# B PHYSICS* 

Frederick J. Gilman
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305


#### Abstract

We review B physics and the motivation for studying B decays, including CP-violating effects in the B meson system.


## Some History

The story of $B$ physics, or more properly the story of the $b$ quark, began at Fermilab ten years ago with the discovery of the upsilon resonance. ${ }^{1}$ By detecting the muon pairs produced in proton-nucleus collisions, the dimuon mass spectrum shown in Figure 1 was observed. As indicated in the figure and in the original paper, the peak near an invariant mass of 9.4 GeV is too broad to be due to a single resonance, given the resolution inherent in the spectrometer: The existence of at least two narrow states, and more probably three, was pointed out. Later experiments ${ }^{2}$ with proton beams in fact were able to resolve the original peak into the $\Upsilon, \Upsilon^{\prime}$ and $\Upsilon^{\prime \prime}$, as shown in Figure 2.

In the meantime, electron-positron experiments were able to study the three narrow upsilon states in some detail. ${ }^{3}$ An example of the tremendous statistical power combined with good energy resolution which is available is shown in Figure 3, where the Crystal Ball scan over the first $\Upsilon$ peak $^{4}$ is displayed. Furthermore, not only do the first three $\Upsilon$ states themselves form a narrow and clean system to study, but they decay into other narrow states. Figure 4 shows Crystal Ball data on the radiative decay ${ }^{5}$ of the $\Upsilon^{\prime}$ into three $\chi_{b}$ states, which must have the opposite behavior under charge conjugation. The last year has seen the discovery of a second set of $\chi_{b}^{\prime}$ states lying in mass between the $\Upsilon^{\prime}$ and $\mathrm{Y}^{\prime \prime}$, as indicated in Figure 5, taken from data of the CUSB collaboration. ${ }^{6}$

Thus there is a whole set of related resonances around 10 GeV in mass. We can understand their quantum numbers, masses and many other properties if they are composed of a spin $1 / 2$ quark, the $b$ (or bottom or beauty) quark bound together with its corresponding antiquark, the $\bar{b}$. Each would have a mass of about 5 GeV . The set of such bottomonium states that would be expected is shown in Figure 6, with a checkmark indicating those that are already found experimentally.

All the states of bottomonium we have discussed so far can decay via the strong or electromagnetic interaction either into a member of the bottomonium family or (by having the $b$ and $\bar{b}$ quarks annihilate) into " $b$-less" matter. When we reach the $\Upsilon^{\prime \prime \prime \prime}$ state, however, another process becomes kinematically available: dissociation (by the usual strong interactions) into two hadrons, one containing the $b$ quark and the other the $\bar{b}$.

[^0]

Fig. 1. The dimuon mass spectrum in $p+$ Nucleus $\rightarrow \mu \bar{\mu}+$ anything, from Ref. 1.


Fig. 2. The dimuon mass spectrum with high resolution from Ref. 2.


Fig. 4. Observation of the three $\chi_{b}$ states in radiative $\Upsilon^{\prime}$ decays from Ref. 5.


Fig. 3. The $\Upsilon$ peak in electron-positron annihilation from Ref. 4.


Fig. 5. Observation of the $\chi_{b}^{\prime}$ states in radiative $\Upsilon^{\prime \prime}$ decays from Ref. 6.

Indeed, the dominant decay mode of the $\Upsilon(4 S)$ is

$$
\Upsilon(4 S) \rightarrow B \bar{B}
$$

where the $B$ mesons which are produced each have a mass of $\sim 5.28 \mathrm{GeV}$ and can be a $B_{d}=\bar{b} d$ or $B_{u}=\bar{b} u$ bound state of a $b$ quark with a light $d$ or $u$ quark. The extra constraint of knowing the energy of the $B$ or $\bar{B}$ when one tunes an electron-positron colliding beam machine to operate on the $\Upsilon(4 S)$ has proven to be an invaluable tool up to now in reconstructing $B$ mesons in exclusive modes. The present world supply of reconstructed $B$ 's is shown ${ }^{7}$ in Figures 7 and 8, due to the CLEO and ARGUS detectors, respectively. At something like 100 to 120 MeV higher in mass we expect to find the state $B_{s}=\bar{b} s$.

Fig. 6. The expected energy levels of the $b \bar{b}$ system. Checkmarks indicate those states which are experimentally observed.


Fig. 7. $B$ mesons reconstructed in exclusive modes at CLEO (from Ref. 7).


Fig. 8. $B$ mesons reconstructed in exclusive modes at ARGUS (from Ref. 7).

## Weak Decays of $B$ Mesons

The lightest particles containing one $b$ or $\bar{b}$ quark will be stable with respect to the strong and electromagnetic interactions. They can decay weakly, since emission of a $W^{-}$ takes one from the lower to the upper components of the weak isospin doublets:

$$
\binom{u}{d^{\prime}}_{L} \quad\binom{c}{s^{\prime}}_{L} \quad\binom{t}{b^{\prime}}_{L}
$$

where the upper components are chosen to be the mass eigenstates $u, c$ and $t$, and the essential complication that the weak and mass eigenstates are not the same is entirely represented in terms of a matrix transformation ${ }^{8}$ operating on the lower components. It takes us from the mass eigenstates ( $d, s$ and $b$ ) to the weak eigenstates ( $d^{\prime}, s^{\prime}$ and $b^{\prime}$ ), and is represented by the unitary ( $\mathrm{K}-\mathrm{M}$ ) matrix:

$$
\left(\begin{array}{c}
d^{\prime}  \tag{1}\\
s^{\prime} \\
b^{\prime}
\end{array}\right)=\left(\begin{array}{lll}
V_{u d} & V_{u s} & V_{u b} \\
V_{c d} & V_{c s} & V_{c b} \\
V_{t d} & V_{t s} & V_{t b}
\end{array}\right)\left(\begin{array}{c}
d \\
s \\
b
\end{array}\right) .
$$

As $V$ is a $3 \times 3$ unitary matrix, it is specified by nine parameters. When we take into account that each of the six quark fields can be changed by a phase with no change in the physics, but that a common phase change of all quark fields would not change the matrix $V$, we are left with $9-(6-1)=4$ parameters. These can be considered as three mixing angles and one phase, with a non-zero value of this phase inducing CP violation. More on this later.

