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THE TAU MISSING DECAY MODES PROBLEM AND LIMITS ON A SECOND TAU NEUTRINO*

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ABSTRACT

This paper quantitatively considers the hypothesis that the tau missing mode problem might be explained by the existence of a second and massive tau neutrino, with mass less than the tau mass. I find that the tau missing mode problem cannot be explained when the hypothesis is formulated according to conventional weak interaction concepts.

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I. INTRODUCTION

The problem of missing decay modes[1,2,3] in τ decays has led to a new examination of our knowledge and understanding of the τ. Experiments show[1,2,3] that the τ topological branching fraction to 1–charged particle is $B_1 = (86.6 \pm 0.3)\%$, but a combination of conventional theory[2] and measurements[1,3] of the branching fractions for individual decay modes does not explain this value of B_1; about 5% is missing.

If the size of the experimental errors on the measured branching fractions is correct, and they appear to be correct,[4] then the cause of the discrepancy must be elsewhere: a widespread systematic error or bias in the experiments; a deviation from conventional elementary particle theory; a contamination from another process; or unknown τ decay modes. Most of these possible causes must produce the discrepancy by contributing to the measurement of B_1, but not contributing as much to the measurement of the individual branching fractions.

In odd moments I had thought about the possibility that the discrepancy could be caused by the existence of a second and massive τ neutrino,[5] N_τ, that neutrino having a mass, m_{N_τ}, less than, but close to, the τ mass. As described qualitatively in Sec. III, the τ decays to such a neutrino might not be counted correctly in most experiments when individual branching fractions are measured. In this paper I finally undertake a quantitative study of this idea. Using average values of the τ branching fractions and the τ lifetime, I reach two conclusions.

(*a*) The missing mode discrepancy cannot be explained by the existence of an N_τ with the properties given in Section III.

(*b*) However, τ decay and lifetime data do not exclude an N_τ when $m_{N_\tau} \gtrsim 1.25$ GeV/c^2.

In Sec. II I summarize the data and arguments leading to the decay mode problem. The second τ neutrino hypothesis is expounded in Sec. III, and an analysis is presented in Sec. IV. Discussion and conclusions are in Sec. V.

I have ignored the astrophysical and cosmological arguments which limit the existence of massive neutrinos. The τ decay mode problem is so perplexing, it is worth using direct data to explore possible solutions.

II. MISSING MODES PROBLEM

The τ missing mode problem arises when three sets of observations are compared:

(*i*) The average measured branching fractions to 1–, 3– and 5–charged particles are $B_1 = (86.6 \pm 0.3)\%$, $B_3 = (13.3 \pm 0.3)\%$, and $B_5 = (0.11 \pm 0.03)\%$,

(*ii*) Much of B_1 comes from the repeatedly measured modes: $e^- \bar{\nu}_e \nu_\tau$, $\mu^- \bar{\nu}_\mu \nu_\tau$, $\pi^- \nu_\tau$, $\rho^- \nu_\tau$, $K^- \nu_\tau$ and $K^{*-} \nu_\tau$; and the recently measured mode $\pi^- 2\pi^\circ \nu_\tau$. The sum of the average measured branching fractions of the first six modes is $B_{e\mu\pi\rho K} = (71.3 \pm 1.3)\%$, the average measured branching fraction[6] for $\pi^- 2\pi^\circ \nu_\tau$ is $B_{\pi 2\pi^\circ} = (7.5 \pm 0.9)\%$.

(*iii*) In conventional τ decay theory, the other modes contributing to B_1 have at least two neutral mesons: $\pi^- \pi^\circ \eta \nu_\tau$, $\pi^- 3\pi^\circ \nu_\tau$, $\pi^- 2\pi^\circ \eta \nu_\tau$, and so forth. The sum of these branching fractions, called $B_{1,multneut \neq 2\pi^\circ}$ is poorly known from direct measurement. Our best knowledge comes from the limits[2,3] derived from conventional theory and other data,[6] specifically $B_{1,multneut \neq 2\pi^\circ} \leq 3.6\%$.

Setting $\delta B_1 = B_1 - [B_{e\mu\pi\rho K} + B_{\pi 2\pi^\circ} + B_{1,multneut\neq 2\pi^\circ}]$, observations (i), (ii) and (iii) lead to

$$\delta B_1 > (4.2 \pm 1.6)\% \ . \tag{1a}$$

This discrepancy increases if conservation of strong isospin is used to set $B_{\pi 2\pi^\circ} \leq B_{\pi^-\pi^+\pi^-} = (6.7 \pm 0.4)$. Then

$$\delta B_1 > (5.0 \pm 1.4)\% \ . \tag{1b}$$

A recent study[4] of the branching fraction measurement errors shows that the errors on the whole are reasonable, assuming the same systematic error does not occur in most experiments. No such error has been found, nor has any other explanation been found for δB_1.

