SLAC – PUB – 4527
July 1988
(N)

DISCRETE PHYSICS:*

Practice, Representation and Rules of Correspondence

H. PIERRE NOYES
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

1. INTRODUCTION

The practice of physics cannot get off the ground without essential agreement among the practitioners as to what they are about, how to go about it, and what constitutes progress in their common effort.

We adopt David McGoveran's modeling methodology [1]. This has three critical elements:

(1) an epistemological framework ("E–frame"), which is a set of loosely defined agreements made explicit by those injecting information into the model formulation—Gefwert [2] would call this a *practical understanding* of physics;

(2) a representational framework ("R–frame"), which is an abstract formalism consisting of a set of symbols and a set of rules for manipulation—to formulate such a frame is, for Gefwert, to practice *syntax*;

(3) a procedural framework ("P–frame"), which is an algorithm that serves to establish *rules of correspondence* between the observations agreed on in the E–frame and the symbols of the R–frame. Gefwert would describe this activity as the practice of *semantics*. Through recursion, the P–frame serves to modify the rules of correspondence, the E–frame and the R–frame, until a sufficient level of agreement concerning accuracy is achieved—or the model fails. Kuhn [3] would call such a failure a "crisis," which in the fullness of time could lead to a "paradigm shift."

Note that we halt the infinite regress of the analysis of terminology in constructive modeling by recognizing the epistemology. We deny the validity and the value of any attempt to analyze "theory-laden" language. Such an analysis lies outside our task when we engage in generating a specific model. Attempting to make such an analysis would require us to generate a model which would contain the specific model as an instance. We *cannot* do so within our methodology. Analysis of that sort would involve nonconstructive methods: the analyst *must* work from a specific model by generalization—having failed to construct the general model first.

The methodology implies iteration in the EPR or ERP sequence, or any interleaving of such sequences. Comparison with our diagram showing how the *participator* engages in a research program in physics [4] is given in Fig. 1. The comparison with McGoveran's modeling methodology is supposed to bring out the fact that the possible legal walks of the diagram are the same, but that the research program is contained *within* the methodology and that the methodology contains routes (arrows)that are *outside* the program. Thus the entry of the participator from a direction outside the box, and of the empirical confrontation (represented by Posiden's pitchfork Ψ) from a different direction, remain the same; so does the fact that corroboration leaves the participator inside, while falsification takes him outside, in yet another direction. The practitioner (and hopefully the reader of this volume) should therefore ask how far we have gone toward meeting his problems with contemporary physics. We assume that we agree on the following criteria:

1. agreement of cooperative communications
   * commonly defined terms as fundamental
   * fundamental vs. derived terms
   * agreement of pertinence
2. agreement of intent
3. agreement on observations
4. agreement of explicit assumptions
5. The Razor
   * agreement of minimal generality
   * agreement of elegance
   * agreement of parsimony

* Work supported by the Department of Energy, contract DE–AC03–76SF00515.

To appear as Article 3 in the Proceedings of the 9th Annual International Meeting of the Alternative Natural Philosophy Association, Department of the History and Philosophy of Science, Cambridge University, September 23–28, 1987

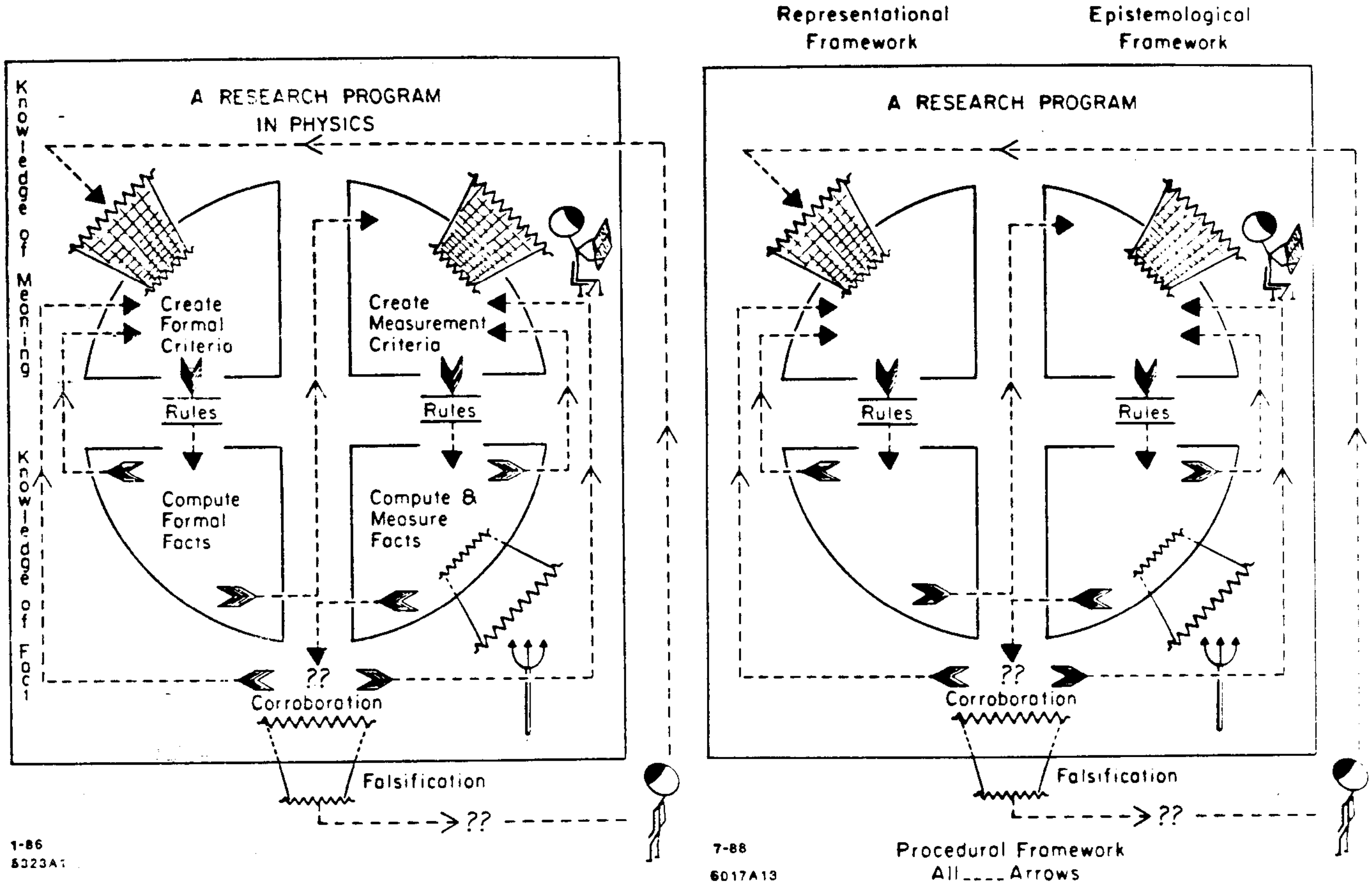

Fig. 1. Comparison between McGoveran's modeling methodology and Gefwert's participator model.

Our agreed upon intent is to model the practice of physics. We take as fundamental the commonly defined terms of laboratory physics, treating terms denoting nonobservables as derived or theoretical terms. We take laboratory events as a sufficient set of observations to be modeled, without requiring the standard theoretical interpretation. We take as understood that an experimental (laboratory) measurement may encompass many acts of observation. In other words, we are not committed to accept the how and why of the observations, only the observations themselves, operationally understood.*

In the next chapter we make a brief historical review of some aspects of modern physics which we find most significant in our own endeavor. In Chapter 3 we discuss the "Yukawa Vertices" of elementary particle theory as used in laboratory practice, second quantized field theory, analytic S–Matrix theory and in our own approach. In Chapter 4 we review the conserved quantum numbers in the Standard Model of quarks and leptons. This concludes our presentation of the "E–frame."

In Chapters 5–8 we try to develop a self-consistent representation of our theory. We have already claimed that this approach provides a discrete reconciliation between the formal (representational) aspects of quantum mechanics and relativity [5].

Chapters 9–13 provide rules of correspondence connecting the formalism to the practice of physics by using the *counter paradigm* and event-based coordinates to construct relativistic quantum mechanics

* Note the distinction between E–terms and R–terms. Von Neumann's "observation" is, at best, only an R–term. One line of criticism of von Neumann starts there, because his R–term is not necessarily consistent with Schrödinger continuity.

in a new way. The process comes to a temporary halt with a sequence of questions which could be answered in this framework.

2. THE HISTORICAL PRACTICE OF PHYSICS

Physics was a minor branch of philosophy until the seventeenth century. Galileo started "physics" in the contemporary sense. He emphasized both mathematical deduction and precise experiments. Some later commentators have criticized his *a priori* approach to physics without appreciating his superb grasp of the experimental method which he created—including reports of his experiments that still allow replication of his accuracy using his methods. He firmly based physics on the *measurement* of *length* and *time*, and established the uniform acceleration of bodies falling freely near the surface of the earth.

A century later, Newton entitled what became the paradigm for "classical" physics, *"The Mathematical Principles of Natural Philosophy,"* recognizing the roots that physics has in both disciplines. He also was a superb experimentalist. To a greater extent than Galileo, Newton had to create "new mathematics" in order to express his insight into the peculiar connection between experience, formalism and methodology that still remains the core of physics. To length and time, he added the concept of *mass* in both its inertial and its gravitational aspect, and tied physics firmly to astronomy through universal gravitation. For philosophical reasons, he introduced the concepts of absolute space and time, and thought of actual measurements as some practical approximation to these concepts.

It is often thought that Einstein's special relativity rejects the concept of absolute space-time, until it is smuggled back in through the need for boundary conditions in setting up a general relativistic cosmology. The concept of the homogeneity and isotropy of space, used by Einstein to analyse the meaning of distant simultaneity in the presence of a limiting signal velocity, in fact is very close to Newton's absolute space and time. What Einstein shows is rather that it is possible to use local, consequential time to *replace* this concept. This was pointed out to me by David McGoveran in the context of our fully finite and discrete approach to the foundations of physics, and our derivation of the Lorentz transformations without using the concept of continuity (cf., Ref. [1]). This same analysis shows that in a discrete physics, the universe has to be multiply connected. The space-like separated "supraluminal" correlations predicted by quantum mechanics—and recently demonstrated experimentally to the satisfaction of many physicists—can be anticipated for spin and for *any* countable degrees of freedom.

Nineteenth century physics saw the triumph of the electromagnetic field theory. That "classical" physics was still firmly based on historical units of mass, length and time; it provided no way to question *scale invariance*. Quantum theory and relativity were born at the beginning of this century. Quantum mechanics did not take on its current form until nearly three decades of work had passed. Although one route to quantum mechanics (that followed by deBroglie and Schrödinger) started from the continuum relativistic wave theory, the currently accepted form breaks the continuity by an interpretive postulate due to von Neumann sometimes called "the collapse of the wave function."

Criticism of this postulate as conceptually inconsistent with the time reversal invariant continuum dynamics of wave mechanics has continued ever since. This criticism was somewhat muted for a while by the near consensus of physicists that Bohr had "won" the Einstein–Bohr debate and the continuing dramatic technical successes of the theory. Scale invariance is gone because of the quantized units of mass, action and electric charge. These specify in absolute (i.e., countable) terms what is meant by "small." Explicitly $r_{Bohr} = \hbar^2/m_e e^2$ (with m_e the electron mass) specifies the atomic scale, $\lambda\!\!\!^{-}_{Compton} = (e^2/\hbar c) r_{Bohr} = \hbar/m_e c$ specifies the quantum electrodynamic scale and the "classical electron radius" $e^2/m_e c^2 = (e^2/\hbar c)\lambda\!\!\!^{-}_{Compton} \simeq 2\hbar/m_\pi c \simeq 14\hbar/m_p c$ specifies the nuclear scale; here m_p is the proton mass, and $m_\pi \simeq 2 \times 137 m_e$ is the neutral pion mass. The elementary particle scale $\hbar/m_p c$ is related to the gravitational scale by $\lambda\!\!\!^{-}_G = (G\hbar^3/c)^{\frac{1}{2}} = \hbar/M_{Planck}c = (Gm_p^2/\hbar c)^{\frac{1}{2}}(\hbar/m_p c)$

The expanding universe and event horizon specify what is meant by "large." Here the critical numbers any *fundamental* theory must explain are: "Age" of the universe as about 15 billion (15×10^9) years; "Mass" of the universe as about $3 \times 10^{76} m_p$—or at least ten times that number if one includes current estimates for "dark matter" ; "Size" of the universe or *event horizon*—naively the maximum radius which any signal can attain (or arrive from) transmitted at the limiting signal velocity c during the Age of the universe. Backward extrapolation using contemporary "laws of physics" to the energy and matter density when the radiation breaks away from the matter (size of the "fireball") is consistent with the observed 2.7°K cosmic background radiation. The cosmological parameters are numerically related to the elementary particle scale by the fact that the visible mass in the currently observable universe is approximately given by $M_{vis.U} \simeq (\hbar c/Gm_p^2)^2 m_p$, and that linearly extrapolating backward from the fireball to the "start of the big bang" gives a time $T_{fireball} \simeq (\hbar c/Gm_p^2)(\hbar/m_p c^2)$ =3.5 million years. It is clear that any theory which can calculate all these numbers has a claim to being a fundamental theory.

For a while it appeared that reconciliation between quantum mechanics and special relativity would resist solution, since the uncertainty principle and second quantization of classical fields gave an infinite energy to each point in space-time! During World War II, Tomonoga, and afterwards Schwinger and Feynman, developed formal methods to manipulate away these infinities and obtain finite predictions in fantastically precise agreement with experiment. Recently the non-Abelian gauge theories have made everything calculated in the "standard model" finite. Weinberg recently asserted at the Schrödinger Centennial in London that there is a practical consensus—but no proof—that second quantized field theory is the *only* way to reconcile quantum mechanics with special relativity. However, he also pointed out that the finite energy due to vacuum fluctuations is then 10^{120} too large compared to the cosmological requirements; the universe should rap itself up and shut itself off almost as soon as it starts expanding [6]. Even if one is willing to swallow this camel, there is no clear way to include strong gravitational fields in the theory. So continued attention to foundations seems fully justified.

The concept on which most of elementary particle physics rests has moved a long way from the mass points of post-Newtonian dynamics. For us, a paraphrase of the concept used by Eddington [7] is more useful: **A PARTICLE** is "A conceptual carrier of conserved 3-momentum and quantum numbers between events." This definition applies in the practice of elementary particle physics, both (1) in the high energy particle physics laboratory and in the theoretical formulations of either (2) second quantized field theory or (3) analytic S-matrix theory. In (1), the experimental application, "events" refer to the detection of any number of incoming and outgoing "particles" localized in macroscopic space-time volumes called "counters," or some conceptual equivalent. In (2), "events" start out as loci in the classical Minkowski 4-space continuum at which the "interaction Lagrangian" acting on a state vector creates and destroys particle states in Foch space. Since this prescription, naively interpreted, assigns an infinite energy and momentum to each space-time point, considerable formal manipulation and reinterpretation is needed before these "events" can be connected to laboratory practice. In (3), "events" refer to momentum-energy space "vertices" which conserve 4-momentum in the "Feynman diagrams" originally introduced in context (2) as an aid to the systematic calculation of renormalized perturbation theory. S-matrix theory makes a strong case for viewing continuous "space-time" as a mathematical artifact produced by Fourier transformation. Like any scattering theory, or any application of second quantized field theory to discrete and finite particle scattering experiments, S-matrix theory includes rules for connecting amplitudes calculated from these diagrams directly to laboratory practice (1).