At the present time, the three angles and one phase of the three-generation $\mathrm{K}-\mathrm{M}$ matrix are limited by direct measurements of the "magnitudes" of the K-M matrix elements $V_{u d}, V_{u s}, V_{c d}, V_{c s}, V_{c b}$, and bounds on the magnitude of $V_{u b}$. This determines two of the angles (or combinations of the angles) fairly well, and bounds a third one. The key experimental restrictions can be stated as ${ }^{9}$

$$
\begin{equation*}
\left|V_{u s}\right|=0.221 \pm 0.002 \tag{2}
\end{equation*}
$$

from strange particle decays, and ${ }^{10}$

$$
\begin{equation*}
\left|V_{c b}\right|=0.046 \pm 0.010 \tag{3}
\end{equation*}
$$

from the $b$ lifetime (see below), and

$$
\begin{equation*}
0.07 \leq\left|V_{u b} / V_{c b}\right| \leq 0.23 \tag{4}
\end{equation*}
$$

where the upper bound comes from the absence of a signal for $b \rightarrow u+\ell \bar{\nu}_{l}$ in semileptonic $B$ decay and the lower bound from the ARGUS observation ${ }^{11}$ of exclusive baryonic $B$ decays which involve $b \rightarrow u+d \bar{u}$ at the quark level.

The small magnitude of $\left|V_{u b} / V_{c b}\right|$ reflects the dominance of decays of the $b$ quark of the form $b \rightarrow c+d \bar{u}$. Typically, each of the exclusive hadronic decay channels which correspond to this process at the quark level has a branching ratio of a few times $10^{-3}$, rather than the few times $10^{-2}$ for charm (see Table 1). There exists some fair theoretical understanding of their rough magnitude. ${ }^{12}$ For processes which involve $b \rightarrow u$ at the quark level, the corresponding typical hadronic branching ratios or limits upon them are at the few times $10^{-4}$ level (see Table 2).

Table 1. B-to-Charm Exclusive Decays (from Ref. 7)

| Mode |  | Events |  | \% Branching Ratio | Experiment |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | sig | bkg |  |  |
| $B^{-} \rightarrow$ | $\psi K^{-}$ | 2.9 | 0.1 | $0.09 \pm 0.06 \pm 0.02$ | CLEO |
|  |  |  |  | $0.07 \pm 0.04$ | ARGUS |
|  | $\psi^{\prime} K^{-}$ |  |  | $0.22 \pm 0.17$ | ARGUS |
|  | $\psi K^{-} \pi^{+} \pi^{-}$ |  |  | $0.11 \pm 0.07$ | ARGUS |
|  | $D^{\circ} \pi^{-}$ | 14.0 | 2.2 | $0.47{ }_{-0.13}^{+0.16+0.08}$ | CLEO |
|  | $\begin{aligned} & D^{+} \pi^{-} \pi- \\ & D^{*+} \pi^{-} \pi^{-} \end{aligned}$ | 1.2 | 0.7 | $0.25{ }_{-0.23}^{+0.41+0.08}$ | CLEO |
|  |  | 2.7 | 1.0 | $0.22{ }_{-0.14-0.05}^{+0.15+0.08}$ | CLEO |
|  |  | 7.0 | 3.0 | $0.5 \pm 0.3 \pm 0.4$ | ARGUS |
|  | $D^{*+} \pi^{-} \pi^{-} \pi^{\circ}$ | 24.0 | 13.0 | $5.0 \pm 1.5 \pm 3.0$ | ARGUS |
| $\bar{B}^{\circ} \rightarrow$ | $\psi \bar{K}^{* 0}$ | 4.5 | 0.5 | $0.41 \pm 0.19 \pm 0.03$ | CLEO |
|  |  | 4.0 |  | $0.33 \pm 0.18$ | ARGUS |
|  | $D^{+} \pi^{-}$ | 4.3 | 0.2 | $0.59_{-0.29-0.14}^{+0.33+0.15}$ | CLEO |
|  | $D^{*+} \pi^{-}$ | 5.3 | 0.3 | $0.33_{-0.14-0.07}^{+0.19+0.11}$ | CLEO |
|  |  | 5.0 | 1.0 | $0.31 \pm 0.16 \pm 0.12$ | ARGUS |
|  | $D^{*+} D_{s}^{-}$ |  |  | < 2.8 | ARGUS |
|  | $D^{*+} D_{s}^{*-}$ |  |  | $4.0 \pm 2.6$ | ARGUS |
|  | $D^{\circ} \pi^{+} \pi^{-}$ | 4.8 | 1.2 | < 3.9 | CLEO |
|  | $D^{*+} \pi^{-} \pi^{\circ}$ | 8.0 | 4.0 | $1.8 \pm 0.9 \pm 0.9$ | ARGUS |
|  | $D^{*+} \pi^{+} \pi^{-} \pi^{-}$ | 6.6 | 4.4 | < 5.0 | CLEO |
|  |  | 27.0 | 12.0 | $3.8 \pm 1.1 \pm 1.8$ | ARGUS |

Table 2. Exclusive $B \rightarrow$ noncharm, nonleptonic two-body decays. Ninety percent confidence level upper limits on numbers of events and percent branching ratios (from Ref. 7).

