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MULTI-BUNCH ENERGY COMPENSATION*

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1. INTRODUCTION

To obtain a luminosity of $10^{34}\text{cm}^{-2}\text{sec}^{-1}$ in a TeV Linear Collider (TLC), it will probably be necessary to accelerate many bunches in one filling of the RF structure. This has the effect of extracting more energy from the structure and thus enhances the overall efficiency of the accelerator. However, this leads to many problems. First, the train of bunches is subject to cumulative beam breakup transversely. This can be controlled by damping the transverse modes with slots in the irises coupled to waveguides.[1,2] In addition, the energy of the bunches must be kept the same to high precision. For the fundamental mode, this entails adjusting the timing of the RF fill and also the bunch spacing. The higher longitudinal modes, although they do not induce instability, also may lead to bunch-to-bunch variations in energy. However, it also seems possible to damp these modes to cure this problem.[1] Of course, there are also problems associated with damping a train of bunches in a damping ring.

In this paper we discuss some of the issues of multi-bunch energy compensation. In the first two sections, we review some basics about energy extraction by a single bunch. In Section 4, multi-bunch energy compensation is treated. In Section 5, we discuss various tolerance issues associated with deviations of amplitude and phase of the RF away from the ideal.

For general information on high energy electron linacs the reader is referred to the review article by P. Wilson in Ref. 3.

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2. A SINGLE BUNCH

Consider a single bunch traversing an RF structure of length L which is partially full of RF. The energy gain of a test charge at the front of the bunch is

$$\Delta E = \int_{o}^{L} \mathcal{E}_z(s)ds \tag{2.1}$$

where $\mathcal{E}_z(s)$ is the envelope of the RF as a function of distance along the structure. The bunch is assumed to be at the crest of the cosine. For off-crest operation this is modified by a factor of the cosine of the phase angle. This factor will be suppressed in the next 3 sections but included where relevant in Section 5.

To calculate energy gain, we need to know the field profile in the structure. As a simple model, consider an initial RF pulse that is square with a width equal to the filling time of the structure. That is, the RF pulse duration is

$$T_f = \frac{L}{v_g} \tag{2.2}$$

where v_g is the group velocity of the structure and L is the length of the structure. As the structure fills, the field level drops due to losses. If the characteristic decay time is

$$T_o = \frac{2Q}{\omega} \quad , \tag{2.3}$$

then the fields decay like

$$\mathcal{E}_z(t) = \mathcal{E}_o e^{-t/T_o} \quad . \tag{2.4}$$

To relate this to the field profile along the structure, we have

$$\begin{aligned} t &= \frac{s}{v_g} \\ t &= \frac{T_f s}{L} \quad . \end{aligned} \tag{2.5}$$

Thus, along the structure, the fields decay like

$$\mathcal{E}_z(s) = \mathcal{E} e^{-s/\ell} \tag{2.6}$$

where

$$\ell = \frac{LT_o}{T_f} \quad . \tag{2.7}$$

Therefore, in this case, the energy gain of a test particle at the front of the bunch is

$$\Delta E = e\mathcal{E}_o \ell (1 - e^{-s_o/\ell}) \tag{2.8}$$

where s_o is the distance which the field has propagated when the particle traverses the structure. For no attenuation, we have

$$\Delta E = e\mathcal{E}_o s_o \quad . \tag{2.9}$$

We assume, throughout the paper, that the particle traverses the structure in a time small compared to the filling time. Thus, during passage there is no additional filling. This is not strictly true. In fact for a structure with $v_g = 0.1c$ the structure fills an additional 10 % while the particle is traversing it. This modifies the particle injection time relative to the time for starting the RF fill and also changes the energy extraction efficiency by about 10 %.

