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B°–$\overline{\mathrm{B}}^\circ$ MIXING — A THEORETICAL EVALUATION AFTER ARGUS*

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ABSTRACT

After an introduction into the phenomenology of B°–$\overline{B}^\circ$ mixing is given, a rather detailed discussion is presented on the theoretical concepts that are involved. An attempt is made to elucidate the discrepancy between different theoretical claims. CP violation is touched only very briefly.

I. Introduction

So far the usual method for studying B°–$\overline{B}^\circ$ mixing has been to search for semileptonic B–decays where the lepton emerges with the "wrong" charge:

$$r_q \equiv \frac{\Gamma(B_q \to l^- \ X)}{\Gamma(B_q \to l^+ \ X)} \tag{1}$$

where $B_q = (\bar{b}q)$, $q = d, \ s$. Two things should be noted right away:

- Nature was kind enough to present us with two types of neutral B–mesons that can oscillate, namely B_d and B_s.
- $r_q \neq 0$ does *not automatically* prove that B_q–$\overline{B}_q$ mixing indeed occurs. In addition, one has to invoke a $\Delta B = \Delta Q_l$ rule that certainly holds in the Standard Model,

$$\begin{aligned} B^\circ \to l^+ \ \nu \ X \not\leftarrow \overline{B}^\circ \\ B^\circ \not\to l^- \ \nu \ X \leftarrow \overline{B}^\circ \ , \end{aligned} \tag{2}$$

  but could be violated by New Physics.

The strength of B°–$\overline{B}^\circ$ mixing is described by

$$x = \frac{\Delta \ M}{\Gamma} \tag{3}$$

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$$y = \frac{\Delta\Gamma}{2\Gamma} , \tag{4}$$

with $2\Gamma = \Gamma_1 + \Gamma_2$ where Γ_1 and Γ_2 denote the width of the two mass eigenstates B_1 and B_2, respectively. Later we will see that $\Delta\Gamma \ll \Gamma$ is estimated; with a $\Delta B = \Delta Q_l$ rule one then has:

$$r \simeq \frac{x^2}{2 + x^2} . \tag{5}$$

The ARGUS findings[1] on like-sign dileptons, etc. imply

$$\frac{\Delta M}{\Gamma} (B_d) \sim 0.4 \; - \; 1 \tag{6}$$

or

$$\frac{\Delta M}{M} (B_d) \sim 10^{-13} . \tag{7}$$

The number given in Eq. (7) can conveniently be used to impress outsiders; the number in Eq. (6), on the other hand, contains the real dynamical information:

$$\Delta \; M_B \sim \mathcal{O}(G_F^2) . \tag{8}$$

B°–$\overline{B}$ mixing thus represents a delicate though, as we will see, important phenomenon.

II. Phenomenology of $\mathbf{B^\circ}$– $\overline{\mathbf{B}}^\circ$ Mixing

Mixing means here that the flavor eigenstates are *not* mass eigenstates; the evolution of neutral B–decays in proper time then takes on a quite complex dependence as can be expressed in the Pais–Treiman notation:

$$\begin{aligned}
|B^\circ(t)\rangle &= g_+(t) \; |B^\circ\rangle_0 + \frac{q}{p} \, g_-(t) \; \left|\overline{B}^\circ\right\rangle_0 , \\
\left|\overline{B}^\circ(t)\right\rangle &= \frac{p}{q} g_-(t) \; |B^\circ\rangle_0 + g_+(t) \; \left|\overline{B}^\circ\right\rangle_0 , \\
g_\pm(t) &= \frac{1}{2} \exp\left[-\frac{1}{2}\Gamma_1 t\right] \exp\left[i m_1 t\right] \left(1 \pm \exp\left[-\frac{1}{2}\Delta\Gamma t\right] \exp\left[i\Delta m t\right]\right) , \\
\Delta\Gamma &= \Gamma_2 - \Gamma_1 , \qquad \Delta m = m_2 - m_1 , \qquad \frac{q}{p} = \frac{1-\bar{\epsilon}}{1+\bar{\epsilon}} .
\end{aligned} \tag{9}$$

This is the most general expression (compatible with CPT invariance). For the present discussion, I will make two simplifying assumptions:

- I assume CP invariance implying $q/p = 1$ (in an appropriate phase convention).
- I ignore $\Delta\Gamma$. Later we will see that $\Delta\Gamma \leq \frac{1}{10}\Delta m$ is a fairly conservative estimate.

The flavor quantum number of neutral B–mesons can, within the Standard Model, most conveniently be traced by studying semileptonic decays since

$$\begin{aligned} \langle \ell^- X | \, \mathcal{L}(\Delta B = 1) \, | B^\circ \rangle_0 &= 0 \\ \langle \ell^+ X | \, \mathcal{L}(\Delta B = 1) \, \left| \overline{B}^\circ \right\rangle_0 &= 0 \,. \end{aligned} \tag{10}$$

Using the simplifications stated above one obtains

$$\begin{aligned} \mathrm{rate}\big(B^\circ(t) \to \ell^- X\big) &\propto |\langle \ell^- X | \mathcal{L} | B^0(t) \rangle|^2 \\ &\propto |g_-(t)|^2 = \frac{1}{2} e^{-\Gamma t}(1 - \cos \Delta m t) \end{aligned} \tag{11}$$

$$\mathrm{rate}\big(B^\circ(t) \to \ell^+ X\big) \propto |g_+(t)|^2 = \frac{1}{2} e^{-\Gamma t}(1 + \cos \Delta m t) \,. \tag{12}$$

It is this deviation from a simple exponential time evolution which is an unambiguous sign of mixing! Present experimental searches cannot resolve any time evolution and are sensitive to *time-integrated* quantities only

$$r \equiv \frac{\Gamma(B^\circ \to \ell^- X)}{\Gamma(B^\circ \to \ell^+ X)} \simeq \frac{x^2}{2 + x^2} \tag{13}$$

$$\chi \equiv \frac{\Gamma(B^\circ \to \ell^- X)}{\Gamma(B^\circ \to \ell^\pm X)} = \frac{r}{1 + r} \,. \tag{14}$$

It is not just academic to remember that an observed $r \neq 0$ *per se* does not prove the existence of mixing. It primarily establishes a violation of a *global* $\Delta B = \Delta Q_\ell$ rule. This would then be interpreted as either due to

- mixing or
- a violation of the $\Delta B = \Delta Q_\ell$ rule that is *local* in time, i.e., a violation of Eq. (10), due to the presence of New Physics!