For "events" generated by *Program Universe* [8] connecting bit strings (see Chapter 6), the "carrier" connects shorter to longer strings, or for strings of the same length connects two "3-events" to form a "4-event." We prove below that in this context the conservation of 3-momentum and quantum numbers consistent with laboratory practice (1) (thanks to the "counter paradigm," Chapter 9) can be derived within our construction of discrete physics, and serves the same purposes as the theoretical constructs in second quantized relativistic field theory (2) or analytic S-Matrix theory (3).

3. YUKAWA VERTICES

With the exception of *gluons*, the standard model of quarks and leptons starts from conventional interaction Lagrangians of the form $g\bar{\psi}\psi\phi$, into which various finite spin, isospin,... operators may be inserted. Here g is the "coupling constant" which measures the strength of the interaction relative to the mass terms in the "free particle" part of the Langrangian, ψ ($\bar{\psi}$) is a fermion (antifermion) second quantized field and ϕ a boson or "quantum" field. All three fields can be expanded in terms of creation and destruction operators in "particle" or "Foch" space states, which in the momentum space representation contain separate 4-momentum vector variables for each fermion, antifermion or quantum.

Fortunately for us, in one of the first successful efforts to tame the infinities in this theory, Feynman introduced a diagrammatic representation for the terms generated by such interaction Lagrangians in a perturbation theory (powers of g) expansion of the terms which need to be calculated and summed in order to obtain a finite approximation for the predictions of the theory. These "Feynman Diagrams" have taken on a life of their own; they bring out the symmetries and conservation laws of the theory in a graphic way. This can be a trap, particularly if they are reified as representing actual happenings in space time; but if used with care, they can short circuit a lot of tedious calculation (or suggest viable additional approximations) and provide a powerful aid to the imagination.

In the usual theory, Minkowski continuum space-time is assumed and any interaction Lagrangian is constructed to be a Lorentz scalar. Consequently, the quantum theory conserves 4–momentum at each 3–vertex. Here one must use care because of the uncertainty principle. If 4–momentum is precisely specified, the uncertainty principle prevents any specification of position, and the vertex can be anywhere in space-time. This is the most obvious way in which the extreme nonlocality of quantum mechanics shows up in quantum field theory. However, if we use a momentum space basis, we can still have precise conservation at the vertices. In practical application of the theory, of course, momentum cannot be precisely known; quasi-localization is allowed as long as the restrictions imposed by the uncertainty principle are respected. In a thorough treatment, this is called "constructing the wave packet"; this requires some care, as can be seen, for instance, by consulting Goldberger and Watson's *Collision Theory*. In practice, one usually works entirely in momentum space, knowing that the orthogonality and completeness of the basis states will allow the construction of appropriate wave packets in any currently encountered experimental situation. We have made a start on the corresponding construction in our theory [4].

Although 4–momentum conservation is now insured in the conventional treatment, this is not the end of the problem. All this insures is that for a particle state with energy ϵ and 3–momentum $\vec{p}$, that $\epsilon^2 - \vec{p}\cdot\vec{p} = M^2$; here M is any invariant with the dimensions of mass and need not correspond to the rest mass of the particle m. In the usual perturbation theory this is simply accepted. The dynamical calculations are made "off mass shell," and the specialization to physical values appropriate to the actual laboratory situations envisaged is reserved to the end of the calculation. S–Matrix theory sticks closer to experiment, in that all amplitudes refer to physical (realizable) processes with all particles "on mass shell." The dynamics is then supposed to be supplied by imposing the requirement of flux conservation ("unitarity")—a nonlinear constraint—and relating particle and antiparticle processes by "crossing." The analytic continuation of the amplitudes for distinct physical processes which gives dynamical content to the theory then makes the problem a self-consistent or "bootstrap" formalism. There is no known way to guarantee a solution of this bootstrap problem, short of including an infinite number of degrees of freedom—if then; of course, it is also well-known that there is no known way to prove that quantum field theory possesses any rigorous solutions of physical interest. Consequently, one again has recourse to finite approximations which may or may not prove adequate to particular situations.

The finite particle number scattering theory [9–12] keeps all particles on mass shell and, hence, has 3–momentum conservation at 3–vertices. This theory then insures unitarity for finite particle number systems by the form of the integral equations; these also provide the dynamics. The uncertainty principle is respected because of the "off-energy-shell" propagator, as it is in nonrelativistic scattering theory; the approximation is the truncation in the number of particulate degrees of freedom.

If we put the "Feynman Diagrams" of the second quantized perturbation theory on mass shell, we can talk about 3–vertices and 4–events using a common language for all three theories. The rules are easy to state, particularly if we do so in the "zero momentum frame." We are justified in using this frame in the mathematical models* because we have restricted ourselves to free-particle, mass-shell kinematics. We can use a corresponding statement in the laboratory because this frame is empirically specified as the frame at rest with respect to the 2.7°K background radiation.† Then the Poincaré invariance of the theories allows us to go from this description to any other convenient Galilean frame.

⋆ This a "Representational framework" statement in McGoveran's terminology.

† That is, again in the language of McGoveran's modeling methodology, we have a rule of correspondence ("Procedural framework" statement) connecting this zero momentum frame to laboratory practice ("Epistemological framework"), including the way calculations are performed in setting up and interpreting experiments.

As we show in Chapter 7, the 3–momenta at a 3–vertex add to zero. Diagrammatically we have three "vectors" which are "incoming" or "outgoing." By putting one of each together we obtain the generic 4–event, as indicated in Fig. 2. Clearly, for 4–events the total momentum of the two outgoing lines has to equal the total momentum of the two incoming lines, but the plane of the outgoing 3–event can be any plane obtained by rotating the outgoing vectors in the planar figure about the axis defined by the single line connecting them. By associating quantum numbers with each line, we can extend this description of 3–momentum conservation in Yukawa vertices and the 4–events constructed from them to the conservation of quantum numbers which "flow" along the lines. The idea of associating physical particles with the lines as carriers of both momentum and quantum numbers which comes from this pictorial representation is almost irresistible. The reader is warned once again to resist this temptation. The diagram is in 3+1 momentum-energy space and *not* in space time. In fact, if we insist on interpreting it as a space-time diagram representing the motion of particles, the quantum theory will blow up! It will force us to assign an infinite energy and momentum to each point of that space time, and simplicity of interpretation becomes elusive.

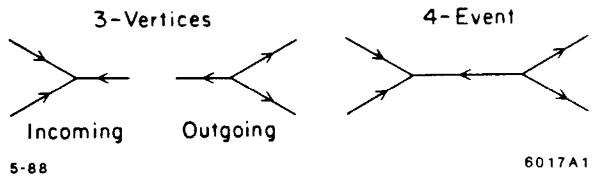

Fig. 2. The connection between 3–vertices and 4–events.

Once we have this picture in hand, "crossing" is easy to define. Since reversing a line and at the same time changing all quantum numbers to their negatives does not alter the conservation laws, the new diagram also represents a possible physical process. The "particle" whose quantum numbers are the negative of another is called its "antiparticle." So "crossing" can also be stated as the requirement that the reversal of a vector and the simultaneous change from particle to antiparticle represents another possible physical process. The manner in which a single diagram in which momenta and quantum numbers add to zero at a general 3–vertex generates emission, absorption, annihilation and decay vertices by this rule is illustrated in Fig. 3. The manner in which a single diagram, in which momenta and quantum numbers add to zero in a general 4–event, generates six physically observable processes by this rule is illustrated in Fig. 4.

Since one of the quantum numbers ("spin") is a pseudovector, "time reversal," which changes the sign of velocity and, hence, the direction, is not the same as the "parity" operation which changes all coordinates to their negatives. In quantum electrodynamics or QED, the theory in which the diagrams originated, the quantum number which distinguishes particle from antiparticle is electric charge; these rules are a consequence of the "CPT invariance" of the theory. They generalize to other types of "charge"; e.g., "color charge" in quantum chromodynamics (QCD). Spin is of great interest since it has a "space-time" significance as well as sharing the discrete, quantized character of other quantum numbers.

Before going on to the other quantum numbers, we note that the form of the Yukawa vertex couples the particle and antiparticle field in such a way that in the "time ordered" interpretation of the diagrams the number of fermions minus the number of antifermions is conserved; this is called the conservation of fermion number. Clearly, the diagrams respect this conservation law; so far as we know, f–number conservation is followed in nature.

4. THE STANDARD MODEL

The fermions encountered in nature fall into two classes: leptons and baryons. So far as we know to date, lepton number and baryon number are separately conserved. The lifetime for the decay of the proton into leptons and other particles has been shown to be greater than 10^{35} years; the experimental upper limit for the value depends on which decay mode was searched for. This fact has already ruled out many proposed schemes for "grand unification." The existence of the enormous underground detectors

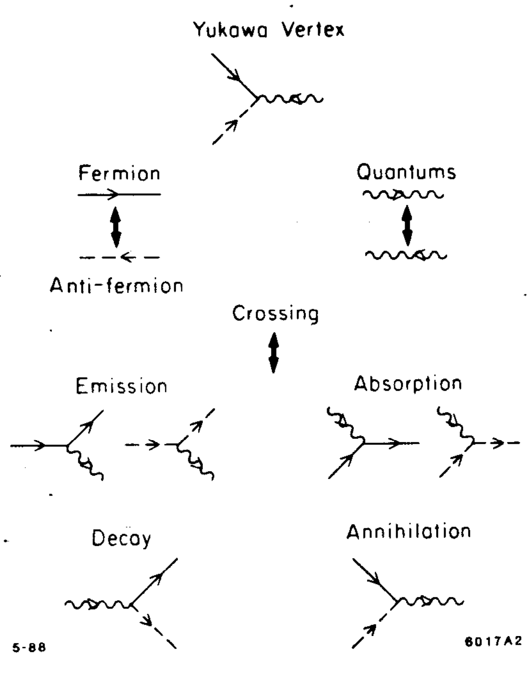

Fig. 3. The generic Yukawa vertex and crossing.

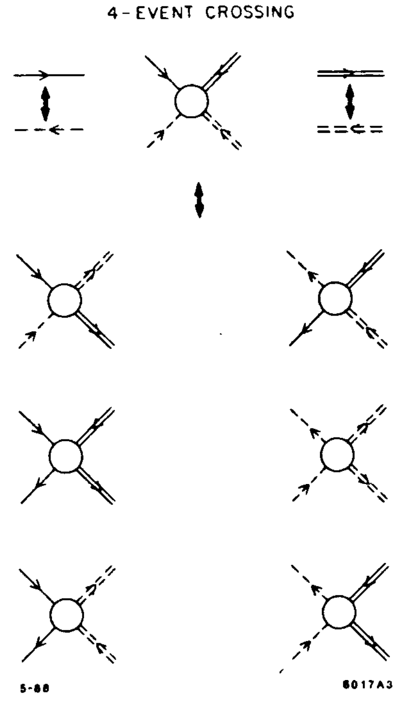

Fig. 4. Four-leg crossing.

constructed to test the hypothesis of proton decay had an unexpected payoff when two of them detected, "simultaneously," neutrino bursts from a supernova explosion 50,000 parsecs (1 parsec = 3.3 light-years) away. Individual neutrinos within the burst were cleanly resolved, but the time spread of the burst itself was so short that no information about the mass of the neutrinos was obtained. Although the time for the actual production of the neutrinos is supposed to be very short, the spread induced by the subsequent diffusion of the neutrinos out through the bulk of the star makes the calculation sensitive to the model used for calculating the explosion. It appears unlikely that limits on how much the neutrino mass might depart from zero better than those already established by terrestrial methods will be forthcoming from the analysis of this exciting event. Empirically, we can take electron-type neutrinos to be massless.

The quanta which couple via elementary Yukawa vertices in the standard model all have spin–1. The earliest coupling explored in quantum field theory was the electromagnetic coupling between electrons (e^-), positrons (e^+) and the massless electromagnetic quanta; the theory, which can be extended to other charged fermions, is called quantum electrodynamics (QED). The masslessness of the electromagnetic quanta is imposed within the second quantized relativistic field theory by requiring the theory to be "gauge invariant." A lower limit to the mass of either fermions or quanta with specified quantum numbers defines a well-understood experimental problem; if all such lower limits had to be finite, this would kill "gauge invariance." The requirement of gauge invariance is not compelling for me prior to some rough consensus as to what additional, independent tests (at an accuracy specified in advance) are relevant. I know of no proposed experimental program that could test gauge invariance within realistic error bounds. However, the upper limits on the mass of electromagnetic quanta are very good; empirically, we can assume photons to be massless.

The skepticism just implied makes my explanatory problem difficult. The current fashion in high energy elementary particle physics starts from "non–Abelian" gauge theories. Their broken "symmetries" generate "mass" from a "spontaneous breakdown of the vacuum." With care, this mechanism is claimed to be a guaranteed way to remove the infinities from a tightly constrained version of second quantized field theory. Without those constraints, which start from the necessity to get rid of the "classical" infinity of the e^2/r potential (infrared divergence) and the "second quantized" infinity of energy–momentum at each space-time point forced on us by the uncertainty principle (ultraviolet divergence), these theories are *prima facie* nonsensical. Self-consistency *within* the mathematical theory (R–frame) is contested by some who take the "rigour" of continuum mathematics seriously.

Following a conventional route in a 4–dimensional formalism one runs into trouble because a massless photon with momentum has only two chiral states (γ_{LL} and γ_{RR}), while the formalism requires four components for a 4–vector. For a massive spin–1 "particle" (i.e., something that can "carry" 3–momentum between two events in any coordinate system, and whose mass defines a rest system) there is no problem. The three states which quantum mechanics requires for spin 1 can be resolved along, against or perpendicular to the direction of motion, while the fourth component of the 4–vector is related to these three components "on-shell" by the invariant mass. When the invariant mass is zero, we are left with only two chiral 3–momentum carrying states. For fermions this is no problem, once parity conservation is abandoned. But for spin–1 massless bosons, the "third" and "fourth" component of the "4–vector" have to combine to yield an undirected $1/r$ "coulomb potential" in a gauge invariant and manifestly covariant manner. In a classical theory with extended sources this was no problem because the transformation between the 4–vector notation and the "coulomb gauge" was always well-defined, although coordinate system dependent; but in second quantized field theory, consistency between the classical substrate and the Feynman rules requires all kinds of technical artifices (indefinite metrics and the like). In a finite particle number theory, one can avoid some of these technical difficulties by always using transverse photons and the coulomb interaction in a well-defined coordinate system, provided the (no longer manifest) "covariance" can be maintained. Of course, this removes some of the (we believe superficial) formal simplicity of the "manifestly covariant" 4–vector formalism. Since the theory we have developed commits us to 3–momentum conservation as fundamental, this is a natural route for us to take.

Once this is understood, the $e_L^-(Q = -e, s_h\hbar = -\frac{1}{2}\hbar)$, $e_R^-(Q = -e, s_h\hbar = +\frac{1}{2}\hbar)$ crossing symmetric Yukawa vertices specifying massive leptonic QED for a single flavor (in this case e) coupled to $\gamma_{LL}, \gamma_{RR}, \gamma_c$ are given in Fig. 5. We note that for electromagnetic coupling, charge and lepton number go together, so the conservation law for one implies the conservation law for the other. We represent the combined conservation laws of $2s_h \in 0, \pm 1, \pm 2$ and $\ell = -Q/e \in 0, \pm 1$, by the vector states in a plane by Fig. 6. A Yukawa (QED) vertex requires three quantum number "vectors" consisting of a fermion, an antifermion and a quantum which add to zero, plus the temporally ordered processes derived from the fundamental diagram by crossing. The field theory notation for this QED coupling is [13] $-i\bar{e}\gamma_\lambda e A_\lambda$, with $Q^2/\hbar c = e^2/\hbar c \simeq 1/137$.

