# A SHORT VADEMECUM ON CP VIOLATION IN HEAVY FLAVOR DECAYS' 

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#### Abstract

A short introduction into the phenomenology of CP asymmetries in beauty (and $\mathrm{D}^{0}$ ) decays is given. Different experimental environments are briefly compared and some semiquantitative estimates listed.


## 1. Introduction

For more than 20 years now we have known CP invariance to be broken in nature; the profound importance of this discovery was immediately realized. However, no real understanding of this phenomenon has emerged yet; one cannot even claim to possess a unique parametrization. I believe that this embarrassing situation will not be overcome unless CP violation can be studied in a dynamical system that is quite different from neutral kaons.

When one relies on the minimal model for implementing CP violation, namely the KM ansatz, one is lead to a quite unequivocal answer to the question where to look for CP violation: the decays of beauty hadrons are the process of choice.

In the KM ansatz it is the interplay of three quark families that makes CP violation observable. Therefore, it is highly advantageous to study beauty decays: $b$-quarks belong to the third family, yet have to decay into members of the lower families.

This general result can easily be made more specific. The requirement that the KM matrix be unitary yields, among others, the following two relations:

$$
\begin{align*}
& V(u d) V^{*}(t d)+V(u s) V^{\prime}(t s)+V(u b) V^{*}(t b)=0  \tag{1}\\
& V(c d) V^{*}(t d)+V(c s) V^{*}(t s)+V(c b) V^{\prime}(t b)=0 \tag{2}
\end{align*}
$$

which simplify considerably when terms of higher order in the small $K M$ angles are ignored $\left(\lambda=\sin \theta_{c}\right)$ :

$$
\begin{align*}
V^{\prime}(t d)+\lambda V^{\prime}(t s)+V(u b) & \simeq 0  \tag{3}\\
-\lambda V^{\prime}(t d)+V^{\prime}(t s)+V(c b) & \simeq 0 \tag{4}
\end{align*}
$$

As first emphasized by Bjorken, Eqs. (3) and (4) are triangle relations that are accessible to intuitive arguments: Eq. (4) describes a "squashed" triangle with $V(t d)=-V(c b)+O\left(\lambda^{2}\right)$. Equation (3) can then be reexpressed as follows:

$$
\begin{equation*}
V^{\prime}(t d)+V(u b)=A \lambda^{3} \tag{5}
\end{equation*}
$$

with $V(c b) \simeq A \lambda^{2}$ in the Wolfenstein notation. According to the data $-\tau_{B}, B^{\circ}-\bar{B}$ mixing and $B \rightarrow p \phi \pi(\pi)-|V(t d)|, V(u b) \mid \sim$ $O\left(\lambda^{3}\right)$; the angles in this triangle are therefore not particularly small, i.e., $V(u b)$ and $V(t d)$ carry sizeable complex phases. They can be probed in B-decays with high sensitivity: this is obviously true for $V(u b)$; it is also correct for $V(t d)$ since it is a crucial element in $B_{d}-\bar{B}_{d}$ mixing. Accordingly, we can be confident that somewhere in $B$-decays large CP asymmetries, say $\sim O(10 \%)$, exist.

The next question is obvious: In which specific $B$-decays does one have the best chance to uncover such CP asymmetries? At present it would be quite premature to attempt a quantitative answer; after all, very few $B_{u, d}$ branching ratios are known, the lifetimes of neutral and charged $B$-mesons have not been determined separately and the

[^0]actual value of the top mass is not known. Therefore, we will present semi-quantitative scenarios that can successively be refined when more data and a better theoretical understanding become available.

