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AN ANALYSIS OF THE ANGULAR MOMENTUM OF THE PROTON

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ABSTRACT

We discuss the contributions to the proton helicity in the infinite-momentum frame from polarized quarks (Δq), polarized gluons (ΔG), and orbital angular momentum (L_Z). We first examine the behavior of the polarized structure function $g_1^p(x)$ at small x, arguing that it is consistent with dominance by the $a_1(1270)$ and related Regge trajectories. This supports the EMC estimate of $\int_0^1 dx\ g_1^p(x)$, and hence the conclusions that $\Delta s < 0$ and $\Delta u + \Delta d + \Delta s \simeq 0$. These conclusions are also supported by recent data on elastic $\overset{(-)}{\nu} p \to \overset{(-)}{\nu} p$ scattering. Next we argue that in the Skyrme model with only pseudoscalar mesons, $\Delta G = 0$ and $L_Z = 1/2$. These results still hold when the chiral Lagrangian is made consistent with the scale transformation properties of QCD by incorporating a scalar gluonium field. Finally, we argue on general grounds and by explicit example that if the parameters of the chiral Lagrangian are adjusted so that gluons carry ~50% of the proton momentum, most of the orbital angular momentum L_Z is carried by quarks.

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ABSTRACT

We discuss the contributions to the proton helicity in the infinite-momentum frame from polarized quarks (Δq), polarized gluons (ΔG), and orbital angular momentum (L_z). We first examine the behaviour of the polarized structure function $g_1^p(x)$ at small x, arguing that it is consistent with dominance by the $a_1(1270)$ and related Regge trajectories. This supports the EMC estimate of $\int_0^1 dx\, g_1^p(x)$, and hence the conclusions that $\Delta s < 0$ and $\Delta u+\Delta d+\Delta s \simeq 0$. These conclusions are also supported by recent data on elastic $\overset{(-)}{\nu}p \to \overset{(-)}{\nu}p$ scattering. Next we argue that in the Skyrme model with only pseudoscalar mesons, $\Delta G = 0$ and $L_z = \frac{1}{2}$. These results still hold when the chiral Lagrangian is made consistent with the scale transformation properties of QCD by incorporating a scalar gluonium field. Finally, we argue on general grounds and by explicit example that if the parameters of the chiral Lagrangian are adjusted so that gluons carry ~50% of the proton momentum, most of the orbital angular momentum L_z is carried by quarks.

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In the naive non-relativistic quark model (NQM),[1] most of the nucleon mass is due to 3 constituent quarks, each weighing about 300 MeV, and the nucleon spin is the vector sum of the 3 quark spins. It has been known for some time that this naive view of the nuclear mass needs modification. The u and d quarks have masses of only a few MeV, and the nucleon would still weigh $\mathcal{O}(1\,\mathrm{GeV})$ even if they were massless.[2] Viewed in the infinite-momentum frame, about a half of the nucleon's momentum is carried by gluons,[3] and the trace of the energy-momentum tensor $\theta^{\mu}_{\mu} = (\beta(g)/2g)\,F_{\mu\nu}\,F^{\mu\nu}$ is pure glue[4] if the quarks are massless. The true decomposition of the nucleon spin has been less clear until recently. The quark contributions Δq to the nucleon spin Σ_{μ} are related to the matrix elements of the quark axial currents by $\langle N|\,\bar{q}_i\gamma_{\mu}\gamma_5 q_i\,|N\rangle = 2\Sigma_{\mu}\cdot\Delta q_i$. Measurements[5] of the axial current matrix elements in neutron and hyperon decays indicated that not all the nucleon spin could be carried by u and d quarks alone, but did not specify all the quark contributions to the nucleon spin, and left open the possibility that the u, d and s quarks together could carry all the nucleon spin. However, this possibility has recently been removed by the EMC measurement of the spin-dependent proton structure function,[6] which indicates[7] that, within errors, the net contribution of all the quarks to the nucleon spin is zero. The positive contribution $\Delta u + \Delta d$ of the u and d quarks to the nucleon spin is actually cancelled by a negative contribution Δs from the s quarks: $\Delta q \equiv \Delta u + \Delta d + \Delta s \simeq 0$. Subsequently, this surprising experimental result has been understood[8] on the basis of chiral symmetry and the $1/N_c$ expansion, as exemplified by the Skyrme model in which the nucleon corresponds to a soliton solution of a chiral Lagrangian.[9] Given that quarks make no net contribution to the nucleon spin ($\Delta q = 0$), the question then arises, what does carry the nucleon spin? There is an angular momentum sum rule[10]

$$\frac{1}{2} = \frac{1}{2}\,\Delta q + \Delta G + L_z\ , \tag{1}$$

analogous to the classic momentum sum rule, with two remaining candidates for giving the nucleon spin: gluons and orbital angular momentum. The main purpose of this paper is to predict values for ΔG and L_z, using chiral symmetry, the $1/N_c$ expansion and scale invariance.