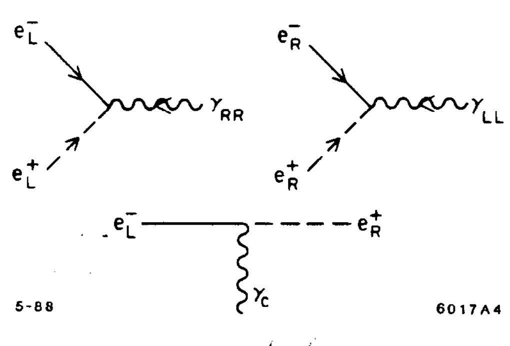

Fig. 5. Quantum electrodynamics.

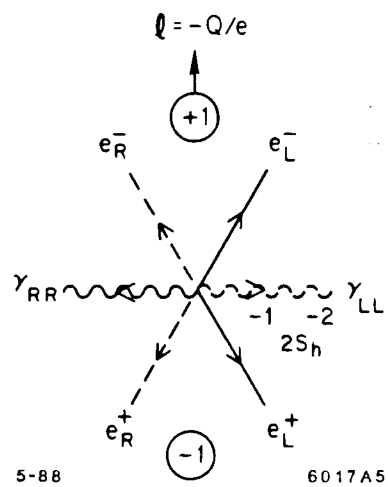

Fig. 6. Quantum electrodynamic conservation laws as planar vectors.

In contrast to the parity conserving electromagnetic vertices, the "weak" interactions violate parity conservation maximally. The easiest way to represent this is to use a massless neutrino (ν_L), conventionally called "left-handed." Consider an arrow in front of you with the head on the right. If you slip your right hand under the arrow to pick it up, your thumb will point in the same direction as the head; if you pick it up by slipping your left hand under the arrow, your thumb will point in the opposite direction to the head. The latter case is called "left-handed." By the Feynman rule, the antineutrino $\bar{\nu}_L$ is then right-handed. The charged quantum which couples to the electron and neutrino is called W (the weak vector boson) and is also chiral, since in the zero momentum frame $e_L^- + \bar{\nu}_L \to W_{LL}^-$; in field theory notation the coupling is

$$-i(G_F M_W^2/\sqrt{2})^{\frac{1}{2}}\ \bar{\nu}\gamma_\lambda(1-\gamma_5)\ eW_\lambda\ .$$

The Weinberg–Salam–Glashow "weak-electromagnetic unification" requires, in addition to this electrically charged weak boson, which was a convenient way to parameterize the parity-nonconserving theory of β–decay, the neutral weak boson Z_0 responsible for "neutral weak currents." The reasons had to do initially with the removal of infinities from the theory, and go through a complicated sequence of arguments that predict, in addition, one or more scalar "Higgs bosons," for which there is at present no laboratory evidence. Since our theory is born finite and cannot produce the infinities of second quantized field theory, we have no need for these hypothetical particles in the first place. If they should be discovered (thanks to current efforts at many laboratories which are now consuming a large fraction of their experimental and computational resources), we will be faced with some difficult conceptual problems in our discrete theory. Fortunately, for the moment, we can ignore them, which makes our presentation of the conservation laws in the leptonic sector considerably simpler.

The coupling of the Z^0 to neutrinos is chiral and is given by

$$(-i/\sqrt{2})(G_F M_Z^2/\sqrt{2})^{\frac{1}{2}}\ \bar{\nu}\gamma_\lambda(1-\gamma_5)\ eZ_\lambda\ .$$

The coupling to electrons is more complicated because it brings in the "weak angle" θ_W that distinguishes the coupling to left- and right-handed electrons in the following way:

$$(-i/\sqrt{2})(G_F M_Z^2/\sqrt{2})^{\frac{1}{2}}\ \bar{e}\gamma_\lambda[R_e(1+\gamma_5)+L_e(1-\gamma_5)]\ eZ_\lambda\ .$$

Here $R_e = 2sin^2\theta_W$, $L_e = 2sin^2\theta_W - 1$. If $sin^2\theta_W = 1/4$, which is not too bad an approximation to the experimental value, Z couples to electrons like a heavy gamma ray, except that it is a pseudovector rather than a vector. The mixing angle is not independent of the masses of the weak bosons, because

$$M_W \sin\theta_W = [\pi e^2/\hbar c G_F\sqrt{2}]^{\frac{1}{2}} = 37.3\ \mathrm{Gev/c^2} = M_Z \sin\theta_W \cos\theta_W\ .$$

Since there were estimates of the weak mixing angle available before the discovery of the weak bosons, their masses could be estimated to be around 84 and 94 Gev/c^2 respectively, which aided greatly in their experimental isolation. Since the W's are charged, they couple to photons and also directly to the Z. These couplings are given in Ref. [13], p. 116. Eventually the more complicated 4–vertices given in the same reference should provide a critical test of the standard model, and conceivably might also distinguish between our theory and the standard model, even in the absence of experimental evidence for the Higgses. We ignore this complexity in what follows.

The conservation law situation is now considerably more complicated than it was for electromagnetic quanta. Charge, lepton number and helicity are still conserved, but the pattern is not easy to follow if written in those terms. Following a strategy that was first introduced into nuclear physics to describe the approximate symmetry between neutron and proton as an "isospin doublet," we form a "weak isospin doublet" from the left-handed electron ($i_z = -\frac{1}{2}$) and left-handed neutrino ($i_z = +\frac{1}{2}$) and, assuming lepton number conservation, can talk about either charge conservation or "z–component of isospin conservation," by introducing an appropriate version of the Gell–Mann–Nishijima formula, namely $Q = \ell/2 + i_z$, for the left-handed doublet. To include the right-handed electron, which does not couple to neutrinos, we make it an isospin singlet. To couple it to γ–rays, we assign it a "weak hypercharge" $Y = -2$ and modify Gell Mann–Nishijima formula to read $Q = Y/2 + i_z$. Our conservation laws are now conveniently described in the 3–space picture given in Fig. 7. The numerical specifications are given in Table 1.

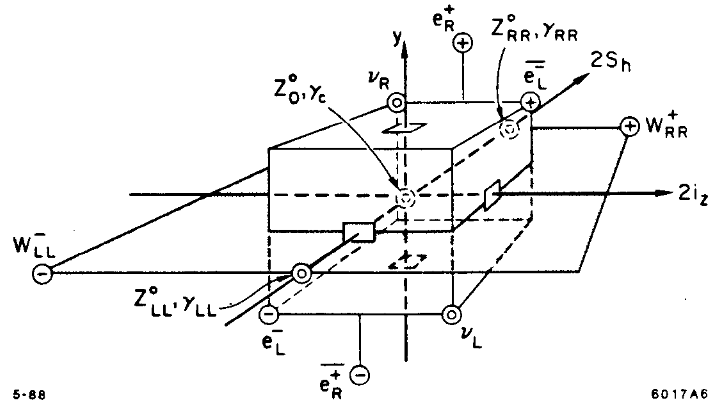

Fig. 7. Weak-electromagnetic unification in terms of weak hypercharge, weak isospin and helicity.

Table 1. Quantum numbers for weak-electromagnetic unification.

	Particle	Q	Y	$2i_z$	ℓ	$2h$	m in Gev/c^2
fermion	ν_L	0	−1	+1	−1	−1	0
	$\bar{\nu}_L$	0	+1	−1	+1	+1	0
	e_L^-	−1	−1	−1	−1	−1	$.511 \times 10^{-3}$
	$\bar{e}_L^-$	+1	+1	+1	+1	+1	"
	e_R^-	−1	−2	0	−1	−1	"
	$\bar{e}_R^-$	+1	+2	0	+1	+1	"
quantum	W_{LL}^-	−1	0	−2	0	−2	$37.3/\sin\theta_W$
	$\bar{W}_{LL}^-$	+1	0	+2	0	+2	"
	Z_{LL}^0, γ_{LL}	0	0	0	0	−2	$37.3/\sin\theta_W\cos\theta_W$,0
	$\bar{Z}_{LL}^0, \bar{\gamma}_{LL}$	0	0	0	0	+2	"
	Z_0^0, γ_c	0	0	0	0	0	"

Although the type of spatial representation of the quantum numbers presented in Fig. 7 suggests that there might be rotational invariance in this space, actually only the values on the axes have precise meaning in terms of conservation laws. Total isospin is only approximately conserved; it is a "broken symmetry." Perhaps this should not be a surprise in a relativistic theory; if we take as the four independent generators of the Poincaré group mass, parallel and perpendicular components of 3-momentum and helicity (or the component of angular momentum along the parallel direction), the total angular momentum cannot be simultaneously diagonalized. People often forget that "total spin" is not a well-defined concept in a relativistic theory.

Now that we have explored in detail the weak-electromagnetic unification of electrons, whose mass is 0.511 Mev/c^2, and their associated massless neutrinos, the full weak-electromagnetic unification scheme is easy to state. In addition to the electrons, we have two systems of leptons with much larger masses, the muon with mass 105.66 Mev/c^2 and the tau lepton with mass 1784 Mev/c^2. Associated with each are left-handed $(\nu_\mu)_L$ and $(\nu_\tau)_L$ neutrinos whose interactions can be experimentally distinguished from those of the electron neutrinos $(\nu_e)_L$ and from each other. They may well be massless, but the upper limits on their masses are much higher than for the electron type neutrinos. The coupling scheme is the same as that we have already discussed above within each "generation" $(e, \mu, \tau = 1^{st}, 2^{nd}, 3^{rd})$ and the coupling between generations, specified by the Kobiyashi–Maskawa mixing angles, is weak.

To complete the scheme for the weak interactions we must bring in the quarks. There are two "flavors" (up and down) for the first (electron) generation, and two (charmed and strange) for the second (muon) generation; there are supposed to be two more in the third (tau) generation to complete the picture. The existence of the beautiful (or bottom) quark is well-established, but searches for the true (or top) quark are still under way. It is the only particle missing from the scheme, other than the Higgses, if you stick to three generations. The quarks are fermions and have electric charge $Q_{u,c,t} = \pm\frac{2}{3}$, $Q_{d,s,b} = \mp\frac{1}{3}$ and baryon number $\frac{1}{3}$. Each forms a weak isodoublet and an isosinglet in the now familiar pattern. This completes the weak-interaction picture at the level we will discuss it here.

The quarks differ markedly from the leptons in several respects. To begin with, they carry a conserved "color charge" with three colors, three anticolors and an eightfold symmetry we will describe in more detail in Chapter 8. They couple strongly at low energy to eight spin-1 colored "gluons." Color conservation is given a vector representation in Fig. 8.

Remarkably, both quarks and gluons are "confined": they show up like internal particulate degrees of freedom in high energy experiments (parton model), but never have been liberated to be studied as free particles. Hence, the definition of their masses is indirect; recent calculations would seem to indicate that the "mass" of an up or down quark is about one-third the mass of a proton at low energy, but falls off like $1/p^2$ as the momentum with which they interact increases [14]. One up quark combined with an up-down pair in a spin-singlet state to forsm an overall color singlet state form a proton with charge 1, while a down quark combined with the pair in the same way forms a neutron with charge 0. Consequently, the β-decay properties of the neutron can be related to the weak isodoublet description given above.

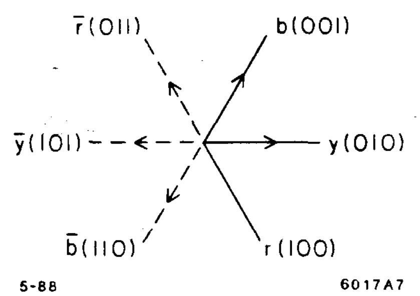

Fig. 8. Colors and anticolors as discrete vectors.

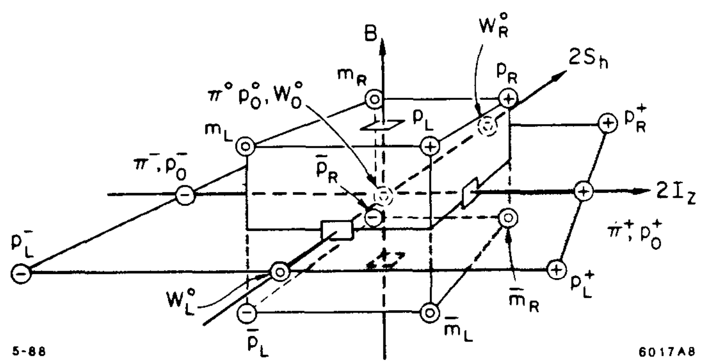

Fig. 9. Spin, isospin and baryon number conservation for color singlet neutrons and protons $p = u(ud), n = (d(ud)$.

So far as quantum number conservation goes, we can talk about baryon number (B) spin and (strong) isospin with charge conservation given by $Q = B/2 + I_z$ in the same way we talked about weak hypercharge and weak isospin conservation above. Quark-antiquark pairs describe the mesons (pions, etc.,) which older theories used to explain nuclear forces, but the details of how the quark-nuclear physics interface actually works quantitatively is a very controversial field of research. The easiest way to picture all this is to write the "color" vertices separately as vectors in a plane and assume that they add to form a color singlet (which can be a neutral colored or anticolored triplet, or any one of the color-anticolor pairs). Then we can return to the familiar picture of neutron, proton, and their antiparticles and associated mesons in the (s_h, I_z, B) space pictured in Fig. 9. Note the symmetry of the diagram for these parity-conserving strong interactions, in contrast to the asymmetric diagram which pictures the parity nonconserving weak-electromagnetic unification.

We will show in Chapter 8 how this whole picture can be reproduced at this level by our discrete physics construction. To get the quantitative details right is obviously a major research program, comparable (until we can find short cuts) to the hard work that is engaging many particle physicists every day in many laboratories. A useful reference that gives some idea of the magnitude of the task is the Proceedings of the 1986 SLAC Summer Institute [15]. Clearly, we must stop at some point short of that effort in this volume; we choose to do so when we have reached the same degree of desciption explained in this chapter.

5. THE COMBINATORIAL HIERARCHY AND THE LABEL–CONTENT SCHEMA

The overall status of the research [16–17], here aimed at providing a common explanatory theory for both quantum mechanics and relativity in a discrete and finite framework, has been provided a historical context in Chapter 2. The early thinking in this program did not approach the problem with such an explicit objective. Bastin realized that when we go to the very large (distant galaxies, early times...) or the very small (quantum events, elementary particles...) the information available to us becomes extremely impoverished compared to the phenomena modeled by classical physics. He concluded that this fact should be reflected in the theory in such a way that this restriction is respected.