One basic classification should be made right from the start: one compares the evolution of decay rates in proper time

$$
\begin{equation*}
\operatorname{rate}(B(t) \rightarrow f)=e^{-\Gamma t} G \leftrightarrow \operatorname{rate}(\bar{B}(t) \rightarrow \bar{f})=e^{-\Gamma t} \bar{G} \tag{6}
\end{equation*}
$$

$G / \bar{G} \neq 1$ establishes CP violation. Such a difference can be realized in two quite distinct ways:

$$
\begin{align*}
& \frac{d}{d t} \frac{G}{\bar{G}} \equiv 0  \tag{7}\\
& \frac{d}{d t} \frac{G}{\bar{G}} \neq 0 \tag{8}
\end{align*}
$$

When $f$ is flavor-specific, i.e., $B(0) \rightarrow f \nmid \bar{B}(0)$, the first situation, Eq. (7), applies. This is always the case when final state interactions (hereafter referred to as FSI) are essential for making a CP asymmetry observable. When $f$ is common to both $B$ and $\bar{B}$-decays-possible only for neutral $B$-decays-then the second scenario, Eq. (8), applies which, as we will see, involves $B^{\circ}-\bar{B}^{\circ}$ mixing.

I will discuss these two cases where I will concentrate on the underlying concepts rather than on the technicalities and details; these can be found in the literature. ${ }^{1}$

## II. $B^{\circ}-\bar{B}^{\circ}$ Mixing and CP Asymmetries

The Pais-Treiman formalism for mixing is applied in a straightforward way:

$$
\begin{gather*}
\left|\bar{B}^{\circ}(t)\right\rangle=g_{+}(t)\left|B^{\circ}\right\rangle_{0}+\frac{q}{p} g_{-}(t)\left|\bar{B}^{\circ}\right\rangle_{0}  \tag{9}\\
\left|\bar{B}^{\circ}(t)\right\rangle=\frac{p}{q} g_{-}(t)\left|\bar{B}^{\circ}\right\rangle_{0}+g_{+}(t)\left|\bar{B}^{\circ}\right\rangle_{0}  \tag{10}\\
g_{ \pm}(t)=\frac{1}{2} e^{-\frac{1}{2} \Gamma_{1} t} e^{i m_{1} t}\left(1 \pm e^{-\frac{1}{2} \Delta \Gamma t} e^{i \Delta m t}\right)  \tag{11}\\
\Delta \Gamma=\Gamma_{2}-\Gamma_{1}, \Delta m=m_{2}-m_{1}
\end{gather*}
$$

The phase of the quantity $q / p$ depends on the phase convention adopted for $\left|\bar{B}^{\circ}\right\rangle_{0} ;$ yet $|q / p|$ does not and therefore represents an observable:

$$
\begin{align*}
\left|\frac{q}{p}\right| & =1+\frac{1}{2} F \sin \phi(\Delta S=2)  \tag{12}\\
F & \simeq\left|\frac{\Gamma_{12}}{M_{12}}\right| \quad \phi(\Delta B=2)=\arg \frac{M_{12}}{\Gamma_{12}}
\end{align*}
$$

A deviation of $|g / p|$ from unity represents a violation of $C P$ invariance.

Semi-leptonic $B^{\circ}$-decays which are flavor-specific allow in principle to search for the corresponding CP asymmetry: the notation

$$
\begin{equation*}
r=\frac{\Gamma\left(B^{\circ} \rightarrow \ell^{-} X\right)}{\Gamma\left(\bar{B}^{\circ} \rightarrow \ell^{+} X\right)} \quad, \quad \bar{r}=\frac{\Gamma\left(\bar{B}^{\circ} \rightarrow \ell^{+} X\right)}{\Gamma\left(B^{\circ} \rightarrow \ell^{-} X\right)} \tag{13}
\end{equation*}
$$

refers to time-integrated rates where $r, F \neq 0$ signals the occurrence of mixing. One then finds

$$
\begin{equation*}
a_{s L}=\frac{r-p}{r+\bar{r}}=\frac{1-\left|\frac{p}{q}\right|^{4}}{1+\left|\frac{p}{q}\right|^{4}} \tag{14}
\end{equation*}
$$

