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CONTROLLING MULTIBUNCH BEAM BREAKUP IN TEV LINEAR COLLIDERS*

K. A. Thompson and R. D. Ruth

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94309

ABSTRACT

To obtain luminosities near $10^{34}\mathrm{cm}^{-2}\mathrm{sec}^{-1}$ in a TeV linear collider, it will probably be essential to accelerate many bunches per RF fill in order to increase the energy transfer efficiency. In this paper we study the transverse dynamics of multiple bunches in a linac, and we examine the effects of several methods of controlling the beam blow-up that would otherwise be induced by transverse dipole wake fields. The methods we study are: (1) damping the transverse modes, (2) adjusting the frequency of the dominant transverse mode so that bunches may be placed near zero-crossings of the transverse wake, and (3) bunch-to-bunch variation of the transverse focusing. We study the utility of these cures in the main linacs of an example of a TeV collider.

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1. INTRODUCTION

The next generation of e^+e^- linear colliders, with center-of-mass energy of about 1 TeV, is in the conceptual design phase. It is believed that it will be possible to build such a collider with moderate extensions of present RF technology. However, the power required for the linac is very high. In order to use the RF as efficiently as possible and also obtain a luminosity close to $10^{34}\text{cm}^{-2}\text{sec}^{-1}$, the acceleration of multiple bunches per RF fill is being considered. Using this technique, it is possible to extract more than 20% of the energy available in the RF cavities, while keeping the bunch-to-bunch energy variation small.[1]

One of the problems with accelerating multiple bunches per RF fill is the cumulative beam breakup instability caused by the transverse dipole wake fields. Each bunch in the train feels the wake produced in the accelerating structure when preceding bunches are slightly off-axis. The spacing between adjacent bunches is only a few RF wavelengths, and the transverse dipole wake in a normal disk-loaded structure continues to ring for many multiples of this spacing. As a result, the transverse amplitudes of the bunches can grow rapidly.

Fortunately, there are several things that can be done to mitigate the effects of the transverse wake. Briefly, the ones which we shall consider in detail here are: (1) damping the transverse dipole modes by means of axial slots through the irises of the RF structures coupled with radial waveguides,[2] (2) tuning the frequency of the fundamental transverse dipole mode to place the bunches as near as possible to zero crossings of the wake fields, and (3) using time-varying quadrupoles to introduce a small change in the focusing for different bunches, so that they are not in resonance with each other.

Cumulative beam break-up in linacs had been clearly identified by 1966,[3] and has been studied by several authors.[4–10] In our approach, we derive integral representations of the solution for each bunch using WKB-type approximations. We also show that a simple "effective length" approximation gives equivalent results up to adiabatic damping factors.

We use our results to study the effectiveness of controlling beam breakup by the two methods described above, in an example using current design parameters for the main linacs of a TeV collider. Additional results for other subsystems of both a TeV linear collider and an intermediate-energy (0.5 Tev in center-of-mass) linear collider are discussed elsewhere.[11]

2. MULTIBUNCH BEAM DYNAMICS

We assume an equal charge of N electrons in each bunch and uniform spacing ℓ between adjacent bunches; the bunch spacing ℓ is of course an integral number of RF wavelengths. We use the smooth-focusing approximation $k_n(s) = 1/\beta_n(s)$ for the focusing function of bunch n, where $\beta_n(s)$ is an average betatron function.

The transverse dipole wake function is a sum of modes of the following form:

$$W_\perp(z) = \frac{1}{pa^2} \sum_m \frac{2L_m}{K_m} \sin(K_m z) \exp(-\frac{K_m z}{2Q_m}) \quad , \tag{2.1}$$

where

$$\begin{aligned} z &= \text{distance behind exciting bunch} \\ K_m = \tfrac{\omega_m}{c} &= \text{wavenumber of mode } m \\ Q_m &= \text{quality factor of mode } m \\ L_m &= \text{loss factor of mode } m \quad \text{[V/Coul/cell]} \\ p &= \text{cell length} \\ a &= \text{iris radius} \quad . \end{aligned}$$

The units of $W_\perp(z)$ are V/Coul/m^2, and $W_\perp(z)$ is to be multiplied by the charge and transverse displacement of the exciting bunch to get the wake field a distance z behind that bunch.