The route into the theory initially followed by Bastin and Kilmister concentrated on the problem of modeling discrete events [18]. Ordered strings of zeros and ones gave a powerful starting point for analyzing this problem. Attention eventually centered on the question of whether bit strings were the same or different. Define a bit string by

$$(a)_n \equiv (\ldots,\ b_i^a, \ldots)_n;\ b \in 0,1;\ i \in 1,2,\ldots,n\ .$$

An economical way to compare an ordered sequence of two distinct symbols with other sequences of the same bit length is to use the operator XOR ("exclusive or," symmetric difference, addition (mod 2) $= +_2, \ldots$). Since we sum (or count) the one's in the string to specify a measure, we must treat the symbols "0," "1" as integers, and only in some contexts can we think of them as bits; hence, our "bit strings" are more complicated conceptually than those encountered in standard computer practice. We therefore use the more general *discrimination* operation "$\oplus$," and a short hand notation for it. Define the symbol $(ab)_n$ and the discrimination operation $\oplus$ by

$$(ab)_n \equiv S^a \oplus S^b \equiv [\ldots,\ (b_i^a - b_i^b)^2, \ldots]_n = [\ldots,\ b_i^a +_2 b_i^b, \ldots]_n\ .$$

The name comes from the fact that the same strings combined by discrimination yield the null string, but when they differ and $n \geq 2$ they yield a third distinct string which differs from either; thus the operation discriminates between two strings in the sense that it tells us whether they are the same or different.

We define the *null string* $(0)_n$ by $b_i^0 = 0, i \in 1,2,\ldots,n$ and the *antinull string* $(1)_n$ by $b_i^1 = 1, i \in 1,2,\ldots,n$. Since the operation $\oplus$ is only defined for strings of the same length, we can usually omit the subscript n without ambiguity. The definition of discrimination implies that

$$(aa) = (0);\quad (ab) = (ba);\quad [(ab)c] = [a(bc)] \equiv (abc)\ ,$$

and so on.

The importance of closure under this operation was recognized by John Amson. It rests on the obvious fact that $[a(ab)] = (b)$, and so on. We say that any finite and denumerable collection of strings, where all strings in the collection have a distinct tag $i, j, k\ \ldots$, are *linearly independent* iff

$$(i) \neq (0) : (ij) \neq (0),\ (ijk) \neq (0), \ldots (ijk\ldots) \neq (0)\ .$$

We define a *discriminately closed subset* of nonnull strings $\{(a),(b),\ldots\}$ as the set with a single string as member or by the requirement that any two different strings in the subset give another member of the subset on discrimination. Then two linearly independent strings generate three discriminately closed subsets, namely

$$\{(a)\},\ \{(b)\},\ \{(a),(b),(ab)\}\ .$$

Three linearly independent strings give seven discriminately closed subsets, namely

$$\{(a)\},\ \{(b)\},\ \{(c)\}\ ,$$

$$\{(a),(b),(ab)\},\ \{(b),(c),(bc)\},\ \{(c),(a),(ca)\}\ ,$$

$$\{(a),(b),(c),(ab),(bc),(ca),(abc)\}\ .$$

In fact, x linearly independent strings generate $2^x - 1$ discriminately closed subsets because this is simply the number of ways one can take x distinct things one, two, three, ... , x at a time. This is critical to the construction of the combinatorial hierarchy, as we now discuss.

The discovery of the combinatorial hierarchy [19] was made by Parker–Rhodes in 1961. The story as I recall hearing it a decade after the facts, which Bastin now informs me is somewhat misleading,* was that the challenge posed to Frederick was how to generate a sequence with one or two small numbers, something of the order of a hundred, some very large number and *stop.*† Frederick (P–R) did indeed generate the sequence $3, 10, 137, 2^{127} + 136 \simeq 1.7 \times 10^{38}$ in suspiciously accurate agreement with the "scale constants" of physics. This was a genuine discovery; the termination is at least as significant!* The sequence is simply $(2 \Rightarrow 2^2 - 1 = 3), (3 \Rightarrow 2^3 - 1 = 7)$ $[3 + 7 = 10], (7 \Rightarrow 2^7 - 1 = 127)$ $[10 + 127 = 137], (127 \Rightarrow 2^{127} - 1 \simeq 1.7 \times 10^{38})$. The real problem is to find some "stop rule" that terminates the construction.

The original stop rule was due to Parker–Rhodes. He saw that if the discriminately closed subsets at one level, treated as sets of vectors, could be mapped by nonsingular (so as not to map onto zero) square matrices having uniquely those vectors as eigenvectors, and if these mapping matrices were themselves linearly independent, they could be rearranged as vectors and used as a basis for the next level. In this way the first sequence is mapped by the second sequence $(2 \Rightarrow 2^2 = 4)$, $(4 \Rightarrow 4^2 = 16)$, $(16 \Rightarrow 16^2 = 256)$, $(256 \Rightarrow 256^2)$. The process terminates because there are only $256^2 = 65{,}536 = 6.5536 \times 10^4$ linearly independent matrices available to map the fourth level, which are many too few to map the $2^{127} - 1 = 1.7016 \ldots \times 10^{38}$ DCsS's of that level. The (unique) combinatorial hierarchy is exhibited in Table 2.

Although this argument proves the *necessity* of the termination (which is no mystery in the sense that an exponential sequence must cross a power sequence at some finite term), it did not establish the existence of the hierarchy. This was first done by me by creating explicit constructions of the

* Quoting a recent letter by Bastin to HPN, 3 March 1988, "Frederick had come very recently into the discussions about hierarchies and level relationships [among Amson, Bastin, Kilmister, Pask], and couldn't come on a second trip with me to the analog computer at Brussels [where they were being explored experimentally?] because of 'flu. When he had his mapping relation giving an 'information preserving' (as we should then have said) relation between levels; and the numbers. He and the rest of us knew that the numbers had to be the primary step to physics, but we planned to avoid any attempt at deduction of them thinking it probably impossible. It was a morning or two later that Frederick arrived very crestfallen because he had found the breakdown of the algorithm." [The "breakdown" referred to is the termination of the sequence at the fourth level, which turns out to be a critical success of the basic theory when we come to explaining gravitation.]

† Continuing the quote from Bastin, "I never proposed that sort of challenge to Frederick, though I can see you may have wanted a quick way to be fair to everyone and hit on that."

* According to Parker–Rhodes, in "Agnosia," *Proc. ANPA 7,* p. 74: "Somewhere around 1962 I hit upon a series of numbers of which Ted Bastin noticed that the last two (the generating procedure could not produce more than four) were close to two well-known physical constants, the reciprocals of the fine-structure constant and the gravitational coupling constant." The somewhat different history given in the "Preface and Acknowledgements" to Parker–Rhodes' *The Theory of Indistinguishables* does not give this credit to Bastin. I know that this preface was an afterthought, and that Frederick did not prepare it with care.

Table 2. The combinatorial hierarchy.

Hierarchy Level ℓ	$B(\ell+1) = H(\ell)$	$H(\ell) = 2^{B(\ell)} - 1$	$M(\ell+1) = [M(\ell)]^2$	$C(\ell) = \Sigma_{j=1}^{\ell} H(j)$
(0)	-	2	(2)	-
1	2	3	4	3
2	3	7	16	10
3	7	127	256	137
4	127	$2^{127} - 1$	$(256)^2$	$2^{127} - 1 + 137$

Level 5 cannot be constructed because $M(4) < H(4)$.

mapping matrices [20] and later more elegantly by Kilmister [21]. That the termination, and indeed the combinatorial hierarchy itself, is much more than the apparently *ad hoc* mapping procedure which first led to it might suggest, can be seen either by Kilmister's latest derivation [22] or by the very different way Parker-Rhodes now gets it out of his *Theory of Indistinguishables* [23]; a useful discussion of that theory entitled "Agnosia" is given in Ref. [17].

For some time, the only operation used in the theory was discrimination. Kilmister eventually realized that one should also think about where the strings came from in the first place. He met this problem by introducing a second operation which he called "generation." As he and I realized, this operation eventually generates a universe which goes beyond the bounds of the combinatorial hierarchy. Once this happens, we can separate the strings into some finite initial segment that represents an element of the hierarchy, which we call the *label*, and the portion of the string beyond the label which we now‡ call the *content*. It is clear that from then on the content ensemble for each label grows in both number and length as the generation operation continues. Since it takes $2+3+7+127 = 139$ linearly independent basis strings to construct the four levels of the combinatorial hierarchy, the labels will be of at least this length; if we use the mapping matrix construction, they will be of length 256. Call this *fixed* length L, the length of any content string n, and the total length at any TICK (see next section) in the evolution of the universe $N_U = L + n$. Then the strings will have the structure $S^a = (L_a)_L \| (A_x^a)_n$ where a designates some string of the $2^{127} + 136$, which provide a representation of the hierarchy, and x designates one of the 2^n possible strings of length n; the symbol "$\|$" denotes string concatenation.

6. PROGRAM UNIVERSE

In order to generate a universe of strings which grows, sequentially, in either number (SU) or length (N_U) Mike Manthey and I created *program universe*. Recently Manthey realized that the criterion used to increase the string length (TICK) was unjustifiably selective. The previously published version of the program [8], called *program universe 1,* is compared with Manthey's new proposal in Fig. 10. The most significant effect of the change, other than simplification (using "The Razor" in McGoveran's terminology), is to allow the bit string universe to contain, ephemerally in many cases, distinct strings which are indistinguishable under discrimination. This will not affect anything in this paper, but might eventually provide alternative cosmological models that make observationally different predictions.

‡ The term Kilmister and I first used was "address" rather than "content." This has turned out to be unfortunate from the point of standard computer science usage. It has been proposed that "address" be replaced by "content," and I adopt that new usage in what follows. Kilmister and I used "address" because we envisaged (as has now happened) the use of this portion of the string to construct our discrete version of "space-time;" thus the address is like that on an envelope, with the label being the name. This has the advantage that neither is meaningful without the other. On the other hand the "contents" of a label describe the relevant states which are occupied at a given TICK of PROGRAM UNIVERSE, and their order of production—if known or knowable—would serve to enumerate them. We should try to stabilize the terminology at ANPA 10.

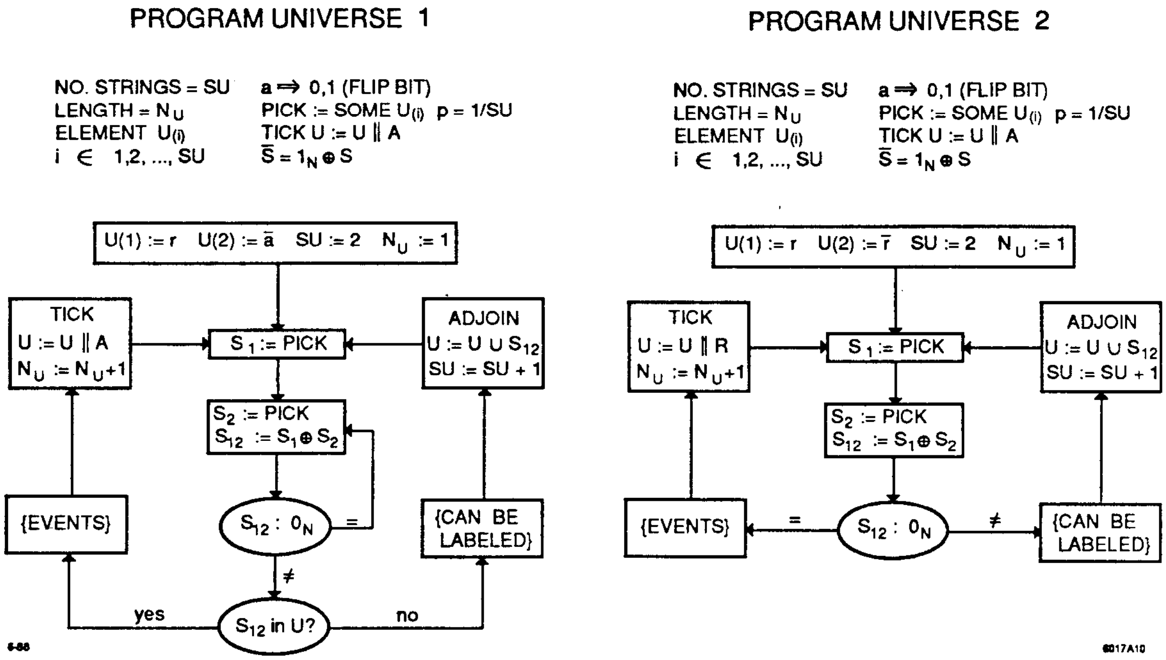

Fig. 10. Program Universe 1 and 2 compared.

The program is initiated by the arbitrary choice of two distinct bits, which become the first two strings in the universe. Whether insisting that one be "0" and the other "1," as in done in the flow chart, rather than allowing both to be arbitrary, will eventually produce a significantly different cosmology (or choice among cosmologies) at our epoch is an open question. Entering the main routine at $PICK$, we choose two strings (i) and (j) and discriminate them: $(ij) \equiv (i) \oplus (j)$. Whenever the two strings picked are identical, $(ij) = (0)_{N_U}$ and we go to TICK. TICK concatenates a single bit, arbitrarily chosen for each string, to the growing end, notes the increase in string length and the program returns to $PICK$. The alternative route, which occurs when discrimination generates a nonnull string, simply ADJOINs the newly created string to the universe and the program returns to $PICK$.

In the older version, we proved that TICK had to be "caused" (in the computer simulation) either by the occurrence of the "3–event" configuration $S^a \oplus S^b \oplus S^c = 0_{N_U}$ or by the configuration $S^a \oplus S^b \oplus S^c \oplus S^d = 0_{N_U}$, which we called a "4–event." But this implied a uniqueness which has no known demonstrable counterpart in nature, as modeled by contemporary physics; there can be many "simultaneous" events. At ANPA 9, I extended the definition of "event" to include all cases in which, at a given string length (or TICK), three or four strings combine under discrimination to produce the null string. This definition is retained here, but in Program Universe 2 is no longer the "cause" of TICK. Instead, we TICK whenever two strings "interact" without producing any novelty. This is as close as we need to get to defining what would be called a "point" in a continuum theory. We will see in Chapter 10 that this construction of a "point" is consistent with our development of Einstein synchronization and, hence, to the extent possible in our discrete theory, consistent with the conventional use of the term "event" in relativity theory.

The constraints $(abc)_{N_U} = (0)_{N_U} = (abcd)_{N_U}$ at each TICK are our model for the unique, nonlocal, yet indivisible and *irreversible* events of quantum mechanics. We have a lot more work to do before we can show that they have the requisite properties. In particular, we have to demonstrate that they can act like the 3–vertices and 4–vertices of the Feynman Diagrams discussed in Chapter 3. When N_U is large, these constraints will be satisfied by many combinations and—because of McGoveran's Principle IV[♮] —*all* must be viewed as "simultaneous" events.

♮ "The theory possesses the property of absolute nonuniqueness," cf., Ref. [1].

The method Manthey and I use to "construct" the hierarchy is much simpler than the original matrix construction given by Parker–Rhodes; in fact, some might call it "simple-minded." We claim that all we have to do is to demonstrate explicitly (i.e., by providing the coding) that any run of PROGRAM UNIVERSE contains (if we enter the program at appropriate points during the sequence) all we need to extract some representation of the hierarchy and the label content scheme from the computer memory *without* affecting the running of the program. The obvious intervention point exists where a new string is generated, i.e., at ADJOIN. The subtlety here is that if we assign the tag i to the string $U[i]$ as a *pointer* to the spot in memory where that string is stored, this pointer can be left unaltered from then on. It is, of course, simply the integer value of $SU + 1$ at the "time" in the simulation [sequential step in the execution of that run of the program] when that memory slot was first needed. Of course, we must take care in setting up the memory that *all* memory slots are of length $N_{max} > N_U$; i.e., can accommodate the longest string we can encounter during the (necessarily finite) time our budget will allow us to run the program. Then, each time the program TICKs, the bits which were present at that point in the sequential execution of the program when the slot $[i]$ was first assigned will remain unaltered; only the growing head of the string will change. Thus, if the strings $i, j, k \ldots$ tagged by these slots are linearly independent at the time when the latest one is assigned, they will remain linearly independent from then on.

