SLAC–PUB–4499
CERN–TH.4931/87
MAD/TH/87–30
December 1987
(T)

GUT MODEL–BUILDING WITH FERMIONIC FOUR–DIMENSIONAL STRINGS*

I. ANTONIADIS‡
CERN,CH-1211 Geneva 23, Switzerland

JOHN ELLIS
CERN, CH–1211 Geneva 23, Switzerland
and
Stanford Linear Accelerator Center,
Stanford University, Stanford, California

J. S. HAGELIN
Maharishi International University, Fairfield, IA 52556

and

D. V. NANOPOULOS
University of Wisconsin, Madison, Wisconsin 53706

ABSTRACT

We report a first attempt at model-building using the fermionic formulation of string theories directly in four dimensions. An example is presented of a supersymmetric flipped SU(5) × U(1) model with three generations and an adjustable hidden sector gauge group. The simplest version of the model contains most of the Yukawa couplings required by phenomenology, but not all those needed to give quark flavor mixing, or tree-level masses for all right-handed neutrinos. These defects may be remedied in a more general version of the model.

Submitted to *Physics Letters B*

---

*Work supported by the Department of Energy, contract DE–AC03–76SF00515.

‡On leave of absence from Centre de Physique Théorique, École Polytechnique, 91128 Palaiseau, France.

It is now well appreciated that the two key ingredients in formulating a consistent string theory are conformal invariance and modular invariance. The former condition fixes the number of degrees of freedom on the world-sheet. It can be satisfied in any number of dimensions $d \leq 26$ (10 for a supersymmetric left- or right-moving sector) if the space-time coordinates $X_\mu : \mu = 0, 1, \ldots, d-1$ are supplemented by internal degrees of freedom contributing $26 - d$ $(15 - 3d/2)$ to the central charge of the Virasoro algebra. Modular invariance then imposes non-trivial constraints on the boundary conditions for these internal degrees of freedom which ensure that counting errors are not made when higher-genus string topologies are summed. It is known that the number of solutions to these modular invariance conditions is restricted, particularly if the number of space-time dimensions d is close to the critical number 26 (or 10). In particular, in $d = 10$ there are only two modular invariant heterotic string theories with $N = 1$ space-time supersymmetry, based on the gauge groups SO(32) and $E_8 \times E_8'$ [1].

The choice of gauge group for string theories formulated directly in $d = 4$ dimensions is much more extensive, and is imperfectly understood as yet. An interesting subclass of models is offered by supersymmetric compactifications of the $E_8 \times E_8'$ heterotic string. Early attention was focused on Calabi-Yau manifold compactifications [2], but has subsequently been extended to other manifolds [3] and to orbifold compactifications [4]. The most systematic approach to this ambiguity in the four-dimensional gauge group would be to use one of the general formulations of string theories directly in $d = 4$, using either bosonic [5] or fermionic [6–9] variables to describe the internal degrees of freedom.

The range of choice in four-dimensional theories is embarrassingly generous, and no systematic enumeration of models has yet emerged. The alternative strategy, followed here, is to start from the bottom up, looking for models which contain phenomenologically favored ingredients such as the Standard Model or a plausible Grand Unified Theory (GUT). Already examples are known of Calabi-Yau [10] and orbifold [11] compactifications which yield $SU(3)_C \times SU(2)_L \times U(1)^n : n \geq 2$ gauge groups. In this paper we look for models containing the flipped supersymmetric SU(5) $\times$ U(1) GUT whose virtues were recently extolled [12,13]: natural doublet-triplet mass splitting, a see-saw mechanism for neutrino masses, no cosmologically embarrassing phase transitions [14], *etc.* Of particular importance is the fact that this model does not need any adjoint Higgs representations. No string model with adjoint chiral superfields can be obtained in the fermionic formulation [9], and this may well be a general property. Other GUTs require adjoint Higgses to break the initial GUT symmetry and/or realize doublet-triplet mass splitting naturally. Therefore, SU(5) $\times$ U(1) is uniquely favored as a GUT group, as has already been stressed in the more limited context of manifold compactification [13].