The bunches are considered to be point macroparticles. Single bunch beam blow-up is a separate question and can be dealt with using different techniques. It is controlled by opening the irises of the structure, by short bunch lengths, and by using "BNS damping" to compensate the wake effects.[12,13] In this paper, we shall only be concerned with the bunch-to-bunch wake fields.

2.1. Multiple Bunches in the Effective–Length Approximation

The standard treatment of beam breakup using two macroparticles[14,15] starts from the equations of motion:

$$x_1'' + k_1^2 x_1 = 0 \tag{2.2}$$

$$x_2'' + k_2^2 x_2 = \frac{Ne^2 W_\perp(\ell)}{E} x_1 \quad . \tag{2.3}$$

Here the x_1 and x_2 are the transverse displacements of the two bunches (assumed to be in a single plane), E is the energy of the electrons in the bunches, and $W_\perp(\ell)$ is the transverse dipole wake function at the second bunch due to the first bunch. Primes denote derivatives with respect to longitudinal distance s.

Suppose $k_1 = k_2 = k$. Then if $x_1(s) = a_1 e^{iks}$ and $x_2(0) = a_1$, the solution $x_2(s)$ for the second bunch satisfies

$$\frac{x_2 - x_1}{a_1} = \frac{Ne^2 W_\perp(\ell)}{2ikE} s e^{iks} \quad . \tag{2.4}$$

Note the linear growth of the envelope of the difference $x_2 - x_1$ with longitudinal distance s. If $k_1 = k$ and $k_2 = k + \Delta k$, with $\Delta k \ll k$, then:

$$\frac{x_2 - x_1}{a_1} = (1 - \frac{Ne^2 W_\perp(\ell)}{2Ek\Delta k})\, 2i \,\sin(\frac{\Delta ks}{2})\, e^{i(k+\frac{\Delta k}{2})s} \quad . \tag{2.5}$$

In this case, the envelope beats with wavelength $4\pi/\Delta k$ instead of growing linearly. If the coefficient in front is made zero by the proper choice of Δk, then there is no growth of the transverse amplitude of the second bunch.

In this approach, acceleration has not been taken explicitly into account; the energy E of the bunches has been assumed constant. However, we may interpret E and the k_n to be the energy and the focusing functions at the beginning of the linac, and s to be not the true distance along the accelerator, but rather an "effective distance". The effective distance is just $\psi(s)/k_0$, where $\psi(s)$ is the phase advance in the actual distance s along the linac and k_0 is the focusing function at the beginning of the linac. The focusing is assumed to vary as

$$k(s) = \left(\frac{\gamma_0}{\gamma(s)}\right)^{1/2} k_0 \tag{2.6}$$

and the acceleration to be linear: $\gamma = \gamma_0 + Gs$ with G a constant. Thus the effective distance is

$$\begin{aligned} s_{eff} &= \frac{\psi(s)}{k_0} = \frac{1}{k_0}\int_0^s k(s)ds \\ &= \frac{2\gamma_0^{1/2}}{G}\left[\gamma^{1/2} - \gamma_0^{1/2}\right] \quad , \end{aligned} \tag{2.7}$$

and if $\gamma(L) \gg \gamma_0$ at $s = L$, the end of the linac, the effective length of the linac is approximately

$$L_{eff} = 2\left(\frac{\gamma_0}{\gamma}\right)^{1/2} L \quad . \tag{2.8}$$

Before treating multiple bunches with adiabatic acceleration, we note that an approach similar to the above may be taken in the case of more than two bunches.