Unfortunately one predicts tiny asymmetries in the KM ansatz (with three families):

$$
\begin{gather*}
a_{s L}\left(B_{d}\right) \leqslant 10^{-3}  \tag{15}\\
a_{s L}\left(B_{s}\right) \leqslant 10^{-4} \tag{16}
\end{gather*}
$$

The smallness of these asymmetries is readily understood:
One estimates

$$
\begin{equation*}
F \simeq \frac{\Delta \Gamma}{\Delta m} \sim O\left(\frac{m_{b}^{2}}{m_{l}^{2}}\right) \ll 1 \tag{17}
\end{equation*}
$$

in contrast to the $K^{\circ}$ case where $F\left(K^{\circ}\right) \simeq 1$ holds and

$$
\begin{equation*}
\phi(\Delta B=2) \sim 0\left(\frac{m_{c}^{2}}{m_{b}^{2}}\right) \ll 1 \tag{18}
\end{equation*}
$$

not dissimilar from the $K^{\circ}$ case.
To observe the kind of CP asymmetry as expressed by $\phi(\Delta B=2)$, Eq. (12), appears therefore to be a rather hopeless enterprise.

This should, however, not drive us into despair about ever observing CP-violation in $B$-decays: there is a second scenario for observable CP violation as characterized by Eq. (8). It applies when a final state $f$ can be reached in both $B^{\circ}$ and $\bar{B}^{\circ}$ decays. There are two types of final states than can satisfy this requirement, namely
(i) CP eigenstates like $B^{\circ} \rightarrow \psi K_{s}, D_{s} \bar{D}_{s}, D \bar{D}, \pi \pi \leftarrow \bar{B}^{\circ}$.

- (ii) Non-CP eigenstates like $B^{\circ} \rightarrow D^{ \pm} \pi^{\mp} \leftarrow \bar{B}^{\circ}$.

The same basic formalism applies in both cases. For this reason, I will restrict myself to discussing CP eigenstates only: our predictions are more reliable there and the physics involved more transparent.

A little theorem can help to illustrate the situation: Let $B_{\text {neut }}$ denote any combination of $B^{\circ}$ and $\bar{B}^{\circ}$-mesons and $f$ a CP eigenstate of definite CP parity. Finding the (proper) time dependence of the decay rate $B_{\text {neut }} \rightarrow f$ to be different from a single, pure exponential, i.e.,

$$
\begin{equation*}
\frac{d}{d t} e^{\Gamma t} \text { rate }\left(B_{n e u t}(t) \rightarrow f\right) \neq 0 \text { for all } \Gamma \tag{19}
\end{equation*}
$$

amounts to an observation of $C P$ violation. The proof is very elementary and can be found elsewhere.

One can be even more specific and show that the most general time evolution is given by four terms:

$$
\begin{align*}
& \operatorname{rate}\left(B_{\text {neut }}(t) \rightarrow f\right) \alpha e^{-\Gamma t}\left(1+A e^{-\Delta \Gamma t}\right.  \tag{20}\\
& \left.\quad+B e^{-\frac{1}{2} \Delta \Gamma t} \cos (\Delta m t)+C e^{-\frac{1}{2} \Delta \Gamma t} \sin (\Delta m t)\right)
\end{align*}
$$

Since one estimates $\Delta \Gamma \ll \Gamma,|q / p| \simeq 1$ one can simplify Eq. (20) considerably

$$
\begin{equation*}
\operatorname{rate}\left(B_{\text {neut }}(t) \rightarrow f\right) \alpha e^{-\Gamma t}\left(1+\frac{N-\bar{N}}{N+\bar{N}} \operatorname{Im} \frac{q}{p} \bar{\rho}_{f} \sin \Delta m t\right) \tag{21}
\end{equation*}
$$

where $\bar{\rho}_{f}=$ Arnpl. $(\bar{B} \rightarrow f) /$ Ampl. $(B \rightarrow f) ; N[\bar{N}]$ denotes the number of $B^{\circ}\left|\bar{B}^{\circ}\right|$-mesons present at $\boldsymbol{t}=\mathbf{0}$.