In this paper, our tool for obtaining this uniquely simple GUT is the fermionic formulation of four-dimensional strings, as developed in refs. [8,9]. We exhibit a choice of boundary conditions for the world-sheet fermions which yields immediately an observable [SU(5) $\times$ U(1)] $\times$ U$(1)^3$ gauge group as well as an *à la carte* hidden sector gauge group. The model contains three generations of quarks and leptons, additional $\underline{10}$, $\overline{\underline{10}}$, $\underline{5}$ and $\overline{\underline{5}}$ representations of SU(5) which could be Higgs fields, and a number of SU(5) $\times$ U(1)-invariant fields as advocated in ref. [12]. Vacuum expectation values of scalar fields break one of the surplus U$(1)^3$ gauge generators via the conventional Higgs mechanism, and another is anomalous. The fermionic formulation also tells us what Yukawa couplings are present, and we find all the ones we want except some of those giving quark flavor mixing and masses for some conjugate neutrinos. These defects may be avoided by a small modification of the simplest consistent choice of fermionic boundary conditions which introduces some additional $\underline{10} + \overline{\underline{10}}$ and $\underline{5} + \overline{\underline{5}}$ representations. While developing our model, we strive to illustrate some general principles of fermionic model-building which may aid subsequent, more inspired, efforts.

We start with a brief review of the main characteristics of four-dimensional heterotic string theories in the fermionic formulation [8,9]. In the light-cone gauge, in addition to the two transverse bosonic coordinates X^μ and their left-moving superpartners $\psi^\mu(z)$, the fermionic content is [6] 44 right-moving and 18 left-moving fermions $\bar{\psi}^A(\bar{z})$: $A = 1, 2, \ldots, 44$ and $\chi^i(z)$, $y^i(z), w^i(z)$: $i = 1, 2, \ldots, 6$, respectively. World-sheet supersymmetry is nonlinearly realized among the latter via the supercurrent

$$T_F(z) \;=\; \psi^\mu \, \partial_z \, X_\mu + \sum_{i=1}^{6} \; \chi^i \, y^i \, w^i \quad . \tag{1}$$

A four-dimensional string model is defined by specifying a set Ξ of boundary conditions for all the world-sheet fermions, constrained by making the world-sheet supercurrent (1) periodic (space-time fermions) or antiperiodic (space-time bosons). When all the boundary conditions are diagonalized simultaneously in some general complex basis $\{f\}$, the elements of Ξ are vectors α such that every complex fermion f picks up a phase

$$f \to -e^{i\pi\alpha(f)} \, f \;:\; \alpha(f) \; \in (-1, 1] \quad , \tag{2}$$

when parallel transported around the string. In this case, Ξ forms a group under addition (mod 2), and can therefore be generated by some basis $B \equiv \{b_1, b_2, \ldots, b_N\}$. It has been shown [8,9] that to every element α of Ξ there corresponds a sector $\mathcal{H}_\alpha$ in the string Hilbert space $\mathcal{H}$, and to every basis element b_i of B a fermion number projection:

$$\mathcal{H} \;=\; \bigoplus_{\alpha\in\Xi} \; \prod_{i=1}^{N} \left[e^{i\pi b_i \cdot F} = \delta_\alpha \; c^* \begin{pmatrix} a \\ b_i \end{pmatrix} \right] \mathcal{H}_\alpha \quad , \tag{3}$$

where F is the vector of all fermion numbers defined: $F(f) = 1 = -F(f^*)$, the dot product is Lorentzian (left minus right), δ_α is the space-time fermion parity and the phases $c\begin{pmatrix}\alpha \\ b_i\end{pmatrix}$ are constrained by multiloop modular invariance.

In order for this paper to be self-contained, we now give the explicit form of the constraints on the basis B and on the phases c for generic rational boundary conditions [9].*

<u>Basis B</u>

$a1$) We choose B to be canonical, *i.e.*, any linear combination $\sum_i m_i b_i = 0$ iff $m_i = 0$ (mod N_i) for some integers N_i (for example, $N_i = 2$ when the fermions are periodic or antiperiodic), and the vector $\mathbf{1} \in B$.