The equation of motion for the transverse displacement of bunch n ($n > 1$) is:

$$x_n'' + k_n^2 x_n = f_n(s) \quad , \tag{2.9}$$

where the driving term is:

$$f_n(s) = \frac{Ne^2}{E} \sum_{j=1}^{n-1} W_\perp((n-j)\ell) x_j(s) \quad . \tag{2.10}$$

We look for a solution of the form $x_n(s) = a_n(s) e^{ik_n s}$, which leads to

$$a_n'' + 2ik_n a_n' = f_n(s) e^{-ik_n s} \quad . \tag{2.11}$$

Assuming the variation of a_n with s is sufficiently slow, we may neglect the a_n'' term. Solving for a_n then yields as the solution for x_n:

$$x_n(s) = \left[x_n(0) + \frac{Ne^2}{2iEk_n} \int_0^s e^{-ik_n s'} \sum_{j=1}^{n-1} W_\perp((n-j)\ell) x_j(s') ds' \right] e^{ik_n s} \quad . \tag{2.12}$$

Again, if there is acceleration we interpret s as the effective length, E as the initial energy, and the k_n as the initial focusing functions. Then the result is essentially equivalent to that in the next section, except for missing adiabatic damping factors.

2.2. Multiple Bunches with Adiabatic Acceleration

Taking acceleration into account, the equation of motion for x_n is:

$$\gamma(s) x_n'' + \gamma'(s) x_n' + \gamma(s) k^2(s) x_n = F_n(s) \quad , \tag{2.13}$$

where we now define

$$F_n(s) \equiv \frac{Ne^2}{mc^2} \sum_{j=1}^{n-1} W_\perp((n-j)\ell) x_j(s) \quad . \tag{2.14}$$

Here m is the rest mass of the electron. The WKB solutions of the homogeneous equation are

$$\begin{aligned} x_n^{\pm}(s) &= x_n^{\pm}(0)\left[\frac{\gamma_0 k_n(0)}{\gamma(s)k_n(s)}\right]^{1/2} \exp\left[\pm i\int_0^s k_n(s')ds'\right] \\ &= x_n^{\pm}(0)\left[\frac{\gamma_0}{\gamma(s)}\right]^{1/4} \exp\left[\pm i\int_0^s k_n(s')ds'\right] \quad , \end{aligned} \tag{2.15}$$

where we have used Eq. (2.6) for the variation of k_n with γ. Now look for a solution to the inhomogeneous Eq. (2.13) of the form:

$$x(s) = u_+(s)x_+(s) + u_-(s)x_-(s) \tag{2.16}$$

(suppressing subscript n for the moment). Without loss of generality we may assume that:

$$u'_+x_+ + u'_-x_- = 0 \quad . \tag{2.17}$$

Substituting into the inhomogeneous equation we obtain

$$\gamma(u'_+x'_+ + u'_-x'_-) = F(s) \quad . \tag{2.18}$$

Thus, we have two simultaneous equations [Eqs. (2.17) and (2.18)] for u'_+ and u'_-, which we may solve and integrate to obtain:

$$\begin{aligned} u_+(s) &= u_+(0) + \int_0^s \frac{Fx_-}{\gamma(x_-x'_+ - x_+x'_-)}ds' \\ u_-(s) &= u_-(0) + \int_0^s \frac{Fx_+}{\gamma(x_-x'_+ - x_+x'_-)}ds' \quad . \end{aligned} \tag{2.19}$$