| Noncharm, Nonstrange Modes ( $b \rightarrow u$ ) |  |  |  |  |  | Strange, Noncharm Modes ( $B \rightarrow s$, penguin |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Mode } \\ & B^{-} \rightarrow \end{aligned}$ | Events | \% | $\begin{aligned} & \text { Mode } \\ & \bar{B}^{\circ} \rightarrow \end{aligned}$ | Events | \% | $\begin{aligned} & \text { Mode } \\ & B^{-} \rightarrow \end{aligned}$ | Events | \% | Mode $\bar{B}^{\circ} \rightarrow$ | Events | \% |
| CLEO |  |  |  |  |  |  |  |  |  |  |  |
| $\pi^{\circ} \pi^{-}$ | 188 | 0.23 | $\pi^{+} \pi^{-}$ | 8 | 0.03 | $\bar{K}^{\circ} \pi^{-}$ | 5 | 0.068 | $K^{-} \pi^{+}$ | 15 | 0.032 |
| $\rho^{\circ} \pi^{-}$ | 2 | 0.02 | $\rho^{ \pm} \pi^{\mp}$ | 376 | 0.61 | $\bar{K}^{* 0} \pi^{-}$ | 7 | 0.026 | $K^{*-} \pi^{+}$ | 2 | 0.070 |
| $\rho^{\circ}{ }^{\circ}{ }_{1}(1270)^{-}$ | 52 | 0.32 | $\rho^{\circ} \rho^{\circ}$ | 9 | 0.05 | $K^{-} \rho^{\circ}$ | 10 | 0.026 | $\bar{K}^{\circ} \rho^{\circ}$ | 3 | 0.080 |
| $\rho^{\circ} a_{2}(1320)^{-}$ | 21 | 0.23 | $\pi^{ \pm} a_{1}(1270)^{\mp}$ | 7 | 0.12 | $K^{-\phi}$ | 4 | 0.021 | $\bar{K}^{\circ} \phi$ | 4 | 0.13 |
|  |  |  | $\pi^{ \pm} a_{2}(1320)^{\mp}$ | 4 | 0.16 | $K^{*-} \boldsymbol{\gamma}$ | 3 | 0.18 | $\bar{K}^{* *} \rho^{\circ}$ | 19 | 0.12 |
|  |  |  | $p \bar{p}$ | 6 | 0.02 |  |  |  | $K^{* *}{ }^{*}{ }^{\circ}$ | 4 | 0.047 |
|  |  |  |  |  |  |  |  |  | $K^{* 0} \gamma$ | 22 | 0.20 |
| ARGUS |  |  |  |  |  |  |  |  |  |  |  |
| $\rho^{\circ} \pi^{-}$ | 8 | 0.07 | $\pi^{+} \pi^{-}$ | 4 | 0.04 | $K^{*-\gamma}$ |  | 0.10 | $\bar{K}^{* 0} \gamma$ |  | 0.04 |

All these decays fall within an overall picture of $b$ decays where the $b$ quark decays through the usual four-fermion weak interaction with QCD corrections. In addition to changing the strength of the usual four-fermion effective weak interaction, there are additional operators introduced by QCD, the "penguins." In bottom decay it is possible to have processes which are Kobayashi-Maskawa (K-M) suppressed where "penguin" diagrams give rise to contributions comparable to, or maybe even larger than, those of ordinary tree level graphs. ${ }^{13}$ Figure 9 shows a possible example. The "penguin" diagram contributes an effective Hamiltonian density:

$$
\begin{equation*}
\mathcal{H}=\frac{G_{F}}{\sqrt{2}} \frac{\alpha_{s}}{3 \pi} V_{t b} V_{t s}^{*} \ln \left(m_{t}^{2} / m_{c}^{2}\right) \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) b \bar{u} \gamma^{\mu} u \tag{5}
\end{equation*}
$$

whereas the usual spectator diagram (aside from short-distance QCD correction factors, $c_{ \pm}$, which are close to unity) corresponds to

$$
\begin{equation*}
\mathscr{H}=\frac{G_{F}}{\sqrt{2}} V_{u b} V_{u s} \bar{u} \gamma_{\mu}\left(1-\gamma_{5}\right) b \bar{s} \gamma^{\mu}\left(1-\gamma_{5}\right) u . \tag{6}
\end{equation*}
$$

 diagram contributing to $\mathrm{K}-\mathrm{M}$ suppressed decays of the $\bar{B}_{d}$ meson.

The "penguin" loses to the spectator graph because of the $\left(\alpha_{s} / 3 \pi\right) \ln \left(m_{t}^{2} / m_{c}^{2}\right)$ that arises from having one loop and the presence of the gluon, but it wins by at least a factor of 20 because of the $\mathrm{K}-\mathrm{M}$ factor $V_{t b} V_{t s}^{*}$, which involves zero and one generation jumps, as compared to $V_{u b} V_{u s}$, which involves two and one generation jumps, respectively. Depending in part
on the matrix elements in particular processes, it could well be that the spectator graph gives the lesser of the two contributions. Then, for example, in the decays $B_{d} \rightarrow K^{+} \pi^{-}$ or $B_{s} \rightarrow \phi \rho$ the "penguin" contribution may be dominant. ${ }^{14}$

A large part of the chance of doing interesting $B$ physics is due to two "surprises." First is the $b$ lifetime. The dominant semileptonic decay of the $b$ quark involves the $c$ quark, as noted above. The decay rate $\Gamma\left(b \rightarrow c e \bar{\nu}_{e}\right)$ can be easily related to that for muon decay, $G_{F}^{2} m_{\mu}^{5} / 192 \pi^{3}$ :

$$
\begin{equation*}
\Gamma\left(b \rightarrow c e \bar{\nu}_{e}\right)=\left|V_{c b}\right|^{2} \frac{G_{F}^{2} m_{b}^{5}}{192 \pi^{3}} F\left(\frac{m_{c}}{m_{b}}\right) \tag{7}
\end{equation*}
$$

where

$$
\begin{equation*}
F(\Delta)=1-8 \Delta^{2}+8 \Delta^{6}-\Delta^{8}-24 \Delta^{4} \ln \Delta \tag{8}
\end{equation*}
$$

The phase space factor, $F(\Delta)$, which is unity for a massless final quark, i.e., $F(0)=1$, drops off rather quickly, so that $F(0.30)=0.52$, a value relevant approximately to the $b \rightarrow c$ transition. Given an inclusive semileptonic branching ratio of $\sim 12 \%$, the whole question of the $b$ lifetime boils down to the value of $\left|V_{c b}\right|$. Before the first measurement, there was a large range of possibilities, but the theoretical guestimates hovered in the neighborhood of several times $10^{-14}$ seconds for $\tau_{b}$. An "upper limit" was about $10^{-12}$ seconds. Sure enough, the $b$ lifetime has turned out to be around $10^{-12}$ seconds. For the theorists this fact turned out to be not so difficult to accommodate after all; the value of $\left|V_{c b}\right|$ was adjusted accordingly [see Eq. (3)]. For experimentalists it has meant that the gaps or tracks between production and decay of hadrons containing a $b$ quark are long enough to be susceptible to present vertex detector technology.