III. HYPOTHESIS

Consider the possibility that the missing 1–charged particle decay modes might be explained by the existence of a second, stable and massive tau neutrino, N_τ, with mass m_{N_τ} less than, but close to the τ mass. The width, Γ_N, for τ decaying to N_τ would be dominated by the 1–charged particle decay modes.

$$\Gamma_{Ne} : \tau^- \rightarrow N_\tau + e^- + \bar{\nu}_e \ , \tag{2a}$$

$$\Gamma_{N\mu} : \tau^- \rightarrow N_\tau + \mu^- + \bar{\nu}_\mu \ , \tag{2b}$$

$$\Gamma_{N\pi} : \tau^- \rightarrow N_\tau + \pi^- \ , \tag{2c}$$

$$\Gamma_{N\rho} : \tau^- \rightarrow N_\tau + \rho^- \ . \tag{2d}$$

I examine a simple form of this two-neutrino hypothesis in which each neutrino has separately the conventional, V–A, weak interaction coupling to the τ.

Each coupling constant is assumed to have the universal strength of the $e - \nu_e$ and $\mu - \nu_\mu$ coupling.

The total decay width is

$$\Gamma = \Gamma_\nu + \Gamma_N , \qquad (3)$$

where Γ_ν is for modes with

$$\tau^- \rightarrow \nu_\tau + \text{ other particles} ,$$

and Γ_N is for

$$\tau^- \rightarrow N_\tau + \text{ other particles} .$$

It is useful to express the various decay widths in ratio to $\Gamma_{\nu e}$, the decay width for

$$\tau^- \rightarrow \nu_\tau + e^- + \bar{\nu}_e .$$

Thus

$$\frac{\Gamma_\nu}{\Gamma_{\nu e}} \approx 5.6 . \qquad (4)$$

Figure 1 gives[7] Γ_N, and its main components from Eq. 1, in terms of $\Gamma_{\nu e}$. The small difference between m_τ and m_{N_τ} causes Γ_N to be much smaller than Γ_ν.

The measured τ lifetime sets an upper limit on Γ_N and hence a lower limit on m_{N_τ} of about 1.0 GeV/c^2 and I take $m_{N_\tau} \geq 1.0$ GeV/c^2, (see Appendix A). In Sec. V I discuss relaxing the coupling constant assumption for N_τ.

The other important effect of the small $m_\tau - m_{N_\tau}$ difference is the shrinking of the energy spectrum of the charged particles produced in the decay. This is illustrated in Fig. 2 for $E_{tot} = 29$ GeV, the total energy of PEP experiments.

I now come to the crucial points:

(*a*) The event selection criteria used to measure B_1 are less restrictive in momentum p than those used to measure the individual branching fractions B_e, B_μ, B_π and B_ρ at the energies used at PEP and PETRA. Most experiments required $p \gtrsim 1.5$ to 2.0 GeV/c to identify μ's and π's, and $p \gtrsim 1.0$ to 1.5 GeV/c to identify e's. Thus e's, μ's and π's from τ decays to N_τ could be counted in B_1 measurements when there is no charged particle identification. They would not be fully or correctly counted in B_e, B_μ, B_π or B_ρ measurements.

(*b*) The efficiencies for selecting events in B_1 measurements are usually in the range of 10 to 50%. The efficiencies for selecting events for individual branching fraction measurements may be a small as a few percent. Both efficiencies, particularly the latter, depend on the momentum spectra assumed for the charged particles in the decay; the presence of N_τ changes these assumptions.

(*c*) A consequence of points (a) and (b) could be that some of the branching fractions B_e, B_μ, B_π and B_ρ are underestimated compared to B_1. This could look like missing, 1–prong decay modes.

IV. ANALYSIS

This hypothesis cannot be fully examined using averages of measured values of the branching fractions. The different experiments use different event selection criteria, hence the presence of N_τ would affect differently the final values of the branching fractions. However, the averages of measured values can be used to argue against this hypothesis being the sole explanation of the missing mode problem, and limits can be set on m_{N_τ}. Experiments with broader e, μ and π identification criteria will be able to do a more exact analysis.

This analysis is based on the following assumptions:

(i) The measurement criteria for B_1 are sufficiently loose that the same efficiency holds for decays with N_τ as for decays with ν_τ. This assumption can lead to an overestimation of the δB_1 I will calculate.

(ii) The measurements of B_e, B_μ and B_π require $p_e > 1.0$ GeV/c, $p_\mu > 2.0$ GeV/c and $p_\pi > 2.0$ GeV/c for the events counted. My δB_1 calculation will be an overestimate for experiments with smaller lower limits.