$a2$) For any pair b_i, b_j of basis elements, one has† $N_{ij} b_i \cdot b_j = 0$ (mod 4) where N_{ij} is the least common multiple of N_i and N_j, and $N_i b_i^2 = 0$ (mod 8) if N_i is even.

$a3$) The number of real fermions which are simultaneously periodic under four boundary conditions b_1, b_2, b_3, b_4 is even.

<u>Phases c</u>

$b1$) We choose the $c\binom{b_i}{b_j}$ for $i < j$ such that they are simultaneously $\delta_{b_i} \times$ (N_j^{th} root of unity) and $\delta_{b_j} \times \exp\{i\frac{\pi}{2} b_i \cdot b_j\} \times$ (N_i^{th} root of unity).

$b2$) The remaining phases are calculated using the properties

---

* We consider here neither some exceptional cases, nor the nontrivial realizations of world-sheet supersymmetry discussed in ref. [9].

† The Lorentzian dot-product counts each real fermion with a factor 1/2.

$$c\begin{pmatrix}\alpha\\ \alpha\end{pmatrix} = -\exp\{i\frac{\pi}{4}\alpha^2\}\, c\begin{pmatrix}\alpha\\ 1\end{pmatrix} \quad ,$$

$$c\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \exp\{i\frac{\pi}{2}\alpha\cdot\beta\}\, c^*\begin{pmatrix}\beta\\ \alpha\end{pmatrix} \quad , \tag{4}$$

$$c\begin{pmatrix}\alpha\\ \beta+\gamma\end{pmatrix} = \delta_\alpha\, c\begin{pmatrix}\alpha\\ \beta\end{pmatrix} c\begin{pmatrix}\alpha\\ \gamma\end{pmatrix} \quad .$$

Physical states from the sector $\mathcal{H}_\alpha$ are obtained by acting on the vacuum $|0\rangle_\alpha$ with bosonic or fermionic oscillators with frequencies $[1 + \alpha(f)]/2$ ($[1 - \alpha(f)]/2$ for f^*) and applying the fermion number projections, eq. (3). The mass formula is

$$M^2 = -\frac{1}{2} + \frac{1}{8}\,\alpha_L^2 + \sum_L \nu_L = -1 + \frac{1}{8}\,\alpha_R^2 + \sum_R \nu_R \quad , \tag{5}$$

where $\alpha_L\,(\alpha_R)$ is the left (right) part of the vector α and the $\nu_L(\nu_R)$ are frequencies. When some fermions are periodic, the vacuum is a spinor in order to represent the Clifford algebra of the corresponding zero modes. For each periodic complex fermion f there are two degenerate vacua $|+\rangle\,,|-\rangle$, annihilated by the zero modes f_0 and f_0^*, and with fermion numbers $F(f) = 0,1$, respectively.

Before starting to build a model, we note a simple but very crucial relation between the world-sheet fermion numbers $F(f)$ and the U(1) charges $Q(f)$ with respect to the unbroken Cartan generators of the four-dimensional gauge group, which are in one-to-one correspondence with the U(1) currents f^*f for each complex fermion f:

$$Q(f) = \frac{\alpha(f)}{2} + F(f) \quad . \tag{6}$$

The charges $Q(f)$ can be shown to be identical with the momenta of the corresponding compactified scalars in the bosonic formulation [15]. The representation

(6) shows that Q is identical with the world-sheet fermion numbers F for states in a Neveu–Schwarz sector ($\alpha = 0$), and is ($F + 1/2$) for states in a Ramond sector ($\alpha = 1$); note that the charges of the $|\pm\rangle$ spinor vacua are $\pm 1/2$.