B–mesons are not produced in isolation since $\Delta B = 0$ holds for the strong and electromagnetic forces. Therefore one has to exercise a certain amount of care in interpreting data on, say, direct leptons attributed to semileptonic B–decays.

(*i*) $B\overline{B}$–production well above beauty threshold can be treated in a simple *probabilistic* way: if the neutral B–meson is produced together with a charged B (or a beauty baryon) which cannot mix one deals in effect with a situation where there is only a single state as far as mixing is concerned. When one encounters $B^\circ\overline{B}^\circ$–production like in

$$e^+e^- \to B^\circ\overline{B}^\circ + X \to \ell\ell + X' ,$$

one can conclude directly, without doing an explicit calculation, for the ratio of such like-sign to opposite-sign dileptons

$$\frac{N(\ell^\pm\ell^\pm)}{N(\ell^+\ell^-)} = \frac{2\chi(1-\chi)}{(1-\chi)^2+\chi^2} = \frac{2r}{1+r^2} . \tag{15}$$

(*ii*) Such a simple probabilistic prescription cannot be followed when one studies a near threshold process like

$$e^+e^- \to \Upsilon(4s) \to B\overline{B} .$$

For the two B–mesons now form a quantum mechanical state of definite orbital angular momentum, namely a p–wave, which is odd under exchange. The requirement of Bose statistics then tells us that at no time can the original $B^\circ\overline{B}^\circ$–system evolve into two identical states $B^\circ(t)B^\circ(t)$ or $\overline{B}^\circ(t)\overline{B}^\circ(t)$. An equivalent statement is the following

$$e^+e^- \to \Upsilon(4s) \begin{array}{l} \nearrow B_1B_2 \\ \not\searrow B_1B_1, B_2B_2 \end{array} ,$$

where $B_{1,2}$ are the two mass eigenstates. Yet even so, there is a simple intuitive argument which immediately yields the correct ratio between like-sign and opposite-sign dileptons; it just goes beyond a purely probabilistic description. Let us visualize neutral B–mesons as vectors in a plane where a B° $[\overline{B}^\circ]$ is denoted by a vector that points perpendicular up [down]. This is exactly the configuration at production time $t = 0$, Fig. 1(a). As time goes on, the two vectors rotate around the origin; the important point here is that they always remain anti-parallel because of Bose statistics, Fig. 1(b). When one of the mesons decays semileptonically, then the quantum coherence is destroyed and one knows immediately the identity of the other meson at that time, Fig. 1(c) — it is like

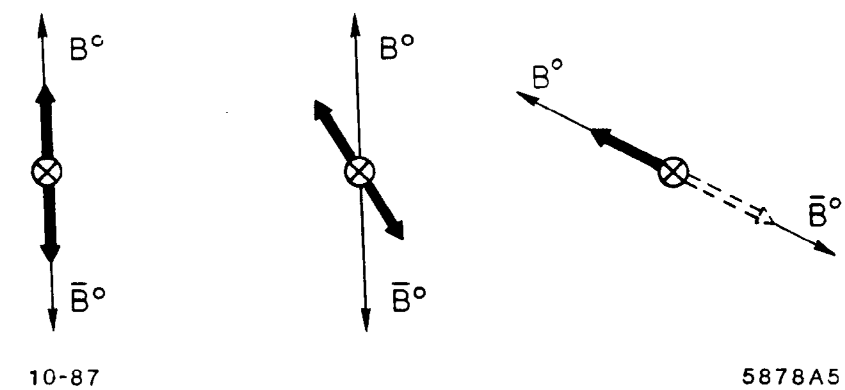

Fig. 1(a), (b), (c): Schematic representation of the time evolution of a pair of neutral B–mesons produced in a p–wave.

an Einstein–Rosen–Podolsky scenario. This situation therefore corresponds to single B–production as far as mixing is concerned:[2]

$$\frac{N(\ell^\pm \ell^\pm)}{N(\ell^+\ell^-)}\left[B^\circ \overline{B}^\circ \text{ in } p \text{ wave}\right] = r \ . \tag{16}$$

(*iii*) $B^\circ \overline{B}^\circ$–mixing affects also the forward-backward asymmetry of beauty jets in e^+e^- annihilation. This asymmetry is calculated for $e^+e^- \to b\,\bar{b}$; the b–quarks then hadronize into beauty jets tracing more or less the direction of flight of the original quarks. The only remaining task then consists in identifying the flavor of the jet — is it B or $\overline{B}$? This can be achieved via semileptonic or any other flavor-specific decays — yet a fundamental problem cannot be circumvented. Any decay can reflect on the flavor of the decaying state only as it was at the time of decay! If mixing occurs, then the flavor at time of decay is not necessarily the flavor at time of production since

$$\bar{b} \ \to \ (\bar{b}d) \ \to \ (b\bar{d})$$

can occur. Thus one necessarily makes an accounting error and the observable forward-backward asymmetry is *smaller* than the one expected on the quark level. For the simple case where only B_d and B_u–mesons are considered, one finds

$$A_{FB}(B_u, B_d) = \frac{1}{1+r} A_{FB}(b \text{ quarks}) \ . \tag{17}$$

The general case can be expressed in an analogous fashion:

$$A_{FB}\big(B_u, B_d, B_s, \Lambda_b\big) = \frac{1}{1+\bar{r}} A_{FB}(b \text{ quarks})$$

$$\bar{r} = \frac{2R\left[r_d + \rho_s r_s(1+r_d)/(1+r_s)\right]}{1 + r_d + R\left[\rho_\Lambda(1+r_d) + 1 - r_d + \rho_s(1-r_s)(1+r_d)/(1+r_s)\right]} \tag{18}$$

where

$$r_i = \frac{\Gamma(B_i \to \ell^- X)}{\Gamma(B_i \to \ell^+ X)} \ , \quad B_i = (\bar{b}i) \ , \quad i = d, s$$

$$R = \frac{b_{SL}(B_d)}{b_{SL}(B_u)} \ \leq \ 1 \ . \tag{19}$$

$\rho_\Lambda\,[\rho_s]$ denotes the $\Lambda_b\,[B_s]$ abundance relative to the number of B^+–mesons.