3. THE WAKEFIELD

The field induced in the fundamental accelerating mode by a short bunch with charge q is given by

$$\mathcal{E}_{\text{wake}} = -2kq \tag{3.1}$$

where k is the fundamental loss parameter per unit length. Ignoring higher mode losses, the efficiency of energy extraction from a no loss structure is

$$\begin{aligned} \eta_o &= 1 - \frac{(\mathcal{E}_z - 2kq)^2}{\mathcal{E}_z^2} \\ &= \frac{4kq}{\mathcal{E}_z} - \frac{4k^2q^2}{\mathcal{E}_z^2} \quad . \end{aligned} \tag{3.2}$$

For small energy extraction, the second term can be neglected.

The particle at the tail of the short bunch feels an accelerating field $\mathcal{E}_z - 2kq$; therefore, for a uniform distribution the average energy gain of the bunch is

$$\overline{\Delta E}/e = (\mathcal{E}_z - kq)s_o - kq(L - s_o) \quad . \tag{3.3}$$

Including the losses in the structure,

$$\overline{\Delta E}/e = \mathcal{E}_z \ell_o (1 - e^{-s_o/\ell_o}) - kqL \quad . \tag{3.4}$$

4. MULTI-BUNCH EFFECTS

First consider the case of two bunches traversing a partially filled accelerating structure. Let the bunches be separated by a time Δt. This allows the field to propagate a distance

$$\Delta s = v_g \Delta t \quad . \tag{4.1}$$

Now consider the energy gained by a particle at the *head* of each of the two bunches. The average energy gain can be calculated with Eq. (3.4). For the first bunch, the result is the same as Eq. (2.8),

$$\Delta V_1 = \mathcal{E}_o \ell (1 - e^{-s_o/\ell}) \tag{4.2}$$

where we have introduced $V \equiv E/e$ to simplify the formulae. For the second bunch, the structure has been filled a bit more and so we find

$$\Delta V_2 = \int\limits_o^{\Delta s} \mathcal{E}_z(s) ds + \int\limits_{\Delta s}^{s_o+\Delta s} (\mathcal{E}_z(s) - 2kq) ds - \int\limits_{s_o+\Delta s}^{L} 2kq ds \quad . \tag{4.3}$$

This expression includes the incoming field and also takes care of the wakefield which propagates out the end of the structure. Performing the integrals we find

$$\Delta V_2 = \mathcal{E}_o \ell (1 - e^{-(s_o+\Delta s)/\ell}) - 2kq(L - \Delta s) \quad . \tag{4.4}$$

Equation (4.4) includes the reduction of accelerating field due to attenuation of the input RF. It does not include the attenuation of the wakefield. This is a rather good approximation since for the case of a short train of bunches the wakefield is in the structure for a much shorter time than the input RF.

To calculate the energy gain of the head of the third bunch, proceed as before to obtain

$$\begin{aligned} \Delta V_3 = & \int\limits_o^{\Delta s} \mathcal{E}_z(s) ds + \int\limits_{\Delta s}^{2\Delta s} (\mathcal{E}_z(s) - 2kq) ds + \int\limits_{2\Delta s}^{s_o+2\Delta s} (\mathcal{E}_z(s) - 4kq) ds \\ & - 4kq(L - s_o - 2\Delta s) \end{aligned} \tag{4.5}$$

which yields

$$\Delta V_3 = \mathcal{E}_o \ell (1 - e^{-(s_o+2\Delta s)/\ell}) - 4kqL + 6kq\Delta s \quad . \tag{4.6}$$

For n bunches, the result is given by

$$\begin{aligned}\Delta V_n =&\mathcal{E}_o\ell(1-e^{-(s_o+(n-1)\Delta s)/\ell})-(n-1)2kqL\\&+n(n-1)kq\Delta s \quad .\end{aligned} \tag{4.7}$$

The coefficient of the last term is actually a sum and is given by

$$2\left[(n-1)^2-\sum_{m=1}^{(n-2)} m\right] \quad . \tag{4.8}$$

After summing the series, this yields the coefficient in Eq. (4.7).