The model we seek in this four-dimensional fermionic string formalism is the supersymmetric flipped SU(5) GUT of ref. [12]. It contains the following chiral matter superfields: Three generations of matter fields $F_i = (\underline{10}, 1/2)$, $\bar{f}_i = (\bar{5}, -3/2)$, and $\ell_i^c = (\underline{1}, 5/2)$; Higgses $H = (\underline{10}, 1/2)$, $\bar{H} = (\overline{10}, -1/2)$, $h = (\underline{5}, -1)$, and $\bar{h} = (\bar{5}, 1)$; and four singlets $\phi_{0,i} = (\underline{1}, 0)$ of SU(5) $\times$ U(1). The superpotential of the model is [12]

$$\begin{aligned} W &= \lambda_1 FFh + \lambda_2 F\bar{f}\bar{h} + \lambda_3 \bar{f}\ell^c h + \lambda_4 HHh \\ &+ \lambda_5 \bar{H}\bar{H}\bar{h} + \lambda_6 F\bar{H}\phi + \lambda_7 h\bar{h}\phi + \lambda_i \phi^3 \quad , \end{aligned} \tag{7}$$

where the Yukawa couplings $\lambda_{1,2,3,6,7,8}$ are matrices in generation space. One component each of the H and $\bar{H}$ acquire large v.e.v.'s breaking SU(5) $\times$ U(1) $\rightarrow$ SU(3)$_\mathrm{C}$ $\times$ SU(2)$_\mathrm{L}$$\times$ U(1)$_\mathrm{Y}$, and SU(2)$_\mathrm{L}$$\times$ U(1)$_\mathrm{Y}$ $\rightarrow$ U(1)$_\mathrm{em}$ via v.e.v.'s of components of the h and $\bar{h}$. Uniquely among all the GUTS known to us, there is no adjoint or larger self-conjugate Higgs representation. The model is also attractive because it solves naturally the Higgs doublet-triplet mass-splitting problem, has a see-saw neutrino mass spectrum, and avoids rapid proton decay [12].

Our string model is generated by the following basis with eight elements: $B = \{1, S, b_1, \ldots, b_5, \alpha\}$ where

$$S \equiv (\underbrace{1, \ldots, 1,}_{\psi^\mu,\, \chi^{1,\ldots,6}} 0, \ldots, 0) , \tag{8a}$$

$$b_1 \equiv (\underbrace{1, \ldots, 1,}_{\psi^\mu,\, \chi^{1,2},\, y^3,\, \bar{y}^3,\, \ldots,\, y^6,\, \bar{y}^6} 0, \ldots, 0; \underbrace{1, \ldots, 1,}_{\bar{\psi},^{1,\ldots,5},\, \bar{\eta}^1} 0, \ldots, 0) , \tag{8b}$$

$$b_2 \equiv (\underbrace{1, \ldots, 1,}_{\psi^\mu,\, \chi^{3,4},\, y^{1,2},\, \bar{y}^{1,2},\, \omega^{5,6}, \bar{\omega}^{5,6}} 0, \ldots, 0; \underbrace{1, \ldots, 1,}_{\bar{\psi}^{1,\ldots,5},\, \bar{\eta}^2}) , \tag{8c}$$

$$b_3 \equiv (\underbrace{1, \ldots, 1,}_{\psi^\mu,\ \chi^{5,6},\ \omega^1,\ \bar\omega^1,\ \ldots,\ \omega^4, \bar\omega^4} \quad 0, \ldots, 0; \underbrace{1, \ldots, 1,}_{\bar\psi^{1,\ldots,5},\ \bar\eta^3} 0, \ldots, 0)\ , \tag{8d}$$

$$b_4 \equiv (\underbrace{1, \ldots, 1,}_{\psi^\mu,\ \chi^{1,2},\ y^{3,6},\ \bar y^{3,6},\ \omega^{4,5},\ \bar\omega^{4,5}} \quad 0, \ldots, 0; \underbrace{1, \ldots, 1,}_{\bar\psi^{1,\ldots,5},\ \bar\eta^1} 0, \ldots, 0)\ , \tag{8e}$$

$$b_5 \equiv (\underbrace{1, \ldots, 1,}_{\psi^\mu,\ \chi^{3,4},\ \omega^{1,5},\ \bar\omega^{1,5},\ y^{2,6},\ \bar y^{2,6}} \quad 0, \ldots, 0; \quad \underbrace{1, \ldots, 1,}_{\bar\psi^{1,\ldots,5},\ \bar\eta^2, \bar\phi^{1,\ldots 4}} \quad 0, \ldots, 0)\ , \tag{8f}$$