Once this is understood, the coding Manthey and I gave for our labeling routine should be easy to follow. We take the first two linearly independent strings and call these the basis vectors for *level 1*. The next vector which is linearly independent of these two starts the basis array for *level 2*, which closes when we have three basis vectors linearly independent of each other and of the basis for *level 1*, and so on until we have found exactly $2 + 3 + 7 + 127$ linearly independent strings. The string length when this happens is then the *label length* L; it remains fixed from then on. During this part of the construction, we may have encountered strings which were *not* linearly independent of the others, which up to now we could safely ignore. Now we make one *mammoth* search through the memory and assign each of these strings to one of the four levels of the hierarchy; it is easy to see that this assignment (if made sequentially passing through *level 1* to *level 4*) has to be unique.

From now on, when the program generates a new string, we look at the first L bits and see if they correspond to any label already in memory. If so, we assign the content string to the *content ensemble* carrying that label. If the new string also has a new label, we simply find (by upward sequential search as before) what level of the hierarchy it belongs to and start a new labeled content ensemble. Because of discriminate closure, the program must eventually generate $2^{127} + 136$ distinct labels, which can be organized by us into the four levels of the hierarchy. Once this happens, the label set cannot change and the parameters i for these labels will retain an *invariant* significance, no matter how long the program continues to TICK. It is this invariance which will later provide us with the formal justification for assigning an invariant mass parameter to each string. We emphasize once more that *what* specific representation of the hierarchy we generate in this way is irrelevant; any "run" of PROGRAM UNIVERSE will be good enough for us.

It should be noted that in a strict sense this way of arriving at the hierarchy is not "constructive." What we do is to go through a procedure which allows us to recognize that the program has generated some bit string representation of the hierarchy. This recognition program is internal to a part of the computer memory, and is not used explicitly in the way we go on to set up rules of correspondence and physical interpretation; it in no way affects the running of the basic program and was coded only in order to show that we could do it. The new *Universe Program* being written by McGoveran will, instead, be strictly constructive and will generate its own stop rule for the label-content separation, rather than putting it in from the outside. This has no immediate consequences other than satisfying the rule of parsimony, but will tie down our cosmology more firmly than the current *Program Universe* does. The *event* definition which we have explained above $[(abc) = (0); (abcd) = (0)]$ will continue to be rigorously applicable.

Each event occurs in a TICK, which increases the complexity of the universe in an irreversible way. Our theory has an ordering parameter (N_U) which is conceptually closer to the "time" in general relativistic cosmologies than to the "reversible" time of special relativity. The arbitrary elements in the algorithm that generates events preclude *unique* "retrodiction," while the finite complexity parameters (SU, N_U) prevent a combinatorial explosion in *statistical* retrodiction. In this sense, we have a *fixed*—though only partially retrodictable—*past* and a necessarily *unknown future* of finite, but arbitrarily increasing, complexity. Only structural characteristics of the system, rather than the bit strings used in computer simulations of pieces of our theory, are available for epistemological correlations with experience.

What was *not* realized when this program was created was that this simple algorithm provides us with the minimal elements needed to construct a finite particle number scattering theory. The increase in the number of strings in the universe by the creation of novel strings from discrimination is our replacement for the "particle creation" of quantum field theory. It is not the same, because it is both finite and irreversible; it also changes the "state space." Note that the string length N_U is simply the number of TICKs that have occurred since the start up of the universe; this order parameter is irreversible and monotonically increasing, like the cosmological "time" of conventional theories. Our events are unique, indivisible and global, in the computer sense; consequently, events cannot be localized and will be "supraluminally" correlated.

7. "VECTOR" CONSERVATION LAWS

So far we have a gross structure based on bit strings, and two operations which generate them via a specific program: (1) ADJOIN, which adjoins a nonnull string produced by discrimination to the extant bit string universe, and (2) TICK which increases the string length by concatenating a single bit, arbitrarily chosen for each string, at the growing end of each string. We have two kinds of connectivity which result from this construction. One is the label-content schema. Once the label basis has closed under discrimination to form 2+3+7+127 linearly independent strings, program universe will necessarily generate some representation of the combinatorial hierarchy at that label length; this will close with $3+7+127+2^{127}-1$ labels of that length. Once the label basis (and label string length) is fixed, program universe assigns each novel content string to a specific label when it is created by discrimination, and augments each content string by an arbitrary bit at each TICK. The second is the connectivity between strings of the same length (i.e., "between ticks") which we have characterized as 3–vertices $(abc)_{L+n} = (0)_{L+n}$ and 4–events $(abcd)_{L+n} = (0)_{L+n}$.

To come closer to what we need for physics in the sense of relating the (R–frame) model to measurement ("counting") in the laboratory, we need to introduce a quantitative measure and a norm for such measures. Once we have done this, we can introduce a third operation connecting bit strings ("inner product") that supports *relative* conservation laws. Define a measure $||x||$ on (x) by

$$||x|| \equiv \Sigma_{i=1}^{n} b_i^x, \qquad x \in a, b, c \dots .$$

This is the usual Hamming measure. $||x||/n$ is McGoveran's normalized attribute distance relative to the reference string (0) ($b_i^0 = 0$ for all i; $||0|| = 0$), and $(n - ||x||)/n$ is the distance relative to the antinull string (1) ($b_i^1 = 1$ for all i; $||1|| = n$).

Consider a *3–vertex* defined by $(abc) = (0)$ or, equivalently, by $||abc|| = 0$.

Theorem 1: The measure $||x||$ is a norm, i.e.,

$$(abc) = (0) \Rightarrow |\; ||a|| - ||b||\; | \leq ||c|| \leq |\; ||a|| + ||b||\; |, \quad \text{cylic on } a, b, c \;.$$

Argument:

From the definition of discrimination, if we consider the three bits at any ordered position i in the three strings of a 3–vertex, we can only have either one zero and two ones's in the three strings, or three zeros. If the single zero is $b_i^a = 0$, call the number of times this occurs n_{bc} (cyclic on a, b, c) and the number of times we have three zero's n_0. Clearly, $n_{bc} + n_{ca} + n_{ab} + n_0 = n$ and $||a|| = n_{bc} + n_{ca}$, cyclic on a, b, c, from which the desired inequalities follow.

Note that this theorem depends on a computer memory. It is *static* in that it depends only on a particular type of configuration that is "wired in" by the program. It is *dynamic*, in the sense that the three strings are brought together as a consequence of past sequences that are *arbitrary* from the point of view of the local vertex. It is *global* in that any single 3–vertex (or 4–event) *could* lead to a TICK which affects the whole bit string universe.

If we now define the inner product $\langle (x)\cdot(y)\rangle$ between two strings $(a),(b)$ connected by a 3–vertex $(abc)=(0)$ with the equality

$$2\langle (a)\cdot(b)\rangle \equiv \|a\|^2+\|b\|^2-\|c\|^2 \;;$$

it follows immediately that

Corollary 1.1:

$$\|ab\|^2 = \langle (a)\cdot(ab)\rangle + \langle (b)\cdot(ab)\rangle = \langle (ab)\cdot(ab)\rangle \;,$$

$$\|a\|^2 = \langle (ab)\cdot(a)\rangle + \langle (b)\cdot(a)\rangle = \langle (a)\cdot(a)\rangle \;,$$

$$\|b\|^2 = \langle (ab)\cdot(b)\rangle + \langle (a)\cdot(b)\rangle = \langle (b)\cdot(b)\rangle \;.$$

If we define a *4–vertex* by $(abcd)=(0)$, or equivalently by $\|abcd\|=0$, with an obvious extension of the notation, it also follows that

Theorem 2:

$$(abcd)=(0) \Rightarrow \|a\| = \|bcd\|, \quad \textit{cyclic on abcd} \;.$$

$$\|ab\| = \|cd\|;\; \|ac\| = \|db\|;\; \|ad\| = \|bc\|$$

Argument:

$(abcd)=(0)\Rightarrow(abc)=(d)$, etc., and $\Rightarrow (ab)=(cd)$, etc., from which the result follows.

Corollary 2.1: For any pair taken from the ensemble *abcd*, the appropriate version of Corollary 1.1 follows.

Corollary 2.2:

$$\langle (a)\cdot(cd)\rangle + \langle (b)\cdot(cd)\rangle \;=\; \|ab\|^2 \;=\; \|cd\|^2 \;=\; \langle (c)\cdot(ab)\rangle + \langle (d)\cdot(ab)\rangle \;,$$

and so on, for any of the three pairs.

Theorem 3:

$$\|abcd\|=0 \Rightarrow \|a\|^2 \;=\; \langle (b)\cdot(a)\rangle + \langle (c)\cdot(a)\rangle + \langle (d)\cdot(a)\rangle \;, \quad \textit{cyclic on abcd} \;.$$

Argument:

This follows by standard (finite!) algebra.

It is tempting to go from these results for the inner product to the conclusion that a 4–vertex defines the vector conservation law

$$\vec{a}+\vec{b}+\vec{c}+\vec{d} \;=\; 0 \;,$$

and that with $\vec{d}=0$, the same is true at a 3–vertex. This, however, depends on a convention. If some vectors are "incoming" and some are "outgoing," the same algebraic relations can be interpreted as also supporting the three interpretations

$$\vec{a}+\vec{b} \;=\; \vec{c}+\vec{d};\; \vec{a}+\vec{c} \;=\; \vec{b}+\vec{d};\; \vec{a}+\vec{d} \;=\; \vec{b}+\vec{c}\,,$$

and the four interpretations

$$\vec{a}+\vec{b}+\vec{c} \;=\; \vec{d}\,; \quad \text{cyclic on } a,b,c,d \;.$$

We can base our version of "crossing invariance on these eight interpretations.

To go from what we have proved above to the usual definition of directions and angles in a "vector space" would require us to derive (among other things) rational fractions for the sines of angles whose cosines are also given by rational fractions. As Pythagorous is often credited with discovering, this problem cannot always be solved in the space of rational fractions (though, of course, he didn't put it that way). We have conservation laws at vertices; they are not *vector* conservation laws in the continuous, directional sense.

We can use "directions" to model experimental (laboratory) facts with reasonable precision. This amounts to a *rule of correspondence*: the *counter paradigm*, Chapter 9. Here we assume that at some string length N, we have either a 3–vertex or a 4–vertex involving a labeled string $(a)_N$ and that we have a second vertex involving the same label and a string $(a)_{N+n}$. From now on, we consider the latest portion of the string of length n and interpret it as a "random walk" in which a "1" represents a step in the + direction and a "0" a step in the − direction. Then the "distance" between the two vertices can be defined as $2\|a\| - n$. The direction is established macroscopically by thinking of the vertices as sequential "counter firings" involving the same "particle" separated by distance L and a (positive) time interval T. Since, empirically, such events always define a velocity $V = L/T$ less than or not measurably different from the limiting velocity c, we relate this type of laboratory fact to our bit string model by taking $V = \beta c$ with $\beta_a = 2\|a\|/n - 1$, and the positive direction along this line defined by the positive sign for β.

Since a 4–vertex $(abcd) = (0)$ can be decomposed in seven different ways, namely

$$(ab) = (cd)\ ; \quad (ac) = (bd)\ ; \quad (ad) = bc)\ ;$$

$$(a) = (bcd)\ ; \quad (b) = (cda)\ ; \quad (c) = (dab)\ ; \quad (d) = (abc\ ;)$$

we can—by appropriate identification of the directions with sequential counter firings in the laboratory—make seven different temporally ordered interpretations of the single 4–vertex given above: three (2,2) channels, four (3,1) channels and the unobservable (4,0) channel. Note that all eight relationships are generated by one 4–vertex.

Our next step is to recall that we can always separate a string into two strings $(a)_{L+n} = (L_a)_L \| (A_a)_n$ where "$\|$" denotes string concatenation. We call the first piece the *label* and the second the *content*. There is a simple correlation between the two pieces. If we take some content string A_a with velocity $\beta_a = 2\|A_a\|/n - 1$, the string $(a1)$ has the opposite velocity. Further, if we use the string (a) as the reference string for a conservation law defined by the inner product relations given above, the reversal of the velocity achieved by discrimination with the antinull string also reverses all the label conservation laws. However any system of bit strings has a *dual* system with all zeros and ones interchanged, but precisely the same algebraic structure. Thus, the theory is invariant under the *arbitrary* choice of reference direction and the *arbitrary* choices of the dichotomous reference symbols in the label, provided they all reverse on this same interchange.

8. THE STANDARD MODEL FOR QUARKS AND LEPTONS USING COMBINATORIAL HIERARCHY LABELS

Physical interpretation of the labels naturally starts with the simplest structures, which are the weak and electromagnetic interactions. We can get quite a long way just by looking at the leading terms in a perturbation theory in powers of $e^2/\hbar c \simeq 1/137$ for quantum electrodynamics and of $G_F \simeq 10^{-5}/m_p^2$ for the low energy weak interactions, such as beta decay. As Lee and Yang saw, if the neutrino is massless and chiral, the Fermi β–decay theory will violate parity conservation maximally; this is still the simplest accurate description of low-energy, weak interactions.

Since ***level 1*** has only two basic entities, we identify these with the neutrino ν and the antineutrino $\bar{\nu}$. Their closure is the zero helicity component of the spin–1 neutral weak boson Z^0, defining the 3–vertex $(\nu\bar{\nu}Z^0)$. If we follow the usual convention of defining the chirality of the neutrino as "left-handed," once we have added content strings and defined directions, we still need a convention as to whether the label is to be concatenated with the string $(1)_n$ with velocity $+c$ or the string $(0)_n$ with velocity $-c$. We can take the bit string state $(\nu_L)_{L+n} = (\nu_\lambda)_L \| (1)_n$ and the right-handed (i.e., anti) neutrino $(\nu_R)_{L+n} = (\bar{\nu}_\rho)_L \| (0)_n$. Then, if we use a representation in which $(\nu_\rho)_L = (1\nu_\lambda)_L$, the Feynman rules will be obeyed. The vertex can be interpreted as representing the physical processes $\nu_L + \nu_R \leftrightarrow Z_0^0$, $\nu_L \leftrightarrow \nu_L + Z_0^0$ or $\nu_R \leftrightarrow \nu_R + Z_0^0$, depending on context. Taking all three particles as incoming (or outgoing), the quantum numbers add to zero—as they should—while, if we reverse the direction of either neutrino to make it outgoing, it becomes the same as the incoming neutrino. Note that for massless particles ($\beta = \pm c$), we cannot specify a direction until we connect them to slower particles whose directions can be assigned. Thus we are forced to adopt a Wheeler–Feynman type of theory, in which all massless "radiation" emitted by charged particles must be absorbed; we will see later that charged particles must be massive and, hence, must have $|\beta| < c$.