4.1. Small Attenuation

In the previous section we have neglected the decay of the wake during the passage of the bunch train. To be consistent we should also neglect the additional decay of the incoming accelerating field during that time. This approximation yields particularly simple formulae and corresponds to the case of

$$n\Delta s/\ell \ll 1 \quad . \tag{4.9}$$

In this case, one can expand the relevant exponential in Eq. (4.7) to yield

$$\Delta V_n = \mathcal{E}_o\ell(1-e^{-s_o/\ell})+(n-1)\mathcal{E}_o e^{-s_o/\ell}\Delta s-(n-1)2kqL+n(n-1)kq\Delta s \tag{4.10}$$

The first two linear terms in n could be used to cancel the variation of V_n exactly were it not for the quadratic effect remaining. Since it is quadratic, it could get quite large. A possible optimum might be to match the energy of the first and last bunch in the train. For N bunches, this is done by setting

$$(N-1)\mathcal{E}_o e^{-s_o/\ell}\Delta s-(N-1)2kqL+N(N-1)kq\Delta s=0 \tag{4.11}$$

which yields

$$\frac{\Delta s}{L}=\frac{2kq}{\mathcal{E}_o e^{-s_o/\ell}+Nkq} \quad . \tag{4.12}$$

In this case, the energy gain of the n^{th} bunch in a train of N bunches is

$$\Delta V_n = \mathcal{E}_o\ell(1-e^{-s_o/\ell})+(n-N)(n-1)kq\Delta s \quad . \tag{4.13}$$

This has its maximum deviation at the center of the bunch train. For an even number of bunches

$$\delta V_{\max} = -\frac{N(N-2)}{4} kq\Delta s. \tag{4.14}$$

It is useful to express these formulae in terms of the single bunch efficiency factor η_o,

$$\eta_o = \frac{4kq}{\mathcal{E}_o} \quad . \tag{4.15}$$

In this case, the bunch spacing is given by

$$\frac{\Delta s}{L} = \frac{\Delta t}{T_f} = \frac{\eta_o e^{s_o/\ell}}{2} \frac{1}{1 + \frac{N}{4}\eta_o e^{s_o/\ell}} \quad . \tag{4.16}$$

Of course, the spacing must also be some multiple of the RF period. After continuous acceleration, we can ignore the injection energy; thus, the fractional variation of the bunch energy is given by

$$\frac{\delta E_{\max}}{E} = \frac{N(N-2)}{32} \left[\frac{L e^{s_o/\ell}}{\ell(1-e^{-s_o/\ell})}\right] \frac{\eta_o^2}{1 + \frac{N}{4}\eta_o e^{s_o/\ell}} \tag{4.17}$$

For small η_o we have

$$\frac{\delta E_{\max}}{E} \simeq \frac{N(N-2)}{32} F\eta_o^2 \tag{4.18}$$

where F is the factor in square brackets in Eq. (4.17). Thus if we have a tolerance on the energy difference between bunches of $\delta E_{\max}$, the energy extraction per bunch is limited by

$$\eta_o \simeq \left[\frac{32}{FN(N-2)} \frac{\delta E_{\max}}{E}\right]^{1/2} \tag{4.19}$$

In the previous analysis the bunch spacing is simply determined by the bunch charge. This is true because we have assumed that all sections in the entire linac are filled in the identical manner. If we wish to have the same bunch spacing, but with say 1/2 the current, then we need to delay the filling of only 1/2 of the sections. The remaining sections must be full when the first bunch arrives and must remain full throughout the bunch train. In this case the RF pulse width must be somewhat greater than the fill time.

This discussion also points the way towards the tailoring of the effective RF pulse by section-to section delays and/or pulse shape changes. In this way it may be possible to correct the droop in energy calculated in this section.