It is easy to show that the denominator $\gamma(x_- x'_+ - x_+ x'_-) = 2i$. Thus the general solution to the inhomogeneous equation for bunch n is

$$x_n(s) = a_n^+ x_n^+(s) + a_n^- x_n^-(s) + \int_0^s G_n(s,s')F_n(s')ds' \quad , \tag{2.20}$$

where a_n^+ and a_n^- are arbitrary constants. The Green function is given by:

$$G_n(s,s') = [\gamma(s)\gamma(s')k_n(s)k_n(s')]^{-1/2} \sin\psi_n(s,s') \quad , \tag{2.21}$$

where:

$$\psi_n(s,s') \equiv \int_{s'}^{s} k_n(s'')ds'' \tag{2.22}$$

is the phase advance for bunch n. Let us take the "positive phase" WKB solution as the motion for the first bunch,

$$x_1(s) = x_1(0)\left(\frac{\gamma_0}{\gamma(s)}\right)^{1/4} \exp\left[\psi_1(s,0)\right] \quad , \tag{2.23}$$

and assume $a_n^- = 0$ for all $n > 1$. Then

$$\begin{aligned} x_n(s) &= x_n(0)\left(\frac{\gamma_0}{\gamma(s)}\right)^{1/4} \exp\left[i\psi_n(s,0)\right] \quad + \\ &\qquad [\gamma(s)k_n(s)]^{-1/2} \int_0^s [\gamma(s')k_n(s')]^{-1/2} \sin\psi_n(s,s')F_n(s')ds' \\ &= x_n(0)\left(\frac{\gamma_0}{\gamma(s)}\right)^{1/4} \exp\left[i\psi_n(s,0)\right] \quad + \\ &\qquad \frac{Ne^2}{\gamma_0 mc^2 k_n(0)}\left(\frac{\gamma_0}{\gamma(s)}\right)^{1/4} \int_0^s \left(\frac{\gamma_0}{\gamma(s')}\right)^{1/4} \sin\psi_n(s,s') \\ &\qquad \times \sum_{j=1}^{n-1} W_\perp((n-j)\ell)x_j(s')ds' \quad . \end{aligned} \tag{2.24}$$

This may be written in the following form for comparison with our previous results and for convenience of numerical computation:

$$
\begin{aligned}
x_n(s) = x_n(0)\left(\frac{\gamma_0}{\gamma(s)}\right)^{1/4} \exp\left[i\psi_n(s,0)\right] \quad + \\
\frac{Ne^2}{2i\gamma_0 mc^2 k_n(0)}\left[\int_0^s \left(\frac{\gamma_0}{\gamma(s')}\right)^{1/4} \exp[-i\psi_n(s',0)]\right. \\
\left.\times \sum_{j=1}^{n-1} W_\perp((n-j)\ell)x_j(s')ds'\right]\left(\frac{\gamma_0}{\gamma(s)}\right)^{1/4} \exp[+i\psi_n(s,0)] \quad + \\
\frac{Ne^2}{2i\gamma_0 mc^2 k_n(0)}\left[\int_0^s \left(\frac{\gamma_0}{\gamma(s')}\right)^{1/4} \exp[+i\psi_n(s',0)]\right. \\
\left.\times \sum_{j=1}^{n-1} W_\perp((n-j)\ell)x_j(s')ds'\right]\left(\frac{\gamma_0}{\gamma(s)}\right)^{1/4} \exp[-i\psi_n(s,0)] \quad .
\end{aligned}
\tag{2.25}
$$

The last term is negligible since the integrand is rapidly oscillating (and indeed we did not even include it when we derived the effective length approximation). The first two terms are equivalent (up to adiabatic damping factors) to the effective length result [Eq. (2.12)] with variables interpreted as discussed there.

3. MINIMIZING THE TRANSVERSE WAKE EFFECTS

We now turn to the study of ways to prevent beam blowup, by damping the transverse wake, by minimizing the wake effects by placing the bunches close to nodes of the wake field, and by varying the focusing to partly cancel the wake force at the bunches.

3.1. Damping the Transverse Dipole Modes

As noted earlier, the modes of the transverse dipole wake can be strongly damped by the use of axial slots through the irises of the RF structure coupled to radial waveguides. The Q of the fundamental transverse mode can be made as low as about 10 in this way, without significant adverse effect on the longitudinal accelerating mode.[2] The $Q's$ obtained for the higher order modes should be at least as low as the Q of the fundamental. For simplicity in the numerical computations, we will take the $Q's$ to be the same for all modes.