(*iv*) UA1 was actually the first to report some positive evidence for mixing averaged over B_d– and B_s–mesons. Their most recent analysis yields[3]

$$\langle\chi\rangle = 0.158 \pm 0.059 \mathrel{\widehat{=}} \langle r\rangle = 0.188 \pm 0.07 \tag{20}$$

which is not in clear conflict with the upper bound reported by MARK II[4]

$$\langle\chi\rangle \leq 0.12 \quad (90\%\ \text{C.L.}) \tag{21}$$

or by JADE which relies on its measurement of the forward-backward asymmetry of beauty jets:

$$\langle\chi\rangle \leq 0.13 \quad (90\%\ \text{C.L.})\ . \tag{22}$$

One should add that MAC has presented some (marginal) evidence for mixing[5]

$$\langle\chi\rangle = 0.21 \begin{array}{l} +\,0.29 \\ \\ -\,0.15 \end{array} \tag{23}$$

which can help to reconcile signals from $p\overline{p}$ collisions and e^+e^- annihilation.

At present there is no unambiguous way to compare $\langle\chi\rangle$ with χ_d since the relative abundance of the various beauty hadrons is not known. Instead one can draw up different "reasonable" scenarios, for example

(*a*) Scenario 1:

$$\text{Prob}(B_u) : \text{Prob}(B_d) : \text{Prob}(B_s) : \text{Prob}(\Lambda_b) \simeq 0.4 : 0.4 : 0.2 : \sim 0 \tag{24}$$

which leads to Fig. 2(a).

(*b*) Scenario 2:

$$\text{Prob}(B_u) : \text{Prob}(B_d) : \text{Prob}(B_s) : \text{Prob}(\Lambda_b) \simeq 0.375 : 0.375 : 0.15 : 0.10 \tag{25}$$

exhibited in Fig. 2(b).

In Scenario 1 one reads off $r_s \leq 0.6$ (90% C.L.) whereas in Scenario 2 even $r_s = 1$ is allowed. A very detailed discussion of such an analysis can be found in Ref. 6.

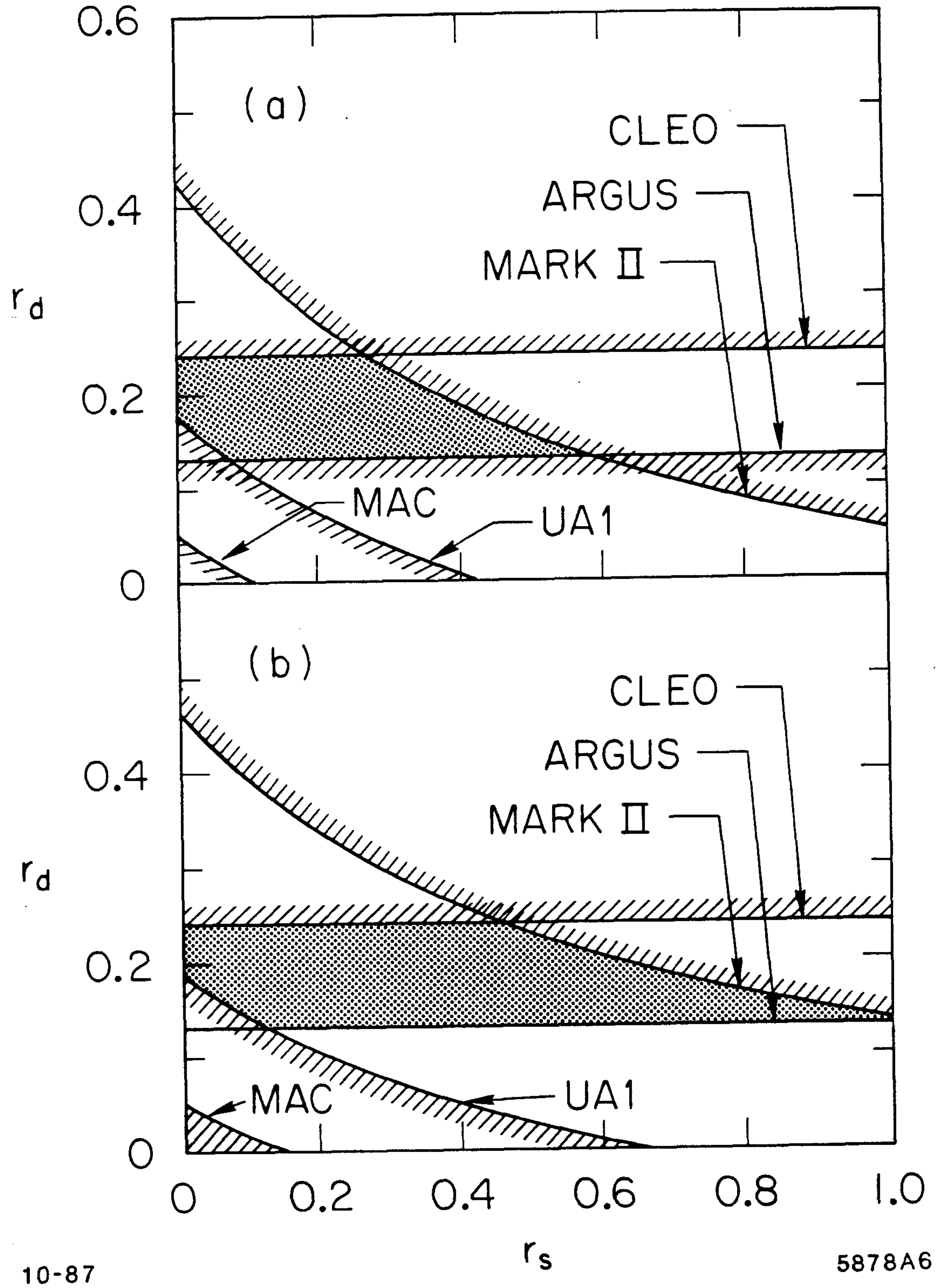

Fig. 2: (a) Experimental (90% C.L.) information on r_d, r_s for $N(B_u) : N(B_d) : N(B_s) : N(\Lambda_b) = 0.4 : 0.4 : 0.2 :\simeq 0$. (b) As in (a), but with $N(B_u) : N(B_d) : N(B_s) : N(\Lambda_b) = 0.375 : 0.375 : 0.15 : 0.10$. (Courtesy of R. Hurst.)

