K^- \to K_s^\circ \pi^\circ K^-$$

and

$$B_u^- \to \bar{D}^\circ K^- \to K_s^\circ \pi^\circ K^- \quad .$$

Another possibility is to have spectator and annihilation graphs contribute to the same process.[39] Still another is to have spectator and "penguin" diagrams interfere. This lat-

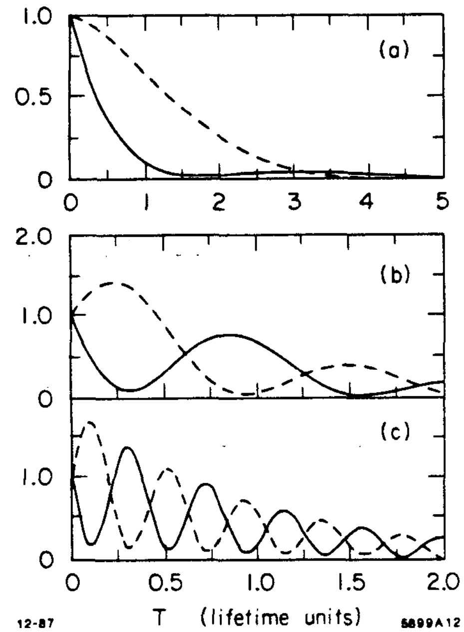

Fig. 11. The time dependence for the quark level process $\bar{b} \rightarrow \bar{u}u\bar{d}$ (dashed curve) in comparison to that for $b \rightarrow u\bar{u}d$ (solid curve). At the hadron level this could be, for example, $B_s \rightarrow \rho K_s^\circ$ (solid curve) in comparison to $\bar{B}_s \rightarrow \rho K_s^\circ$ (dashed curve) (the curves are interchanged for the ρK_s° final state because it is odd under CP) for values of (a) $\Delta m/\Gamma = 1$, (b) $\Delta m/\Gamma = 5$, and (c) $\Delta m/\Gamma = 15$, from Ref. 38.

ter possibility is the analogue of the origin of the parameter ϵ' in neutral K decay, but as discussed previously, there is no reason to generally expect a small asymmetry here. Indeed, with a careful choice of the decay process, large CP violating asymmetries are expected.

Note that not only do these routes to obtaining a CP violating asymmetry in decay rates not involve mixing, but they do not require one to know whether one started with a B or $\bar{B}$, i.e., they do not require "tagging." These decay modes are in fact "self-tagging" in that the properties of the decay products (through their electric charges or flavors) themselves fix the nature of the parent B or $\bar{B}$.

Even with potentially large asymmetries, the experimental task of detecting these effects is a monumental one. When the numbers for branching ratios, efficiencies, *etc.* are put in, it appears that 10^7 to 10^8 produced B mesons are required to end up with a significant asymmetry (say, 3σ), depending on the decay mode chosen.[33] This is beyond the samples available today (of order a few times 10^5) or in the near future ($\sim 10^6$).

The Outlook

I look at the next several years as being analogous to reconnaissance before a battle: We are looking for the right place and manner to attack CP violation in the B meson system. We need:

- Information on branching ratios of "interesting" modes down to the $\sim 10^{-5}$ level in branching ratio. For example, we would like to know the branching ratios for $B_d \rightarrow \pi\pi, p\bar{p}, K\pi, \psi K, D\bar{D}$ + three body modes + ... and for $B_s \rightarrow \psi\phi, K\bar{K}, D\pi, \rho K, \ldots$.
- Accurate $B\bar{B}$ mixing data, first for B_d, but especially verification of the predicted large mixing of B_s.
- A look at the "benchmark" process of rare decays, $B \rightarrow K\mu\bar{\mu}$.
- Experience with triggering, secondary vertices, tertiary vertices, "tagging" B versus $\bar{B}$, distinguishing B_u from B_d, distinguishing B_d from B_s, ...
- Various "engineering numbers" on cross sections, x_F dependence, B versus $\bar{B}$ production in hadronic collisions, ...

Many of these things are worthy, lesser goals in their own right, and may reveal their own "surprises." But the major goal is to observe CP violation. With all the possibilities, plus our past history of getting some "lucky breaks," over the next few years we ought to be able to find some favorable modes and a workable trigger and detection strategy. While the actual observation of CP violation may well be five or more years away, this is a subject whose time has come.

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37. Here we hold the K–M angles and phase fixed in comparing the situations with different amounts of $B - \bar{B}$ mixing. This neglects the effect that the measurement of the mixing has had on the favored values of the K–M angles and especially on the phase, which was discussed in the section on the Kobayashi–Maskawa matrix. Very roughly, larger mixing indirectly tends to decrease $\sin\delta_{KM}$ (with other parameters held fixed). Allowing this variation makes the comparison in the figures of the situation before and after the ARGUS observations less dramatic.

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