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ON SUPERSYMMETRY BREAKING IN SUPERSTRINGS*

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ABSTRACT

We prove that the existence of a slightly massive gravitino or gaugino in a class of Gaussian string compactifications, implies the existence of an *entire tower* of such states below M_{planck}, signaling the approach to a limit of decompactification.

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Introduction

Unless a completely different mechanism is responsible for protecting the gauge hierarchy, we expect that in a realistic string vacuum space-time supersymmetry is broken at a scale near M_{weak}, i.e., infinitesimal in units of M_{planck}. The purpose of this letter is to prove that, at least in the restricted class of (Gaussian) four-dimensional models, constructed with free world-sheet fields in refs. [1–5], the existence of a gravitino or gaugino of infinitesimal tree-level mass ($M_{3/2}/M_{planck} \ll 1$) implies the existence of an *entire tower* of such states, with masses equal to $(2N+1)M_{3/2}$, $(4N+1)M_{3/2}, \ldots$, where N is some, not too large, integer. This is a signal of decompactification, meaning that one or more internal radii become huge or, by duality, infinitesimal in units of M_{planck}^{-1}, so that the corresponding momenta become quasicontinuous.

The fact that the scale of tree-level supersymmetry breaking is necessarily linked to the size of some internal dimension is not a *priori* obvious. This is, for instance, not the case if instead of supersymmetry one considers the breaking of gauge symmetries [1,6]; neither is it true if instead of superstrings one considers traditional Kaluza–Klein supergravities [7]. It is thus tempting to consider this phenomenon as a characteristic stringy signature at low energy. However, even though there is no direct experimental evidence against the existence of some extra dimension at say 100 TeV, such a scenario faces at least one serious difficulty:* couplings of order one at low energies should, by naive dimensional analysis, become huge at the unification scale, invalidating the semiclassical description of the string and creating a new hierarchy problem. For this reason, it is more cautious to interpret our result as ruling out small tree-level supersymmetry breaking, at least in this restricted class of Gaussian models.

Our interest in these issues was spurred by attempts to construct string solutions with supersymmetry broken by a Scherk–Schwarz mechanism [7,9].

⋆ There is of course also the perennial cosmological constant problem, which was studied in a similar context in ref. [8].

Similar results to ours were derived by Dine and Seiberg [10] and Banks and Dixon [11]. Their arguments are not restricted to the Gaussian compactifications considered here, but their conclusions are weaker: they show that supersymmetry cannot be broken continuously by sliding the vacuum expectation value of a scalar field at an analytic point and in a flat direction of its potential. This does not, however, rule out a number of interesting possibilities: for instance, there could exist vacua with hierarchically suppressed supersymmetry breaking, which cannot be continuously connected to supersymmetric ones. Or, the scale of supersymmetry breaking could be proportional to some internal radii, but with a constant of proportionality of the order, say, of 10^{-16}. Or else, broken supersymmetry could characterize only the nearly massless, but not all of the massive string modes. Last, but not least, it is plausible, but by no means yet established, that the only continuous string parameters are vacuum expectation values of scalar fields; it is in fact noteworthy that in many interesting supergravity models [7,12] the scale of supersymmetry breaking is not tuned by a Higgs field. In contrast to the proofs of refs. [10] and [11], our proof would, if extended to arbitrary string compactifications, exclude all of these possibilities, since we make no assumption about the mechanism producing a slightly massive gravitino or gaugino.

Finally, let us note that if supersymmetry is not broken at tree level, then nonrenormalization theorems [13] guarantee that it will not break through radiative corrections. A possible exception could occur if the gauge group contains anomalous U(1) factors [14], but as we will here also show the chiral charge asymmetry can be bounded from below by a not too small number. This makes it unlikely that supersymmetry will break, if at all, at a scale near M_{weak} by such a mechanism, leaving nonperturbative effects as the final alternative. These difficulties in breaking space-time supersymmetry should, among other things, make us rethink about the necessity of having it as an approximate low energy symmetry in string theories.