3.2. Tuning the Frequency of the Fundamental Transverse Mode

The transverse dipole wake for the accelerating structure considered here is strongly dominated by its fundamental mode and has zero crossings that are approximately equally spaced. Figure 1 shows a typical dipole wake which was computed using the program TRANSVRS[16] for a disk-loaded structure designed to operate at 17.1 GHz. The structure has a cell length of 5.83 mm, internal cell radius of 7.47 mm, and a relatively large iris radius of 3.47 mm. This structure has no slots to damp the transverse modes. However, assuming that such slots damp higher order transverse modes at least as much as they damp the fundamental transverse mode and that this fundamental mode dominates the others, the slotted structures will also have periodic wake zero crossings.

Therefore it is possible to place all the bunches in a train near zero crossings of the wake field, if the ratio of the frequency of the fundamental dipole mode to the frequency of the accelerating RF is appropriately tuned. The condition that this be so is just

$$\frac{1}{2} n\lambda_{W_\perp} = m\lambda_{rf} = \ell \quad , \tag{3.1}$$

where ℓ is the bunch spacing, m and n are integers, and λ_{rf} and $\lambda_{W_\perp}$ are the wavelengths of the RF and the fundamental dipole wake mode.

3.3. Bunch-to-Bunch Variation of Transverse Focusing

By the use of a system of time-varying quadrupoles in addition to the main system of quadrupoles, we could introduce a small spread in the focusing functions k_n of the bunches. If the focusing increment at a given bunch is chosen appropriately, one can at least partially cancel the wake force due to the preceding bunches [*cf.* Eq. (2.5)]. It is not practical to use this method by itself to control the wake field effects of multiple bunches because, for the parameter regimes we will be considering, the required spread in the values of the k_n would be too large. However, in some cases it may be a useful adjunct to other methods.

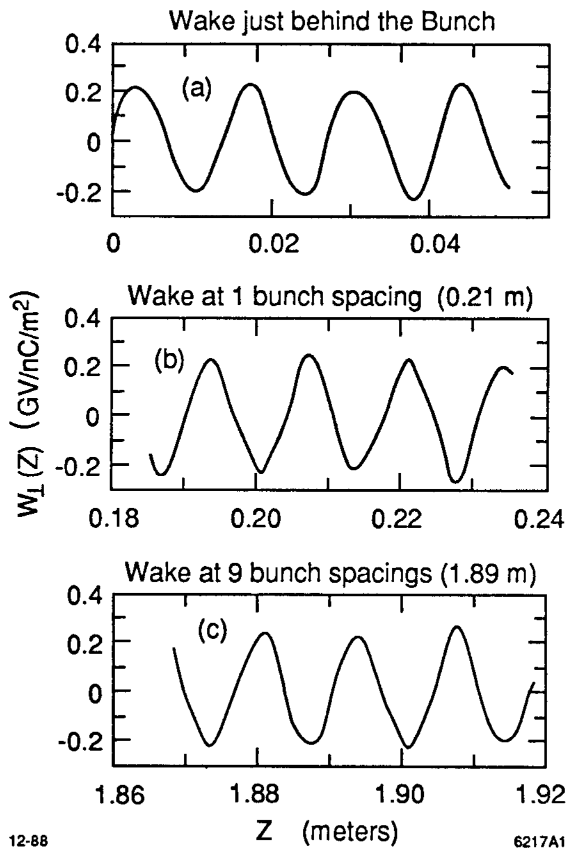

Fig. 1. A typical dipole wake for a disk-loaded structure designed to operate at 17.1 GHz. Ninety modes have been included. The wavenumber of the fundamental mode is about 462 m^{-1} and the zero crossings are nearly equally spaced at half the corresponding wavelength. (a) shows the wake immediately behind the first bunch, (b) is centered at the second bunch, and (c) is centered at nine bunch spacings. The bunch spacing is 21.0 cm (twelve RF wavelengths at 17.1 GHz).