Interpretation of *level 2* as modeling the vertices of quantum electrodynamics for electrons, positrons and photons is almost as easy. We take as the linearly independent basis strings $(e_\lambda^+), (e_\lambda^-), (\Gamma_{\lambda\lambda})$ and define the nonnull string which guarantees their independence as $(\Gamma_c) = (e_\lambda^+ e_\lambda^- \Gamma_{\lambda\lambda})$. The remaining three label strings which close essi level 2 are then defined by

$$(e_\rho^+) = (\Gamma_c e_\lambda^-) \; ; \qquad (e_\rho^-) = (\Gamma_c e_\lambda^+) \; ; \qquad (\Gamma_{\rho\rho}) = (\Gamma_c \Gamma_{\lambda\lambda}) \; .$$

We take the same convention for positive direction and chirality as we did for *level 1*, using the negative, left-handed electron as our reference string and the velocity $\beta_{e_L^-} = 2k_{e_L^-}/n - 1$ as positive when this number is positive. The physical states, where we omit the subscripts on β, are then given by

$$(\gamma_c)_{L+n} = (\Gamma_c)_L \| (1)_n \; ; \quad (e_L^-) = (e_\lambda^-) \| (-\beta)_n \; ; \quad (e_L^+) = (e_\lambda^+) \| (-\beta)_n \; ,$$

$$(e_R^+) = (e_\rho^+) \| (\beta)_n = (\gamma_c e_L^-) \; ; \quad (e_R^-) = (e_\rho^-) \| (\beta)_n = (\gamma_c e_L^+) \; ,$$

$$(\gamma_{RR}) = (\Gamma_{\rho\rho}) \| (1)_n \; ; \quad (\gamma_{LL}) = (\Gamma_{\lambda\lambda}) \| (0)_n = (\gamma_c \gamma_{RR}) \; ,$$

and the Feynman rules are obeyed for all 3–vertices.

The 4–vertex $(e\bar{e}\gamma\gamma_c) = (0)$ cannot be readily discussed until we have the configuration space theory nailed down. It is related to our finite treatment of Bremstrahlung in a "coulomb field." The vertex $(\gamma_{LL}\gamma_{RR}\gamma_c) = (0)$ would seem to imply an interaction between photons and the "coulomb field,"—a vertex that vanishes in the conventional theory because of the masslessness of the photon and gauge invariance.

A related problem arises with the vertices implied by our connection between particles and antiparticles, namely

$$(\nu\bar{\nu}\,1) = (0) \; ; \quad (e\bar{e}\,1) = (0) \; ; \quad (\gamma\bar{\gamma}\,1) = 0 \; .$$

A little thought shows that such vertices will occur for *any* particle-antiparticle pair. Hence, the antinull label string "interacts" with everything and must be assigned to *level 4*. This unique label string, which occurs with probability $1/(2^{127} + 136)$, is identified with Newtonian gravitation. It leads to the bending of light in a "gravitational field," as we will show at a later stage in the development of the theory. Of course, to get the experimentally observed result, we will have to identify the "spin–2" gravitons as well, and show that they double this deflection.

These problems will have to be deferred until we have articulated the theory further. We conclude this article by identifying the *level 3* structure with the quarks and gluons of quantum chromodynamics. This discussion follows along the lines already laid down in discussing the first two levels. We take as our basis label strings a quark part $(u^+), (u^-), (d^+)$ or (d^-) concatenated with a color part $(r), (y), (b)$, which gives us the seven independent strings needed to form *level 3*. The color strings are linearly independent, so we can define (analogous to what we did at *level 2*)

$$(ryb) = (w)\ ; \quad (\bar{r}) = (rw)\ ; \quad (\bar{y}) = (yw)\ ; \quad (\bar{b}) = (bw)\ ,$$

from which it follows that

$$(ry\bar{b}) = (0)\ ; \quad (r\bar{y}b) = (0)\ ; \quad (\bar{r}yb) = (0)\ ; \quad (\bar{r}\bar{y}\bar{b}) = (0)\ .$$

Similarly, the linear independence of the quark parts allows us to define

$$(u^+u^-d^+d^-) = (Q)\ ; \quad (\bar{q}) = (qQ)\ , \qquad q \in u^+, u^-, d^+, d^-\ .$$

Then a colored quark label $(q_c^{\pm}) = (q^{\pm})\|(c)$ and a colored gluon label $(g_c) = (Q)\|(c)$, $c \in r, y, b$ allow us to recognize the label part of the Yukawa vertex for QCD as $(q_{c_1}\bar{q}_{c_2}g_{c_3}) = (0)$. The essential point here is that, as proved above, $(c_1\bar{c}_2c_3) = (0)$ for any three distinct colors. We can then attach content labels and helicity in the same way we did in QED, and once again the Feynman rules apply. Any one familiar with lowest order QCD can now immediately derive from our formalism the "valence quark" structure of the proton and neutron in terms of three quarks, and the structure of the π, ρ and ω in terms of quark-antiquark pairs. In contrast to the *level 2* situation, the 3–gluon vertex does not vanish and implies a 4–gluon vertex, so we find that we have constructed *all* the lowest order vertices of QCD with the correct conservation laws.

The problem of "color confinement" is solved, in principle, by *McGoveran's Theorem* [24,25]; i.e., the conclusion that in any finite and discrete theory there can be no more than three "homogeneous and isotropic dimensions" that remain indistinguishable as the (finite and discrete) cardinals and ordinals keep on increasing. (We discuss this theorem with more care in Chapter 9.) Because our labels are tied to contents and, hence, via the counter paradigm to macroscopic directions, we can only have three quantum number "dimensions" asymptotically. These are saturated by the three absolutely (so far as we know currently) conserved quantum numbers: lepton number, baryon number and charge (or "z–component" of isospin), leaving no room for free quarks or gluons conserving asymptotic "color charge." They can occur at short distance as degrees of freedom in the scattering theory—as we showed above—but eventually they have to "compactify" and become distinguishable from free particle quantum numbers. We can conclude this immediately without any detailed dynamical argument.

9. THE COUNTER PARADIGM

Bastin has insisted for decades that the primal contact between a (computable) formalism and the empirical "world" can only be made once. This was a basic reason why he and Kilmister [18,19] fastened on steps of a scattering process as a likely point at which to investigate the connection between finite mathematics and physical theory. I started thinking of the elementary scattering process as fundamental, thanks to my early involvement in Chew's S–Matrix theory; for me this gave specific content to Bridgman's operationalism and Heisenberg's very early ideas. At ANPA 2 and 3 some of us saw that Stein's "random walk" derivation of the Lorentz transformation and the Uncertainty Principle [26] must somehow connect to scattering processes; others recognized the seminal nature of his work because of his ontological viewpoint.

The specific genesis of the "counter paradigm" occurred after my presentation [27] at the conference honoring deBroglie's 90^{th} birthday. Fortunately, I had an opportunity to start working on the final version of that paper [28] in consultation with Ted Bastin before it was published. I realized that if I thought of Stein's "random walk" as a model for two sequential events in two spatially separated laboratory counters with the discrete step length being the deBroglie relativistic phase wavelength, that by representing Stein's random walks as bit strings with the bit 1 taken as a step toward the final counter and the bit zero a step away from it, I had the right point of contact between the bit strings used in the *combinatorial hierarchy* and the start of a scattering theory.

So far we have only discussed 3– and 4–vertices for a fixed value of n, but each time program universe TICKs, each content string in each labeled ensemble acquires an arbitrary bit at the growing end. In the absence of further information, each content string therefore represents a sequence of Bernoulli trials, with 0 and 1 representing the two possibilities. This has an extremely important consequence, which we call *McGoveran's Theorem* [24,25]. As has been noted by Feller [29], if we have D *independent* sequences of Bernoulli trials, the probability that after n trials we will have accumulated the same number (k) of one's is $p_D(n) = (\frac{1}{2^{nD}})\Sigma_{k=0}^{n}\binom{n}{k}^D$. He then shows that the probability that this situation will repeat N times is strictly bounded by

$$P_D(N) \; = \; \Sigma_{n=1}^{N} \, p_D(n) < \left[\frac{2}{\pi D}\right]^{-\frac{1}{2}} \; \Sigma_{n=1}^{N} \, n^{\frac{1}{2}(D-1)} \; .$$

Consequently, for $D = 2,3$ where $p_D(n) < n^{-\frac{1}{2}}, n^{-1}$, such repetitions can keep on occurring with finite probability; but for four or more independent sequences, this probability is strictly bounded by zero in the sense of the law of large numbers.

McGoveran uses finite attributes, which can always be mapped onto ordered strings of zeros and ones, as the starting point for his ordering operator calculus. As is discussed in more detail in Ref. [1], these can be used to construct a finite and discrete metric space. In order to introduce the concept of *dimensionality* into this space, he notes that we need some metric criterion that does not in any way distinguish one dimension from another. (In a continuum theory, we would call this the property of "homogeneity and isotropy"; we need it in our theory for the same reason Einstein does in his development of special relativity.) McGoveran discovered that by interpreting the coincidences $n = 1, 2, \ldots, N$ in Feller's construction as "metric marks," the metric space so constructed has precisely the discrete property corresponding to "homogeneity and isotropy" as just defined. Consequently, Feller's result shows that in *any* finite and discrete theory, the number of independent "homogeneous and isotropic" dimensions is bounded by three! If we start from a larger number of independent dimensions using *any* discrete and finite generating process for the attribute ensembles, we find that the metric will, for large numbers, continue to apply to only three of them, and that what may have looked like another dimension is not; the probability of generating the next "metric" mark in any of the others (let alone all of them) is strictly bounded by $1/N_{MAX}$!

Of course, the argument depends on the theory containing a *universal ordering operator* which is isomorphic to the ordinal integers. Further—since we know empirically that "elementary particles" are *chiral*—we will need three, rather than two "spatial" dimensions. Thus *any* discrete and finite theory such as ours, when applied to physics, must be globally described by three dimensions and a monotonically increasing order parameter. Consequently, we are justified in constructing a "rule of correspondence" for our theory which connects the large number properties of our R–frame to *laboratory* (E–frame) 3 + 1 space-time. Earlier treatments of the "counter paradigm" simply took this possibility for granted. McGoveran's Theorem fills this serious logical gap.

We begin with the paradigmatic case of a single particle entering a space-time volume (detector) $\Delta V \Delta T$, causing a count and a time T, later entering a second detector with similar resolution a macroscopic distance L from the first and causing a second count. We then say that the (average) velocity of the particle between the two detectors is $V = L/T$; empirically, this number is always less than or indistinguishable from the limiting velocity c.

This language is well-understood by the particle physics experimentalist, but raises a number of problems for others. To begin with he uses "cause" in a philosophically vague but methodologically precise sense, which includes a host of practical experience about "background," "spurious counts," "real counts," "goofs," "GOK's" (i.e., "God only knows"), ...

The actual practice of experimental particle physics implies the concept of *indistinguishability* in a critical way; the experimentalist uses, often without conscious analysis, finite collections whose cardinal number may exceed their ordinal number; this fact is diagnostic for *sorts* that are not reducible to *sets* [24]. To put it more formally in terms of "background" and "counts," in the absence of a constructive definition of the two subsets—which is often unavailable in practice, and in our theory we would claim can be unavailable in principle—the two collections are *sorts* rather than *sets*.

The rule of correspondence in the counter paradigm case (two sequential counts spatially separated) applies to a labeled string with label L_a, which at the TICK with the content string length n_0 was part of a 3- or 4-vertex and, again, part of a vertex at content string length n_0+n_a, AND WHICH IS APPROPRIATELY ASSIGNED TO THEORETICALLY RELEVANT DATA RATHER THAN TO BACKGROUND. We ask how many one's were added to the content string; we call these k_a. We identify the (average) laboratory velocity of the particle ($V = L/T$) with the R-frame quantity by the equation $V = [(2k_a/n_a) - 1]\,c$. The sign of this velocity defines the positive or negative sense of the direction between the counters in the laboratory (or visa versa: a choice must be made *once*). Since the evolution of the bit string universe will provide many candidates for the strings which meet these criteria within the time and space resolution of the counters, we will have to provide more and more precise definitions of these criteria as the analysis develops.

10. EVENT-BASED COORDINATES AND THE LORENTZ TRANSFORMATIONS

As is discussed with much more care in Ref. [1], any theory satisfying our principles can be mapped onto ensembles of bit strings simply because, with respect to *any* attribute, we can say whether a collection has that attribute or does not. To introduce a metric, we need a distance function *relative* to some reference ensemble. Because of our finite and discrete principles, any allowed program can only take a finite number of steps to bring any ensemble into local isomorphism with the reference ensemble *in respect to that attribute.* Note that there can be many attributes, many distance functions, and that the space can be multiply connected. Note also that this definition also provides a (dichotomous, e.g., $\pm$) *sense* to the computation steps: they must increase the attribute distance or decrease it. Calling the number of increments I and the number of decrements D, using a well-defined computational procedure, the attribute distance is, clearly, $D_A = I - D$, and the total number of steps $N = I + D$. Then we can also define the *attribute velocity* with which the two ensembles are "separating" or "coming together," $V_A = (I - D)/(I + D)$. Thus, there always is a "limiting velocity" for each attribute, which is attained when all steps are taken in the same direction.

If we wish to model the events of which contemporary physics takes cognizance, we know that all physical attributes are directly or indirectly coupled to electromagnetism. Therefore, the limiting velocity of physics, c, will be the *smallest* of these limiting attribute velocities, simply because it refers to the attribute with the maximum cardinality. Any ensemble of attributes specified by a more limited description involves a "supraluminal" velocity, without allowing supraluminal communication of information. Hence, we can expect to find correlation between and synchronization of events in space-like separated regions; from our discrete point of view, the existence of the effects demonstrated in Aspect's and other EPR-Bohm experiments is anticipated and in no way paradoxical. We guarantee Einstein locality for *causal* events; that is, for those initiated by the transfer of *physical* information [30].

In order to go from this general proof of the limiting velocity to the laboratory practice of relativistic particle quantum mechanics, we need a more specific formalism than the general derivation given in Ref. [1]. We start from the 3- and 4-vertices already mentioned and consider how they can be used to model the "laboratory" situation given in Fig. 11. The initial 4-vertex $(abcd)_{L+n_0} = 0$ is followed sequentially by five vertices involving "soft" photons. In the laboratory neither vertices, nor elementary events, nor soft photons can be observed; limiting cases in which the disturbance caused by the firing of counters connected with these five events is negligibly small are easy to envisage. We use a specific

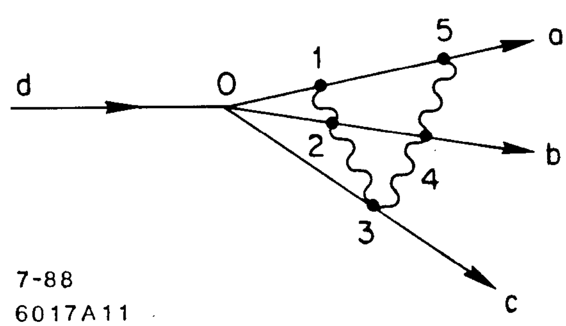

Fig. 11. A 4-event followed by five events involving limiting velocity signals which can be used to establish the Lorentz transformations for Event 3.

example of labels that can, if we wish, be given a specific interpretation in which particles a, b, c have spin–1/2 and the photons have left or right spin–1 helicity.