Fig. 10. Box diagram contributing an off-diagonal element to the $B-\bar{B}$ mass matrix. The important (short-distance) contribution comes from a $t$ quark in the loop.

$$
\begin{equation*}
B^{\circ}(t)=e^{-i\left(m_{1}+m_{2}\right) t / 2} e^{-\Gamma t / 2}\left[\cos \left(\frac{\Delta m}{2} t\right) B^{\circ}+i \sin \left(\frac{\Delta m}{2} t\right) \bar{B}^{\circ}\right] \tag{9}
\end{equation*}
$$

where we have assumed $\Gamma_{1} \approx \Gamma_{2}=\Gamma$ and set $\Delta m=m_{1}-m_{2}$. The initial $B^{\circ}$ oscillates to a $\bar{B}^{\circ}$ and then back again to a $B^{\circ}$ in a time $2 \pi / \Delta m$ (or $2 \pi \Gamma / \Delta m$ in lifetime units). The question then is: What is $\Delta m / \Gamma$ ? Before the fact, a theoretical guestimate on the high
end for $(\Delta m / \Gamma)_{B_{d}}=x_{d}$ was $\sim 0.2$. This past year the ARGUS collaboration found ${ }^{15}$ $x_{d}=0.73 \pm 0.18$; the mixing time is not so different from the lifetime. For theorists this has meant an upward adjustment in the combination of a hadronic matrix element, a K-M matrix element, and, most importantly, the value of $m_{t}$. For experimentalists, this together with the $b$ lifetime means that in some situations not only will $B$ mesons live long enough to leave a measurable gap, but that in this time there is a nonnegligible chance that they will oscillate into the corresponding antiparticle state.

## Rare B Decays

The benchmark process in rare B decays is $B \rightarrow K \mu \bar{\mu}$. In the standard model this decay proceeds through an "electromagnetic penguin" diagram and should occur with a branching ratio of a few times $10^{-6}$. There does not seem to be any reason to expect important competition from long-range effects and this process should be a clean test of one-loop effects in the standard model. ${ }^{16}$ The presence of a fourth generation ${ }^{17}$ could increase the branching ratio appreciably to perhaps a few times $10^{-5}$.

The same basic one-loop diagram can lead to a real photon and result in the decay $b \rightarrow s+\gamma$ at the quark level, or $B \rightarrow K^{*}+\gamma, B \rightarrow K^{* *}+\gamma$, etc. at the hadron level. Here QCD corrections are absolutely critical: They change the GIM suppression in the amplitude from being in the form of a power law, $\left(m_{t}^{2}-m_{c}^{2}\right) / M_{W}^{2}$, to the softer form of a logarithm, $\ln \left(m_{t}^{2} / m_{c}^{2}\right)$. This corresponds to an enhancement by one to two orders of magnitude ${ }^{18-20}$ over the rate expected from the simplest one-loop electroweak graph. ${ }^{21}$ The inclusive process at the quark level, $b \rightarrow s \gamma$, should occur with a branching ratio of roughly ${ }^{16} 10^{-3}$; exclusive modes like $B \rightarrow K^{*} \gamma$ and $B \rightarrow K^{* *} \gamma$ are estimated at 5 to $10 \%$ of this. ${ }^{18}$ Again, a fourth generation could enhance this rate by an order of magnitude or so. ${ }^{22}$ The extension to a supersymmetric world is more interesting. The obvious new diagrams come from putting the supersymmetric partners of the quarks and the $W$ in the loop of the "electromagnetic penguin" diagram. Much more important, ${ }^{23}$ however, is the transition from a "penguin" to a "penguino," the "penguin" diagram involving a gluino and a squark. Because it involves strong interaction couplings rather than weak ones, it competes (and interferes) with the QCD enhanced "electromagnetic penguin" and produces a branching ratio of order a few times $10^{-3}$.

Turning away from one-loop processes, the decay $\bar{B}^{-} \rightarrow \tau^{-} \nu_{\tau}$ is predicted to occur at the level of a few times $10^{-5}$. It would permit the direct measurement of the parameter $f_{B}$, which is an ingredient of the theoretical expression for $\Delta M_{B}$ (which results in $B-\bar{B}$ mixing).

Other potential rare decays that are commonly considered are those that are forbidden in the standard model. ${ }^{24}$ Whereas most limits on flavor changing neutral currents involve first and second generation quarks and/or leptons, $B \rightarrow \mu \tau$ and $B \rightarrow K \mu \tau$ involve flavor changing neutral currents which connect the second and third generations. Some attempts to understand the origin of generations of quarks and leptons and/or the size of the elements of the $\mathrm{K}-\mathrm{M}$ matrix predict the existence of these processes. For example, with horizontal gauge bosons it is possible to build a model where some of these processes
occur at the level of $\sim 10^{-5}$ in branching ratio without contradicting existing experimental data. ${ }^{16}$ However, something below $10^{-9}$ seems a more typical level at which to expect them, if they occur at all.

## CP Violation

The standard model allows for CP violation in the form of a phase originating in the quark mixing matrix, the $\mathrm{K}-\mathrm{M}$ matrix. ${ }^{8}$ When there are three generations of quarks and leptons, there is one CP-violating phase and any difference of rates between a given proccss and its CP conjugate process has the form

$$
\begin{equation*}
\Gamma-\bar{\Gamma} \propto \text { coef. } \times s_{1}^{2} s_{2} s_{3} s_{\delta} c_{1} c_{2} c_{3} \tag{10}
\end{equation*}
$$

where we employ for definiteness the original parametrization of the matrix ${ }^{8}$ in terms of three angles $\theta_{i}$ with $i=1,2,3$, plus a phase $\delta$ and $s_{\delta}=\sin \delta, s_{i}=\sin \theta_{i}$ and $c_{i}=\cos \theta_{i}$. Our present experimental knowledge allows us to make the approximation: $c_{1} c_{2} c_{3} \approx 1$, which is good to an accuracy of a few percent.

The combination of sines and cosines of $K-M$ angles that occurs in Eq. (10) is mandatory for a CP-violating effect with three generations. It is precisely this combination of factors that occurs in the determinant of the commutator of mass matrices introduced by Jarlskog, ${ }^{25}$ to formulate a general condition for CP violation, if her basis-independent condition is restated in the $\mathrm{K}-\mathrm{M}$ parametrization. We see explicitly from Eq. (10) that the presence of non-zero mixing for all three generations is required in order to have a CP-violating effect. This is not surprising; we know that with only two generations there is no $C P$ violation from the quark mixing matrix (all the potential phases can be absorbed into the quark fields) and this is exactly the situation we would be in if we set one of the mixing angles to 0 or $\pi / 2$ and decoupled one of the generations from the other two.