4. RESULTS FOR MAIN LINACS OF A TEV COLLIDER DESIGN

We have in effect a three-dimensional parameter space to explore, while holding all other parameters fixed. The three parameters to be varied are:

1. The Q value of the modes of the transverse dipole wake (taken to be the same for all the modes).
2. The frequency of the fundamental transverse dipole mode (in our computations, the frequencies of the other modes will be assumed unchanged).

3. The total spread in the values of the focusing function (which can be distributed linearly or in some other way among the bunches).

For illustration, we consider a main linac accelerating frequency of 17.1 GHz.

Table 1: Parameters for Main Linacs at 17.1 GHz	
Number of bunches	10
Number of particles per bunch	1.67×10^{10}
Bunch spacing ℓ	21.0 cm
Initial energy of linac	18 GeV
Final energy of linac	500 GeV
Linac length	3000 m
Initial beta function	3.2 m ($k_0 = 0.3125\ m^{-1}$)

The parameter set used is shown in Table 1. Each linac accelerates 10 bunches per RF fill, to an energy of 0.5 TeV. The spacing of 12 RF wavelengths between bunches (at the assumed RF frequency) is chosen in order to match the energy extracted by the bunch train to the energy input from the RF. This gives very nearly the same acceleration for every bunch in the bunch train.[1]

Let us first examine the effectiveness of lowering the $Q's$ and tuning the frequency of the fundamental transverse dipole mode. The RF wavelength at 17.1 GHz is 1.75 cms, and the wavelength of the fundamental mode of the unmodified transverse dipole wake (Fig. 1) is 1.36 cms. If the frequency of the fundamental mode is shifted slightly, so that its wavelength is 1.31 cms, then Eq. (3.1) is satisfied with n=32, and we have

$$\frac{\lambda_{rf}}{\lambda_{W_\perp}} = \frac{4}{3} \quad . \tag{4.1}$$

When this relation is satisfied, the frequency of the fundamental transverse mode is 477.85 m^{-1}, which we shall denote by K_{W_0}. In Fig. 2, we show "tuning curves" of the maximum transverse amplitude x_{max} in the bunch train as a function of the frequency of the fundamental transverse dipole mode, for values of Q = 30, 40, 50, and 60. The value of x_{max} is the maximum of the amplitudes reached by all bunches as they travel down the linac, normalized by dividing out the adiabatic damping factor $(\gamma_0/\gamma)^{1/4}$. The central frequency, at which $\lambda_{rf}/\lambda_{W_\perp} = 4/3$, is

477.85 m^{-1}. The range about the central frequency shown in the figure is only $\pm 0.1\%$. Thus the tolerances on tuning the frequency of the dipole mode are rather tight. If we take $x_{max} \leq 2$ as a figure of merit, then for $Q = 60$ we would have to tune to within about 0.04% of the central frequency K_{W_0}.

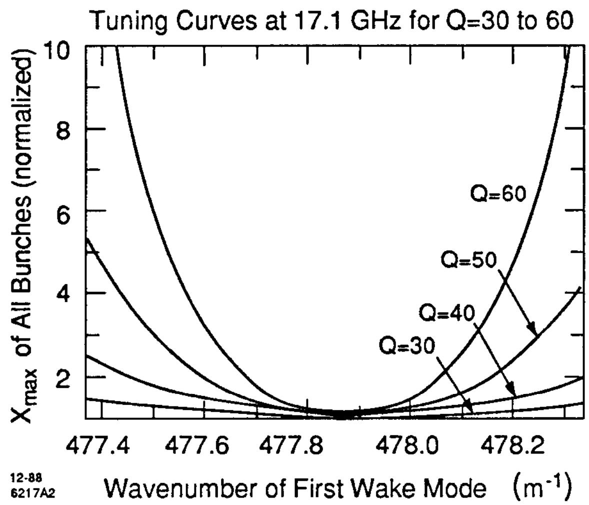

Fig. 2. Maximum transverse amplitude x_{max} (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of Q = 30, 40, 50, and 60, at 17.1 GHz accelerating frequency. The central frequency, where $\lambda_{rf}/\lambda_{W_\perp} = 4/3$, is 477.85 m^{-1}. The spread shown about K_{W_0} is $\pm$ 0.1%.