First we study in more detail the behavior of the spin-dependent structure function $g_1^p(x)$ at small x, verifying that it is consistent with the expected Regge behavior[11] and extracting the effective $a_1(1270)/f_1(1285)/f_1(1420)$ trajectory intercept. We give examples of the polarized s quark distribution $\Delta s(x)$ which are consistent with this Regge behavior and the known magnitude of the strange sea. These results support the EMC extrapolation of $g_1^p(x)$ to $x=0$, its determination of $\int_0^1 dx\, g_1^p(x)$, and the deduction that $\Delta s < 0 \,:\, \Delta q \equiv \Delta u + \Delta d + \Delta s \simeq 0$. We also point out that a negative value of Δs is supported by recent elastic $\overset{(-)}{\nu}\, p \rightarrow \overset{(-)}{\nu}\, p$ scattering data.[12] After recalling our previous argument[8] using chiral symmetry and the $1/N_c$ expansion to explain why $\Delta q \simeq 0$, we then use the same approach to estimate ΔG and L_z in Eq. (1). We include gluon degrees of freedom in the chiral Lagrangian via a scalar gluonium field coupled in such a way as to reproduce the anomalous scale invariance Ward identities of QCD.[13,14] As in the case of the previous chiral Lagrangian without gluonium fields, all the angular momentum of the nucleon is orbital:

$$L_z \simeq \frac{1}{2}, \qquad \Delta G \simeq 0 . \tag{2}$$

One can go further and ask what fraction of L_z is carried by gluons, and what fraction by quarks. When the parameters of the scale-invariant chiral Lagrangian model are adjusted so that gluons carry 50% of the nucleon's linear momentum,[3] we find that they carry less than 40% of the nucleon's orbital angular momentum.

We start by discussing the small x behavior of $g_1^p(x)$ and the magnitude of Δs. According to standard Regge lore,[11,10] the high-energy behavior of the imaginary part of the forward $\gamma^* + N \rightarrow \gamma^* + N$ scattering amplitude $G_i\ (\nu, q^2)$, where q is the γ^* momentum and $\nu \equiv p_N \cdot q$, is

$$G_1^N(\nu, q^2) \simeq \sum_i \beta_i^\gamma(q^2)\ \nu^{\alpha_i(0)-1}\ \beta_i^N \tag{3}$$

with the $\beta_i^\gamma(q^2)$ Regge residues of the γ^*, the β_1^N Regge residues of the nucleon, the $\alpha_i(0)$ the intercepts of Regge trajectories, and the index i represents the relevant $a_1(1270)$, $f_1(1285)$ and $f_1(1420)$ trajectories. The spin-dependent structure function $g_1^N(x) \equiv \lim_{\substack{\nu\to\infty,\\ x\equiv -q^2/2\nu \text{ fixed}}} \nu\, G_1^N(\nu,q^2)$. If one assumes that the Regge residues $\beta_i^\gamma(q^2) \simeq \bar\beta_i^\gamma(-q^2/2)^{-\alpha_i(0)}$ as $q^2 \to -\infty$, then the Regge terms [Eq. (3)] have non-trivial scaling limits[15] and

$$g_1^N(x) \simeq \sum_i \bar\beta_i^\gamma\, \beta_i^N\, x^{-\alpha_i(0)}\ . \tag{4}$$

Since all meson Regge trajectories are expected to have equal slopes α', one expects the intercepts of the $a_1(1270)$ and $f_1(1285)$ trajectories to be almost equal, with the intercept of the $f_1(1420)$ trajectory slightly lower. Accordingly, we have fitted the data on $g_1^p(x)$ at low x with a single power of x: $g_1^p(x) \simeq B\,x^{-\alpha}$. We have made fits to the lowest 8, 7, 6 and 5 data points, as seen in Fig. 1. All the fits are of good quality and consistent with one another. For example, using the seven points in $x < 0.2$ one finds

$$\alpha = -0.07^{+0.42}_{-0.32}\ , \qquad B = 0.30^{+0.44}_{-0.17}\ . \tag{5}$$

There is no hint in the data of the behavior $g_1^p(x) \sim 1/x\ln^2 x$ which was once suggested on the basis of the Pomeron-Pomeron cut,[16,17] but it is no longer favored on theoretical grounds[10,11,18]

Very little is known experimentally about the intercept of the $a_1(1270)$ trajectory, and still less about the $f_1(1285)$ and $f_1(1420)$ intercepts. Values of $\alpha_{a_1}(0)$ in the range (-0.5,0.0) have generally been assumed. In ref. [19], $\alpha_{a_1}(0) \sim 0$ was favored, leading to a small unnatural polarization component in $\pi^- p \to \rho^0 n$, whilst another analysis[20] found that $\alpha_{a_1}(0) = 0$ gave too little target polarization effects in the reaction $\pi^- p \to \pi^+\pi^- n$, and favored $\alpha_{a_1}(0) \in (-0.4, -0.2)$. An analysis of polarized pp scattering[21] suggested $\alpha_{a_1}(0) \sim -0.5$, but smaller values of $\alpha_{a_1}(0)$ were also found to give too little target polarization effects in $\pi p \rightarrow \pi^+\pi^- n$.[19] Theoretically, one would expect $\alpha_{a_1}(0) \simeq 1 - \alpha' m_{a_1}^2$, etc., if the Regge trajectories were linear. Taking $m_{a_1} = 1.27$ GeV (1.10 GeV) and $\alpha' = 0.9$ GeV^{-2}, one estimates $\alpha_{a_1}(0) = -0.45(-0.1)$. Within these ranges, higher values of $\alpha_{a_1}(0)$ are slightly favored by our fits [Eq. (5)]: perhaps the a_1 trajectory is not linear, or perhaps m_{a_1} is less than the 1.27 GeV now quoted by the Particle Data Group[22]. An indication in this direction comes from the DELCO collaboration[23], which quotes $m_{a_1} \simeq 1.06$ GeV.