4.2. An Example

Say that we allow the bunches to vary by a full energy spread of

$$\left(\frac{\delta E}{E}\right)_{\max} = 2 \times 10^{-3} \quad , \tag{4.20}$$

and consider the case of 10 bunches. To calculate F and $e^{-s_o/\ell}$ we need to know s_o. As a first approximation we can take $s_o = L$. Then the factor F is only a function of the attenuation parameter $\tau = L/\ell$. For this example we take $\tau = .6$, which means the energy extraction per bunch could be

$$\eta_o \simeq 1.82\% \quad . \tag{4.21}$$

This in turn determines the bunch spacing to be

$$\frac{\Delta t}{T_f} = 1.53 \times 10^{-2} \quad . \tag{4.22}$$

If we consider a structure with an RF frequency of 11.4 GHz, then a filling time of 70 nsec yields a bunch spacing of about 12 RF periods (1.1 nsec). In this case, the first bunch is injected when the structure is 84.7% full. Thus the 10 bunch train has an energy which is 15.3% lower than a single bunch would have if injected into the full structure. In general, the percentage loss of average acceleration gradient in the structure is simply $N\Delta t/T_f$.

5. TOLERANCES FOR MAINTAINING BUNCH-TO-BUNCH ENERGY

In the next few short sections we discuss several different effects which can destroy the careful energy match calculated in the previous section. In these next few sections the attenuation parameter τ is set to zero and thus losses are neglected.

5.1. Phase Shift Due to Wakefields

As discussed in the first section, the head of the bunch sees a different field than the tail. For a short uniform bunch and considering only the fundamental longitudinal mode, the wakefield induced by the bunch causes the tail particles to be depressed in energy by

$$\Delta V_{\text{tail}} = -2kqL \quad . \tag{5.1}$$

This yields a linear slope across the bunch given by

$$\frac{dV_{\text{wake}}}{d\phi} = \frac{2kqL}{\Delta\phi} \tag{5.2}$$

where $\Delta\phi$ is the full bunch length in RF phase. This is typically cancelled by shifting the phase of the drive relative to the bunch arrival so that the slope of the RF cancels that due to the longitudinal wakefield. In this case the energy gain of the n^{th} bunch is

$$\begin{aligned}\Delta V_n(\phi) = \mathcal{E}_o s_o \cos(\phi) + (n-1)\mathcal{E}_o \Delta s \cos(\phi) + \\ [-(n-1)2kqL + n(n-1)kq\Delta s]\cos(\phi - \phi_o)\end{aligned} \tag{5.3}$$

while the slope of the RF at the n^{th} bunch is given by

$$\frac{d\Delta V_n}{d\phi}|_{\phi=\phi_o} = -\mathcal{E}_o s_o \sin\phi_o - (n-1)\mathcal{E}_o \Delta s \sin\phi_o \tag{5.4}$$

where ϕ_o is the location of the bunch on the RF. Increasing ϕ corresponds to movement ahead of the bunch. We would like to cancel the linear slope for all bunches; however, Eq. (5.4) makes that impossible.

For a single bunch we cancel the wakefield slope which yields

$$\sin\phi_o = \frac{2kqL}{\mathcal{E}_o L \Delta\phi} \quad . \tag{5.5}$$

Written in terms of the η_o parameter this becomes

$$\sin\phi_o = \frac{\eta_o}{2\Delta\phi} \quad . \tag{5.6}$$

For many bunches, one might cancel the slope at the middle bunch, $\frac{N}{2}$. This yields

$$\sin\phi_o = \frac{\eta_o}{2\Delta\phi} \frac{L}{s_o + \frac{(N-2)}{2}\Delta s} \quad . \tag{5.7}$$

Simplifying by using the results of the previous section we find

$$\sin\phi_o \simeq \frac{\eta_o}{2\Delta\phi} \frac{1}{1 - \frac{(N+2)}{4}\eta_o} \quad . \tag{5.8}$$