Fermionic Proof

The idea of our proof is that *world-sheet* supersymmetry severely restricts the form of massless, or infinitesimally massive space-time spinors. Together with modular invariance, this places strong constraints on the allowed superstring spectra. Consider the fermionic construction [2,4,5] of four-dimensional heterotic string theories. In addition to the four space-time coordinates X^μ and their left-moving superpartners ψ^μ, one has an extra 18 left-moving and 44 right-moving fermions χ^a and η^A, respectively. The world-sheet supercharge is [15]:

$$T_F = \psi^\mu \partial_z X_\mu + i f_{abc}\ \chi^a \chi^b \chi^c \ , \tag{1}$$

with f_{abc} the structure constants of some semisimple Lie group G. Since G has dimension 18, it is either SU(2)^6, or SU(3) $\times$ O(5) or finally SU(2) $\times$ SU(4). A space-time spinor must belong to a Hilbert–space sector $H_{\mathcal{A}}$, in which the fermionic fields have the following boundary conditions when transported around the string:

$$\psi^\mu \to \psi^\mu \ , \tag{2a}$$

$$\chi^a \to \mathcal{A}_G{}^{ab} \chi^b \ , \tag{2b}$$

$$\eta^A \to \mathcal{A}_{Right}{}^{AB} \eta^B \ , \tag{2c}$$

where $\mathcal{A}_{Right}$ is an orthogonal matrix, while $\mathcal{A}_G$ must, in addition, belong to the group of automorphisms [5] of G: $\mathcal{A}_G \in \ Aut(G)$. This last requirement is due to the fact that the supercurrent must be periodic.

We may always choose some, generally complex basis $f^{(1)}, \ldots, f^{(K)}$ for the fermions that diagonalizes the boundary conditions (2):

$$f^{(i)} \to -e^{i\pi\alpha^{(i)}} f^{(i)} \quad . \tag{3}$$

We denote by $\alpha = (1,\ \alpha_G;\ \alpha_{Right})$ the corresponding vector of phases, restricted so that $-1 < \alpha^{(i)} \leq 1$. States in $H_{\mathcal{A}}$ are, as usual, constructed by acting on the vacuum $|0\rangle_{\mathcal{A}}$ with positive-frequency oscillators, where the frequencies of the $f^{(i)}$ oscillators are equal to

$$\frac{1+\alpha^{(i)}}{2} + \text{integer} \quad . \tag{4}$$

The mass of a state is determined by the zeroth-order Virasoro gauge conditions:

$$M^2 = \sum_L (\text{frequencies}) - \frac{1}{2} + \frac{1}{8} + \frac{\alpha_G \cdot \alpha_G}{8} \ , \tag{5a}$$

$$= \sum_R (\text{frequencies}) - 1 + \frac{\alpha_{Right} \cdot \alpha_{Right}}{8} \quad . \tag{5b}$$

In eq. (5a) the factor 1/8 is the contribution of the two real, or one complex, transverse fermions ψ^μ which are periodic, since we are considering a Ramond sector.

Next, we examine the phase-vector length $\alpha_G \cdot \alpha_G$ as $\mathcal{A}_G$ ranges over $Aut(G)$. This is minimized for some special automorphisms $\mathcal{A}_G^0$ with the following two crucial properties:

(*i*) $\alpha_G^0 \cdot \alpha_G^0 = \frac{1}{6}\, dim\ G = 3$, which is precisely the value necessary to make the left-moving vacuum in the sector $H_{\mathcal{A}}$ massless, and

(*ii*) $N \cdot \alpha_G^0 = 0$ (mod 2) for some small integer N ($N \leq 6$ for the groups that interest us), which means that $\mathcal{A}_G^0$ is some small root of the identity.

These properties were established in ref. [5], where all the minima $\mathcal{A}_G^0$ were classified exhaustively. Here we will restrict ourselves to inner automorphisms, and give an alternative elegant proof which we owe to Peter Goddard [16]. An inner automorphism is a group element in the adjoint representation; up to a conjugation it can be written as:

$$\mathcal{A}_G = e^{i\pi \vec{\theta} \cdot \vec{H}} \ , \tag{6}$$

with H_l the mutually commuting Cartan generator. The eigenvalues of this matrix are 1 for each of the $rank(G)$ commuting generators, and $e^{i\pi \vec{\theta}\cdot\vec{\rho}}$ for every generator corresponding to the root vector $\vec{\rho}$. Thus, we find:

$$\alpha_G \cdot \alpha_G = \frac{1}{2}\,\text{rank}\ (G) + \sum_{\substack{+ve \\ \text{roots}}} (\vec{\theta}\cdot\vec{\rho} - 1)^2 \quad . \tag{7}$$