(iii) The measurement of B_ρ does not require π identification. Many experiments identify the ρ by reconstructing the π–π° mass, not requiring π identification. If π identification is required or there is a lower limit on p_π, my calculation of δB_1 is an underestimate.

(iv) The τ energy is 14.5 GeV, the energy used at PEP. My calculation of δB_1 is an overestimate for higher energies.

Let α stand for e, μ or π indicating the charged particle in the decay mode. Let $B_{\nu\alpha}$ or $B_{N\alpha}$ be the true branching fractions for the decays containing ν_τ or N_τ, and let $f_{\nu\alpha}$ and $f_{N\alpha}$ be the corresponding efficiencies. Suppose n τ's are produced in a data sample, the number of observed decays to mode α are

$$n_{\alpha,obs} = (f_{\nu\alpha} B_{\nu\alpha} + f_{N\alpha} B_{N\alpha})\ n\ .$$

Note the definitions

$$B_{\nu\alpha} = \frac{\Gamma_{\nu\alpha}}{\Gamma_\nu + \Gamma_N}\ , \qquad B_{N\alpha} = \frac{\Gamma_{N\alpha}}{\Gamma_\nu + \Gamma_N}\ .$$

The f efficiencies take account of the momentum restrictions on the α as well as many other criteria, such as the solid angle of acceptance and the efficiency for particle identification. Defining

$$\begin{aligned} B_{\alpha,obs} &= \frac{n_{\alpha,obs}}{n} , \\ B_{\alpha,obs} &= f_{\nu\alpha}\, B_{\nu\alpha} + f_{N\alpha}\, B_{N\alpha} . \end{aligned} \tag{5}$$

It is useful to define

$$r_\alpha = \frac{B_{N\alpha}}{B_{\nu\alpha}} ;$$

then the true total branching fraction of the α mode is

$$B_\alpha = B_{\nu\alpha} + B_{N\alpha} = B_{\alpha,obs} \left(\frac{1 + r_\alpha}{f_{\nu\alpha} + r_\alpha\, f_{N\alpha}} \right) . \tag{6}$$

If there were an N_τ, but the experimenter were unaware, the experimenter would derive a branching fraction

$$B_{\alpha,derived} = \frac{B_{\alpha,obs}}{f_{\nu\alpha}} . \tag{7}$$

The difference between B_α and $B_{\alpha,derived}$ gives δB_1 due to the α decay mode under the assumptions listed earlier. Then

$$\delta B_1 = \sum_\alpha \left[B_{\alpha,derived}\, r_\alpha \left(\frac{1 - \frac{f_{N\alpha}}{f_{\nu\alpha}}}{1 + r_\alpha \frac{f_{N\alpha}}{f_{\nu\alpha}}} \right) \right] . \tag{8}$$

Thus δB_1 depends on the product of two factors: r_α and

$$F_\alpha = \frac{1 - \frac{f_{N\alpha}}{f_{\nu\alpha}}}{1 + r_\alpha \frac{f_{N\alpha}}{f_{\nu\alpha}}} . \tag{9}$$

Here r_α is the ratio of $\Gamma_{N\alpha}$ to $\Gamma_{\nu\alpha}$. The factor F_α differs from zero to the degree that the efficiency $f_{N\alpha}$ is less than the efficiency $f_{\nu\alpha}$. Figure 3 gives the ratio $f_{N\alpha}/f_{\nu\alpha}$; Table I gives F_α and r_α. Both extend below $m_{N_\tau} = 1.0$ GeV/c^2 to show the range of values.

As m_{N_τ} increases above 1.0 GeV/c^2, F_α increases and r_α decreases. Their product is always less than about 0.02. This means that the error due to the presence of N_τ in deriving B_α will be about 2% or less. I needed a 5 to 10% effect for the N_τ hypotheses to be worth pursuing, so the idea doesn't work.

Completing the calculation, Table I gives

$$\delta B_{1\alpha} = B_{\alpha,derived}\, r_\alpha\, F_\alpha ,$$

and

$$\delta B_1 = \delta B_{1e} + \delta B_{1\mu} + \delta B_{1\pi} .$$

I use[3]

$$B_{e,derived} = 0.177 ,$$

$$B_{\mu,derived} = 0.177 ,$$

$$B_{\pi,derived} = 0.109 .$$

The largest value of δB_1 is at the lower limit $m_{N_\tau} = 1.0$ GeV/c^2, and is still ten times too small compared to the values of δB_1 given in Eqs. (1).

V. DISCUSSIONS AND CONCLUSIONS

Is there any way to change the hypothesis of Sec. III to increase the calculated value of δB_1? Either F_α or r_α or both must be increased. Consider F_α first.