III. Theoretical Interpretation and Expectations

A) Fundamentals

It is fairly straightforward to convince oneself that within the Standard Model the quark box contribution is by far the most dominant term for Δm_B:

$$\Delta m_B|_{\text{theor}} \simeq \Delta m_B|_{\text{box}} \ . \tag{26}$$

There are various lines of argument all leading to the same conclusion:

- There are no clear resonances anymore at high mass scales $\sim m_B$. It makes good sense then to invoke the duality argument that the quark description expressed in the box diagram represents an appropriate average over the contributing hadronic channels.
- The dominant mass scale for Δm_B is set by the top mass — $\Delta m_B \propto m_t^2$ to first approximation — which is much larger than the ≤ 1 GeV scale ruling long distance dynamics. Resonance effects will then have only a small impact on Δm_B (it could be somewhat different for $\Delta\Gamma_B$, see later) since its domain $\sim$ (1 GeV)^2 is tiny compared to m_t^2 and small even relative to m_B^2. The dynamical situation is thus quite different for the K°–$\bar{K}^\circ$ and the B°–$\overline{B}^\circ$ case.

$\Delta m_B|_{\text{box}}$ depends on three crucial input parameters as apparent from Fig. 3:

- m_t, the top mass (m_b, m_c are relevant for $\Delta\Gamma_B$);
- the KM parameter $V(tq)$ (assuming $|V(tb)| \simeq 1$);
- the hadron wave function $B_B f_B^2$ defined in complete analogy to the K° case:

$$\left\langle \overline{B}^\circ \right| (b\bar{q})_{V-A}(b\bar{q})_{V-A} \left| B^\circ \right\rangle \equiv \frac{4}{3} B_B\, f_B^2\, m_B^2 \ . \tag{27}$$

More specifically, when ignoring m_c^2 and m_B^2 relative to m_t^2 — which amounts to a very good approximation for Δm_B — one finds

$$M_{12} = \eta_{\text{QCD}} \left(\frac{G_F}{4\pi}\right)^2 \frac{4}{3} B_B\, f_B^2\, \xi_t^2\, m_B\, E\left(\frac{m_t^2}{M_W^2}\right) \ , \tag{28}$$

where $\xi_t = V(tb)V^*(tq)$ and[7]

$$E(x) = x\left(\frac{1}{4} + \frac{9}{4(1-x)} - \frac{3}{2(1-x)^2}\right) - \frac{3}{2}\left(\frac{x}{1-x}\right)^3 \log x \ . \tag{29}$$

η_{QCD} contains the radiative QCD corrections. I will drop this factor in the following anticipating that it is not significant numerically when one considers

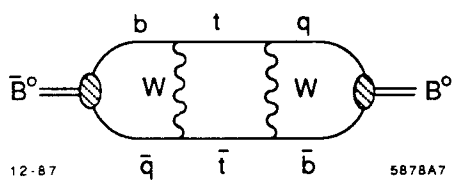

Fig. 3: Box diagram for $\Delta m(B_q)$.

the other uncertainties we are going to discuss. Nevertheless I want to make one comment on it: There has been a recent claim[8] that η_{QCD} represents a significant suppression, i.e., by a factor of roughly two, for $m_t^2 \sim \mathcal{O}(M_W^2)$! Unfortunately this claim is *misleading:* Firstly the authors of Ref. 8 include QCD corrections in $\mathcal{L}(\Delta B = 2)$ that involve $\alpha_S(m_b m_d)$, i.e., with an explicit dependence on the light quark mass m_d; secondly they evaluate the quark masses at the scale ~ 1 GeV. This produces the lion's share of the stated suppression.

There has to be considerable concern whether this is a reliable procedure numerically since $\sqrt{m_b m_d} \sim 230$ MeV. However, the real criticism of their procedure goes deeper: one has to form the matrix element of the local operator $\mathcal{L}(\Delta B = 2)$, see Eq. (27), which introduces the decay constant f_B. Yet the natural scale for defining f_B is M_b since it can in principle be observed in $B \to l\nu$. The $\Delta B = 2$ operator should therefore be taken at scale M_B and the dependence on m_d relegated to f_B. As discussed in detail in Ref. 9 (where the term "hybrid logarithm" is coined) these renormalizations of $\mathcal{L}(\Delta B = 2)$ and f_B then *cancel* if the weak currents are purely lefthanded.

Three limiting cases can help to illustrate the function $E(x)$ in Eqs. (28) and (29):

$$E\left(\frac{m_t^2}{M_W^2}\right) \simeq \frac{m_t^2}{M_W^2} \times \begin{pmatrix} 1 \\ \frac{3}{4} \\ \frac{1}{4} \end{pmatrix} \text{ for } m_t^2 \begin{pmatrix} \ll \\ = \\ \gg \end{pmatrix} M_W^2 \ . \tag{30}$$

At first sight the last row in Eq. (30) appears quite paradoxical, if not outright nonsensical: $E(x)$ — far from getting smaller — actually increases *ad infinitum* when top quarks become more or more massive. This seems to contradict various decoupling theorems. Yet closer scrutiny reveals that these decoupling theorems are evaded in the one legal way, namely by invoking nongauge couplings: for it is the *longitudinal* W–bosons that create this effect and they are the reincarnation of the original charged Higgs fields! It is also for this reason mainly that a process like B°–$\overline{B}^\circ$–mixing has a high sensitivity to heavy quarks, unless the corresponding KM parameters are highly suppressed.