We assume that it takes n_i TICKs of program universe beyond $L+n_0$ to generate the strings involved in the i^{th} event. Since all strings will have the portion through content string length n_0 unaltered, we need use only these *relative* values: $n_i = N_U(i) - L - n_0$ and the corresponding terminal pieces of the strings for our contents. For Event 1, we take the three strings to be

$$(a) = (1000)\|(A_1^a)_{n_1} ; \quad (a') = (0100)\|(A_1^a)_{n_1} ; \quad (\bar{\gamma}) = (1100)\|(0)_{n_1} .$$

Hence, $(aa'\bar{\gamma}) = (0)$ defines a 3–vertex in which the velocity of a does not change; we could call it a "soft photon" vertex. By crossing (cf., Chapters 3 and 7 above), this also can be interpreted as a vertex in which a flips its spin and emits a photon with the appropriate helicity; i.e., $(\gamma) = (0011)\|(1)_{n_1}$. The laboratory direction between Events 1 and 2 then defines the reference direction for all subsequent discussion. The remaining vertices can be consistently represented by using

$$(b) = (1000)\|(A_2^b)_{n_2} ; \quad (\gamma) = (0011)\|(1)_{n_2} ; \quad (b') = (0111)\|(A_2^b)_{n_2} ;$$

$$(\gamma') = (1100)\|(1)_{n_2} ;$$

$$(c) = (1000)\|(A_3^c)_{n_3} ; \quad (\gamma') = (1100)\|(1)_{n_3} ; \quad (c') = (0111)\|(A_3^c 1)_{n_3} ;$$

$$(\bar{\gamma}') = (0011)\|(0)_{n_3} ,$$

$$(b') = (0111)\|(A_4^{b'})_{n_4} ; \quad (\bar{\gamma}') = (0011)\|(0)_{n_4} ; \quad (b'') = (1000)\|(A_4^{b'})_{n_4} ;$$

$$(\bar{\gamma}) = (1100)\|(0)_{n_4} ;$$

$$(a') = (0100)\|(A_5^{a'})_{n_5} ; \quad (a'') = (1000)\|(A_5^{a'})_{n_5} ; \quad (\bar{\gamma}) = (1100)\|(0)_{n_5} .$$

We now trust that our rule of correspondence between 3– and 4–vertices and a standard "laboratory" situation used in the derivation of the Lorentz transformations is clear.

For simplicity, we consider here that particle a is, *on the average*, "at rest" between Events 0, 1 and between Events 1, 5:

$$k_0^a = \frac{n_0}{2} ; \qquad k_1^a = \frac{n_1}{2} ; \qquad k_5^a = \frac{n_5}{2} .$$

We also assume, again *on the average*, that b and c have constant velocity over the appropriate intervals:

$$\beta_b = 2\frac{k_0^b}{n_0} - 1 = 2\frac{k_2^b}{n_2} - 1 = 2\frac{k_4^b}{n_4} - 1 ,$$

$$\beta_c = \beta = 2\frac{k_0^c}{n_0} - 1 = 2\frac{k_3^c}{n_3} - 1 .$$

Our next simplification is to assume that all the events lie on a single "line," reducing this to a 1+1 dimensional problem. None of these simplifications are needed, as can be seen from the general discussion in Ref. [1].

In conventional terms, we are asking the question of how the coordinates of an event at $x = \beta ct$ in one coordinate system (the one in which particle a is at rest) transform to the coordinate system in which particle b is at rest. We assume, as in conventional treatments, that the velocity of light is the same in all coordinate systems and that the time at which Event 3 occurs is the average between when the light signal that defines Event 3 was emitted by a and returns to it. Introducing a parameter with the dimensions of length, whose value we will discuss later, these statements follow immediately from the definitions of attribute distance and velocity, since

$$\frac{x}{\lambda} = 2k - n ; \qquad \frac{ct}{\lambda} = n ; \qquad \beta = \frac{2k}{n} - 1 ,$$

for any particle, and $k = 0$ or n specifies a connection with the limiting velocity for any set of strings. This is even clearer when we introduce "light cone" coordinates:

$$d_+ = n + (2k - n) = 2k ; \qquad d_- = n - (2k - n) = 2(n - k) .$$

The relationship between the two descriptions is illustrated in Fig. 12.

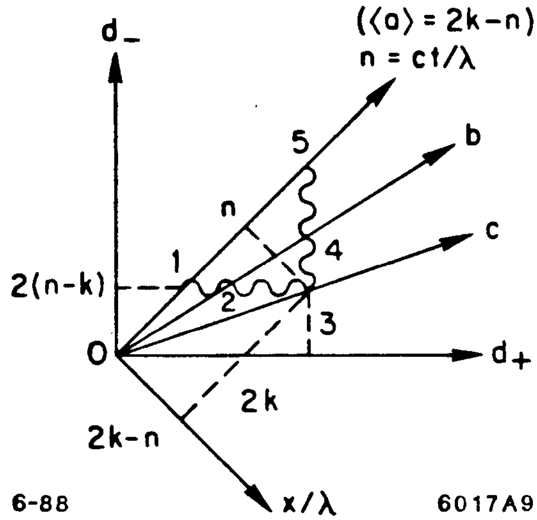

Fig. 12. The connection between space-time and light-cone coordinates in terms of bit string distances and velocities for the physical situation envisaged in Fig. 1.

One way to derive the Lorentz transformations is to require that the interval s between Events 0 and 3 be invariant, where

$$\frac{s^2}{\lambda^2} = \frac{c^2t^2 - x^2}{\lambda^2} = n^2 - (2k - n)^2 = 4k(n - k) .$$

In light cone coordinates this relationship becomes

$$d_+d_- = 4k(n - k) = \frac{s^2}{\lambda^2} ,$$

which makes one way of insuring the invariance requirement particularly simple, namely

$$k' = \rho k , \quad n' - k' = \rho^{-1}(n - k) \Rightarrow 4k'(n' - k') = 4k(n - k) .$$

Note that if we are to compare the integer *bit string* coordinates, this restricts k' to be a rational multiple of k. One of the great successes of our theory is precisely this restriction that keeps events an integral number of deBroglie wavelengths apart. A fundamental explanation of why our theory can contain "interference" phenomena starts here.

If we now note that

$$d_{\pm} = (1 \pm \beta)\, n \ ,$$

the invariance requirement gives us

$$\frac{k'}{k}\,\frac{n-k}{n'-k'} \;=\; \rho^2 \;=\; \frac{1+\beta'}{1+\beta}\,\frac{1-\beta}{1+\beta'} \ .$$

Hence,

$$\beta_\rho \;=\; \frac{\beta'-\beta}{1-\beta\beta'} \iff \rho^2 \;=\; \frac{1+\beta_\rho}{1-\beta_\rho} \ .$$

From the fact that when transforming from a system at rest $d_+/d_- = 1$, we see that the relative velocity between the two systems is simply β_ρ; we have derived the velocity composition law for rational fraction velocities in any system. Tom Etter arrived at this composition law for attribute velocities on general grounds, as is discussed in Ref. [1]. With

$$\gamma \;=\; \frac{1}{2}\left[\rho + \rho^{-1}\right] \ ,$$

we have that

$$x' \;=\; \gamma(x + \beta_\rho ct) \ ; \qquad t' \;=\; \gamma(ct + \beta_\rho x) \ .$$

QED

11. QUANTUM MECHANICS

Program universe provides an ***invariant*** significance for the label strings, once they close (in some length, with at least 139 bits) to form some basis for some realization of the combinatorial hierarchy. For each of the $2^{127} + 136$ labels L_ℓ, we can assign a dimensional parameter λ_0^ℓ, which is the step length when the particle is "at rest"; i.e., when on the average $2k_\ell = n_\ell$. Since program universe increases the string length one arbitrary bit at a time, this requirement can at best be satisfied only at every other step. We have seen that when all steps are in the same direction (i.e., when the content string is either the null string or the antinull string), this corresponds to a "light signal." In any string evolution, all steps are executed at the limiting velocity c—a finite and discrete "zitterbewegung." The invariance of λ_0^ℓ allows us to associate with each label an invariant parameter with the dimensions of mass m_0^ℓ, and relate the two by $\lambda_0^\ell = h/m_0^\ell c$, where h is a universal constant with the dimensions of action. We will now show that h can indeed be identified with Planck's constant.

The extension of our Lorentz transformations to momentum space is now immediate. We simply define $E = \gamma m_0 c^2$, $p = \gamma\beta m_0 c$. For $p_\pm = E/c \pm p$, we have $p_+p_- = m_0^2c^2$, $p_+/p_- = k/(n-k)$ and $(p_+x_- + p_-x_+)/2 = Et - px$. The justification of calling this "momentum" is more than definitional; we showed above that 3– and 4–vertices support "vector" conservation laws and "crossing symmetry." We have 3–momentum conservation in any allowed event-based reference frame. Clearly, $m_0c\lambda_0 = h = E\lambda/c$ in any allowed coordinate system, and we have recovered the initial identification of the step length in the "random walk" as $\lambda = hc/E$, the deBroglie phase wavelength with which our initial statement of the "counter paradigm" began. We can now ***derive*** the quantum mechanical commutation relations from our model.

We note that if we consider a system that evolves with constant velocity $\beta_0 \equiv 2k_0/n_0 - 1$, strings which grow subject to this constraint, i.e., $n = n_T n_0, k = n_T k_0, 1 \leq n_T \leq n/n_0$, will have a periodicity $T \equiv n_T\Delta t = n_T\lambda/c$ specifying the events in which this condition can be met. Hence, in more complicated situations where there can be more than one "path" connecting strings with the same velocity to a single event, this event can occur only when the paths differ by an integral number of "d–wavelengths" λ. Thus, our construction already contains the seeds of "interference" and a conceptual explanation of the "double slit experiment."

We have already seen that any system with "constant velocity"—at those "ticks" when events can occur—evolves by discrete steps $\pm\lambda_a$ in $x = q_a$ *between* ticks. McGoveran's ordering operator calculus [1] which specifies the connectivity between events allows these discrete happenings to occur in a *void* where space and time are meaningless. Since $\lambda/\Delta t = c$, each step occurs forward or backward with the limiting velocity; thus, we deduce a discrete *zitterbewegung* from our theory. If we think of this as a "trajectory" in the pq phase space, each time-step induces a step $\pm\lambda$ in q correlated with a step $\pm mc$ in p. Even in the case of a particle "at rest," this must be followed by two steps of the opposite sign to return the system to "rest." Thus there is, minimally, a 4-fold symmetry to the "trajectory" in phase space corresponding to the generation periodicity we discovered above.

If we now recall from classical mechanics [31] that for any momentum which is a constant of the motion, we can transform to angle and action variables, with $\oint p_J dq_J = J$ where J has the dimensions of action, $p_J = J/2\pi$ and q_J is cyclic, we have an immediate interpretation. In the classical case, the "period" goes to infinity for a free particle; for us, we have already seen that we have a *finite* period $T = \lambda/c$. Therefore, we can immediately identify $m_a c\lambda_a = J = n_T h$; we have constructed Bohr–Sommerfeld quantization within our theory.

To go on to the commutation relations, we can replace the geometrical description of periodic trajectories in phase space by using complex coordinates $z = (q, ip)$ [or by $(q_J, in_T h/2\pi)$, where q_J is restricted to $2n+1$ values with $-n_T \leq n \leq +n_T$]. Then the steps around the cycle in the order $qpqp$ are proportional to $\pm 2\pi\,(1, i, -1, -i)$, where $\pm$ depends on whether the first step is in the positive or negative direction or, equivalently, whether the circulation is counterclockwise or clockwise. We have now shown why $qp - pq = \pm i\hbar$ for free particles in our theory; this result holds for any theory satisfying our principles which uses a discrete free particle basis.

In order to go to a detailed 3–dimensional description, we must supply three linearly independent reference strings, define inner products with respect to them (cf., Chapter 7) and go to a "coordinate" description. There will then be three independent periodicities (velocities and momenta) which will commute with each other but not with their conjugate position variable. The commutation relations for angular momentum follow immediately. Since this has already been shown in quite general terms in Ref. [1], we will leave the details to future publications. An alternative is to develop the "radial coordinate" (n, l, m) description using "bound states" as the basis.

Now that we have two ($\hbar$ and c) of the 3–dimensional constants needed to connect a fundamental theory to experiment in the 3–space in which physics operates, and which we have proved must be the asymptotic space of our theory, all that remains is to determine the unit of mass. This has already been done for us by the combinatorial hierarchy result $2^{127} + 136 \simeq 1.7 \times 10^{38} \simeq \hbar c/Gm_p^2 = (M_{Planck}/m_p)^2$, which tells us that we can either identify the unit of mass in the theory as the proton mass, in which case we can calculate (to about 1% in this first approximation) Newton's gravitational constant or—if we take the Planck mass as fundamental—calculate the proton mass. From now on, we have to compute everything else. If we fail to agree with experiment to the appropriate accuracy (one of the rules of correspondence!), we must either revise or abandon the theory.

12. A DISCRETE MODEL FOR THE BOHR ATOM

We have seen that any bit string has the deBroglie periodicity h/mc^2 for each digital "time step" $\Delta n = 1$ and that, when it evolves with "constant velocity," also has the longer digital period n_0 connected to the velocity by $\beta = 2k_0/n_0 - 1$ at each finite "position" $N_{ph} n_0 \beta = N_{ph}(2k_0 - n_0)$, where an event can (but need not) occur after the initial vertex at $N_{ph} = 0$. Note that we are not interested in particles "at rest." We define $\Delta k_0 = k_0 - n_0/2$ and, hence, $\beta = \Delta k_0/n_0$. Only one integer can be added to the string at each step. This must happen Δk_0 times before the periodic pattern can be completed. Therefore, the number of step lengths in the periodic pattern—the *coherence length*—is $n_0 = 1/\beta$. Since, as we saw above, the step length is $\lambda = hc/E$, we find that the coherence length required for periodic phenomena at constant velocity is $\lambda_g = hc/\beta E = h/p$.

By adding a constraint representing a second periodicity, we can now model the periodicity representing a "closed orbit around some fixed center." Clearly, this periodicity must use the coherence length derived above if we are to have a stable, repeating pattern that starts from some "origin" and closes after N_B coherence lengths. This model, which only describes the average "motion," will persist from the time when we start the model off to the time when some vertex—for example the absorption of a "hard" photon—ends the finite sequence of periods. Of course, this can only occur at one of the positions allowed for events. In the average sense, we can image this "trajectory" as a regular polygon with N_B sides of length λ_g. With the usual "geometrical" image in mind, we call the distance traversed in this period "$2\pi R$" $= N_B\lambda_g$ and, hence, $mvR = N_B\hbar$. Afficionados of the early history of quantum mechanics will recognize that we have constructed a digital version of deBroglie's analysis of the geometry of the Bohr atom and produced a reason for angular momentum quantization. For the meaning of "π" in a discrete and finite theory, refer to the discussion in Ref. [1].

Although this part of the derivation of the Bohr atom should be reasonably familiar, our introduction of the "electromagnetic interaction" will be radically different from the conventional approach. We have seen above that the coulomb interaction is represented by only one out of 137 labels in the combinatorial hierarchy construction, and that strings evolve by the arbitrary selection of strings from memory to calculate the vertices (thanks to the counter paradigm, these vertices have now become "events"). In the case at hand, 136 of these choices can only provide a "background" which will cause fluctuations of the position of our particle; on the average these must cancel out. Only once in 137 times will the step correspond to the vertex that serves to keep the particle in its orbit. We can think of this as happening at the vertices of the polygon; i.e., N_B times in one full period. So, compared to the basic evolution time, we find that for this electromagnetic orbit, $\beta = 1/137N_B$. Making the hierarchy identification $137 = \hbar c/e^2$, our quantization condition derived above then gives us the standard result $R = N_B^2\hbar^2/me^2$ and an explanation of the old puzzle of why the Bohr radius is 137 times the Compton wavelength!