For the example shown in Section 4.2, this yields a variation in slope of about $\pm 8\%$. If we consider a full bunch length of 125 μm, this yields a $\Delta\phi$ of 3×10^{-2}

for 11.4 GHz. For the η_o given in the example, we find

$$\sin \phi_o \simeq .30 \tag{5.9}$$

or

$$\phi_o \simeq 18^o \quad . \tag{5.10}$$

In this simple model, the full energy spread of the head and tail bunch is

$$\frac{\delta E}{E} = \pm \frac{(N-1)\Delta s}{2 s_o} \sin \phi_o \Delta \phi \simeq \pm 7 \times 10^{-4}. \tag{5.11}$$

Since we expect the final focus to have a relative energy bandwidth of about $\pm 2 \times 10^{-3}$, this variation of slope is probably acceptable.

5.2. Tolerance on Phase of the RF

For a single bunch, phase variations over the RF pulse are not that important since the bunch integrates over them. For many bunches, phase variations lead to bunch-to-bunch energy differences. To model this, consider first a systematic phase variation $\delta\phi$ over the train of bunches which occurs during the last part of the fill of the structure. The phase variation during the RF pulse up to s_o is averaged over by all bunches. Then the energy gained by the last bunch is

$$\begin{aligned} E_N/e =& \mathcal{E}_o s_o \cos\phi_o + \mathcal{E}_o (N-1)\Delta s \cos(\phi_o + \delta\phi) \\ & - (N-1) 2kqL + N(N-1)kq\Delta s \quad . \end{aligned} \tag{5.12}$$

The deviation from the gain of the first bunch is

$$\begin{aligned} \frac{\Delta E_N}{E_0} &= -\delta\phi \tan\phi_o \frac{(N-1)\Delta s}{s_o} \\ \frac{\Delta E_N}{E_0} &\simeq -\delta\phi \tan\phi_o \frac{N\eta_o}{2} \quad . \end{aligned} \tag{5.13}$$

Notice that the tolerance is less severe for smaller phase angles and that the effect is reduced by the ratio of the length of the bunch train to the initial fill.

Random phase variations from section to section are calculated in a similar fashion. In this case if M is the number of independently powered sections, the variation in energy of the tail relative to the head grows proportional to $\sqrt{M}$ while the energy grows proportional to M. Thus the energy variations due to random phase errors are reduced relative to the systematic by a factor of $\frac{1}{\sqrt{M}}$.

For the example shown in Section 4.2 the tolerance for systematic phase variation over the bunch train is

$$\delta\phi \simeq 4^\circ. \tag{5.14}$$

For a large number of sections the random phase variation tolerance is much larger ($\propto \sqrt{M}$); however, Eq. (5.13) is not accurate in this case since we have assumed small $\delta\phi$.

5.3. Tolerance on q

If for some reason the charge per bunch is different than design, the relative energy of the bunches in the bunch train will be shifted.

Using Eq. (4.7), the shift in energy of the n^{th} bunch due to a change Δq in the charge of each bunch is given by

$$\Delta V_n = [-(n-1)2kL + n(n-1)k\Delta s]\Delta q \quad . \tag{5.15}$$

The relative energy change after acceleration is given by

$$\frac{\Delta E_n}{E_n} \simeq \frac{\Delta E_n}{\mathcal{E}_o s_o} \quad . \tag{5.16}$$

Substituting and rewriting in terms of η_o we find

$$\frac{\Delta E_n}{E_n} \simeq -\frac{(n-1)\eta_o}{2}\frac{\Delta q}{q} \quad . \tag{5.17}$$

If only one bunch changes charge by Δq_{no}, then the change in energy is

$$\frac{\Delta E_n}{E_n} \simeq -\frac{\eta_o}{2}\frac{\Delta q_{no}}{q_{no}} \quad . \tag{5.18}$$

For the example shown in Section 4.2 the tolerance on the systematic variation of charge in all bunches is

$$\frac{\Delta q}{q} \simeq 2.5 \times 10^{-2}. \tag{5.19}$$

Random variations could perhaps be a factor of 3 larger while the allowable variation of a single bunch in the train is a factor of 10 larger.