Changing from $V-A$ coupling to another form will not change F_π because the π always has a flat energy spectrum. Looking back at Fig. 2, F_e and F_μ can be increased by distorting the energy spectra to smaller energies. However, there is no accepted coupling form which does this sufficiently to substantially increase F_e and F_μ. Thus I see no way within conventional ideas to get a large increase in the F_α's.

Of course, r_α can be increased by setting the τ–N_τ coupling constant to be larger than the τ–ν_τ coupling constant. This is an ugly variation and is limited by the discussion of Appendix A. I am reluctant to investigate this variation.

Therefore, my first conclusion is that within the bounds of the hypothesis in Sec. III of a second and massive τ neutrino, and within the limits of the simple analysis in Sec. IV, I cannot explain the τ missing mode problem.

Although I cannot find a believable way to explain the missing modes problem using the N_τ hypothesis, I note that as m_{N_τ} approaches m_τ, it becomes impossible to exclude the existence of an N_τ. Figure 4 gives Γ_N/Γ_ν using $\Gamma_\nu/\Gamma_{\nu e} = 5.6$. Once $\Gamma_N/\Gamma_\nu \lesssim 0.02$, measurement errors would prevent discerning the effect of the N_τ modes on τ branching fractions or τ decay spectra or the τ lifetime. This corresponds to $m_{N_\tau} \gtrsim 1.25$ GeV/c^2. Thus my second conclusion is that at present the existence of such an N_τ cannot be excluded using existing τ data and the hypothesis in Sec. III. Of course, experiments on neutrino pair production can look for an N_τ in the course of counting the number of types of neutrinos. C. Hawkins[8] has suggested a special method of looking for N_τ with m_{N_τ} very close to m_τ.

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TABLE I.

Values of the parameters F_α, r_α, $\delta B_{1\alpha}$ and δB_1 defined in the text. The calculations were carried to more decimal places than are given in the table. δB_1 is also given in percent for direct comparison with Eqs. 1.

m_{N_τ} (GeV/c^2)	0.6	0.8	1.0	1.2	1.4	1.6
F_e	0.008	0.051	0.097	0.175	0.329	0.755
r_e	0.441	0.233	0.096	0.028	0.004	0.000
$r_e F_e$	0.004	0.012	0.009	0.005	0.001	0.000
δB_{1e}	0.001	0.002	0.002	0.001	0.000	0.000
F_μ	0.022	0.104	0.193	0.349	0.605	1.000
r_μ	0.434	0.226	0.090	0.024	0.003	0.000
$r_\mu F_\mu$	0.010	0.024	0.018	0.009	0.002	0.000
$\delta B_{1\mu}$	0.002	0.004	0.003	0.001	0.000	0.000
F_π	0.011	0.025	0.053	0.107	0.231	0.760
r_π	0.694	0.504	0.315	0.156	0.050	0.003
$r_\pi F_\pi$	0.007	0.013	0.017	0.017	0.011	0.003
$\delta B_{1\pi}$	0.001	0.001	0.002	0.002	0.001	0.000
δB_1	0.003	0.008	0.007	0.004	0.002	0.000
δB_1 (%)	0.3	0.8	0.7	0.4	0.2	0.0

Appendix A

N_τ AND THE τ LIFETIME

It is common to compare the measured value of the τ lifetime, τ_τ with a value calculated from B_e or B_μ. Reference 4 uses

$$\tau_\tau \text{ (measured)} = (3.02 \pm 0.09) \times 10^{-13} \text{ s} . \qquad (A1)$$

and calculates

$$\tau_\tau \text{ (predicted)} = (2.87 \pm 0.05) \times 10^{-13} \text{ s} . \qquad (A2)$$

The calculation uses measured values of B_e and B_μ constrained by e–μ–τ universality.

If N_τ exists the predicted liftime is to be calculated from

$$\tau_\tau \text{ (predicted)} = 16.002(\Gamma_{\nu e}/\Gamma) \times 10^{-13} \text{ s} \qquad (A3)$$

where $\Gamma = \Gamma_\nu + \Gamma_N$. Since

$$B_{e,\text{derived}} = \left(1 + r_e \frac{f_{Ne}}{f_{\nu e}}\right)\left(\frac{\Gamma_{\nu e}}{\Gamma}\right) \qquad (A4)$$

then

$$\tau_\tau \text{ (predicted)} = \frac{1.6002 \; B_{e,\text{derived}}}{\left(1 + r_e \frac{f_{Ne}}{f_{\nu e}}\right)} \times 10^{-13} \text{ s} . \qquad (A5)$$