Equation (21) contains three crucial elements:
(i) $\operatorname{Im} \frac{q}{p} \tilde{\rho}_{f}:$ It is this quantity that is intrinsically connected with CP violation which suggests the following notation:

$$
\begin{equation*}
\frac{q}{p} \bar{\rho}_{f} \equiv\left|\frac{q}{p} \bar{\rho}_{f}\right| e^{i \phi(\Delta R=1 \& 2)} \tag{22}
\end{equation*}
$$

The phase $\phi(\Delta B=1 \& 2)$ represents the strength of CP violation and combines the effects of the $\Delta B=2$ mixing process- $q / p$-and the $\Delta B=1$ decay $-\bar{\rho}_{f}$.
(ii) $\sin \Delta m t$ : This factor explicitly exhibits the need for mixing to occur- $\Delta m \neq 0$-to have an observable CP asymmetry. Yet it should be noted that its dependence on $\Delta m$ is quite different from the time-integrated quantity $r$ usually employed to express mixing:

$$
\begin{equation*}
r=\frac{\Gamma\left(B^{\circ} \rightarrow \ell^{-} X\right)}{\Gamma\left(B^{\circ} \rightarrow \ell^{+} X\right)} \simeq \frac{x^{2}}{2+x^{2}}, x=\frac{\Delta m}{\Gamma} \tag{23}
\end{equation*}
$$

(iii) $N-\bar{N}$ : If one starts from an equal population of $B^{\circ}$ and $\bar{B}^{\circ}$ in the sample under study (and if as expected $\Delta \Gamma \ll \Gamma$ ) no asymmetry can emerge. The reason for that is quite obvious: since these final states are common to $B^{\circ}$ as well as $\bar{B}^{\circ}$-decays, they can by themselves not reveal whether they came from a $B^{\circ}$ or a $\widehat{B}$; thus no CP asymmetry can be defined.
These quantities will now be discussed in more detail:
$a d(i i i)$ The required flavor tagging can be provided by Nature, i.e., through a production asymmetry like the forwardbackward asymmetry in $e^{+} e^{-} \rightarrow b \bar{b}$ or through associated production or leading particle effects in hadronic collisions; or it can be imposed by human intervention, i.e., by identifying the flavor of the hadron that was produced in conjunction with the neutral $B$-meson whose decay one is studying.
$a d(i i)$ The time dependence of the signal is quite unique and striking. Therefore, one has to place a high premium on the ability to resolve the time evolution. If that cannot be achieved, i.e., if one can observe only time-integrated rates, one has to keep three complications in mind:

- Since

$$
\begin{equation*}
\int_{0}^{\infty} d t \operatorname{rate}(B(t) \rightarrow f) \alpha \frac{x}{1+x^{2}} \operatorname{Im} \frac{q}{p} \bar{\rho}_{f} \tag{24}
\end{equation*}
$$

one encounters large suppression for large mixing, i.e., $x \gg 1$.

- The reaction

$$
e^{+} e^{-} \rightarrow \Upsilon(4 s) \rightarrow B \bar{B}
$$

produces the $B \bar{B}$ pair in a configuration that is odd under charge conjugation. Then one obtains

$$
\begin{align*}
& \iint d t d \bar{t}\left\{\operatorname{rate}\left(B^{\circ}(t) \bar{B}^{\circ}(\bar{t}) \rightarrow\left(\ell^{+} X\right) \bar{f}\right)\right. \\
& \left.\quad-\operatorname{rate}\left(B^{\circ}(t) \bar{B}^{\circ}(\bar{t}) \rightarrow\left(\ell^{-\circ} X\right) f\right)\right\} \alpha  \tag{25}\\
& \iint d t d \bar{t} e^{-\Gamma(t+\bar{t})} \sin \Delta m(t-\bar{t}) \operatorname{Im} \frac{q}{p} \bar{\rho}_{f}=0
\end{align*}
$$

i.e., no asymmetry can be observed.

