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A COMMENT ON THE QUANTIZATION OF CHIRAL BOSONS*

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In a recent *Physical Review Letters*, Floreanini and Jackiw[1)] suggest an action suitable for the quantization of a two-dimensional chiral boson. Several years ago W. Seigel proposed an apparently unrelated action for the same system. In this comment we point out a connection between these two approaches.

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The classical Lagrangian density for a left-moving chiral scalar, as introduced by Siegel [1], is

$$\mathcal{L} = \partial_-\phi\partial_+\phi + \lambda(\partial_-\phi)^2 \qquad , \tag{1}$$

where the Lagrange multiplier $\lambda(x,t)$, transforms as the $++$ component of a second rank tensor. Classically, one finds, upon varying ϕ and λ, the following equations of motion

$$\partial_-(\partial_+\phi + \lambda\partial_-\phi) = 0 \qquad\qquad \partial_-\phi = 0 \qquad , \tag{2}$$

with the solution (as desired) $\phi_c = \phi(x^+)$. The Hamiltonian density is

$$\mathcal{H}_c = \frac{1}{2}\frac{1}{1+\lambda}(\Pi + \lambda\phi')^2 + \frac{1}{2}(1-\lambda)(\phi')^2 \qquad . \tag{3}$$

The vanishing of the canonical momentum conjugate to λ, $\Pi_\lambda(x)$, is a first class constraint; for consistency, we demand that this constraint be preserved in time, so that $\dot{\chi}_1 = \frac{d\Pi_\lambda}{dt}(x) = \{\Pi_\lambda(x), H_c\} = 0$. This gives the additional constraint $\tilde{\chi}_2 = \frac{1}{2}(\frac{1}{1+\lambda})^2(\Pi - \phi')^2 = 0$ which is also first class. However, the second class constraint $\chi_2(x) = \Pi(x) - \phi'(x) = 0$ is classically equivalent to $\tilde{\chi}_2$ and is just the canonical transcription of $\partial_-\phi = 0$. The quantization of the system using Faddeev-Popov and BRST methods was previously considerd.[3,4] This approach led to some puzzling features: Only a pair of chiral bosons could be consistently quantized[4] and the necessary additional Liouville term could not be coupled to gravity [5]. Moreover, recently it was argued[6] that this approach is inapplicable for the Lagrangian (1) because the first class constraint is a square of a second class one. It is not clear to us at this stage whether this last statement is justified. In any event, we use here the alternative approach of adopting the second class constraint χ_2. Following Dirac's quantization procedure[7], we add now a third constraint $\chi_3(x) = \lambda(x) - f(x)$ with an arbitrary function $f(x)$. Now all the constraints are second class. The non-zero matrix elements of the constraints' algebra[8] $C_{ij}(x,y) = \{\chi_i(x), \chi_j(y)\}$ are $C_{22} = -2\delta'(x-y)$, $\quad C_{13} = -C_{31} = -\delta(x-y)$. We now use the Dirac brackets to pass to the quantum theory via the usual definition of the commutator of two operators F, G:

$[F(x), G(y)] \iff i\{F(x), G(y)\}^*$. This leads to the following operator algebra

$$[\phi(x), \phi(y)] = \frac{1}{4i}\epsilon(x-y) \quad . \tag{4}$$

Since the constraints are now true operator identities, we are free to evaluate the Hamiltonian (3) on the constraint surface, giving

$$\mathcal{H}(x) = \Pi^2(x) = \frac{1}{4}\left(\Pi(x) + \phi'(x)\right)^2 \quad , \tag{5}$$

The theory is now explicitely independent of $\lambda(x)$.

We proceed now to the path integral quantization. For a Hamiltonian system with first and second class constraints, path integral quantization was discussed in Ref. [9]. Applying the general formalism to our case yields the generating functional

$$\begin{aligned} Z[J] = \int [d\phi][d\pi][d\lambda][d\Pi_\lambda]\,\delta(\chi_1)\delta(\chi_2)\delta(\chi_3) \\ \text{Det}\left[C_{ij}(x,y)\right] exp\left\{i\int d^2x\left(\Pi\dot{\phi} + \Pi_\lambda\dot{\lambda} - \mathcal{H}_C - J\phi\right)\right\} \end{aligned} \quad , \tag{6}$$

where $\chi_i (i = 1,2,3)$ were defined above, and $\mathcal{H}_C$ is the canonical Hamiltonian (3) (a normalization by $Z[0]^{-1}$ is implied in (6)). Using the delta-functionals, and noting that $Det[C]$ is a field-independent constant, one obtains

$$Z[J] = \int [d\phi]\, exp\left\{i\int d^2x\mathcal{L}'\right\}, \qquad \mathcal{L}' = \dot{\phi}\phi' - (\phi')^2 - J\phi \tag{7}$$

This Lagrangian, $\mathcal{L}'$, coincides with the local form of the Lagrangian given in ref. [1]. The classical equation of motion for this Lagrangian, with $J = 0$, is $\frac{d}{dx}\partial_-\phi = 0$ which has the general solution $\partial_-\phi = g(t)$. However, the functional integral (6) or (7) is not specified completely until we include boundary conditions; thus, by requiring $\partial_-\phi = 0$ at the spatial boundaries, we set $g = 0$ and recover the correct classical equation.

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