$$\alpha \equiv (\underbrace{1, \ldots, 1,}_{\chi^{1,\ldots,4},\ y^1,\ \bar y^1\, \omega^2,\ \bar\omega^2\, y^4,\ \bar y^4\, \omega^5,\ \bar\omega^5} \quad 0, \ldots, 0; \underbrace{\frac{1}{2}, \ldots, \frac{1}{2},}_{\bar\psi^{1,\ldots,5},\ \bar\eta^{1,2,3}} \ \underline{A})\ , \tag{8g}$$

In the notation used above, all the left-movers ψ^μ, χ^i, $y^i\,\omega^i$, as well as twelve right-movers $\bar y^i$, $\bar\omega^i$ are real fermions, while the remaining right-movers $\bar\psi^{1,\cdots,5}$, $\bar\eta^{1,2,3}$ and $\bar\phi^{1,\cdots,8}$ are complex, and the semicolons separate real and complex fermions. A few preliminary comments are in order. (a) The presence of the vectors S, b_1 and b_2 guarantees $N = 1$ space-time supersymmetry [8]. (b) The restricted set of basis elements $\{1, S, b_1, b_2, b_3\}$ alone would yield an SO(10) $\times$ SO(6)^3 observable gauge group, together with 3×2 copies of massless chiral fields in ($\underline{16}$, $\underline{4}$, 1, 1) + ($\underline{16}$, $\bar{\underline{4}}$, 1, 1) representations [one for each SO(6)] and an E_8 hidden gauge group. (c) The basis elements b_4, b_5 and α break SO(6)^3 to U(1)^3 and SO(10) to SU(5) $\times$ U(1) . The reduction in the rank is achieved by using the real fermions y^i, $\bar y^i$, ω^i and $\bar\omega^i$, and is the maximum possible in this approach, whilst the unitary group appears thanks to the complex fermions $\bar\psi^{1,\cdots,5}$ in α. The choice of 1/2 for their boundary conditions is the only one whose α–projection leaves in the massless physical spectrum the full spinor of SO(10), as will become clear below. (d) By choosing appropriately the vector $\underline{A}$ in eq. (8g) describing twists of $\bar\phi^{1,\cdots,8}$ which are consistent with condition (a2) above, it is possible to guarantee that the only massless states transforming nontrivially under the observable gauge group are those from the sectors S, b_1, b_2, b_3, b_4 (space-time fermions) and 0, $b_1 + S$, $b_2 + S$, $b_3 + S$, $b_4 + S$ (space-time bosons). The particular form of $\underline{A}$ determines at the same time the final form of the hidden gauge group. The conditions (a2) impose the following constraints on the vector $\underline{A}$. Defining N_α to be the smallest integer for which $N_\alpha \alpha = 0 \pmod 2$, simple inspection of the vector α, eq. (8g), shows that $N_\alpha = 4n$, where n is an integer depending on $\underline{A}$. One must impose (i) $n(\underline{A}^2) = 0 \pmod 2$, corresponding to $N_\alpha \alpha^2 = 0 \pmod 8$ and (ii) $n\underline{A} \cdot \underline{B} = 0 \pmod 1$, corresponding to $N_{\alpha b_5}\, \alpha \cdot b_5 = 0$ (mod 4), where $\underline{B} \equiv (1, 1, 1, 1; 0, 0, 0, 0)$ for $\bar{\phi}^{1,\ldots,4}$ and $\bar{\phi}^{5,\ldots,8}$, respectively. One must in addition choose $\underline{A}$ so as to guarantee that no extra massless states transforming nontrivially under the observable gauge group are introduced.