- In

$$
\begin{equation*}
\mathrm{e}^{+} e^{-} \rightarrow B^{\circ} \bar{B}^{\circ \star}+\text { h.c. } \rightarrow B^{\circ} \bar{B}^{\circ} \gamma \tag{26}
\end{equation*}
$$

one finds after complete time integration a factor $2 x /\left(1+x^{2}\right)^{2}$ in the asymmetry which acts like a $1 / x^{3}$ suppression for $x \gg 1$.
A value $x=\Delta m / \Gamma \sim 1-$ similar to the ARGUS findings on $B_{d}-\bar{B}_{d}$ mixing-is quite optimal for these studies.
$\operatorname{ad}(i)$ As already mentioned one predicts $|q / p| \simeq 1$ with a high degree of confidence. For decays like $B \rightarrow \psi K_{s}$ where only one isospin amplitude contributes $\left|\bar{\rho}_{f}\right|=1$ holds. In those cases $q / p \bar{\rho}_{f}$ represents a unit vector in the complex plane whose phase $-\phi(\Delta B=1 \& 2)$ - is given in terms of KM parameters.
Decays like $B_{s} \rightarrow \psi \phi, D_{s} \bar{D}_{s}$ which involve $(\bar{b} s) \rightarrow \bar{c} c \bar{s} s$ transitions on the quark level are expected to exhibit relatively small CP asymmetries:

$$
\begin{equation*}
\operatorname{Im} \frac{q}{p} \bar{\rho}_{f}(\bar{b} s \rightarrow \tau c \bar{s} s) \sim \mathcal{O}\left(\lambda^{2}\right) \lesssim \text { few } \% \tag{27}
\end{equation*}
$$

This is not surprising at all, since on the leading level only quarks of the second and third families contribute. More specifically, this situation is described by the triangle of Eq. (4).

The quark level transitions $(\bar{b} d) \rightarrow \bar{c} c \bar{s} d,(b d) \rightarrow \bar{u} u d d$ and $(\bar{b} s) \rightarrow \bar{s} u \bar{u} s$, on the other hand, probe the Bjorken triangle, Eq. (5). More precisely, for the decays

$$
B_{d} \rightarrow \psi K_{s}, B_{d} \rightarrow \pi^{+} \pi^{-}, B_{s} \rightarrow K^{+} K^{-}
$$

one finds

$$
\begin{equation*}
\operatorname{Im} \frac{q}{p} \bar{\rho}_{f} \sim \sin 2 \varphi_{1}, \sim-\sin 2 \varphi_{2}, \sim \sin 2 \varphi_{2} \tag{28}
\end{equation*}
$$

where

$$
\begin{equation*}
-V(t d)=|V(t d)| e^{i \varphi_{1}}, V(u b)=|V(u b)| e^{i \varphi_{2}} . \tag{29}
\end{equation*}
$$

Any violation of Eq. (27) or (28)-like $\operatorname{Im} q / p \bar{p}_{f}(\bar{b} s \rightarrow$ $\bar{c} c \bar{s} s) \gtrsim 0.1$ or $\varphi_{1}+\varphi_{2}+\varphi_{3} \neq 180^{\circ}$, i.e., a "nonplanar geometry"-would show the existence of New Physics, most likely a fourth family.