Note that at M bunch spacings behind a bunch, the wake field has damped by a factor of about

$$\exp\left(-\frac{K_{W_0}\ell M}{2Q}\right) \approx \exp\left(-\frac{50M}{Q}\right) \quad , \tag{4.2}$$

and so a given bunch can feel a significant wake field from several bunches ahead of it, unless it is near a node of these wakes and/or the value of Q is well below 50. For smaller $Q's$, there will be very little interaction beyond immediately adjacent bunches. Figure 3 shows tuning curves for the smaller values $Q = 15$, 20, and 25. The spread around K_{W_0} in this figure is $\pm 1\%$, and as can be seen from the figure, for $Q = 15$, we have $x_{max} < 2$ for frequencies within this range.

Of course, as Q is lowered, the frequency of the fundamental transverse mode becomes less sharply defined; the full width at half-maximum of the resonance

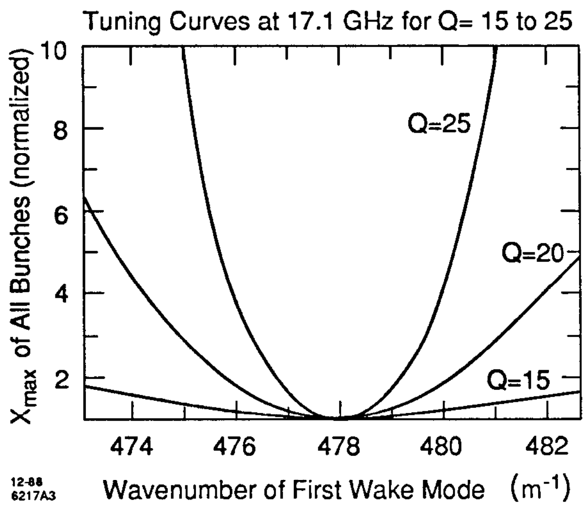

Fig. 3. Maximum transverse amplitude x_{max} (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of $Q = 15$, 20, and 25, at 17.1 GHz accelerating frequency. The central frequency, where $\lambda_{rf}/\lambda_{W_\perp} = 4/3$, is 477.85 m^{-1}. The spread shown about K_{W_0} is $\pm$ 1%.

around the central frequency K_{W_0} is $\Gamma \equiv \frac{K_{W_0}}{Q}$ (and the central frequency is shifted slightly from that of the undamped mode). So it is also of interest to compare the ratio R of the tuning tolerance for a given Q to the full width Γ of the resonance at that Q:

$$R \equiv \frac{\Delta K_{W_0}}{\Gamma} \quad . \tag{4.3}$$

In Table 2, we show the full-width tuning tolerance ΔK_{W_0} for the criterion $x_{max} \leq 2$, the full-width Γ of the resonance peak, the tuning tolerance expressed as a percentage of the undamped central frequency, and the ratio R. The parameters used and the values of Q tabulated are those used in Figs. 2 and 3. The lower values of Q, say, up to 30 or so, seem to be the most desirable in that the tolerance on tuning is at least 10% of the bandwidth of the resonance; this should be straightforward to do. Thus, since these $Q's$ have already been achieved in models, it appears that the beam blowup can be controlled without resorting to the third method, namely the introduction of a spread in focusing over the different bunches. However, for illustration we show in Fig. 4 the same case as in Fig. 3, except that a nonzero spread in the focusing functions has been introduced. From Eq. (2.5), we see that there will be exact cancellation of the part of the wake at a