Finally

$$\Delta M_B \simeq 2 \ \mathrm{Re}|M_{12}| = 2|M_{12}| \ . \tag{31}$$

B) $\Delta m(B_d)$ vs. $\Delta m(B_s)$

From Eqs. (28) and (31) one reads off immediately

$$x_s = x_d \frac{|V(ts)|^2}{|V(td)|^2} \frac{(Bf_B[B_s])^2}{(Bf_B[B_d])^2} \geq x_d \frac{|V(ts)|^2}{|V(td)|^2} , \tag{32}$$

where we have already anticipated $Bf_B[B_s] \geq Bf_B[B_d]$.

Since t–quarks have not been observed yet, there exists no direct information on the KM parameters $V(td), V(ts)$ [or $V(tb)$ for that matter]. However, with only three families, one can employ unitarity to constrain them quite considerably. I find the Wolfenstein parametrization[10] of the KM matrix most convenient for this and latter purposes:

$$V_{\rm KM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} , \tag{33}$$

plus terms of higher order in λ. As expected, there are four independent parameters: λ, A, ρ, η.

The first one, λ, is basically the Cabibbo angle

$$\lambda \simeq 0.22 . \tag{34}$$

A is estimated from the beauty lifetime[23]

$$A \simeq 1.0 \pm 0.3 , \tag{35}$$

with considerable systematic uncertainties. Using

$$0.1 \leq \left| \frac{V(ub)}{V(cb)} \right| \leq 0.25 , \tag{36}$$

leads to

$$0.3 \leq \sqrt{\rho^2 + \eta^2} \leq 1.14 . \tag{37}$$

The unitarity of the 3×3 KM matrix then implies

$$|V(ts)| \simeq |V(cb)| = A\lambda^2 . \tag{38}$$

The dependence on the KM parameters actually drops out from $x_s = \Delta m/\Gamma \propto |V(ts)|^2/|V(cb)|^2$.

Unfortunately there is no such simple relation between $|V(td)|$ and $|V(ub)|$, $|V(cb)|$; $|V(td)|$ in particular depends on the sign of ρ, in contrast to $|V(ub)|$, and becomes maximal for $\rho < 0$.

For $|\rho| \leq 1$ — it cannot be significantly larger and still satisfy Eq. (37) and reproduce the observed CP violation in K_L–decays — and $\rho < 0$ one finds

$$|V(td)| \lesssim 0.02 \tag{39}$$

$$\frac{|V(ts)|^2}{|V(td)|^2} = \frac{1}{\lambda^2\left((1-\rho)^2+\eta^2\right)} \gtrsim 5 \tag{40}$$

and therefore a quite *conservative* bound

$$x_s > 5x_d \gtrsim 2.2 \tag{41}$$

$$r_s > 0.71 \tag{42}$$

i.e., the mixing rate — Δm — is considerably larger than the decay rate — Γ — for B_s–mesons. This has two consequences:

(α) $B_s - \bar{B}_s$–mixing is thus expected to approach its maximal value $r_s = 1$. An observation of slower mixing — say $r_s < 0.7$ — is thus a sign of New Physics — like a fourth family or an isoscalar quark or flavor-changing neutral currents, etc., — that contributes *destructively* to $B_s - \overline{B}_s$–mixing. I have already mentioned that combining the lower bound χ_d from ARGUS with the upper bound on $\langle\chi\rangle$ from Mark II implies $r_s < 0.6$ — *if* the production probabilities of B_d, B_s, etc., states hold as stated in Eq. (24).

(β) The real test of mixing consists of observing the special *time* evolution given in Eqs. (11) and (12) where the exponential is modulated by a cos function, as shown in Fig. 4 for the two "typical" values $x = 0.75$ and $x = 5$. One realizes immediately that very good time resolution is required to observe very fast mixing.

C) $\Delta m(B_d)$ and m_t

Unfortunately, it is much harder to make an absolute prediction on Δm as a function of m_t: there is the "hard" input parameter $V(tb)V^*(td)$ and the "soft" one $B_B f_B^2$. I have already stated that we have some nontrivial constraints on $|V(tb)V^*(td)|$ obtained via unitarity from $V(cb)$ and $V(ub)$. Since it is the