None of the angles $\varphi_{i}=1,2,3$, has a particular propensity to have a value close to $0^{\circ}$ or $90^{\circ}$. Overall one can say (details can be found in the literature):

$$
\begin{equation*}
\operatorname{Im} \frac{q}{p} \bar{\rho}_{f} \simeq O(0.1) \tag{30}
\end{equation*}
$$

is quite realistic and even values like

$$
\begin{equation*}
\operatorname{Im} \frac{q}{p} \bar{\rho} \simeq 0.5 \tag{31}
\end{equation*}
$$

though being optimistic are attainable.
Since the branching ratios for the most promising modes are nothing to brag about-for instance,

$$
\begin{aligned}
& B R\left(B_{d} \rightarrow \psi K_{d}\right) \sim 5 \times 10^{-4} \\
& B R\left(B_{d} \rightarrow \pi^{+} \pi^{-}\right) \sim O\left(10^{-5}\right)
\end{aligned}
$$

is expected theoretically-the question arises quite naturally whether one can gain in statistics by analyzing inclusive decays without jeopardizing the signal, i.e., the CP asymmetry. The answer is yes-but only under certain carefully maintained circumstances. For the sign of the asymmetry depends-among other things-on the CP parity of the final state. Therefore,

$$
\begin{equation*}
\text { Asymm. }\left(B \rightarrow \psi K_{s}\right)=- \text { Asymm. }\left(B \rightarrow \psi K_{L}\right) . \tag{32}
\end{equation*}
$$

Accordingly

$$
\begin{equation*}
\text { Asymm. }\left(B_{d} \rightarrow \psi X\right)=0 . \tag{33}
\end{equation*}
$$

A similar concern has to be addressed in $B^{\circ} \rightarrow p \bar{p}$-decays. For $p \bar{p}$ can form a $p$ - or an $s$-wave and

$$
\begin{equation*}
\text { Asymm. }\left(B \rightarrow[p \bar{p}]_{p}\right)=- \text { Asymm. }\left(B \rightarrow[p \bar{p}]_{s}\right) . \tag{34}
\end{equation*}
$$

For the same reason one can state quite generally that adding a $\pi$ to a final state will flip the sign of the CP asymmetry since $\mathrm{CP}\left|\pi^{\circ}\right\rangle=-\left|\pi^{\circ}\right\rangle$.

There is one meaningful test of CP invariance that can be performed in $e^{+} e^{-} \rightarrow \Upsilon(4 s) \rightarrow B \bar{B}$ even without any capability to resolve decay vertices: one searches for the reaction

$$
e^{+} e^{-} \rightarrow \Upsilon(4 s) \rightarrow B^{\circ} \bar{B}^{\circ} \rightarrow f_{1} f_{2}
$$

where $f_{1}, f_{2}$ denote two CP eigenstates of the same CP parity. A single event of this type (in principle) establishes $C P$ violation. For the initial state is CP even, the final state CP odd:

$$
\begin{equation*}
\mathrm{CP}[\Upsilon(4 s)]=+1 ; \operatorname{CP}\left[f_{1} f_{2}\right]=\operatorname{CP}\left[f_{1}\right] \operatorname{CP}\left[f_{2}\right](-1)^{l}=-1 \tag{35}
\end{equation*}
$$ since $B \bar{B}$ are produced in a $p$-wave.

Quantitatively one finds

$$
\begin{align*}
& B R\left(\left.B^{\circ} \bar{B}\right|_{\Upsilon(4 s)} \rightarrow f_{1} f_{2}\right) \sim F B R\left(B \rightarrow f_{1}\right) B R\left(B \rightarrow f_{2}\right) \\
& F=\frac{x^{2}}{1+x^{2}}\left(2 \operatorname{Im} \frac{q}{p} \bar{\rho}_{f_{1}}\right)\left(2 \operatorname{lm} \frac{q}{p} \bar{\rho}_{f_{2}}\right) \sim \frac{1}{4}-1 . \tag{36}
\end{align*}
$$