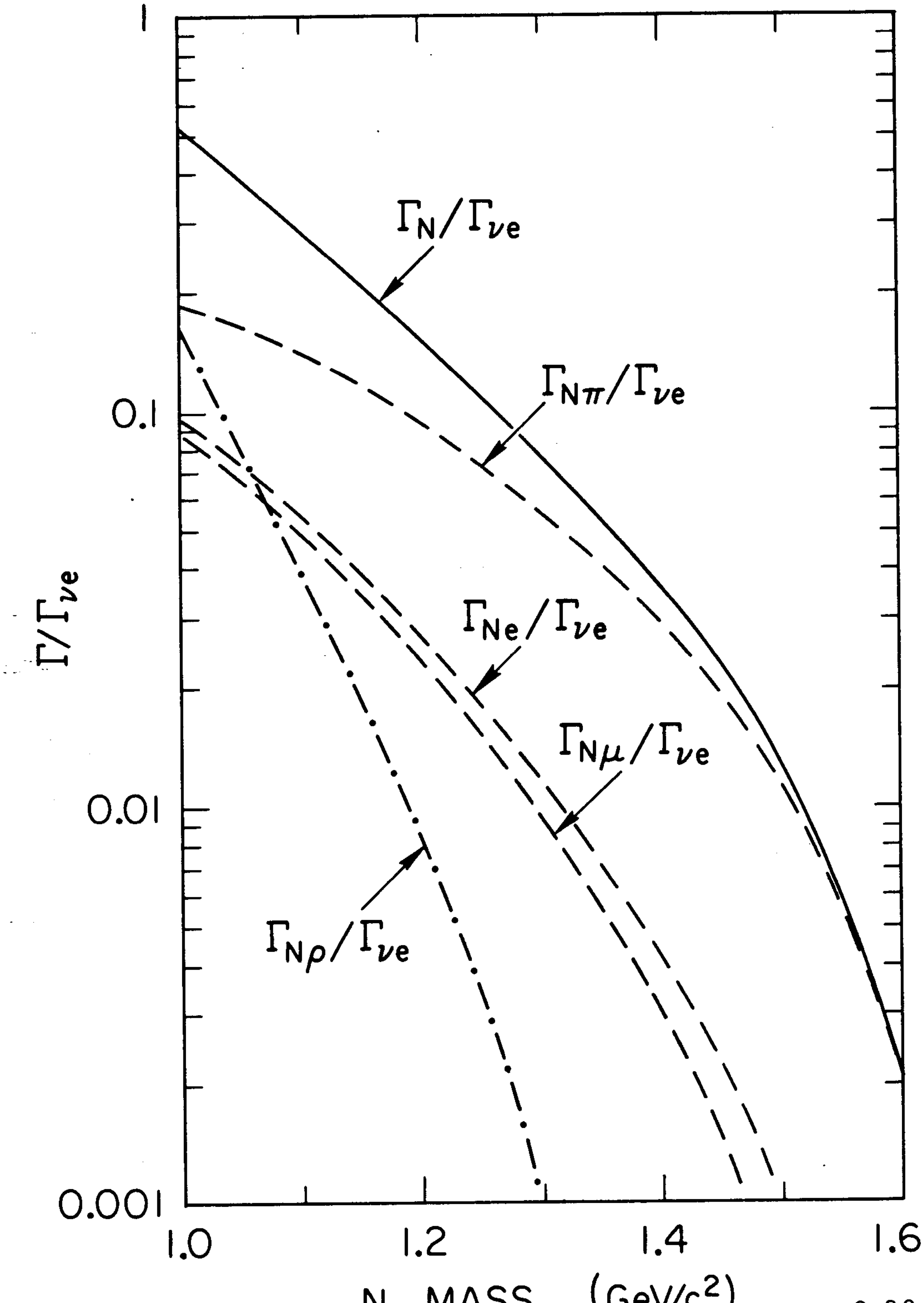
When $m_{N_\tau} = 1.0$ GeV/c^2, the denominator in Eq. (A5) is 1.086; smaller values of m_{N_τ} increase the denominator.

Suppose there is an N_τ with 1.0 GeV/c^2, mass, and set $B_{e,\text{derived}}$ equal to the current average measured value: $B_e = 0.177$. Then Eq. (A5) gives τ_τ (predicted) $= 2.61 \times 10^{-13}$ s, which is four standard deviations below τ_τ (measured), Eq. (A1). Hence I have used 1.0 GeV/c^2 for the lower limit on m_{N_τ} in this paper.

This lower limit argument is weaker if one assumes the current average measured value of B_e is wrong, and should be larger, say 0.19 or 0.195. Then conventional theory requires proportional increases in B_μ, B_π, and B_p. Such increases would reduce δB_1 in Eqs. 1 to zero and there would be no need for a second τ neutrino hypothesis in the first place.