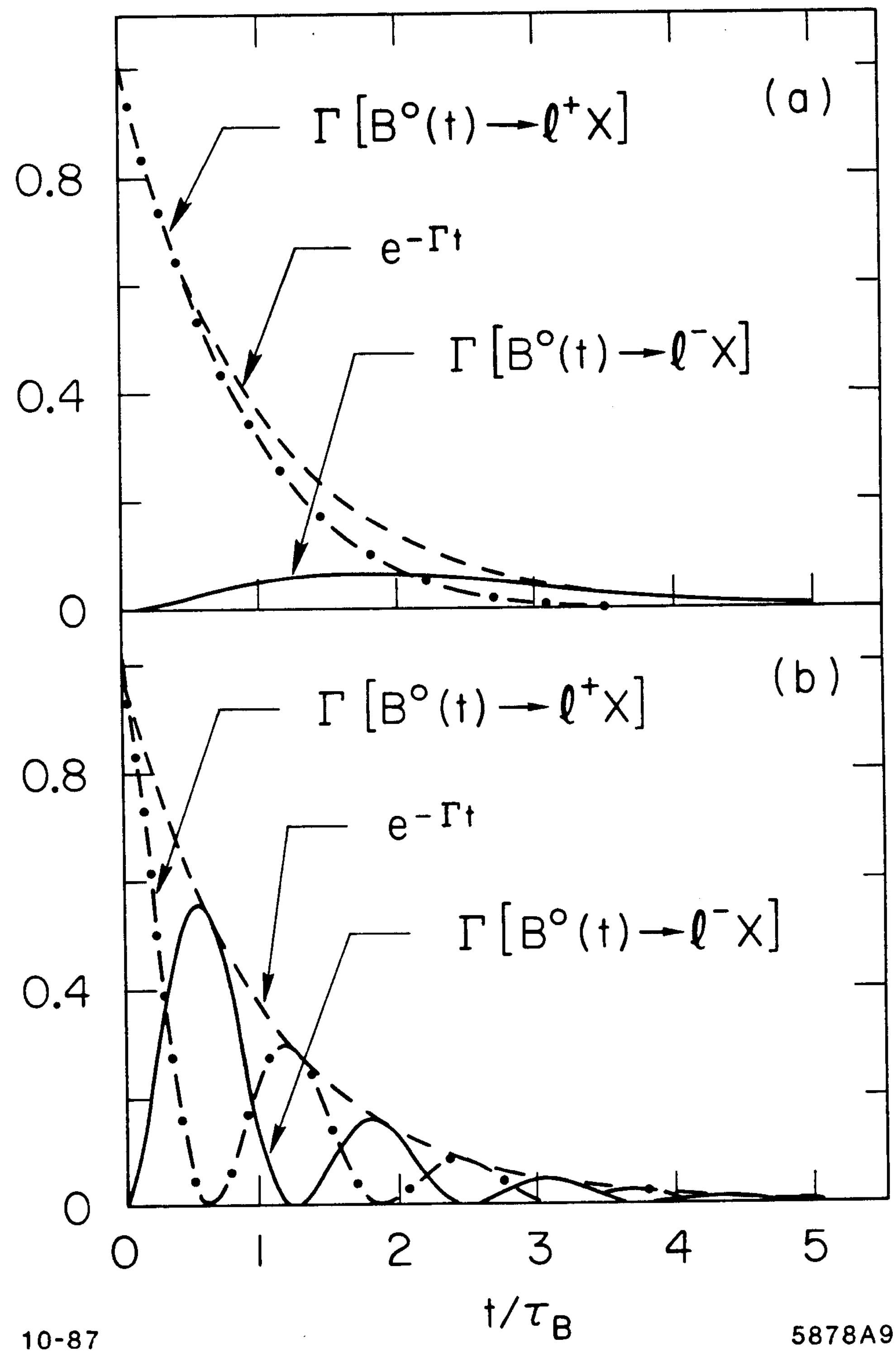

Fig. 4: Proper time evolution of semileptonic B^0–decays with (a) $\frac{\Delta m}{\Gamma} = 0.75$ and with (b) $\frac{\Delta m}{\Gamma} = 5.00$.

ordinary strong interactions that are responsible for B and f_B we can conclude immediately

$$\begin{aligned} B_B &\simeq \mathcal{O}(1) \\ f_B &\simeq \mathcal{O}(f_\pi, f_K) \simeq \mathcal{O}(150\ \text{MeV})\ . \end{aligned} \tag{43}$$

Thus we find ourselves not in a position of complete ignorance concerning these parameters — the problem is that our understanding is numerically not sufficiently precise. In looking at Eqs. (28) and (31) we realize that Δm_B depends on the square both of f_B and $|V(td)|$! Varying f_B by a factor of two which is perfectly consistent with Eq. (43) has an unpleasantly large impact. Δm_B changes by a factor of four and the real mixing observable $r = x^2/(2+x^2)$ by an order of magnitude! There are actually two sides to this coin, namely the strong dependence of $\Delta m_B|_{\text{theor}}$ on certain input parameters:

- No precise prediction for or interpretation of Δm_B can be given as long as more than one of these inputs is unknown or only purely known.
- Numerically precise statements can however be made as soon as our ignorance has been narrowed down to only one (or better still, zero) input parameter.

Various theoretical models have been employed over the years to compute or at least estimate the relevant parts of the B–meson wave function. The results are tabulated below:[12–16]

for B_d–mesons

$$B_B f_B^2 \sim \begin{cases} (60 - 130\ \text{MeV})^2 & \text{MIT bag models} \\ (100 - 150\ \text{MeV})^2 & \text{Potential models} \\ (115 \pm 15\ \text{MeV})^2, (190 \pm 30\ \text{MeV})^2 & \text{QCD sum rules} \\ (120\ \text{MeV})^2/\alpha_s & B^* - B \text{ mass splitting} \\ \lesssim (220\ \text{MeV})^2 & \text{Scaling from } f_D \end{cases} \tag{44}$$

and for B_s–mesons

$$B_B f_B^2 \sim \begin{cases} (140 - 200\ \text{MeV})^2 & \text{MIT bag models} \\ (140 - 200\ \text{MeV})^2 & \text{Potential models}\ . \\ (140 \pm 20\ \text{MeV})^2, (210 \pm 30\ \text{MeV})^2 & \text{QCD sum rules} \end{cases} \tag{45}$$

A few comments are in order:

- Comparing Eqs. (44) and (45) exhibits the general feature

$$B_B\, f_B^2\, [B_s] \;\geq\; B_B\, f_B^2\, [B_d]$$

as expected intuitively, for in a nonrelativistic ansatz

$$f_B^2 \;=\; \frac{12|\varphi(0)|^2}{M_B}\,, \tag{46}$$

where $\varphi(0)$ denotes the meson wave function at the origin.

This wave function is controlled by the reduced mass μ_B which is $m_s\,[m_d]$ for $B_s\,[B_d]$–mesons; the wave function is then more concentrated at the origin for B_s than for B_d–mesons and despite $M(B_d) < M(B_s)$ one expects quite generally $f_B^2[B_s] > f_B^2[B_d]$. More explicitly — and therefore also in a more model dependent way — one finds

$$f_B \;\propto\; \mu_B\,.$$

A fairly similar pattern holds also when relativistic effects are included.[13]

- It is rather easy to prove that in potential models $B \equiv 1$ always holds[17] — it amounts to a nice homework problem actually.

- B_B has not been calculated via the QCD sum rule approach yet. The values I have quoted there refer to f_B only. The two numbers for f_{B_d} are from an identical ansatz — the numerical difference is due completely to the usage of a different b–quark mass. (One further remark can be made in passing: it is obviously highly dangerous and therefore inadvisable for a theorist to quote an error on his/her results. If they had not done that in this instance, one would be speaking of an uncertainty instead of a discrepancy.)

- The B^*–B mass splitting yields at best on order of magnitude estimate on f_B (and nothing on B_B) since it is quite unclear which is the appropriate value for α_s: is it $\alpha_s \sim 1/2$ or $\alpha_s \sim 1$?