When we form a CP-violating asymmetry we divide a difference in rates by their sum:

$$
\begin{equation*}
\text { Asymmetry }=\frac{\Gamma-\bar{\Gamma}}{\Gamma+\bar{\Gamma}} \tag{11}
\end{equation*}
$$

If we do this for K decay, the decay rates for the dominant hadronic and leptonic modes all involve a factor of $s_{1}^{2}$, i.e., essentially the Cabibbo angle squared. A CP-violating asymmetry will then have the general dependence on $\mathrm{K}-\mathrm{M}$ factors:

$$
\begin{equation*}
\text { Asymmetry }_{K \text { Decay }} \propto s_{2} s_{3} s_{\delta} \tag{12}
\end{equation*}
$$

The right-hand side is of order $10^{-3}$ (see the discussion below). This is both a theoretical plus and an experimental minus. The theoretical good news is that CP-violating asymmetries in the neutral K system are naturally at the $10^{-3}$ level, in agreement with the measured value of $|\epsilon|$. The experimental bad news is that, no matter what the K decay process, it is always going to be at this level, and therefore difficult to get at experimentally with the precision necessary to sort out the standard model explanation of its origin from other explanations.

Note also that because CP violation must involve all three generations while the $K$ has only first and second generation quarks in it (and its decay products only involve first generation quarks), CP-violating effects must come about through heavy quarks in loops. There is no CP violation arising from tree graphs alone.

This is not the case in B decay (or B mixing and decay). First, the decay rate for the leading decays is very roughly proportional to $s_{2}^{2}$, which happens to be much smaller than the corresponding quantity $\left(s_{1}^{2}\right)$ in K decay. But more importantly, we can look at decays which have rates that are $\mathrm{K}-\mathrm{M}$ suppressed by factors of $\left(s_{1} s_{2} s_{3}\right)^{2}$ or $\left(s_{1} s_{3}\right)^{2}$, just to choose two examples. By choosing particular decay modes, it is then possible to have asymmetries which behave like

$$
\begin{equation*}
\text { Asymmetry }_{B \text { Decay }} \propto s_{\delta} \tag{13}
\end{equation*}
$$

With luck, this could be of order unity! Note, though, that we have to pay the price of CP violation somewhere. That price, the product $s_{1}^{2} s_{2} s_{3} s_{\delta}$, is given in the CP-violating difference of rates in Eq. (10). The K-M factors either are found in the basic decay rate, resulting in a very small branching ratio, or they enter the asymmetry, which is then correspondingly small. This is a typical pattern: the rarer the decay, the bigger the potential asymmetry. The only escape from this pattern comes from outside of K-M factors: to find a decay mode where the coefficient of the right-hand side of Eq. (10) is large. A good example of this is provided by $x_{d}$, which is big because of the combination of the value of a hadronic matrix element, a $\mathrm{K}-\mathrm{M}$ matrix element, and the value of $m_{t}$.

The fact that asymmetries in K and B decay can be different by orders of magnitude is part and parcel of the origin of CP violation in the standard model. It "knows" about the quark mass matrices and can tell the difference between a $b$ quark and an $s$ quark. This is entirely different from what we expect in general from explanations of CP violation that come from very high mass scales, as in the superweak model or in left-right symmetric gauge theories. Then, all quark masses are negligible compared to the new, very high mass scale. Barring special provisions, there is no reason why such theories would distinguish one quark from another; we expect all CP-violating effects to be roughly of the same order, namely that already observed in the neutral K system.

As the present year has proceeded the standard model "explanation" of CP violation has looked better and better. In particular, there have been two important new experimental results for $\epsilon^{\prime} / \epsilon$. First came the preliminary result from a test run of the Fermilab experiment: ${ }^{26}$

$$
\epsilon^{\prime} / \epsilon=3.5 \pm 3.0 \pm 2.0 \times 10^{-3}
$$

and then this past summer, the preliminary result from the CERN experiment ${ }^{27}$

$$
\epsilon^{\prime} / \epsilon=3.5 \pm 0.7 \pm 0.4 \pm 1.2 \times 10^{-3}
$$

Both experiments have the capability of eventually decreasing both their statistical and systematic error bars below the $10^{-3}$ level. If $\epsilon^{\prime} / \epsilon \sim 3.5 \times 10^{-3}$, then CP-violating effects
from heavy quark loops is a likely interpretation and, especially if $m_{t}$ is large, the result is not wildly different than expectations. It would seem that the wind is blowing in the direction of the standard model and the explanation of CP violation in terms of the $\mathrm{K}-\mathrm{M}$ phase.

## CP Violation in B Decay

The possibilities for observation of CP violation in B decays are much richer than for the neutral $K$ system. The situation is even reversed, in that for the $B$ system the variety and size of CP-violating asymmetries in decay amplitudes far overshadows that in the mass matrix. ${ }^{28}$

To start with the familiar, however, it is useful to consider the phenomenon of CP violation in the mass matrix of the neutral $B$ system. Here, in analogy with the neutral K system, one defines a parameter $\epsilon_{B}$. It is related to $p$ and $q$, the coefficients of the $B^{\circ}$ and $\bar{B}^{\circ}$, respectively, in the combination which is a mass matrix eigenstate by

$$
\frac{q}{p}=\frac{1-\epsilon_{B}}{1+\epsilon_{B}}
$$

The charge asymmetry in $B^{\circ} \bar{B}^{\circ} \rightarrow \ell^{ \pm} \ell^{ \pm}+X$ is given by ${ }^{29}$

$$
\begin{gather*}
\frac{\sigma\left(B^{\circ} \bar{B}^{\circ} \rightarrow \ell^{+} \ell^{+}+X\right)-\sigma\left(B^{\circ} \bar{B}^{\circ} \rightarrow \ell^{-} \ell^{-}+X\right)}{\sigma\left(B^{\circ} \bar{B}^{\circ} \rightarrow \ell^{+} \ell^{+}+X\right)+\sigma\left(B^{\circ} \bar{B}^{\circ} \rightarrow \ell^{-} \ell^{-}+X\right)}=\frac{\left|\frac{p}{q}\right|^{2}-\left|\frac{q}{p}\right|^{2}}{\left|\left.\right|_{q} ^{p}\right|^{2}+\left|\frac{q}{p}\right|^{2}}  \tag{14}\\
=\frac{I m\left(\Gamma_{12} / M_{12}\right)}{1+\frac{1}{4}\left|\Gamma_{12} / M_{12}\right|^{2}} \tag{15}
\end{gather*}
$$

where we define $<B^{\circ}|H| \bar{B}^{\circ}>=M_{12}-\frac{i}{2} \Gamma_{12}$. The quantity $\left|M_{12}\right|$ is measured in $B-\bar{B}$ mixing and we may estimate $\Gamma_{12}$ by noting that it gets contributions from $B^{\circ}$ decay channels which are common to both $B^{\circ}$ and $\bar{B}^{\circ}$, i.e., $\mathrm{K}-\mathrm{M}$ suppressed decay modes. This causes the charge asymmetry for dileptons most likely to be in the ballpark of a few times $10^{-3}$, and at best $10^{-2}$. For the foreseeable future, we might as well forget it experimentally.