5.4. Tolerance on $\mathcal{E}(s)$

If there are variations in the amplitude of the RF pulse in the last part of the pulse which is just entering the structure as the bunches are extracting energy, this will lead to bunch-to-bunch changes in energy. To model this effect, consider that at the m^{th} RF section, the RF pulse is modified by $\Delta\mathcal{E}_m$ over a distance d_m. Then those bunches which arrive after this perturbation has entered the structure, receive an additional energy gain

$$\delta V = \Delta\mathcal{E}_m d_m \quad . \tag{5.20}$$

If this effect is random and uncorrelated from section to section, then the relative energy change induced in M sections is

$$\begin{aligned} \frac{\delta E_{rms}}{E} &= \frac{\Delta\mathcal{E}_{rms}}{\mathcal{E}} \frac{d_{rms}}{s_o} \frac{1}{\sqrt{M}} \\ &\leq \frac{\Delta\mathcal{E}_{rms}}{\mathcal{E}} \frac{N\eta_o}{2\sqrt{M}} \quad . \end{aligned} \tag{5.21}$$

Systematic variations would yield

$$\frac{\delta E_{rms}}{E} \lesssim \frac{\Delta\mathcal{E}_{sys}}{\mathcal{E}} \frac{N\eta_o}{2} \quad . \tag{5.22}$$

For the example shown in Section 4.2, the tolerance on systematic variation of the accelerating field over the bunch train is

$$\frac{\Delta\mathcal{E}_{sys}}{\mathcal{E}} \simeq 0.02. \tag{5.23}$$

Once again random variations can be much larger for many sections.

5.5. The Effect of a Leading Edge on $\mathcal{E}_o(s)$

It is easy to calculate the effect of a non-square RF pulse on the overall energy gain by superposition. It causes no problem in the previous analysis unless it begins to propagate out the end of the structure during the bunch train. In this case, it induces a large bunch-to-bunch energy spread. The simple method of dealing with this problem is to avoid it; that is, inject the bunch train in such a way so that the last bunch traverses the structure just before the leading edge begins to propagate out the end of the structure. The analysis in Section 4 applies in this case; however, the energy of the bunch train is once again reduced somewhat.

Let the leading edge be triangular in shape and be of length αL. Then the reduction in energy gain is simply

$$\frac{\Delta E}{E} = \frac{\alpha L}{2s_o} \simeq \frac{\alpha}{2}$$

6. Conclusion

In this paper we have discussed the control of the bunch-to-bunch energy spread for a short train of intense bunches. We have seen that it is indeed possible under ideal conditions to control the energy spread while extracting as much as 20% of the RF energy. Unfortunately conditions are rarely ideal, so tolerances have also been discussed here. The tolerances do not look that bad, but it is likely that in any real system we would need a strategy for measuring the bunch-to-bunch energy differences. In this way by adjusting the fill timing of several special RF sections, it is probably possible to correct errors which do not vary from pulse to pulse. Pulse-to-pulse systematic variations are rather unlikely but do have tight tolerances while the more likely random variations which might happen pulse to pulse have much looser tolerances.

Finally, one should note that the transverse motion is coupled to the energy variation through the chromaticity of the lattice. This probably means that energy discrepancies between bunches must be corrected locally where they develop or they could lead to separate trajectories for each bunch in the bunch train.

To conclude, recall that we have assumed that the higher order longitudinal modes are damped between bunches. Structures which accomplish this have been suggested in Ref. 1 and there is presently a program at SLAC to do more detailed measurements. It is also important to continue the studies discussed here to develop a realistic strategy for controlling bunch to bunch energy in the face of errors. In spite of all these difficulties the result is most likely well worth the effort since it leads to much higher luminosities for future linear colliders.

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