To calculate the binding energy, consider the energy change between this average motion and the particle at rest caused, for example, by the emission or absorption of a photon. We must use the average velocity because, in the absence of other information, we cannot know "where" in the orbit the interaction occurs. Our theory can readily accommodate emission and absorption of photons—conserving both momentum and energy—as we have seen in our derivation of the Lorentz transformations, and can include the usual recoil correction, if we so desire. Thus, we argue that the binding energy ϵ_{N_B} is related to the velocity $\beta_{N_B} = 1/137N_B$ by $(\epsilon_{N_B} + m_0c^2)^2 = m_0^2c^4/(1-\beta_{N_B}^2)$, from which all the usual results for the Bohr atom follow to order β^2.

13. SCATTERING THEORY

To construct a scattering theory, we need to provide the connectivity between events. To obtain a statistical connection between events, we start from our counter paradigm and note that, because of the macroscopic size of laboratory counters, there will always be some uncertainty $\Delta\beta$ in measured velocities, reflected in our integers k_a by $\Delta k = \frac{1}{2}N\Delta\beta > 0$. A measurement which gives a value of β outside this interval will have to be interpreted as a result of some scattering that occurred among the TICK's that separate the event (firing of the exit counter in the counter telescope that measures the initial value of $\beta = \beta_0$ to accuracy $\Delta\beta$), which defines the problem and the event which terminates the "free particle propagation"; we must exclude such *observable* scatterings from consideration.

What we are interested in is the probability distribution of finding two values k, k', within this allowed interval, and how this correlated probability changes as we TICK away. If $k = k'$, it is clear that when we start, both lie in the interval of integral length $2\Delta k$ about the central value $k_0 = \frac{N}{2}(1+\beta_0)$. When $k \neq k'$, the interval in which both can lie will be smaller and will be given by

$$[(k+\Delta k) - (k' - \Delta k)] \;=\; 2\Delta k - (k' - k) \;,$$

when $k' > k$, or by $2\Delta k + (k' - k)$ in the other case. Consequently, the correlated probability of encountering both k and k' in the "window" defined by the velocity resolution, normalized to unity when they are the same, is $f(k,k') = [2\Delta k \mp (k'-k)]/[2\Delta k \pm (k'-k)]$, where the positive sign corresponds to $k' > k$. The correlated probability of finding two values k_T, k_T' after T TICKs in an event with the same labels and same normalization is $[f(k_T, k'_T)]/[f(k,k')]$. This is one if $k' = k$ *and* $k'_T = k_T$. However, when $k' \neq k$, a little algebra allows us to write this ratio as

$$\frac{1 \pm \frac{2(\Delta k - \Delta k_T)}{(k'-k)} + \frac{4\Delta k \Delta k_T}{(k'-k)^2}}{1 \mp \frac{2(\Delta k - \Delta k_T)}{(k'-k)} + \frac{4\Delta k \Delta k_T}{(k'-k)^2}} .$$

If the second measurement has the same velocity resolution $\Delta\beta$ as the first, since $T > 0$, we have that $\Delta k_T < \Delta k$. Thus, if we start with some specified spread of events corresponding to laboratory boundary conditions and tick away, the fraction of connected events we need consider diminishes. If we now ask for the correlated probability of finding the value β', starting from the value β for the sharp resolution approximation (i.e., ignoring terms smaller than $1/T$ or proportional to $1/T$ and smaller), this is one if $\beta = \beta'$, and bounded by $\pm 1/T$ otherwise. That is, we have shown that in our theory a free particle propagates with constant velocity with overwhelming probability—our version of Newton's first law, and Descartes' principle of inertia.

Were it not for the $\pm$, the propagator in a continuum theory would simply be a δ–function. In our theory, we have already established relativistic "point particle" scattering kinematics for discrete and finite vertices connecting finite strings. We also showed that the order in which we specify position and velocity introduces a sign that depends on which velocity is greater, which in turn depends on the choice of positive direction in our laboratory coordinate system and, hence, in terms of the general description on whether the state is incoming or outgoing. In order to preserve this critical distinction in our propagator and keep away from the undefined (and undefinable for us) expression *const.*/0, we write the propagator as

$$P(\beta, \beta') = \left[\frac{-i\eta\lambda}{\beta' - \frac{\beta \mp i\eta}{T}} \right] ,$$

where η is a positive constant less than T. The normalization of the propagator depends on the normalization of states, and is best explored in a more technical context, such as the relativistic Faddeev equations for a finite particle number scattering theory in the momentum space continuum approximation, being developed elsewhere [9–12].

14. A TEMPORARY HALT

Each paper I have written for *ANPA Proceedings* has had to stop at an unsatisfactory point for me, and I fear for any reader who has persisted to the end. In the past, I have clobbered together a synopsis—fortunately, often prophetic—of where we might be headed. This time I wish to put the burden on the reader. I ask some questions which I believe might be answered by pursuing the lines already laid down. I am working on all of them, and would appreciate some company!

Queries

We take $\hbar, c$ and G as measured by current *scale invariant* techniques, and define our dimensional units of mass [M], length [L] and time [T] by

$$[M] \equiv \left(\frac{\hbar c}{G}\right)^{1/2} ; \qquad [L] \equiv \frac{\hbar}{[M]c} ; \qquad [T] \equiv \frac{[L]}{c} .$$

It is taken as understood in our work that a fundamental theory such as ours must compute everything else as pure numbers in terms of ratios to these units, and provide rules of correspondence consistent with the current practice of physics that will enable us to say how successful we have been in making such calculations.

Query 1:

To what extent do you agree or disagree with this statement? What arguments would you advance in support of it? What experimental or logical evidence would convince you that this is a bad starting point for a fundamental theory?

It is often thought by people who have followed the ANPA program that we have, by now, predicted up to a factor of $[1 \pm O(1/137)]$, the following physical consequences, where the symbols have their usual significance:

$$[M] = (2^{127} + 136)\ m_p\ ; \qquad \frac{\hbar c}{e^2} = 137 = 2^2 - 1 + 2^3 - 1 + 2^7 - 1\ .$$

Query 2:

What arguments would you advance to support this conclusion? What experimental or logical evidence would convince you that these results are wrong or misleading?

Query 3:

Can you explain why you believe in, or do not believe in, the Parker–Rhodes formula for the proton electron mass ratio

$$\frac{m_p}{m_e} = \frac{137\pi}{\frac{3}{14}\left(1 + \frac{2}{7} + \frac{4}{49}\right)\frac{4}{5}}\ .$$

Query 4:

Using the recent results establishing momentum conservation, can you

(*a*) calculate the "center-of-mass" correction to the Bohr formula $[m_e \to m_e/(1 + m_e/m_p)]$, and

(*b*) see if a consistent discrete calculation provides a new route to the Parker–Rhodes formula?

Query 5:

Can you, by using the relativistic discrete theory including angular momentum and "elliptical orbits," obtain the Bohr–Sommerfeld fine structure splitting for the hydrogen spectrum and, by using—instead—the spin degree of freedom, show that this is consistent with the Dirac calculation of the same quantity?

Query 6:

By treating the $(1)_{N_L}$ label (i.e., the unique label in the full, 4–level $2^{127}+136$ bit string representation of the hierarchy which interacts with everything) as the Newtonian "quantum" in the same way that the coulomb "quantum" is treated in the previous exercises, can you solve the Kepler problem?

Query 7:

Can you show that our theory predicts the gravitational red shift for light emitted from any massive object?

Query 8:

Can you show that Newtonian gravitation in our theory predicts only half the observed deflection of apparent stellar positions by the sun? Can you extend the gravitational theory to provide spin–2 gravitons, in addition to the Newtonian term, and show that one can then get the experimental result?

Query 9:

By using spin–2 gravitons in the Kepler problem (see Query 6)—in analogy to the Dirac version of the Bohr–Sommerfeld problem (see Query 5)—can you calculate the precession of the perihelion of Mercury?

Query 10:

Can you show that the mass of the neutral pion is approximately 274 times the electron mass (137 electron-positron pairs), and calculate the binding energy?

Query 11:

Is the identification of $(2^{127}+136)^2$ as an estimate of the baryon number (and charged lepton number) of the universe, which seems natural in the context of *program universe*, a necessary consequence of theories of the type we are constructing?

Query 12:

Is the fact that particles currently known can only be identified with reasonable assurance at *level 3*, that all such particles are "visible" (interact electromagnetically, either directly or indirectly) and that, from the statistical point of view, labels that close on the first two levels will be 127/10 times more prevalent, an indication that there should be roughly ten times as much "dark" as "visible" matter in the universe? Realize that although these labels are not identified, they, like *any* label in the scheme, must interact gravitationally.

Query 13:

Does the success of the Noyes–Dyson argument for the mass of the neutral pion (see Query 10) take us far enough to calculate the 2–gamma decay lifetime of this particle (0.87×10^{-16} seconds)?

Query 14:

How do we calculate the mass of the W and the Z_0? If we can do this, the $\pi^{\pm} - \pi^0$ and neutron-proton mass splittings should follow.

Query 15:

Can we calculate some approximation to the "gluon condensate" which allows Namyslowski to get "running masses" for quarks and gluons? If so, most of strong interact physics should follow, in due course.

Query 16:

Are there quantum geons?

REFERENCES

[1] D. O. McGoveran, "Foundations for a Discrete Physics," in *Proc ANPA 9*, H. P. Noyes, ed.

[2] C. Gefwert and D. O. McGoveran, "Prephysics" in *Proc ANPA 9*, H. P. Noyes, ed.

[3] T. S. Kuhn, *The Structure of Scientific Revolutions*, Univ. of Chicago Press, 1962.

[4] H. P. Noyes, C. Gefwert and M. J. Manthey, "A Research Program with No 'Measurement Problem'," *Annals of the N. Y. Acad. Sci.* **480**, 553–563 (1986).

[5] H. P. Noyes, *A Discrete Reconciliation between Quantum Mechanic and Relativity*, Schaldming, February 1987.

[6] A. Linde, in *Physics Today*, p. 61, September, 1987, estimates this same number as 10^{125}; his attempt to "save the phenomena" is more speculative than ours!

[7] Kilmister supplies the following reference for Eddington's version: "in *Fundamental Theory*, p. 30: 'The term "particle" survives in modern physics, but very little of its classical meaning remains. A particle can best be defined as *the conceptual carrier of a set of variates*. [FOOTNOTE. It is also conceived as *the occupant of a state* defined by the same set of variates.]' That is, of course, CUP 1946. But already in *Proc. Camb. Phil. Soc.* **40**, 37 (1944) he says: 'A "conceptual carrier of variates" is generally called a particle.' "

[8] M. J. Manthey, "Program Universe," in *Proc. of ANPA 7* (1986), Dr. F. Abdullah, Room E517, The City University, Northampton Square, London EC1V 0HB; see also SLAC–PUB–4008 and references and appendices therein.

[9] H. P. Noyes and J. V. Lindesay, *Australian J. Phys.* **36**, 601 (1983).

[10] J. V. Lindesay, A. J. Markevich, H. P. Noyes and G. Pastrana, "A Self–Consistent, Covariant, and Unitary Three–Particle Scattering Theory," *Phys. Rev.* **D 33**, 2339 (1986).

[11] H. P. Noyes, "A Minimal Relativistic Model for the Three Nucleon System," in *The Three Body Forces in the Three Nucleon System*, B. L. Berman and B. F. Gibson, eds., Springer–Verlag, Berlin., 1986, pp. 495–499.

[12] J. V. Lindesay and H. P. Noyes, "Zero Range Scattering Theory II. Minimal Relativistic Three–Particle Equations and the Efimov Effect," submitted to *Phys. Rev.* **D** and J. V. Lindesay, Ph.D. Thesis, Stanford (SLAC Report No. 243, 1981).

[13] C. Quigg, *Gauge Theories of the Strong,DWeak and Electromagnetic Interactions*, Benjamin, Menlo Park, 1983, p. 113.

[14] J. M. Namyslowski, *Phys. Lett.* **B192**, 170 (1987).

[15] *Probing the Standard Model*, Eileen C. Brennan, ed., SLAC Report No. 312.

[16] For a summary of the early history, cf., H. Pierre Noyes, Christoffer Gefwert, and Michael J. Manthey, "Toward a Constructive Physics," SLAC–PUB–3116 (rev. Sep 1983).

[17 For the status of the research at the last meeting (ANPA 8) and the progress following that reported in the preceding reference, see "H. P. Noyes, "On the Construction of Relativistic Quantum Mechanics," in *Proc. of ANPA 7* (1986), Dr. F. Abdullah, Room E517, The City University, Northampton Square, London EC1V 0HB; see also SLAC–PUB–4008, and references and appendices therein.

[18] E. W. Bastin and C. W. Kilmister, *Proc. Roy. Soc.*, **A 212**, 559 (1952);–*Proc. Camb. Phil. Soc.*, **50**, 254, 278 (1954), **51**, 454 (1955);**53**, 462 (1957); **55**, 66 (1959).

[19] T. Bastin, *Studia Philosophica Gandensia*, **4**, 77 (1966).

[20] T. Bastin, H. P. Noyes, J. Amson and C. W. Kilmister, "On the Physical Interpretation and Mathematical Structure of the Combinatorial Hierarchy," *International Journal of Theoretical Physics* **18**, 455-488 (1979).

[21] C. W. Kilmister, "The Mathematics needed for Self–Organization," pp. 11–17 of *Disequilibrium and Self Organization* (C.W.K. ed.,) Reidel, Dordrecht, 1986; see also Appendix II.1 "Brouwerian Foundations for the Hierarchy," (based on a paper presented at ANPA 2, King's College, Cambridge, 1980) and Appendix II.2 "On Generation and Discrimination," (based on a paper presented at ANPA 4, King's College Cambridge, 1982) in Ref. [16].

[22] C. W. Kilmister, "A Final Foundational Discussion?" in Ref. [17].

[23] A. F. Parker–Rhodes, *The Theory of Indistinguishables,*Synthese Library **150**, Reidel, Dordrecht, 1981.

[24] Ref. [1] gives McGoveran's current development of this point

[25] H. P. Noyes, D. McGoveran, T. Etter, M. Manthey and C. Gefwert, in "A Paradigm For Discrete Physics?," *Abstracts, 8th International Congress of Logic, Methodology and Philosophy of Science*, complied by V. L. Rabinovich, Inst. Phil. Acad. Sci. USSR, Moscow, 1987, Vol. 2, Sec. 8, pp. 98–101.

[26] I. Stein, Seminar at Stanford 1978, and presentations at ANPA 2 and 3, Kings' College, Cambridge, 1980 and 1981; the latest version of this work has been submitted to *Physics Essays*, 1988.

[27] H. P. Noyes, "A Finite Particle Number Approach to Quantum Physics," SLAC–PUB–2906 (April 1982).

[28] H. P. Noyes, "A Finite Particle Number Approach to Physics," in *The Wave–Particle Dualism*, S. Diner *et al.,* eds., Reidel, Dordrecht, 1984.

[29] W. Feller, *An Introduction to Probability Theory and its Applications*, Vol. I, 3rd Ed., p. 316; For more detail see the first edition, p. 247 *et seq.*, Wiley, New York, 1950.

[30] This argument was made by Noyes, McGoveran, Etter, Manthey and Gefwert in an abstract submitted to (but not published by) the 2nd *INTERNATIONAL SYMPOSIUM ON FOUNDATIONS OF QUANTUM MECHANICS—In the Light of New Technology*, Tokyo, September 1–4, 1986; the text is given in footnote 4, p. 50 of *Proc. ANPA 7.*

[31] H. Goldstein, *Classical Mechanics*, 2nd ed., Addison–Wesley, 1980, pp. 445–471.