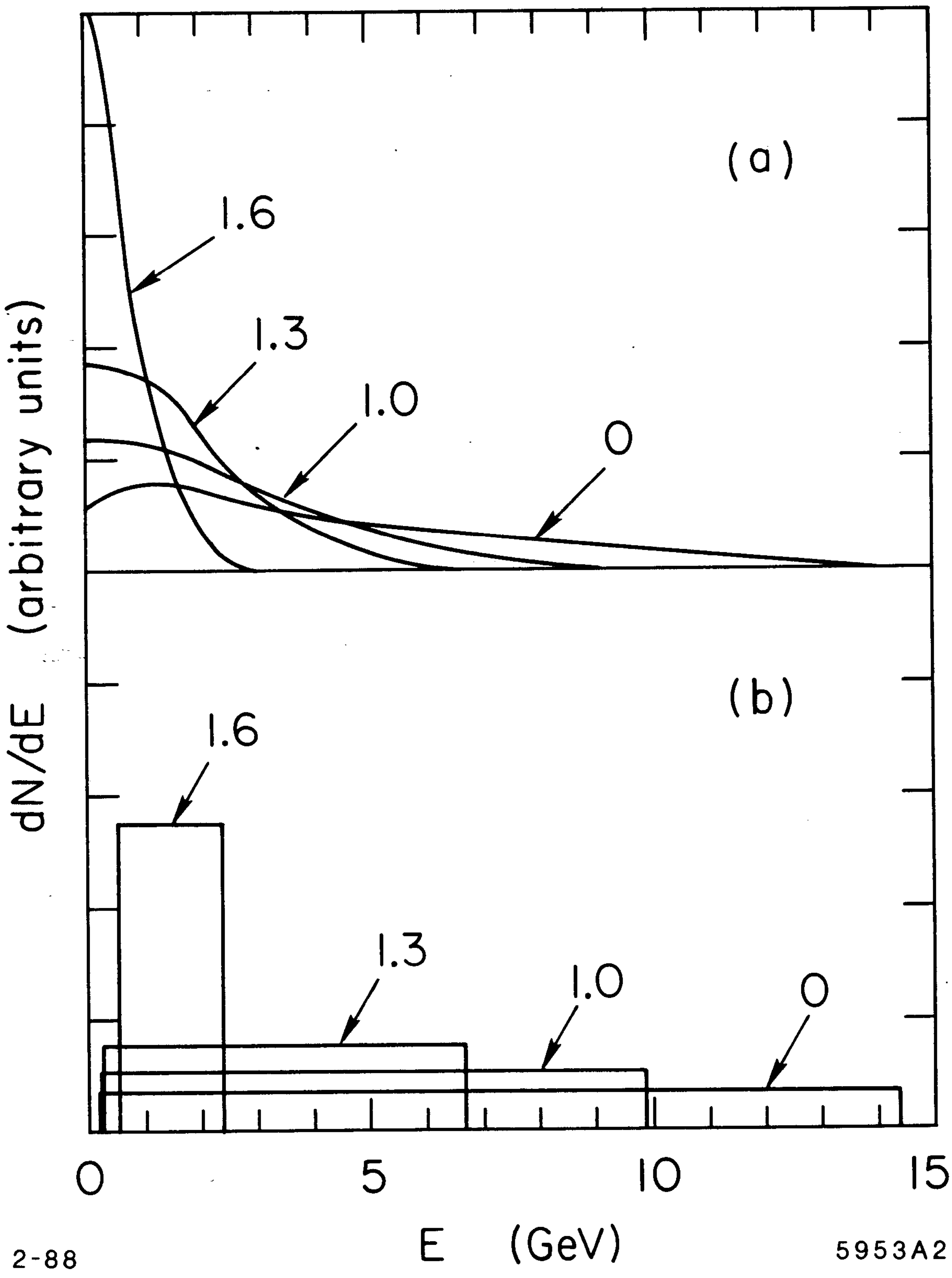
FIGURE CAPTIONS

1. The ratios $\Gamma_N/\Gamma_{\nu e}$, $\Gamma_{N\pi}/\Gamma_{\nu e}$, $\Gamma_{Ne}/\Gamma_{\nu e}$, $\Gamma_{N\mu}/\Gamma_{\nu e}$ and $\Gamma_{N\rho}/\Gamma_{\nu e}$, as a function of the N_τ mass.
2. The energy spectra for $m_{N_\tau} = 0.0$, 1.0, 1.3 and 1.6 GeV/c^2 for (a) the e in $\tau^- \to N_\tau + e^- + \bar{\nu}_e$ and (b) the π in $\tau^- \to N_\tau + \pi^-$. The energy of the τ is 14.5 GeV.
3. The ratio $f_{N\alpha}/f_{\nu\alpha}$.
4. The ratio Γ_N/Γ_ν.