Table 2: Tuning parameters for the fundamental transverse dipole mode for TLC main linacs				
Q	$\Delta K_{W_0}(\mathrm{m}^{-1})$	$\Gamma = \frac{K_{W_0}}{Q}(\mathrm{m}^{-1})$	$\frac{\Delta K_{W_0}}{K_{W_0}}$	$\frac{\Delta K_{W_0}}{\Gamma}$
15	>11.0	31.9	>2.3%	>34%
20	4.37	23.9	0.91%	18%
25	2.56	19.1	0.54%	13%
30	1.60	15.9	0.33%	10%
40	0.87	11.9	0.18%	7.3%
50	0.57	9.56	0.12%	6.0%
60	0.38	7.96	0.08%	4.8%

given bunch that is due to the immediately preceding bunch when

$$\Delta k_{adj} = \frac{Ne^2 W_\perp(\ell)}{2Ek_0} \quad . \tag{4.4}$$

Here Δk_{adj} is the difference between the focusing functions of adjacent bunches. Multiplying this by 9 gives the total spread Δk_{tot} over all 10 bunches. For $Q = 20$ and for a frequency of the fundamental transverse mode 0.6% above K_{W_0}, we have $\Delta k_{tot} = 6.24\%$. We show the results for this value of Δk_{tot} in Fig. 4(b). However, the phase advance difference due to a focusing spread can introduce complications. In our example, the total phase advance in the main linacs is about 60π. Thus for a focusing spread of 1% or so, the spread in phase advance is becoming significant compared to 2π. In such a case, the amplitude of betatron oscillations must be smaller than the transverse bunch dimensions or there must be position control of individual bunches at the end of the linac, to keep the bunches from missing each other at the interaction point. Figure 4(a) shows the case of a total spread of 1% linearly distributed over the bunches. We see that for the smaller value of Δk_{tot} shown in Fig. 4(a), there is no appreciable increase in the tuning tolerance compared to Fig. 3, and so this method of alleviating the wake field effects does not seem as useful as the other two methods we have investigated.

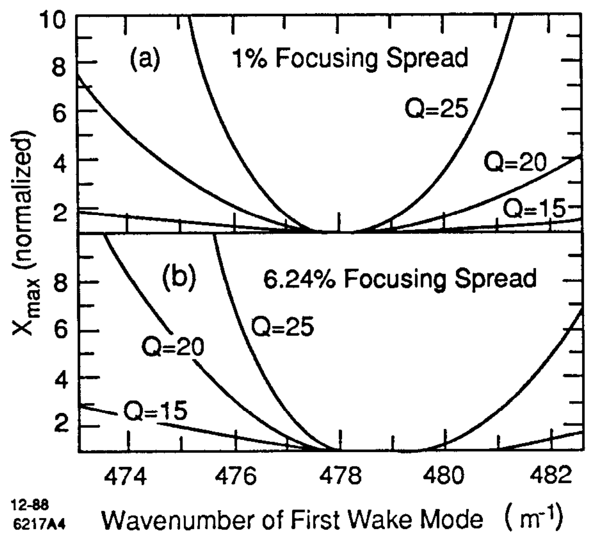

Fig. 4. Maximum transverse amplitude x_{max} (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of Q = 15, 20, and 25, with nonzero, linearly distributed spread in the focusing functions over the bunches. In (a), $\Delta k/k_0 = 1\%$, and in (b), $\Delta k/k_0 = 6.24\%$.

5. CONCLUSIONS AND ACKNOWLEDGMENTS

We have demonstrated that it is possible to control the multibunch beam breakup due to the transverse dipole wake field by using an experimentally realizable combination of (1) damping the wake field (lowering the Q's of the modes), and (2) tuning the frequency of the fundamental mode so that the bunches ride near zero crossings of the wake field. The fundamental transverse dipole wake mode needs to be damped to a Q below about 30, and experiments done by Palmer[2] indicate that this should be feasible.

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