The fit parameters Eq. (5) are consistent with the extrapolation of $g_1^p(x)$ to $x = 0$ assumed by the EMC, and support their estimate $\int_0^1 dx\, g_1^p(x) = 0.114 \pm 0.012 \pm 0.026$.[6] As discussed elsewhere, when combined with measurement of g_A and of the SU(3) F/D ratio in hyperon β–decay,* this EMC result suggests that $\Delta s = -0.23 \pm 0.08$ and $\Delta q \simeq \Delta u + \Delta d + \Delta s = 0.00 \pm 0.24$.[6,7,27] This conclusion can be strengthened slightly by combining the EMC and SLAC-Yale data, which are consistent at values of x where the data overlap. Taken together, the two data sets give[27]

$$\int_0^1 dx g_1^p(x) = 0.112 \pm 0.009 \pm 0.019 \tag{6}$$

when combined with the values of g_A and the F/D ratio, this leads to

$$\Delta u = 0.73 \pm 0.07, \qquad \Delta d = -0.52 \pm 0.07, \qquad \Delta s = -0.24 \pm 0.07, \tag{7}$$

and

$$\Delta q = \Delta u + \Delta d + \Delta s = -0.01 \pm 0.21 \tag{8}$$

confirming the startling result of ref. 7.

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* Recently, new data from Fermilab on polarized Σ^- β–decay have become available.[24] A new global fit[25] which also includes a new estimate of neutron lifetime has the net effect of bringing down the F/D ratio by no more than one standard deviation relative to the previous estimate,[5] i.e. from 0.63 to 0.61.

It has been pointed out previously[28] that elastic $\overset{(-)}{\nu}\, p \to \overset{(-)}{\nu}\, p$ scattering is sensitive to Δs. Assuming the Weinberg-Salam model, the cross sections involve the Z^0 coupling to the $(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d - \bar{s}\gamma_\mu\gamma_5 s)$ axial current, and hence are sensitive to $\Delta u - \Delta d - \Delta s$ in our notation. New information on Δs can be obtained from recent data on elastic $\overset{(-)}{\nu}\, p \to \overset{(-)}{\nu}\, p$ scattering.[12] The result of a fit to their data can be parametrized as

$$\Delta u - \Delta d - \Delta s = g_A\,(1+\eta) \tag{9}$$

where $\eta = 0.12 \pm 0.07$.[12] Using $\Delta u - \Delta d = g_A = 1.25$ this result corresponds to†

$$\Delta s = -0.15 \pm 0.09 \tag{10}$$

This is not sufficient by itself to prove that $\Delta s < 0$, but it is consistent with the value of Δs extracted from polarized μp scattering, and supports the subsequent deduction that $\Delta q = \Delta u + \Delta d + \Delta s \simeq 0$.

It has recently been observed[30,31] that the Δu, Δd, Δs appearing in the parton model expression for $\int_0^1 dx g_1^p(x)$ and elsewhere acquire QCD radiative corrections and should be replaced[31] by $\widetilde{\Delta u} = \Delta u - (\alpha_s/2\pi)\Delta G$, etc... There are additional QCD radiative correction to the extraction of $\Delta u - \Delta d - \Delta s$ from $\overset{(-)}{\nu}\, p \to \overset{(-)}{\nu}\, p$ scattering due to heavy quark loops[32], but these are $\mathcal{O}(\alpha_s/\pi)^2$ and it is consistent to neglect them in the first next-to-leading order in $\mathcal{O}(\alpha_s/\pi)$.‡ It has been suggested[31] that perhaps $\Delta s = 0$ and the discrepancy between the EMC result for $\int_0^1 dx g_1^p(x)$ and the previously expected value of 0.19[33] might be entirely due to ΔG. This would require $\Delta G \simeq 8 \pm 2$ at $Q^2 \simeq 10\,\mathrm{GeV}^2$, where $\alpha_s \simeq 0.2$, and $L_z \simeq -8$, surprisingly large values. We will in fact argue in the following, on the basis of the Skyrme model, that $\Delta G \simeq 0$.

† This result is also obtained in ref. 29.

‡ Our numerical estimate indicates that their effect is in any case much smaller than the experimental error in η.

It is *a priori* surprising that Δs is so large and negative, and one might query its consistency with our knowledge of the strange quark momentum distribution $s(x)$ and the expected Regge behavior at small x. We have investigated this question by constructing possible s quark polarization densities $\Delta s(x)$ of the form

$$\Delta s(x) = \begin{Bmatrix} Cx^{-\alpha} & \text{for } x < x_0 \\ s(x) & \text{for } x > x_0 \end{Bmatrix} : \int_0^1 \Delta s(x)\, dx = -0.23 \tag{11}$$

where $\alpha = -0.25$ or -0.5 (*cf.* the earlier discussicn), $s(x)$ is taken from Ref. 34, and x_0 is defined by requiring continuity in $\Delta s(x)$. Two polarization densities of this form, multiplied by the coefficient 1/18 with which $\Delta s(x)$ contributes to $g_1^P(x)$, are compared with the data on $g_1^p(x)$ in Fig. 2. We find consistency between the data on $g_1^p(x)$, the inferred magnitude of Δs, the independent measurements of $s(x)$ and the expected Regge behavior of $\Delta s(x)$ at small x. Our faith in the canonical interpretation of the EMC data that $\Delta s < 0$ and $\Delta q = \Delta u + \Delta d + \Delta s \simeq 0$ is thereby reinforced.