Turning now to CP violation in decay amplitudes, in principle this can occur whenever there is more than one path to a common final state. For example, let us consider decay to a CP eigenstate, f , like $\psi K_{g}^{\circ}$. Since there is substantial $B^{\circ}-\bar{B}^{\circ}$ mixing, one can consider two decay chains of an initial $B^{\circ}$ meson:

$$
\begin{aligned}
& B^{\circ} \rightarrow B^{\circ} \\
& B^{\circ} \rightarrow \bar{B}^{\circ}
\end{aligned} \quad \nearrow,
$$

where $f$ is a CP eigenstate. The second path differs in its phase because of the mixing of $B^{\circ} \rightarrow \bar{B}^{\circ}$, and because the decay of a $\bar{B}$ involves the complex conjugate of the K M factors involved in $B$ decay. The strong interactions, being CP invariant, give the
same phases for the two paths. The amplitudes for these decay chains can interfere and generate non-zero asymmetries between $\Gamma\left(B^{\circ}(t) \rightarrow f\right)$ and $\Gamma\left(\bar{B}^{\circ}(t) \rightarrow f\right)$. Specifically,

$$
\begin{equation*}
\Gamma\left(\bar{B}^{\circ}(t) \rightarrow f\right) \sim e^{-\Gamma t}\left(1-\sin [\Delta m t] \operatorname{Im}\left(\frac{p}{q} \rho\right)\right) \tag{16a}
\end{equation*}
$$

and

$$
\begin{equation*}
\Gamma\left(B^{\circ}(t) \rightarrow f\right) \sim e^{-\Gamma t}\left(1+\sin [\Delta m t] \operatorname{Im}\left(\frac{p}{q} \rho\right)\right) \tag{16b}
\end{equation*}
$$

Here we have neglected any lifetime difference between the mass matrix eigenstates (thought to be very small) and set $\Delta m=m_{1}-m_{2}$, the difference of the eigenstate masses, and $\rho=A(B \rightarrow f) / A(\bar{B} \rightarrow f)$, the ratio of the amplitudes, and we have used the fact that $|\rho|=1$ when $f$ is a CP eigenstate in writing Eq. (11). From this we can form the asymmetry:

$$
\begin{equation*}
A_{\mathrm{CP} \text { Violation }}=\frac{\Gamma(B)-\Gamma(\bar{B})}{\Gamma(B)+\Gamma(\bar{B})}=\sin [\Delta m t] \operatorname{Im}\left(\frac{p}{q} \rho\right) \tag{17}
\end{equation*}
$$

In the particular case of decay to a CP eigenstate, the quantity $\operatorname{Im}\left(\frac{p}{q} \rho\right)$ is given entirely by the K-M matrix and is independent of hadronic amplitudes. However, to measure the asymmetry experimentally, one must know if one starts with an initial $B^{\circ}$ or $\bar{B}^{\circ}$, i.e., one must "tag."

We can also form asymmetries where the final state $f$ is not a CP eigenstate. Examples are $B_{d} \rightarrow D \pi$ compared to $\bar{B}_{d} \rightarrow \bar{D} \bar{\pi} ; B_{d} \rightarrow \bar{D} \pi$ compared to $\bar{B}_{d} \rightarrow D \bar{\pi} ;$ or $B_{s} \rightarrow D_{s}^{+} K^{-}$compared to $\bar{B}_{s} \rightarrow \bar{D}_{s}^{-} K^{+}$. These is a decided disadvantage here in theoretical interpretation, in that the quantity $\operatorname{Im}\left({ }_{q}^{p} \rho\right)$ is now dependent on hadron dynamics.

It is instructive to look not just at the time-integrated asymmetry between rates for a given decay process and its CP conjugate, but to follow the time dependence, ${ }^{30}$ as given in Eqs. (16a) and (16b). As a first example, Figures 11, 12 and 13 show $^{31}$ the time dependence for the process $\bar{b} \rightarrow \bar{c} u \bar{d}$ (solid curve) in comparison to that for $b \rightarrow c \bar{u} d$ (dashed curve). At the hadron level this could be, for example, $B_{d} \rightarrow \bar{D}^{-} \pi^{+}$in comparison to $\bar{B}_{d} \rightarrow D^{+} \pi^{-}$. The direct process is very much K-M favored over that which is introduced through mixing, and hence the magnitude of the ratio of amplitudes, $|\rho|$, is very much greater than unity. Figures 11,12 and 13 show the situation for $\Delta m / \Gamma=0.2$ (at the high end of theoretical prejudice before the ARGUS result ${ }^{15}$ for $B_{d}$ mixing), $\Delta m / \Gamma=\pi / 4$ (near the central value from ARGUS), and $\Delta m / \Gamma=5$ (roughly the minimum value expected for the $B_{s}$ in the three generation standard model, given the central value of ARGUS for $B_{d}$ ). In none of these cases are the dashed and solid curves distinguishable within "experimental errors" in drawing the graphs. This is simply because $|\rho|$ is so large that even with "big" mixing the second path to the same final state has a very small amplitude, and hence not much of an interference effect.


Fig. 11. The time dependence for the quark level process $\bar{b} \rightarrow \bar{c} u \bar{d}$ (solid curve) in comparison to that for $b \rightarrow c \bar{u} d$ (dashed curve). At the hadron level this could be, for example, $B_{d} \rightarrow \bar{D}^{-} \pi^{+}$in comparison to $\bar{B}_{d} \rightarrow D^{+} \pi^{-}$. $\Delta m / \Gamma=0.2$.


Fig. 12. Same as Figure 11, but with $\Delta m / \Gamma=\pi / 4$.


Fig. 13. Same as Figure 11, but with $\Delta m / \Gamma=5$.