Let us normalize the length of the long roots to 2. Then $\sum_{+ve} \rho^i\,\rho^j = h\,\delta^{ij}$ where h is the (integer) dual Coxeter number ($h = n$ for SU(n) and $h = n-2$ for O(n) with $n \geq 5$). Defining

$$\vec{\theta_0} = \sum_{+ve} \vec{\rho}/h \ , \tag{8}$$

and using the Freudenthal–de Vries strange formula:

$$\vec{\theta_0}\cdot\vec{\theta_0} = \frac{dim\ G}{3h} \quad , \tag{9}$$

we may rewrite eq. (7) as follows:

$$\alpha_G \cdot \alpha_G = \frac{1}{6}\ dim\ G + h(\vec{\theta} - \vec{\theta_0})^2 \quad . \tag{10}$$

It follows immediately that the minimum phase-vector length is $\frac{1}{6}\ dim\ G$, as advertized, and it is obtained for the special automorphism

$$\mathcal{A}_G^0 = e^{i\pi \vec{\theta}_0\cdot\vec{H}} \ , \tag{11}$$

which, among inner automorphisms, is unique modulo the choice of the subset of positive roots. Furthermore, it is straightforward to check that $\mathcal{A}_G^0$ is an N-th root of the identity, with $N = h$ for simply laced group, while $N = 2h$ for $O(5)$; N is therefore a small integer. The special automorphisms, eq. (11), have been studied extensively by Coxeter, and also arise in some other amusing contexts [17].

For our purposes, they are special because they can yield massless space-time gravitinos:*

$$\overline{\partial} X_1^\mu |0>_S \ , \tag{12}$$

with $S = (1,\ \alpha_G^0;\ \alpha_{Right} = 0)$. Using the mass formula, eq. (5), and the fact that near S there are no almost periodic fermions with infinitesimal-frequency oscillators, it is now easy to see that the only candidate for a slightly massive gravitino is of the form:

$$\overline{\partial} X_1^\mu \ |0>_{S+\delta S} \ , \tag{13}$$

where $\delta S = (0\ \delta\alpha_G;\ \delta\alpha_{Right}) \ll 1$, and the mass is

$$M_{3/2}^2 = \frac{1}{8}(\delta\alpha_G)^2 = \frac{1}{8}(\delta\alpha_{Right})^2 \quad . \tag{14}$$

Here, the absence of a linear term is due to the fact that $\alpha_G^0 \cdot \delta\alpha_G = 0$ since α_G^0 minimized vector length. Suppose now that this state belongs to the Hilbert space of a four-dimensional fermionic string model. This means that $S + \delta S$ is in the group Ξ of allowed boundary conditions, and that the state (13) survives the generalized GSO-type projections:

$$e^{i\beta \cdot F} = -c \begin{bmatrix} S+\delta S \\ \beta \end{bmatrix} = \begin{cases} 1 & \text{if} \quad \delta_\beta = 1 \\ \\ \pm 1 & \text{if} \quad \delta_\beta = -1 \end{cases} \tag{15}$$

for all $\beta \epsilon \Xi$. We are here using the notation of ref. [5]. F is the vector of fermion-number operators, the dot product is Lorentzian, left minus right, $c \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ is the coefficient with which the corresponding spin-structure contributes to the one-loop amplitudes, and $\delta_\beta = 1$ or -1 according as β is a Neveu–Schwarz or Ramond boundary condition. Equation (15) follows from the fact that on the state (13), $e^{i\beta \cdot F}$ acts, respectively, as the identity or as a chirality operator.

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* As a result they can be used to generate $N = 2$ superconformal models, as well as Θ-function identities.

Now Ξ is a group under addition of phase vectors modulo 2. Furthermore, $2N \cdot S = 0 \bmod 2$, from which we conclude that $S+(2N+1)\delta S,\ S+(4N+1)\delta S \ldots$, etc. are also in Ξ. We therefore have an entire tower of candidate low-mass gravitinos

$$\partial X_1^\mu |0>_{S+(2N+1)\delta S}\ ; \quad \partial X_1^\mu |0>_{S+(4N+1)\delta S} \cdots \ ,$$

with mass differences equal to $2N \cdot M_{3/2}$. These states satisfy left-right level matching due to the absence of a linear term in eq. (14) . We must still show that they survive all GSO–type projections, i.e., that the coefficients:

$$c \begin{bmatrix} S+(2N+1)\delta S \\ \beta \end{bmatrix} ; \ c \begin{bmatrix} S+(4N+1)\delta S \\ \beta \end{bmatrix} \cdots$$

also satisfy eq. (15). To prove this one must use the duality and factorization conditions [2,4,5]:

$$\begin{aligned} c \begin{bmatrix} \alpha \\ \beta \end{bmatrix} &= e^{\frac{i\pi}{2}\alpha\cdot\beta}\ c \begin{bmatrix} \beta \\ -\alpha \end{bmatrix} \\ c \begin{bmatrix} \alpha \\ \beta+\gamma \end{bmatrix} &= \delta_\alpha\ c \begin{bmatrix} \alpha \\ \beta \end{bmatrix} c \begin{bmatrix} \alpha \\ \gamma \end{bmatrix} \end{aligned} , \qquad (16)$$

and the fact that

$$NS \cdot \beta = \begin{cases} \text{even integer} \quad \text{if } \delta_\beta = 1 \\ \\ \text{integer} \quad \text{if } \delta_\beta = -1 \end{cases} , \qquad (17)$$

which helps get rid of phases. Equation (17) follows from the fact that $\alpha_G^0 + \beta_G + 1$, or $\alpha_G^0 + \beta_G$ are the phase vectors of an automorphism, if β_G is an automorphism ($\delta_\beta = -1$) or antiautomorphism ($\delta_\beta = 1$), respectively, and that $N\alpha_G^0 = 0 \bmod \mathrm{Z}$, and finally that α_G^0 minimizes vector length over the group of automorphisms. This then completes our proof, which goes through similarly for outer automorphisms. Note that the integer N governing the mass differences between the tower of gravitinos is at most equal to six.

Bosonic Proof

It is instructive to translate this proof in the bosonic language. The proof is, as we will see simpler and more general, but we are unable to obtain a very tight bound on N. Recall that to a given state in $H_{\mathcal{A}}$, with fermion numbers F, we assign a momentum-winding number vector [2]:

$$p = \frac{1}{2}\alpha + F \quad .$$

This has ten left and 22 right components. Allowed momenta belong to a shifted lattice $\Delta + \Gamma$ where the shift vector is $\Delta = (1\ 0 \ldots 0)$, the entry 1 corresponding to the bosonized transverse ψ^μ. Modular invariance dictates that $p^2 = p_L^2 - p_R^2$ be an odd integer, and that the lattice Γ be integer and self-dual; this means that elements of Γ have integer dot products with each other, and that any q such that $(q - \Delta)$ has integer dot products with Γ is an allowed momentum [2]. These constraints generalize the usual conditions of an even, integer, self-dual Lorentzian lattice [1], with the extra complications arising because one treats the bosonized world-sheet fermions on the same footing as the internal-space coordinates.

Now the most general Lorentz-invariant supercharge is of the form [18]:

$$T_F = \psi^\mu \partial_z X_\mu + \sum a(r) e^{ir\cdot\varphi} + \sum b(t) \cdot \partial_z \varphi e^{it\cdot\varphi} \ , \tag{19}$$

where the vectors r, t and b entering in the sums have nonvanishing components only along the nine internal bosonized left-moving coordinates. T_F is a good weight 3/2 conformal field provided $r^2 = 3,\ t^2 = 1$ and $b(t) \cdot t = 0$. Further conditions must be imposed in order that the anticommutator of two supercharges yield the energy-momentum tensor, but the solutions to these complicated equations have not yet been completely classified.

Consider next the transformation properties of T_F as one moves around the string parametrized by $0 < \sigma \leq 2\pi$. Using the fact that $\varphi(\sigma + 2\pi) = \varphi(\sigma) \pm 2\pi P^{(op)}$, where the momentum-winding number operator obeys canonical commutation relations with φ and the sign depends on whether φ is the left- or right-moving, one finds that:

$$e^{iq\cdot\varphi(\sigma+2\pi)} \left|p\right\rangle = e^{2i\pi q\cdot p + i\pi q^2} \cdot e^{iq\cdot\varphi(\sigma)} \left|p\right\rangle \quad . \tag{20}$$

Since T_F must be either antiperiodic (Neveu–Schwarz) or periodic (Ramond), we conclude that:

$$\Delta \cdot p = r \cdot p = t \cdot p = \begin{cases} 0 \quad \mathrm{mod}\ Z \quad (\mathrm{NS}) \\ \\ \frac{1}{2} \quad \mathrm{mod}\ Z \quad (\mathrm{R}) \end{cases} \tag{21}$$

for all r, t entering in expression (19), and for any momentum p. It follows by "self-duality" that all the vectors r and t are themselves allowed momenta of the theory.