We differ on this point from Ref. 35, where an attempt was made to use the data on the $s(x)$ to set a bound on $\Delta s(x)$. This cannot be done directly, for $s(x)$ contains a singular contribution from Pomeron exchange. Ref. 35 assumed that the Pomeron contribution to $s(x)$ could be isolated in a reliable fashion for all values of x, leaving $\Delta s(x) = B(1-x)^p$, $B \simeq 0.53, p \simeq 8.25$, which is much smaller than (11) near x_0, and does not reproduce the Regge-like behavior x^α, $\alpha \sim 0$, as seen in $g_1(x)$ for $x \lesssim 0.2$. Thus the "bound" on Δs quoted in Ref. 35 is only a statement about the integral of a hypothetical parametrization of $\Delta s(x)$ and may well be violated. The same objection applies to similar arguments of Ref. 36. To illustrate this point, Fig. 2 shows a sample distribution $\Delta s(x)$ which violates the "bound" of Refs. 35 and 36, yet is consistent with our knowledge of $s(x)$ and the expected Regge behavior, and is in good agreement with the experimental data.

As we discussed elsewhere,[8] the result that $\Delta q \simeq 0$ can be understood using chiral symmetry and the $1/N_c$ expansion as embodied in the Skyrme model of nucleons as soliton solutions of a chiral Lagrangian.[9] It was first shown[8] by explicit computation that $\langle p|A^0_\mu(x)|p\rangle = 0$ in the limit of chiral symmetry and large N_c, where $A^0_\mu \equiv \sqrt{\frac{2}{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s)$. This calculation was confirmed by a generalized Goldberger–Treiman relation relating $\langle p|A^0_\mu|p\rangle$ to the η_0-proton coupling, where η_0 is the ninth (SU(3)-singlet) pseudoscalar meson.[8] The baryonic soliton exists because Π_3 (SU(3)) $= \mathbb{Z}$, and contains no η_0 component. The baryon contains non-trivial $SU(3)$-octet $\underset{\sim}{\pi}, K_a$ and η_8 fields, but these decouple from the $SU(3)$ singlet η_0 in the large-N_c limit. Hence the η_0-nucleon coupling vanishes in the chiral limit, and so also does $\langle p|A^0_\mu|p\rangle \propto \Delta q$. Note that the argument that $\Delta q \simeq 0$ is independent of the specific form of the chiral Lagrangian chosen.§

We now use the same philosophy of chiral symmetry and the $1/N_c$ expansion to analyze the origin of the nucleon spin, arguing that it is due to L_z and not to ΔG. We start from the conventional chiral Lagrangian

$$\mathcal{L} = \frac{f_\phi^2}{16}\left\{Tr\partial_\mu U\partial^\mu U^\dagger - \frac{A}{N_c}(-i\ln\det U)^2\right\} + \frac{1}{32e^2}Tr\left([U_{\mu R}, U_{\nu R}][U^\mu_R, U^\nu_R]\right)$$
$$+ N_c\mathcal{L}_{WZ} \tag{12}$$

where $U_{\mu R} \equiv \partial_\mu U U^\dagger$ with

$$U(x) = \exp\left(\frac{2i}{f_\phi}\sum_{i=0}^{8}\lambda_i\phi_i(x)\right) \tag{13}$$

where $\left\{\phi_i \equiv \eta_0, \underset{\sim}{\pi}, K_a, \eta_8\right\}$, f_ϕ is the pseudoscalar nonet decay constant, A gives $m^2_{\eta 0}$ of order $1/N_c$, e is determined by fitting the static properties of the baryons, and the Wess–Zumino part of the Lagrangian is given in Refs. [9, 37]. The nucleon mass to leading order in $1/N_c$ is obtained from the Lagrangian (12) by

§ The vanishing of Δq is therefore a model-independent result (like $F/D = 5/9$), unlike the specific numerical value of g_A, which is highly model-dependent.

looking first for a Skyrme solution $U_0(\mathbf{x}) = \exp[iF(r)\boldsymbol{\tau}\cdot\mathbf{x}]$ where F(r) $= \pi$ at $r \equiv |\mathbf{x}| = 0$ and $F(r) \to 0$ as $r \to \infty$. The shape function F(r) is then chosen to minimize the mass $M = \int d^3\mathbf{x}\theta_{00}(\mathbf{x})$. For any given solution U_0, $U = AU_0A^{-1}$, where A is an arbitrary constant SU(3) matrix, is also a finite-energy solution, but is not an eigenstate of spin and isospin.[9] To obtain such eigenstates, we rotate $U_0(\mathbf{x})$ by some time-dependent matrix A(t), which rotates the soliton as a solid non-relativistic body with angular frequency $\omega_i : \theta_{0i}(\mathbf{x}) = \theta_{00}(r)\epsilon_{ijk}\omega_j x_k$ and the angular momentum of the spinning body is

$$L_i = \int d^3\mathbf{x}\epsilon_{ijk}x_j\theta_{0k}(\mathbf{x}) = \omega_i J; \qquad J = \frac{2}{3}\int d^3\mathbf{x}\theta_{00}(r)r^2 \tag{14}$$

The angular momentum $\mathbf{L}$ is quantized: $\mathbf{L}^2 = \frac{3}{4}$ for the nucleon and $\mathbf{L}^2 = \frac{15}{4}$ for the Δ, and the physical masses $m_{N,\Delta} = M + \mathbf{L}^2/2J$, where the second term is just the kinetic energy of a slowly-rotating body.[9] It is clear from this analysis that the gluons make no contribution, and that all the spin of the nucleon is due to orbital angular momentum: $\Delta G = 0$ and $L_z = \frac{1}{2}$.