As a final remark: The same phenomenology can be applied to $D^{\circ}$-decays like $D^{\circ} \rightarrow K^{+} K^{-}$:

$$
\begin{align*}
& \operatorname{rate}\left(D^{\circ}(t) \rightarrow K^{+} K^{-}\right) \alpha e^{-\Gamma t}\left(1-\sin \Delta m t \operatorname{Im} \frac{q}{p} \bar{\rho}_{f}\right)  \tag{37}\\
& \operatorname{rate}\left(\bar{D}^{\circ}(t) \rightarrow K^{+} K^{-}\right) \alpha e^{-\Gamma t}\left(1+\sin \Delta m t \operatorname{Im} \frac{q}{p} \bar{\rho}_{f}\right) \tag{38}
\end{align*}
$$

Such a study is greatly helped by two very beneficial circumstances:

- The branching ratio is quite decent:

$$
B R\left(D^{\circ} \rightarrow K^{+} K^{-}\right) \sim 0.5 \%
$$

- Flavor tagging can effectively be achieved via $D^{* \pm} \rightarrow$ ${ }_{D}^{(-)} \pi^{ \pm}$decays.

There is of course a double caveat:
(i) The Standard Model predicts very little $D^{\circ}-\bar{D}^{\circ}$ mixing and no observable CP violation. This makes it a unique hunting ground for New Physics.
(ii) The E691 collaboration has placed a very stringent upper bound on $D^{\circ}-\bar{D}^{\circ}$ mixing

$$
\begin{equation*}
r_{D} \equiv \frac{\Gamma\left(D^{\circ} \rightarrow \bar{D}^{\circ} \rightarrow f\right)}{\Gamma\left(D^{\circ} \rightarrow f\right)}<0.5 \% \tag{39}
\end{equation*}
$$

Yet one has to keep in mind that

$$
\begin{equation*}
r_{D} \sim \frac{x^{2}}{2+x^{2}} \tag{40}
\end{equation*}
$$

Therefore, $r_{D}=0.5 \%$ corresponds to $x=\Delta m / \Gamma=0.1$ and accordingly in this case
$\operatorname{rate}\left(D^{\circ}(t) \rightarrow K^{+} K^{-}\right) \alpha e^{-\Gamma t}\left(1-0.1 \times \frac{t}{\tau_{D}} \operatorname{Im} \frac{q}{p} \bar{\rho}_{f}\right)$,
i.e., $C P$ asymmetries of order $5-10 \%$ are still allowed in principle and should be searched for.

## III. Final State Interactions and CP Violation

When two different amplitudes contribute to the decay of a bottom hadron $B$ into a final state $f$, one writes for the matrix element

$$
\begin{align*}
M_{f} & =\langle f| \mathcal{L}(\Delta B=1)| \rangle \\
& =\langle f| \mathcal{L}_{1}|B\rangle+\langle f| \mathcal{L}_{2}|B\rangle  \tag{42}\\
& =g_{1} M_{1} e^{i \alpha_{1}}+g_{2}, 2 e^{i \alpha_{2}}
\end{align*}
$$

where $M_{1}, M_{2}$ denote the matrix elements for the weak transition operators $\mathcal{L}_{1}, \mathcal{L}_{2}$ with the KM parameters $g_{1}, g_{2}$ and the strong (or electromagnetic) phase shifts $\alpha_{1}, \alpha_{2}$ factored out. For the CP conjugate decay $\bar{B} \rightarrow \bar{f}$ one then finds

$$
\begin{align*}
\bar{M}_{f} & =\langle\bar{f}| \mathcal{L}(\Delta B=1)|\bar{B}\rangle \\
& =g_{1}^{*} M_{1} e^{i \alpha_{1}}+g_{2}^{*} M_{2} e^{i \alpha_{2}} . \tag{43}
\end{align*}
$$

The same phase shifts $\alpha_{1}, \alpha_{2}$ (instead of $-\alpha_{1},-\alpha_{2}$ ) have been written down in Eq. 43 since CP invariance is obeyed by the strong and electromagnetic forces. Comparing Eq. 42 with Eq. 43 one obtains

$$
\begin{equation*}
\Gamma(B \rightarrow f)-\Gamma(\bar{B} \rightarrow \bar{f}) \propto \operatorname{Im} g_{1}^{*} g_{2} \sin \left(a_{1}-a_{2}\right) M_{1} M_{2} . \tag{44}
\end{equation*}
$$