Let now E^d be the Euclidean space, of dimension $d \leq 10$, spanned by r, t and Δ, and let Γ_E be the lattice of vectors in E^d which have half-integer or integer dot products with all r, t and Δ. For any given theory the elements of Γ_E do not belong in general to the lattice Γ of shifted momenta, but there exists some integer N bounded from above by:

$$N \leq 3^9 \times 2 \tag{22}$$

such that $2Nq \in \Gamma$ whenever $q \in \Gamma_E$. To see why, choose among all r, t and Δ some linearly independent basis $e_1, \ldots, e_d$ spanning E^d. Then for any momentum p we have:

$$2Nq \cdot (p - \Delta) = 2N \sum_{I,J=1}^{d} (q \cdot e_I)(g^{-1})_{IJ}(e_J \cdot (p - \Delta)) \ , \tag{23}$$

where $g_{IJ} = e_I \cdot e_J \in Z$. Now $q \cdot e_I$ and $(p - \Delta) \cdot e_J$ are half-integers or integers, while the components of g^{-1} are rationals with a common denominator that divides $det(g)$. Choosing N to be twice this denominator we see that $2Nq \cdot (p - \Delta)$ is always integer, proving that $2Nq$ belongs to the self-dual lattice Γ. The bound (22) follows from the fact that $det(g)$ is the square volume of the parallelepiped $(e_1, \ldots e_d)$ which is less than 3^9, since $e_I^2 = 1$ or 3 for all I.

The next, crucial observation is that a would-be massless gravitino must have a momentum p_0 lying entirely in E^d, and hence belonging to the lattice Γ_E. Indeed recall the mass formula:

$$\begin{aligned} M^2 &= \frac{1}{2}p_L^2 - \frac{1}{2} + \sum_L (\text{frequencies}) \ , \\ &= \frac{1}{2}p_R^2 - 1 + \sum_R (\text{frequencies}) \ . \end{aligned} \tag{24}$$

Since to create a gravitino we must act on the right with a $\overline{\partial} X_1^\mu$ oscillator, p_0 can have only left nonvanishing components: $p_{0,R} = 0$. On the other hand, it follows from the super–Virasoro algebra alone that in a Ramond sector:

$$M^2 = T_F(0)^2 \geq 0 \ ,$$

where $T_F(0)$ is the zeroth moment of the supercurrent. This means that for a massless fermionic state $p_{0,L}$ must minimize vector length, subject to the constraints (21) of having half-integer dot products with r, t and Δ. Thus it cannot have a component normal to all these vectors simultaneously, and must hence lie in E^d.

We are now almost done. Indeed, consider a theory containing a slightly massive gravitino. Its momentum is necessarily of the form $p = p_0 + \delta p$ with $p_0 \in \Gamma_E$ and $\delta p \ll 1$. But from our previous discussion we conclude that if p is an allowed momentum, then so are:

$$p_0 + (2Nk + 1)\delta p \ , \qquad k = 1, 2, 3 \ldots \ ,$$

giving an entire tower of spin-3/2 states with masses much less than M_{planck}. A second consequence of our discussion is that, if there is a chiral charge-asymmetry it is bounded from below by $1/\sqrt{N}$, and hence cannot be made arbitrarily small. The reason is that if p and $\tilde{p}$ stand for the momenta of two chiral, massless fermions, then $p_L^2 = \tilde{p}_L^2 = 1/2$ and $p_R^2 = \tilde{p}_R^2 = 1$ or 0, and from our previous arguments $2Np_L \cdot \tilde{p}_L \in Z$. But since $p \cdot \tilde{p} = p_L \cdot \tilde{p}_L - p_R \cdot \tilde{p}_R$ is an integer, we conclude that so is $N(p_R \pm \tilde{p}_R)^2$. Hence the charge asymmetry $|p_R \pm \tilde{p}_R|$, where the sign depends on the relative helicity, cannot be less than $1/\sqrt{N}$. The induced one-loop Fayet–Iliopoulos D–term can thus be made arbitrarily small only if the string coupling constant is infinitesimal.

Let us conclude with some remarks. Firstly, the proof of the existence of a tower of states below M_{planck} goes through identically if, instead of a slightly massive gravitino, we assume the existence of a slightly massive gaugino corresponding to the U(1) [22] maximal abelian subgroup of the gauge group. Secondly, the proof in the bosonic language encompasses a wider class of supercurrents [18], but the bound (22) is, for all the examples we know, overestimating N by three orders of magnitude. Without a complete classification of supercharges of type (19) we do not however know how to significantly lower this bound. Finally, we believe the arguments presented here can be extended with some modifications to generic orbifold constructions [19]; we do not have however an exhaustive proof, because the complete rules of the game are not yet known explicitly.

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