However, it is not immediately clear that this analysis is realistic, since the Lagrangian (12) contains no hint of the gluonic degrees of freedom. Before concluding that the gluons do not contribute to the nucleon spin, we should at least give them a chance to contribute!

We do this by modifying the chiral Lagrangian (12) to be consistent with the anomalous scale invariance properties of QCD. This is done by introducing a scalar field χ which can be regarded as a scalar gluonium field and acts as a pseudo-dilaton of broken scale invariance.[13,14] The parenthesized part of $\mathcal{L}$ is replaced by

$$\begin{aligned}\mathcal{L}_\chi = \frac{f_\phi^2}{8}\left\{(\chi/\chi_0)^2\,\mathrm{Tr}\,\left(\partial_\mu U\partial^\mu U^\dagger\right) - \frac{A}{N_c}(\chi/\chi_0)^4\left(i\ln\det U\right)^2\right\} \\ + 8a\partial_\mu\chi\partial^\mu\chi + \chi^4\ln(\chi/\Lambda)\end{aligned} \tag{15}$$

where $\chi_0 \equiv \langle 0|\,\chi\,|0\rangle = \Lambda/e^{1/4}$ and the Skyrme and Wess-Zumino terms are left unchanged. The energy-momentum tensor density for the modified Lagrangian is

$$\begin{aligned}\theta_{\mu\nu} = \frac{f_\phi^2}{8}&\left\{(\chi/\chi_0)^2\left[2\mathrm{Tr}\left(\partial_\mu U \partial_\nu U^\dagger\right) - g_{\mu\nu}\,\mathrm{Tr}\left(\partial_\rho U \partial^\rho U^\dagger\right)\right]\right\} \\ &+ 16a\left[\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}g_{\mu\nu}\left(\partial_\rho\chi\partial^\rho\chi\right)\right] + \mathcal{O}\left((\partial_\mu U)^4\right)\end{aligned} \tag{16}$$

where we have neglected the A term which is $\mathcal{O}(1/N_c)$ and the higher order terms in (12), which do not contribute to subsequent equations. Using the equation of motion for χ, the trace of the energy-momentum tensor (16) can be written as

$$\theta_\mu^\mu = -4\chi^4 \ln\chi/\Lambda \tag{17}$$

corresponding to $\theta_\mu^\mu = \beta(g)/2gF_{\mu\nu}F^{\mu\nu}$ in QCD. Disregarding the slowly-varying time derivatives in (16), we can write

$$\theta_{00} \simeq \frac{f_\phi^2}{8}\left[(\chi/\chi_0)^2\,\mathrm{Tr}\left(\partial_i U\partial_i U^\dagger\right)\right] + \left[8a\left(\partial_i\chi\partial_i\chi\right)\partial_\rho\chi\partial_\rho\chi + \chi^4\ln\chi/\Lambda\right] + \mathcal{O}\left((\partial_\mu U)^4\right) \tag{18}$$

and

$$\begin{aligned}\theta_{33} \simeq \frac{f_\phi^2}{8}&\left\{(\chi/\chi_0)^2\left[2\mathrm{Tr}\left(\partial_3 U\partial_3 U^\dagger\right) - \mathrm{Tr}\left(\partial_i U\partial_i U^\dagger\right)\right]\right\} \\ &+ \left[16a\left(\partial_3\chi\partial_3\chi\right) - 8a\left(\partial_i\chi\partial_i\chi\right) - \chi^4\ln\chi/\Lambda\right] + \mathcal{O}\left((\partial_\mu U)^4\right)\end{aligned} \tag{19}$$

Also, we define

$$\theta_{++} \equiv \theta_{00} + \theta_{33} = \frac{f_\phi^2}{8}\left\{\frac{2}{3}(\chi/\chi_0)^2\,\mathrm{Tr}\left(\partial_i U\partial_i U^\dagger\right)\right\} + \frac{16a}{3}\left(\partial_i\chi\partial_i\chi\right) \tag{20}$$

with a view to later use.

The modified, scale-invariant chiral Lagrangian also has a soliton solution, with mass

$$M = \int d^3\mathbf{x}\theta_{00} \tag{21}$$

Forms of the chiral soliton shape function $F(r)$ for various values of the model parameters are given in ref. 14, along with the corresponding forms of $\hat{\chi}(r) \equiv \chi(r)/\langle 0|\chi|0\rangle$. We show in Fig. 3 the profile functions $F(r)$ and $\hat{\chi}(r)$ for our own preferred solution, to be discussed later. The angular momentum of the modified soliton is still given by equation (14), but now using the expression (18) for θ_{00}. It is clear that all the angular momentum is still orbital, so we conclude that

$$\Delta G = 0\,, \qquad L_z = \frac{1}{2} \tag{22}$$

It is hardly surprising that $\Delta G = 0$, since the scalar gluonium field χ corresponds naively to a pair of gluons whose helicities are precisely antiparallel. This naive argument was not applicable via the anti-correlated $\bar{q}q$ helicities in the octet pseudoscalar mesons to the flavor non-singlet combinations of Δu, Δd and Δs, because the baryon is a topologically non-trivial soliton $[\Pi_3(SU(3)) = \mathbb{Z}]$, which mixes external spin and internal flavour symmetry. However, no such mixing can occur for the isosinglet pseudoscalar (implying that $\Delta q = 0$) or for the isosinglet gluonium state (implying that $\Delta G = 0$).*