- The last line in Eq. (44) is obtained using the nonrelativistic expression Eq. (46) to relate f_B to the (90% C.L.) upper limit $f_D < 340$ MeV obtained by Mark III in its search for $D^+ \to \mu^+\nu_\mu$.

The uncertainties on the KM parameters and hadronic wave functions can be expressed quite conveniently in units of a calibration factor F

$$F = \frac{|V(td)|^2}{(0.01)^2} \, \frac{B\, f_B^2}{(150 \text{ MeV})^2} \,. \tag{47}$$

Our preceding discussion leads to the range

$$F \;\sim\; 0.5 - 7 \;, \tag{48}$$

as a realistic one, even with a certain touch of conservatism — nevertheless not one canonized by completely hard facts and/or calculations.

Figure 5 shows a comparison of x_d as a function of m_t with the ARGUS numbers; I conclude[22]

$$m_t \;\gtrsim\; 50 \text{ GeV} \quad \text{if} \quad r_d \;\geq\; 0.1 \;, \tag{49}$$

with m_t quite possibly much closer to 100 GeV!

A violation of Eq. (49) would indicate the presence of New Physics — a fourth family, a nonminimal Higgs sector, etc. — yet before such a conclusion would be finalized, one would of course reanalyze — with much more effort and impetus — whether the intrinsic theoretical uncertainties are truly reflected in Eq. (48)!

Recently a strong criticism[11] has been raised against a bound like Eq. (49). Among other things those authors used quite different input parameters than most other researchers:

In particular, they use considerably larger $|V(td)|$ than is generally accepted, namely

$$|V(td)| \lesssim 0.03 \;, \tag{50}$$

rather than $|V(td)| \lesssim 0.02$, as derived in Eq. (39). However, Eq. (50) is inconsistent with present data on semileptonic B–decays:

- It is true that even

$$|V(bc)| \sim 0.06 \;-\; 0.07 \;, \tag{51}$$

has to be allowed for in *interpreting* the data; in particular the BSW scheme[18] of hadronization yields

$$|V(cb)| \sim 0.055 \pm 0.01 \;, \tag{52}$$

i.e., significantly larger than the value quoted for some obscure reason by the Particle Data Group!

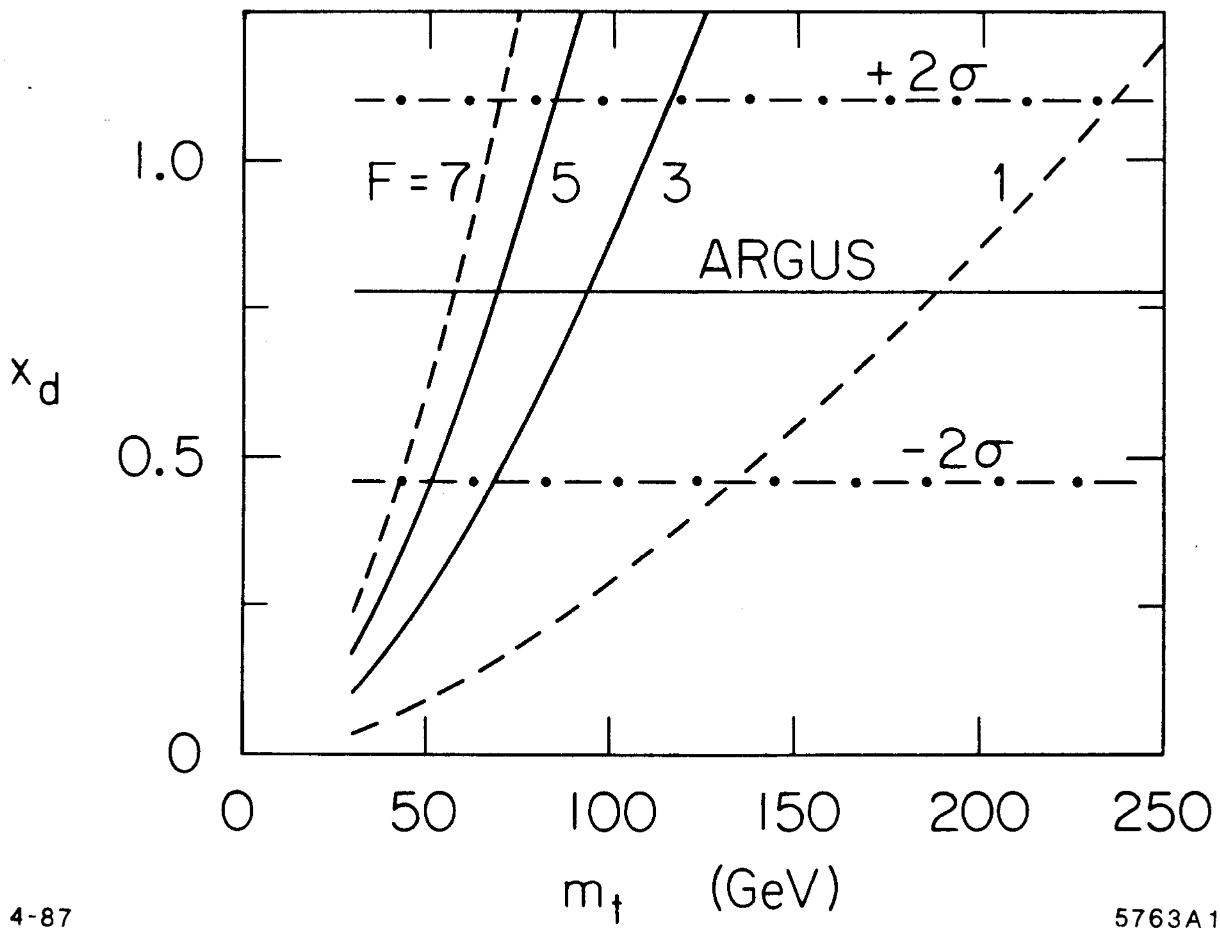

Fig. 5: $\frac{\Delta m}{\Gamma}(B_d)$ as a function of m_t compared with the ARGUS findings.