A much more interesting case is shown in Figures 14, 15 and 16 for the time dependence at the quark level for the process $\bar{b} \rightarrow \bar{c} c \bar{s}$ (solid curve) in comparison to that for $b \rightarrow c \bar{c} s$ (dashed curve). At the hadron level this could be, for example, $B_{d}$ in comparison to $\bar{B}_{d}$ decaying to the same, (CP self-conjugate) final state, $\psi K_{s}^{\circ}$. As discussed before, $|\rho|=1$ in this case. The advantages of having $\Delta m / \Gamma$ for the $B_{d}^{\circ}$ system as suggested by ARGUS (Figure 15) rather than previous theoretical estimates (Figure 14) are very apparent. When we go to mixing parameters expected for the $B_{s}^{\circ}$ system (Figure 16), the effects are truly spectacular.


Fig. 14. The time dependence for the quark level process $\bar{b} \rightarrow \bar{c} c \bar{s}$ (solid curve) in comparison to that for $b \rightarrow c \bar{c} s$ (dashed curve). At the hadron level this could be, for example, $B_{d} \rightarrow \psi K_{g}^{\circ}$ (dashed curve) in comparison to $\bar{B}_{d} \rightarrow \psi K_{g}^{\circ}$ (solid curve). (The curves are interchanged for the $\psi K_{s}^{\circ}$ final state because it is odd under CP.) $\Delta m / \Gamma=0.2$.


Fig. 15. Same as Figure 14, but with $\Delta m / \Gamma=\pi / 4$.


Fig. 16. Same as Figure 14, but with $\Delta m / \Gamma=5$.

Figures 17, 18 and 19 illustrate the opposite situation to that in Figures 11 to 13; mixing into a big amplitude from a small one. We are explicitly comparing the quark level process $\bar{b} \rightarrow \bar{u} c \bar{d}$ (solid curve) to $b \rightarrow u \bar{c} d$ (dashed curve). At the hadron level this could be, for example, $B_{d} \rightarrow D^{+} \pi^{-}$in comparison to $\bar{B}_{d} \rightarrow \bar{D}^{-} \pi^{+}$. The direct process is very much $\mathrm{K}-\mathrm{M}$ suppressed compared to that which occurs through mixing and hence the magnitude of the ratio of amplitudes, $|\rho|$, is very much less than unity. Here we have an example where too much mixing can be bad for you! As the mixing is increased (going from Figure 17 to 19), the admixed amplitude comes to completely dominate over the original amplitude, and their interference (leading to an asymmetry) becomes less important in comparison to the dominant term.


Fig. 17. The time dependence for the quark level process $\bar{b} \rightarrow \bar{u} c \bar{d}$ (solid curve) in comparison to that for $b \rightarrow u \bar{c} d$ (dashed curve). At the hadron level this could be, for example, $B_{d} \rightarrow D^{+} \pi^{-}$in comparison to $\bar{B}_{d} \rightarrow \bar{D}^{-} \pi^{+}$. $\Delta m / \Gamma=0.2$.


Fig. 18. Same as Figure 17, but with $\Delta m / \Gamma=\pi / 4$.


Fig. 19. Same as Figure 17, but with $\Delta m / \Gamma=5$.

A more likely example of the situation for $B_{s}$ mixing is shown ${ }^{32}$ in Figure 20(c). The oscillations are so rapid that even with a very favorable difference in the time dependence for an initial $B_{s}$ versus an initial $\bar{B}_{s}$, the time integrated asymmetry is quite small. Measurement of the time dependence becomes a necessity for CP violation studies.

A second path to the same final state could arise in several other ways besides through mixing. For example, one could have two cascade decays that end up with the same final state, such as:

$$
B_{u}^{-} \rightarrow D^{\circ} K^{-} \rightarrow K_{s}^{\circ} \pi^{\circ} K^{-}
$$

and

$$
B_{u}^{-} \rightarrow \bar{D}^{\circ} K^{-} \rightarrow K_{s}^{\circ} \pi^{\circ} K^{-}
$$



Fig. 20. The time dependence for the quark level process $\bar{b} \rightarrow \bar{u} u \bar{d}$ (dashed curve) in comparison to that for $b \rightarrow u \bar{u} d$ (solid curve). At the hadron level this could be, for example, $B_{s} \rightarrow \rho K_{s}^{\circ}$ (solid curve) in comparison to $\bar{B}_{s} \rightarrow \rho K_{s}^{\circ}$ (dashed curve) (the curves are interchanged for the $\rho K_{s}^{\circ}$ final state because it is odd under CP) for values of (a) $\Delta m / \Gamma=1$, (b) $\Delta m / \Gamma=5$, and (c) $\Delta m / \Gamma=15$.

Another possibility is to have spectator and annihilation graphs contribute to the same process. ${ }^{33}$ Still another is to have spectator and "penguin" diagrams interfere. This latter possibility is the analogue of the origin of the parameter $\epsilon^{\prime}$ in neutral K decay, but as discussed previously, there is no reason to generally expect a small asymmetry here. Indeed, with a careful choice of the decay process, large CP-violating asymmetries are expected.

Note that not only do these routes to obtaining a CP-violating asymmetry in decay rates not involve mixing, but they do not require one to know whether one started with a $B$ or $\bar{B}$, i.e., they do not require "tagging." These decay modes are in fact "self-tagging" in that the properties of the decay products (through their electric charges or flavors) themselves fix the nature of the parent $B$ or $\bar{B}$.

Even with potentially large asymmetries, the experimental task of detecting these effects is a monumental one. When the numbers for branching ratios, efficiencies, etc. are put in, it appears that $10^{7}$ to $10^{8}$ produced $B$ mesons are required to end up with a significant asymmetry (say, $3 \sigma$ ), depending on the decay mode chosen. ${ }^{28}$ This is beyond the samples available today (of order a few times $10^{5}$ ) or in the near future ( $\sim 10^{6}$ ).