The one remaining question is how the non-zero orbital angular momentum L_z (22) is shared between the gluon and quark constituents in a light-cone Fock space decomposition of the nucleon wave function. We recall that deep inelastic scattering experiments have established that about 50% of the nucleon momentum in the infinite-momentum frame is carried by gluons.[3] In the language of the chiral Lagrangian (12), (15), this statement can be interpreted as one of approximate equality for the U (quark) and χ (gluon) contributions to $\int d^3x\theta_{++}$

* Note that the peculiar QCD behavior hypothesized in ref. 38, with contributions to the spin growing indefinitely $\propto \ln Q^2$, is avoided here because both Δq and $\Delta G = 0$.

as given by equation (20):

$$\frac{f_\phi^2}{12}\int d^3\mathbf{x}\,(\chi/\chi_0)^2\,\mathrm{Tr}\left(\partial_i U \partial_i U^\dagger\right) \quad \simeq \quad \frac{16a}{3}\int d^3\mathbf{x}\,(\partial_i\chi\partial_i\chi) \tag{23}$$

Since the angular momentum L_i (14) is given by the moment of inertia, which weights the space integral of θ_{00} with a multiplicative factor of r^2, contributions to L_z from large r are favoured. In the chiral limit $\partial_i U(\mathbf{x}) \sim r^{-3}$ as $|r| \to \infty$, whereas $\partial_i\chi \sim \exp(-m_\chi r)$, where $m_\chi \sim 1$ GeV is the scalar gluonium mass. Therefore the gluonium contribution to θ_{00} is concentrated at smaller r than the density of pseudoscalar meson fields, and we would accordingly expect the glue to make less than a 50% contribution to L_z. This intuition is borne out by our own explicit numerical calculations.

We solve numerically the equations of motion for the scale-invariant chiral Lagrangian (12),(15), using $\Lambda = 0.22$ GeV, $f_\phi = 0.132$ GeV, and $e = 4.5$. We have evaluated the percentage of the linear momentum and of the orbital angular momentum carried by the quarks for different values of a in the chiral limit. Stable numerical solutions could not be found for $a < 0.0025$. We find that the gluons carry $\sim$ 50% of the linear momentum when $a = 0.0025$. In this case they carry about 36% of the orbital angular momentum. This orbital angular momentum fraction varies by $\sim$ 1% for $0.0025 < a < 0.0040$, for which the linear momentum fraction changes by 2 or 3%. The chiral profile function F and the scaled gluonium field $\hat{\chi}(r)$ for our preferred value of $a = 0.0025$ are shown in Fig. 3. The corresponding contributions to the moment of inertia, J_q and J_g, are plotted in Fig. 4 as functions of r. At large r, $J_q(r) = J_q(\infty) - const/r$. The asymptotic value $J_q(\infty)$ is obtained from a fit which is also shown in Fig. 4.

It is interesting to note that $\hat{\chi}(0) \lesssim 10^{-3}$ for values of a giving $\sim$ 50% of the linear momentum to the gluons. This independent phenomenological constraint therefore favors the suggestion in ref. 14 that a bag may appear in a chiral scale-invariant soliton.

In this paper we have continued our previous analysis[8] of the different contributions of quarks, gluons and orbital angular momentum to the spin of the proton. Previously we argued that in the limit of chiral symmetry and large N_c, $\Delta q \equiv \Delta u + \Delta d + \Delta s = 0$. In the first part of this paper, we presented a more detailed Regge analysis of the spin-dependent structure functions $g_1^p(x)$ at small x, which supported the interpretation of the EMC data that indeed $\Delta s < 0$ and $\Delta q \simeq 0$. These conclusions were also supported by $\overset{(-)}{\nu}\, p \rightarrow \overset{(-)}{\nu}\, p$ scattering data. In the second part of this paper, we argued that the proton spin is not due to gluon polarization but is in fact orbital angular momentum. Furthermore, we argued that most of this orbital angular momentum is carried by quark constituents of the proton, rather than by gluon constituents.

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FIGURE CAPTIONS

1) Fits to the EMC data[6] on $g_1^p(x)$ of the form $Bx^{-\alpha}$. The data points at the 8,7,6 and 5 lowest values of x are used.

2) Three possible forms of the s quark polarization density contribution $\Delta s(x)/18$ to $g_1^p(x)$ are compared to the EMC data.[6] The parametrizations are those of Eq. (11) with $\alpha = 0$, 0.25 or 0.5.

3) Profile functions of $F(r)$ (for the chiral field) and $\hat{\chi}(r)$ (for the scalar gluonium field) for $\Lambda = 0.22$ GeV, $f_\phi = 0.132$ MeV, $e = 4.5$ and $a = 0.0025$. With this choice of parameters, gluons carry 50% of the proton's linear momentum and 36% of the orbital angular momentum.

4) Quark and gluon contributions, J_q and J_g, to the moment of inertia J for $a = 0.0025$.