- However, in that same scheme one deduces from the data

$$\left|\frac{V(ub)}{V(cb)}\right| \lesssim 0.11 \tag{53}$$

or

$$|V(ub)| \lesssim 0.007 , \tag{54}$$

and thus

$$|V(td)| \lesssim 0.021 . \tag{55}$$

- In other schemes like that of GIW[19] one infers instead

$$|V(cb)| \sim 0.033 \; - \; 0.053 \tag{56}$$

$$\left|\frac{V(ub)}{V(cb)}\right| \lesssim 0.19 , \tag{57}$$

and again

$$|V(td)| \lesssim 0.021 . \tag{58}$$

There is an important lesson contained in this apparently simple arithmetics: one has to be aware of correlations that are introduced by the theoretical procedures employed in extracting the KM parameters from the data. Therefore it is inconsistent to use

$$|V(cb) \sim 0.065 \tag{59}$$

$$\frac{V(ub)}{V(cb)} \sim 0.25 , \tag{60}$$

simultaneously: these two values are incompatible with the data on $B-$ decays, unless the various theoretical models used to *interprete* the data are all misleading (a not altogether inconceivable possibility).

D) $\Delta\Gamma$

Comparing the box diagrams of Fig. (6) one obtains

$$\left|\frac{\Delta\Gamma}{\Delta M}\right| \simeq \frac{3\pi}{2} \frac{\left(\frac{m_b}{M_W}\right)^2}{E\left(\frac{m_t^2}{M_W^2}\right)} , \tag{61}$$

where $E(x)$ is the function defined in Eq. (29) which decreases quickly for increasing x.

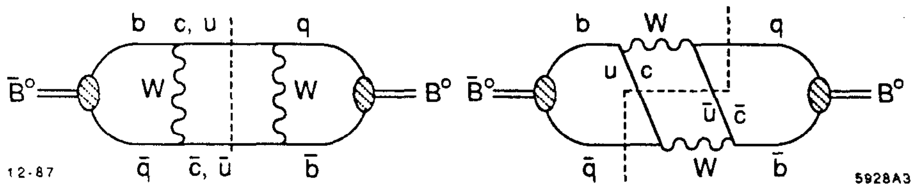

Fig. 6: Box diagram for $\Delta\Gamma(B_q)$.

For $m_t = 50$ GeV one obtains from Eq. (61)

$$\left|\frac{\Delta\Gamma}{\Delta\, m}\right| \sim 0.05\ . \tag{62}$$

Using the ARGUS number

$$\frac{\Delta\, m}{\Gamma}\,(B_d) \sim 0.5\ -\ 1 \tag{63}$$

and its "extrapolation" to the B_s–system

$$\frac{\Delta\, m}{\Gamma}\,(B_s) \gtrsim 2.8\ , \tag{64}$$

one concludes

$$\frac{\Delta\Gamma}{\Gamma}\,(B_d) \lesssim \mathcal{O}(1\%) \tag{65}$$

$$\frac{\Delta\Gamma}{\Gamma}\,(B_s) \sim 15\%\ . \tag{66}$$

One arrives at essentially the same estimates also in a different way:

- The channels that are common to B_d and $\overline{B}_d$–decays are Cabibbo–suppressed, like B_d, $\overline{B}_d \rightarrow D\overline{D}$. Their branching ratios will therefore be of order 10^{-3} at best. There are certainly many of those, yet they contribute with alternating signs, like in

  $$B_d \rightarrow D^+D^- \rightarrow \overline{B}_d \tag{67}$$

  vs.

  $$B_d \rightarrow D^+D^-\pi^\circ \rightarrow \overline{B}_d\ ; \tag{68}$$

  Eq. (65) thus represents a rather conservative bound.

- B_s, $\overline{B}_s$–decays share modes that are *not* Cabibbo–suppressed like B_s, $\overline{B}_s \rightarrow D_s^{(*)}\ D_s^{(*)}$. These two-body modes are expected to command quite large branching ratios; e.g.,

  $$BR(B_s \rightarrow D_s^*\ D_s^*) \sim 0.05\ .$$

  This channel by itself would then produce a nonnegligible lifetime difference

  $$\frac{\Delta\Gamma}{\Gamma}\,(B_s) \sim 2\ BR(B_s \rightarrow D_s^*\ D_s^*) \sim 0.1\ .$$

These estimates[20] are meant to show that $\Delta\Gamma/\Gamma$ could be as large as 10–20% for B_s–mesons. It could be significantly smaller, too, due to cancellations taking place between

$$B_s \to D_s^* \ D_s^* \to B_s$$

and

$$B_s \to D_s^* \ \overline{D}_s + \text{h.c.} \to B_s$$

transitions.

IV. Impact on CP asymmetries in B–decays

The apparently large B_d–$\overline{B}_d$–mixing has three consequences for CP asymmetries in beauty decays:

(1) All CP asymmetries that require the presence of mixing, obviously have a much better chance to reach the level of observability.

(2) Since in the absence of New Physics B_s–$\overline{B}_s$–mixing proceeds quite rapidly, one has to place a high premium on the capability to resolve evolutions in proper time when searching for CP asymmetries in B_s–decays.

(3) Large B_d–$\overline{B}_d$–mixing strongly suggests that top quarks are rather heavy. This in turn *decreases* the size of the KM phase required to reproduce ϵ_K. Accordingly this will decrease the size for those CP asymmetries in particular that do not involve B°–$\overline{B}^\circ$–mixing.

A detailed discussion of this rather complex topic can be found in the literature.[21]

V. Summary

A confirmation of B_d–$\overline{B}_d$–mixing as observed by ARGUS would be highly exciting:

- It is given by a one-loop process in the Standard Model and thus represents a true quantum effect.
- It is very sensitive to the presence of New Physics: the existence of the latter would be signaled when relatively light top quarks were observed or B_s–$\overline{B}_s$–mixing were found not to be rapid.
- It will change for good the environment in which the search for CP violation in beauty decays has been discussed. What used to be considered as the toyland for imaginative souls is beginning to be viewed as worthy the attention of serious people.

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