## The Outlook

I look at the next several years as being analogous to reconnaissance before a battle: We are looking for the right place and manner to attack CP violation in the $B$ meson system. We need:

- Information on branching ratios of "interesting" modes down to the $\sim 10^{-5}$ level in branching ratio. For example, we would like to know the branching ratios for $B_{d} \rightarrow \pi \pi, p \bar{p}, K \pi, \psi K, D \bar{D}+$ three body modes $+\ldots$ and for $B_{s} \rightarrow \psi \phi, K \bar{K}, D \pi$, $\rho K, \ldots$
- Accurate $B \bar{B}$ mixing data, first for $B_{d}$, but especially verification of the predicted large mixing of $B_{s}$.
- A look at the "benchmark" process of rare decays, $B \rightarrow K \mu \bar{\mu}$.
- Experience with triggering, secondary vertices, tertiary vertices, "tagging" b versus $\bar{B}$, distinguishing $B_{u}$ from $B_{d}$, distinguishing $B_{d}$ from $B_{s}, \ldots$
- Various "engineering numbers" on cross sections, $x_{F}$ dependence, $B$ versus $\bar{B}$ production in hadronic collisions, . . .
Many of these things are worthy, lesser goals in their own right, and may reveal their own "surprises." But the major goal is to observe CP violation. With all the possibilities, plus our past history of getting some "lucky breaks," over the next few years we ought to be able to find some favorable modes and a workable trigger and detection strategy. While the actual observation of CP violation may well be five or more years away, this is a subject whose time has come.


## REFERENCES

1. S. W. Herb et al., Phys. Rev. Lett. 39, 252 (1977).
2. E605 Collaboration, to be published and C. Brown, private communication.
3. K. Berkelman, Phys. Rep. 98, 145 (1983); P. Franzini and J. Lee-Franzini, Ann. Rev. Nucl. Sci. 33, 1 (1983).
4. Crystal Ball Collaboration, to be published; E. D. Bloom, private communication.
5. R. Nernst et al., Phys. Rev. Lett. 54, 2195 (1985).
6. J. Lee-Franzini, invited talk at the XVth SLAC Summer Institute on Particle Physics, August 10-21, 1987 (unpublished).
7. K. Berkelman, invited talk at the International Symposium on the Production and Decay of Heavy Flavors, Stanford, California, September 1-5, 1987 and Cornell preprint CLNS 97/102, 1987 (unpublished) and references therein.
8. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
9. Particle Data Group, Phys. Lett. 170B, 74 (1986), and references therein.
10. Most of the error quoted in Eq. (3) is not from the experimental uncertainty in the value of the $b$ lifetime, but in the theoretical uncertainties in choosing a value of $m_{b}$ and in the use of the quark model to represent inclusively semileptonic decays which, at least for the $B$, are dominated by a few exclusive channels. We have made the error bars larger than they are often stated to reflect these uncertainties.
11. W. Schmidt-Parzefall, invited talk at the 1987 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, Germany, July 27-31, 1987.
12. See, for example, B. Stech, invited talk at the International Europhysics Conference, Uppsala, Sweden, June 25-July 1, 1987 and Heidelberg preprint HD-THEP-87-19, 1987 (unpublished); I. I. Y. Bigi, invited talk at the International Symposium on the Production and Decay of Heavy Flavors, Stanford, California, September 1-5, 1987 and SLAC preprint SLAC-PUB-4455, 1987 (unpublished).
13. B. Guberina, R. D. Peccei and R. Ruckl, Phys. Lett. 90B, 169 (1980).
14. For a more recent calculation of "penguin" effects in exclusive $B$ decays see M. B. Gavela et al., Phys. Lett. 154B, 425 (1985).
15. H. Albrecht et al., Phys. Lett. 192B, 245 (1987).
16. N. G. Deshpande, invited talk at the International Symposium on the Production and Decay of Heavy Flavors, Stanford, California, September 1-5, 1987 (unpublished) reviews radiative $B$ decays and references earlier work on the subject.
17. G. Eilam, J. L. Hewett and T. G. Rizzo, Phys. Rev. D34, 2773 (1986); W.-S. Hou, R. S. Willey and A. Soni, Phys. Rev. Lett. 58, 1608 (1987).
18. N. G. Deshpande, P. Lo and J. Trampetic, University of Oregon preprint OITS-352, 1987 (unpublished); N. G. Deshpande et al., Phys. Rev. Lett. 59, 183 (1987).
19. S. Bertolini, F. Borzumati and A. Masiero, Phys. Rev. Lett. 59, 180 (1987).
20. B. Grinstein, R. Springer and M. B. Wise, Caltech preprint CALT-68-1451, 1987 (unpublished) consider explicitly the QCD corrections when $m_{t} \sim M_{W}$.
21. M. A. Shifman et al., Phys. Rev. D18, 2583 (1978).
22. W.-S. Hou, A. Soni and H. Steger, Phys. Lett. 192B, 441 (1987).
23. S. Bertolini, F. Borzumati and A. Masiero, Phys. Lett. 192B, 437 (1987).
24. N. G. Deshpande and A. Soni, Proceedings of the 1986 Summer Study on the Physics of the Superconducting Supercollider, edited by R. Donaldson and J. Marx (Fermilab, Batavia, 1987), p. 58 and references therein.
25. See C. Jarlskog, invited talk at the International Symposium on the Production and Decay of Heavy Flavors, Stanford, California, September 1-5, 1987 (unpublished) and references therein.
26. B. Winstein, invited talk at the Division of Particles and Fields Meeting, Salt Lake City, Utah, January 14-17, 1987 (unpublished).
27. I. Mannelli, invited talk at the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, Germany, July 27-31, 1987 and CERN preprint CERN-EP/87-177, 1987 (unpublished).
28. K. J. Foley et al., to be published in the Proceedings of the Workshop on Experiments, Detectors and Experimental Areas for the Supercollider, Berkeley, California, July 7-17, 1987 and SLAC preprint SLAC-PUB-4426, 1987 (unpublished) review the current status of CP violation in $B$ decay and give references to previous work.
29. A. Pais and S. B. Treiman, Phys. Rev. D12, 2744 (1975); L. B. Okun et al., Nuovo Cim. Lett. 13, 218 (1975).
30. The importance of this has been particularly emphasized by I. Dunietz and J. L. Rosner, Phys. Rev. D34, 1404 (1986).
31. These graphs were constructed by R. Kauffman, in accord with the paper of Dunietz and Rosner, Ref. 30, but with somewhat different parameters: $s_{1}=0.22, s_{2}=$ $0.09, s_{3}=0.05$ and $\delta=150^{\circ}$ and the values of $\Delta M / \Gamma$ given in the text and figure captions.
32. I. Dunietz, University of Chicago Ph.D thesis, 1987 (unpublished).
33. This possibility has been particularly emphasized by L. L. Chau and H. Y. Cheng, Phys. Lett. 65B, 429 (1985).

[^0]:    *Work supported by the Department of Energy, contract DE-AC03-76SF00515.
    Invited talk at the Workshop on High Sensitivity Beauty Physics at Fermilab, Batavia, Illinois, November 11-14, 1987