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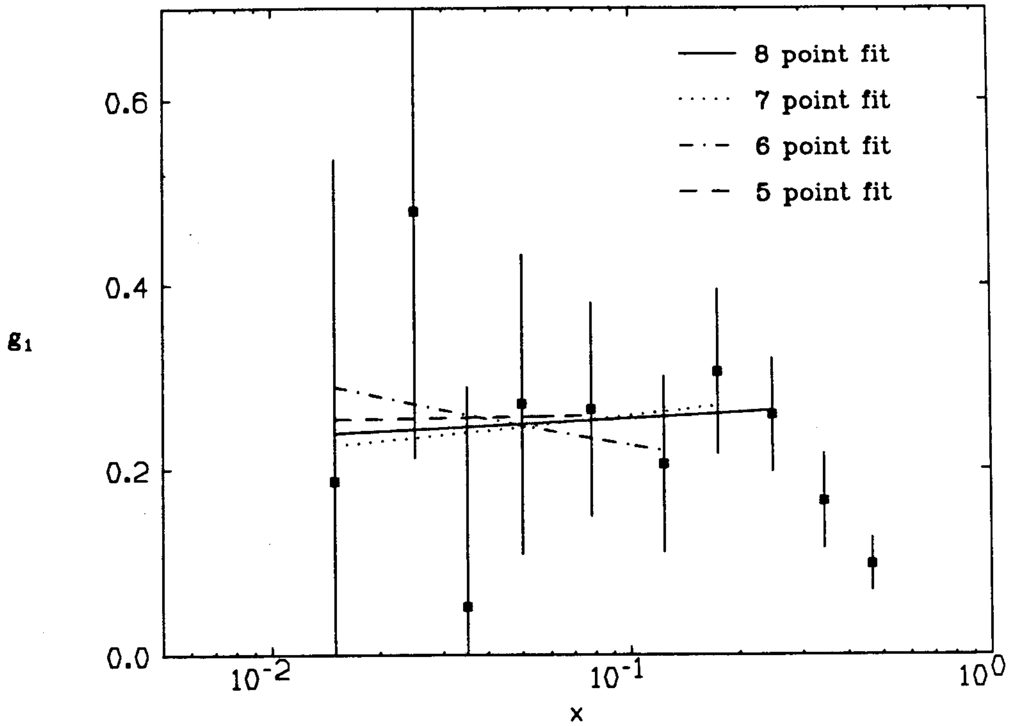
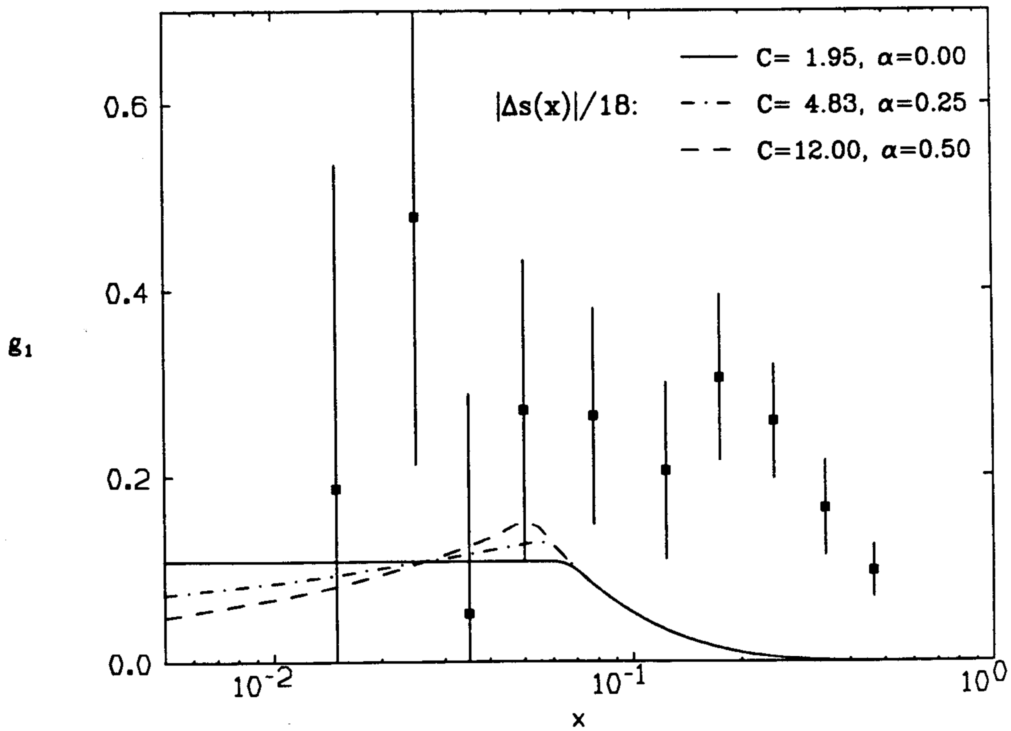


8 point fit
7 point fit
6 point fit
5 point fit
0.6
0.4
0.2
0.0
g1
10-2
10-1
100
x

Fig. 1



|Δs(x)|/18:
C= 1.95, α=0.00
C= 4.83, α=0.25
C=12.00, α=0.50
0.6
0.4
0.2
0.0
g1
10-2
10-1
100
x