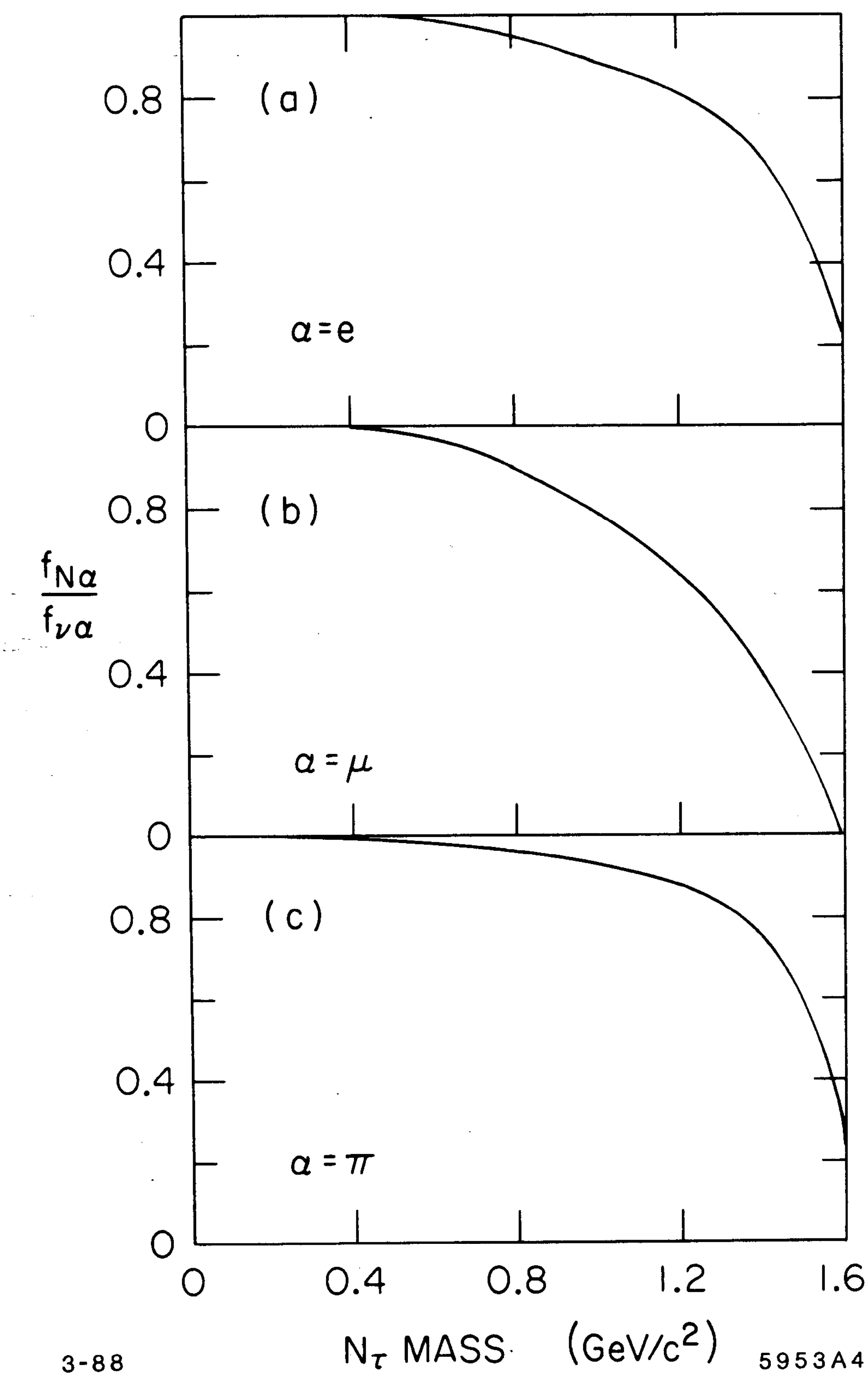
1
0.1
0.01
0.001
1.0
1.2
1.4
1.6
$\Gamma/\Gamma_{\nu e}$
$\Gamma_N/\Gamma_{\nu e}$
$\Gamma_{N\pi}/\Gamma_{\nu e}$
$\Gamma_{Ne}/\Gamma_{\nu e}$
$\Gamma_{N\mu}/\Gamma_{\nu e}$
$\Gamma_{N\rho}/\Gamma_{\nu e}$
N_τ MASS (GeV/c^2)
5953A1
2-88