Thus two conditions have to be met simultaneously for such an " asymmetry to show up:
( $\alpha$ ) The weak couplings $g_{1}$ and $g_{2}$ have to possess a relative complex phase; therefore small KM angles have to be involved.
( $\beta$ ) Nontrivial phase shifts $\alpha_{1} \neq \alpha_{2}$ have to be generated from the strong (or electromagnetic) forces.
Condition ( $\beta$ ) does not, in principle, pose a severe restriction; in practice it introduces considerable uncertainties into numerical predictions. An interesting scenario-in my judgmentis provided by invoking Penguin contributions. ${ }^{2}$ The phase shift $\alpha_{1}-\alpha_{2} \neq 0$ is produced by the loop diagram with charm as the internal quark-which does not yield a local, though maybe a short-distance operator. Doing detailed calculation one finds

$$
\begin{array}{r}
B R\left(B \rightarrow K^{ \pm} \pi^{\mp}\right) \sim O\left(10^{-5}\right) \\
\frac{\Gamma\left(B^{\circ} \rightarrow K^{+} \pi^{-}\right)-\Gamma\left(B^{\circ} \rightarrow K^{-} \pi^{+}\right)}{\Gamma\left(B^{\circ} \rightarrow K^{+} \pi^{-}\right)+\Gamma\left(B^{\circ} \rightarrow K^{-} \pi^{+}\right)} \sim 1-10 \% \tag{45}
\end{array}
$$

The nice feature of this decay mode is that it is flavorspecific: $K^{+} \pi^{-}$can come only from a $B^{\circ}$ whereas $K^{-} \pi^{+}$is necessarily produced in a $\bar{B}$-decay.

## IV. Conclusions

There is one basic unequivocal statement: The KM scheme of implementing CP violation leads to relatively large CP asymmetries in beauty decays. Theoretical uncertainties enter only into questions on the exact size of such asymmetries and on the best modes to search for them.

Improved experimental information on branching ratios, the top mass and on $V(u b)$ will help in an essential way to refine our predictions or expectations.

When CP violation becomes observable due to $B^{\circ}-\bar{B}^{\circ}$ mixing, the following rather general statements can be made:
$(+)$ Large asymmetries of order $10 \%$ or more are expected.
$(+)$ The predictions are relatively reliable.
$(+)$ The very special dependence on proper time that is introduced by mixing should provide a striking signature in searches for asymmetries.
(-) Typically one has to identify exclusive modes; otherwise substantial cancellations can occur as far as the CP asymmetries are concerned. In particular, one does not want to lose $\pi^{\circ}$-mesons.
$(-)$ Flavor tagging is essential.
$(-)$ The reaction

$$
e^{+} e^{-} \rightarrow \Upsilon(4 s) \rightarrow B \bar{B}
$$

is quite ill-suited for any such analysis as long as no information on the $B$-decay vertices is available.
The scorecard looks quite different when it is the final state interactions that make CP violation observable:
$(+)$ No flavor tagging is required.
$(+)$ One can study it also in $\Upsilon(4 s) \rightarrow B \bar{B}$.
$(-)$ One has to rely on number counting since no special time dependence is introduced.
$(-)$ The branching ratios are quite low and it is very hard to see how such a CP asymmetry could ever reach or exceed the $10 \%$ level.
(-) The predictions are less than compelling or reliable.

## References

1. For an up-to-date review see, I. I. Bigi, V. A. Khoze, N. G. Uraltsev and A. I. Sanda, The Question of CP Noninvariance-as seen through the Eyes of Neutral Beauty, SLAC-PUB-4476.
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