Fig. 2

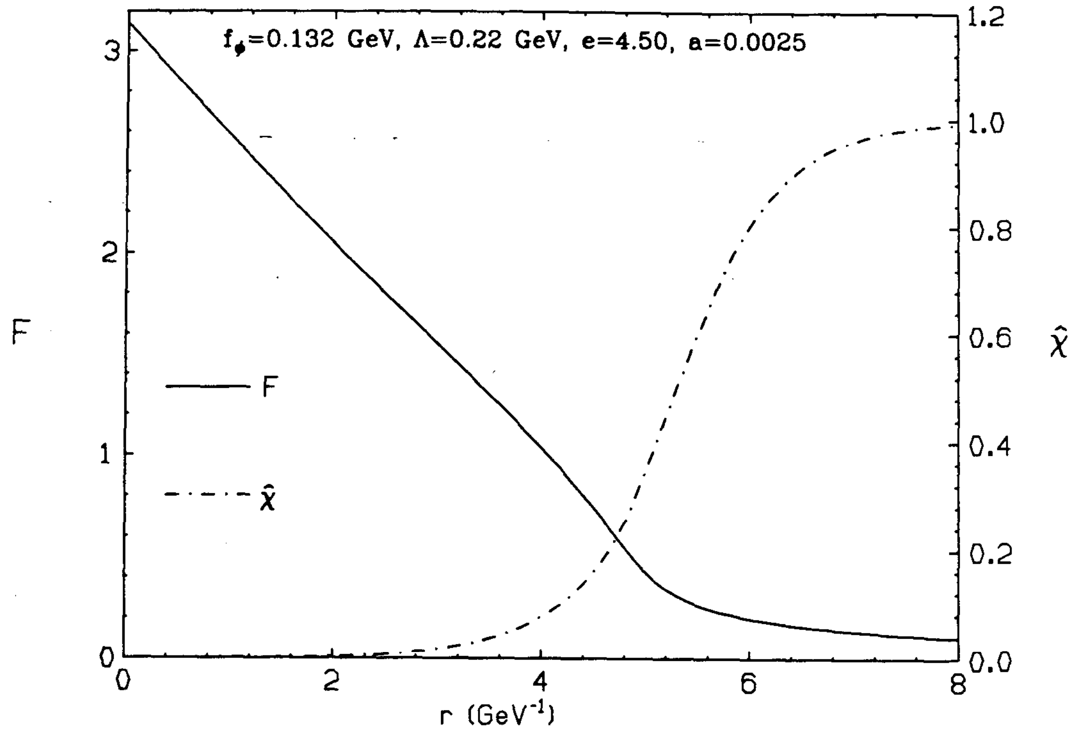

f_ϕ=0.132 GeV, Λ=0.22 GeV, e=4.50, a=0.0025
F
$\hat{\chi}$
r (GeV^{-1})

Fig. 3

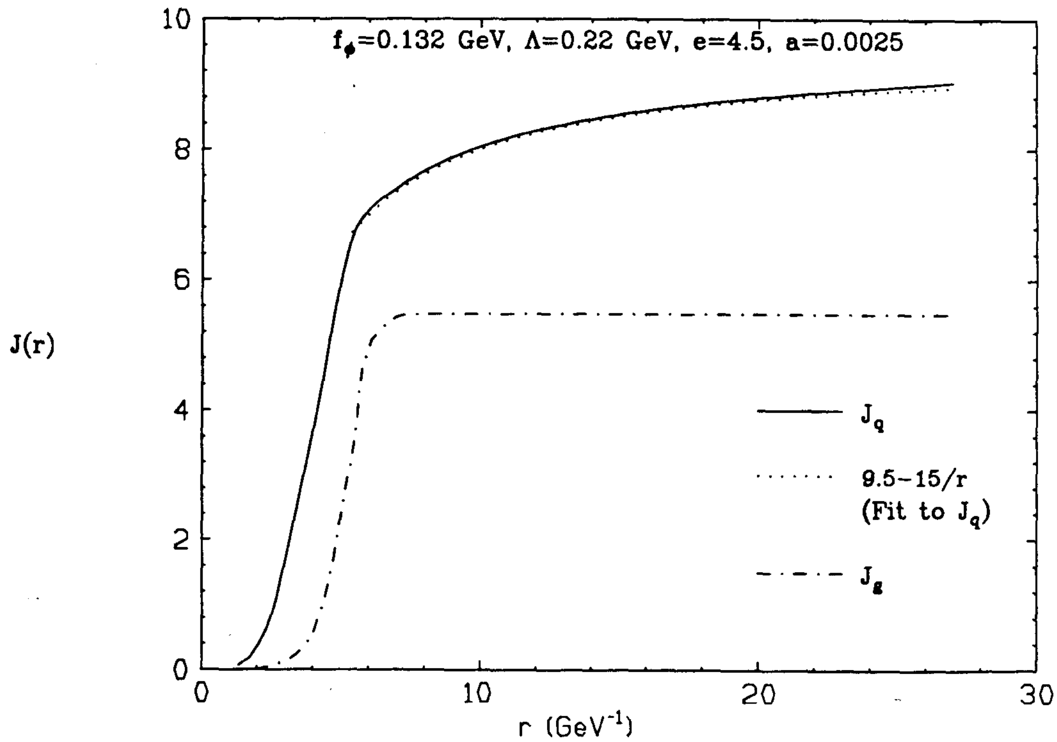

f_ϕ=0.132 GeV, Λ=0.22 GeV, e=4.5, a=0.0025
J(r)
J_q
9.5−15/r
(Fit to J_q)
J_g
r (GeV^{-1})

Fig. 4