A possible choice of the phases c is

$$c\begin{pmatrix} b_i \\ b_j \end{pmatrix} = c\begin{pmatrix} b_i \\ S \end{pmatrix} = c\begin{pmatrix} \alpha \\ b_i \end{pmatrix} = c\begin{pmatrix} b_i \\ 1 \end{pmatrix} = -1 (i \neq j;\, i,j = 1, \ldots, 5) \quad . \tag{9a}$$

The first relation ensures the same chirality for all the SO(10) spinors, while the others are a matter of consistent choice. Using property (b2) above, one finds

$$c\begin{pmatrix} b_i \\ \alpha \end{pmatrix} = \begin{cases} +1 & \text{for } i \neq 3,5 \ , \\ -i & \text{for } i = 3 \ , \end{cases} \tag{9b}$$

and determines other phases not needed here.

It is straightforward to derive the massless spectrum of the model (8). In the Neveu–Schwarz sector (0) one has the graviton, dilaton and two-index antisymmetric tensor states: $\psi^{\mu}_{1/2}\, \bar{\partial} X^{\nu}_{1}\, |0\rangle_0$, the gauge bosons and some matter fields transforming only under the hidden gauge group [whose specific form depends on the choice of $\underline{A}$ in eq. (8g)], which we will not write explicitly here. One also has:

(a) The gauge bosons of the observable gauge group SU(5) $\times$ U(1) $\times$ U(1)^3 : $\psi^{\mu}_{1/2}\, \bar{\psi}^{a}_{1/2}\, \bar{\psi}^{b*}_{1/2}\, |0\rangle_0$ with $a, b = 1, \ldots, 5$ and $\psi^{\mu}_{1/2}\, \bar{\eta}^{\alpha}_{1/2}\, \bar{\eta}^{\alpha*}_{1/2}\, |0\rangle_0$ with* $\alpha = 1, 2, 3$ which we denote by U(1)$_\alpha$.

* The first U(1) factor is the combination $\psi^{\mu}_{1/2} \sum_a \bar{\psi}^{a}_{1/2}\, \bar{\psi}^{a*}_{1/2}\, |0\rangle_0$. The other U(1) charges are model-dependent, and may be altered by other choices of the vector α.

(b) Three candidate Higgses h_α, $\bar{h}_\alpha$: $\chi^i_{1/2}\,\bar{\psi}^{a*}_{1/2}\,\bar{\eta}^{\alpha*}_{1/2}\,|0\rangle_0$ and $\chi^i_{1/2}\,\bar{\psi}^a_{1/2}\,\bar{\eta}^\alpha_{1/2}\,|0\rangle_0$ with $i = 1, 2$ or $3, 4$ with $\alpha = 1$ or 2, respectively, and $\chi^i_{1/2}\,\bar{\psi}^{a*}_{1/2}\,\bar{\eta}^3_{1/2}\,|0\rangle_0$ and $\chi^i_{1/2}\,\bar{\psi}^a_{1/2}\,\bar{\eta}^{3*}_{1/2}\,|0\rangle_0$ with $i = 5, 6$. Thus, their quantum numbers are†

$$\left.\begin{array}{lllll} h_1 & = & (\underline{5}, -1)_{-1,0,0} & \bar{h}_1 & = (\underline{\bar{5}}, 1)_{1,0,0} \\ h_2 & = & (\underline{5}, -1)_{0,-1,0} & \bar{h}_2 & = (\underline{\bar{5}}, 1)_{0,1,0} \\ h_3 & = & (\underline{5}, -1)_{0,0,1} & \bar{h}_3 & = (\underline{\bar{5}}, 1)_{0,0,-1}\ , \end{array}\right\} , \tag{10}$$

in a self-explanatory notation.

(c) Six SU(5) $\times$ U(1) singlets $\chi^i_{1/2}\,\bar{\eta}^\alpha_{1/2}\,\bar{\eta}^\beta_{1/2}\,|0\rangle_0$, $\chi^i_{1/2}\,\bar{\eta}^{\alpha*}_{1/2}\,\bar{\eta}^{\beta*}_{1/2}\,|0\rangle_0$ $(\alpha \neq \beta)$ with $i = 1, 2$ or $3, 4$ when $\alpha, \beta = 2, 3$ or $1, 3$, respectively, and $\chi^i_{1/2}\,\bar{\eta}^1_{1/2}\,\bar{\eta}^{2*}_{1/2}\,|0\rangle_0$, $\chi^i_{1/2}\,\bar{\eta}^{1*}_{1/2}\,\bar{\eta}^2_{1/2}\,|0\rangle_0$ with $i = 5, 6$. Thus their quantum numbers are:

$$\left.\begin{array}{lllll} \phi_{23} & = & (1, 0)_{0,1,1} & \bar{\phi}_{23} & = (1, 0)_{0,-1,-1} \\ \phi_{13} & = & (1, 0)_{1,0,1} & \bar{\phi}_{13} & = (1, 0)_{-1,0,-1} \\ \phi_{12} & = & (1, 0)_{1,-1,0} & \bar{\phi}_{12} & = (1, 0)_{-1,1,0}\ , \end{array}\right\} , \tag{11}$$

(d) The sectors b_α with $\alpha = 1, 2, 3$ produce out of the corresponding vacua $|0\rangle_\alpha$ three families of quarks and leptons which transform as spinors of 0(16). After making the various fermion projections and decomposing with respect to the final gauge group, they are seen to transform as

$$M_1 \equiv M_{1/2,0,0}\ , \qquad M_2 = M_{0,1/2,0}\ , \qquad M_3 = M_{0,0,-1/2}\ , \tag{12}$$

where each $M_i \equiv F_i + \bar{f}_i + \ell^c_i = (\underline{10},\ 1/2) + (\underline{\bar{5}},\ -3/2) + (\underline{1},\ 5/2)$ is a complete Weyl spinor of SO(10).

† Note that the four U(1) charges are related to the following world-sheet U(1) currents: $\sum_{a=1}^{5}\ \bar{\psi}^{a*}\,\psi^a$ and $\bar{\eta}^{\alpha*}\,\eta^\alpha$ with $\alpha = 1, 2, 3$.

(*e*) The sector b_4 gives rise to the Higgses H_1 and $\bar{H}_1$:

$$H_1 = (\underline{10},\ 1/2)_{1/2,0,0}\,, \qquad \overline{H}_1 = (\overline{10},\ -1/2)_{-1/2,0,0}\,. \tag{13}$$

Let us also mention the following massive states in the b_4 sector. They are produced by acting on the various spinor components of $|0\rangle_{b_4}$ with the oscillators $\chi^i_{1/2}$, $\bar{\eta}^\alpha_{1/2}$ or $\bar{\eta}^{\alpha *}_{1/2}$ with $i = 3, 4$ or $5, 6$ when $\alpha = 2$ or 3, respectively:

$$\left.\begin{array}{llll} H_{12} &= (\underline{10}, 1/2)_{-1/2,1,0} & \bar{H}_{12} &= (\overline{10}, -1/2)_{1/2,-1,0} \\ H_{13} &= (\underline{10}, 1/2)_{-1/2,0,-1} & \bar{H}_{13} &= (\overline{10}, -1/2)_{1/2,0,1} \end{array}\right\}\,, \tag{14}$$

These will play an important role in the doublet-triplet mass-splitting.

Using the above spectrum and quantum numbers, we now describe some of the properties of the model and compare them with the desired properties of the corresponding field theory model [12], eq. (7).

(a) The $H_1, \bar{H}_1$ scalars, eq.(13), are candidate Higgs fields. If they acquire v.e.v.'s, they break SU(5) $\times$ U(1) spontaneously to the Standard Model $\mathrm{SU(3)_C}\times$ $\mathrm{SU(2)_L}\times$ $\mathrm{U(1)_Y}$. At the same time, they break $\mathrm{U(1)}^3$ to $\mathrm{U(1)}^2$. The presence of $H_1 H_1 h_1 + \bar{H}_1 \bar{H}_1 \bar{h}_1$ superpotential couplings, which are allowed by the U(1) charges, eqs. (10,13), and derived on general grounds in ref. [16], would then realize the elegant doublet-triplet mass-splitting mechanism of ref. [12] by giving a large mass to the triplet components of h_1, $\bar{h}_1$. The massive states H_{12}, $\bar{H}_{12}$, H_{13}, $\bar{H}_{13}$, eq. (14), have $H_1 H_{12} h_2 + \bar{H}_1 \bar{H}_{12} \bar{h}_2$ and $H_1 H_{13} h_3 + \bar{H}_1 \bar{H}_{13} \bar{h}_3$ superpotential couplings which would likewise provide through mixing large masses of order $M^2_{\mathrm{GUT}}/M_P \gtrsim 10^{13}$ GeV for the triplet components of $h_{2,3}$ and $\bar{h}_{2,3}$.*

⋆ As in ref. 17, these compensate for the effects of light Higgs doublets in the renormalization group equations, so that acceptable values of $\sin^2\theta_W$, *etc.*, are obtained.