Fig. 1



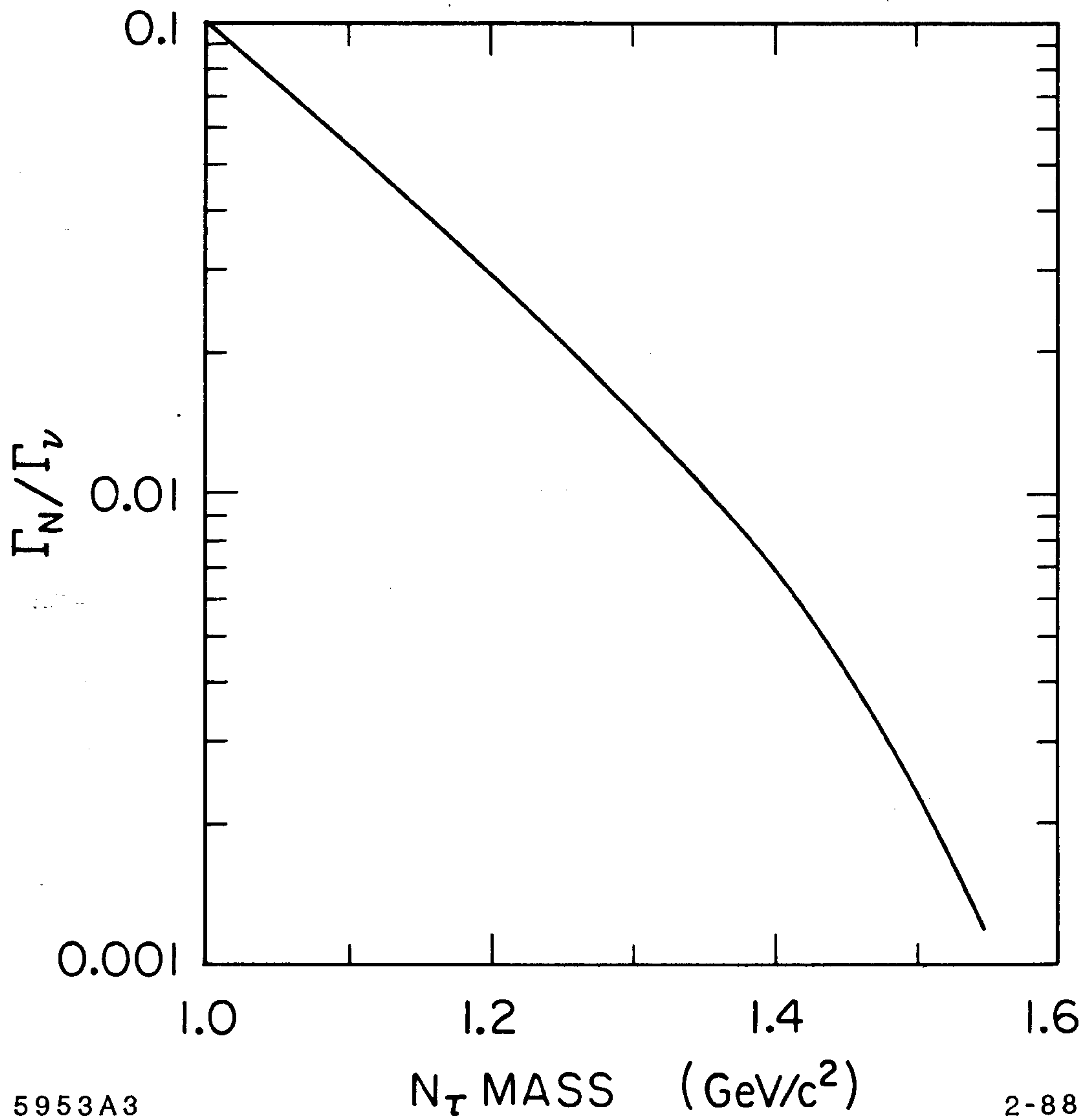
(a)
1.6
1.3
1.0
0
(b)
1.6
1.3
1.0
0
dN/dE (arbitrary units)
0
5
10
15
E (GeV)
2-88
5953A2

Fig. 2



0.8
(a)
0.4
α=e
0
0.8
(b)
$\frac{f_{N\alpha}}{f_{\nu\alpha}}$
0.4
α=μ
0
0.8
(c)
0.4
α=π
0
0
0.4
0.8
1.2
1.6
N_τ MASS (GeV/c^2)
3-88
5953A4

Fig. 3



0.1
0.01
0.001
Γ_N/Γ_ν
1.0
1.2
1.4
1.6
N_τ MASS (GeV/c^2)
5953A3
2-88

Fig. 4