(b) The h_α, $\bar{h}_\alpha$ scalars, eq.(10), are candidates to be the Higgses of the Standard Model. They correspond to flat directions of the effective scalar potential at the string tree level, and in the presence of soft supersymmetry breaking they would acquire nonzero v.e.v.'s which would break $\mathrm{SU(3)_C}\times \mathrm{SU(2)_L}\times \mathrm{U(1)_Y}$ to $\mathrm{SU(3)_C}\times \mathrm{U(1)_{em}}$ at the weak scale. Among the desired superpotential couplings of the form $M_i M_j h$, those which are allowed by the U(1) charges are $(\underline{10}, 1/2)_\alpha\,(\underline{10}, 1/2)_\alpha\, h_\alpha$ and $(\underline{1}, 5/2)_\alpha\,(\underline{\bar{5}}, -3/2)_\alpha\, h_\alpha$ which would provide masses to the charge -1/3 quarks and leptons. Thus far, a big defect of the model is that the couplings which could give masses to the charge +2/3 quarks and conjugate neutrinos are forbidden by the extra U(1) charges. This difficulty is partially resolved by the next observation.

(c) The three extra U(1)'s are anomalous [18]. It has been shown [19] that consistency of the string theory implies that string loop corrections generate D–breaking of such an anomalous U(1). The latter can lead to large v.e.v.'s of order M_P, but does not in our case lead to the spontaneous breaking of space-time supersymmetry close to the Planck scale. In our case, it is the combination $\mathrm{U(1)_1} + \mathrm{U(1)_2} - \mathrm{U(1)_3}$ which is broken by this mechanism. If this were the only combination of U(1)'s to be broken, still no charge +2/3 quark masses would be generated. However, since the expected v.e.v.'s for H_1 and $\bar{H}_1$ break spontaneously $\mathrm{U(1)_1}$, the only U(1) remaining down to low energies is $\mathrm{U(1)_2} + \mathrm{U(1)_3}$. In principal, this residual U(1) allows the following superpotential couplings of the matter fields: $M_1M_1h_1$, $M_2M_2h_2$, $M_2M_3h_1$, $M_3M_3h_3$ and $M_1M_1\bar{h}_1$, $M_2M_2\bar{h}_3$, $M_2M_3\bar{h}_1$, $M_3M_3\bar{h}_2$. These would be sufficient to give nonzero masses to all the charge +2/3 quarks. However, they would only give Cabibbo–Kobayashi–Maskawa mixing between two generations. It would be natural to conjecture that the odd one out may be identified as a first approximation with the (t, b) quarks. In practice, using only nonrenormalizable couplings involving the singlet fields introduced above, all the above Yukawa couplings except those coupling M_2 and M_3 are generated, and there is no Cabibbo-Kobayashi–Maskawa mixing at all. Moreover, the residual $\mathrm{U(1)_2} + \mathrm{U(1)_3}$ does not allow large masses for all

the conjugate neutrinos at the tree level. In principle, one could generate v.e.v.'s for other scalars with nonzero residual U(1) charges which would remove these last remaining phenomenological defects. Indeed, one could modify slightly the above model, eqs. (8), to become more realistic, at the price of the appearance of extra Higgses. For example, if one modifies the vector b_5, eq. (8e), by making $\bar{\phi}^{1,\dots,4}$ antiperiodic, one would obtain an extra $(\underline{10}, 1/2) + (\overline{\underline{10}}, -1/2)$ pair as well as extra $\underline{5}$'s and $\overline{\underline{5}}$'s. Their extra superpotential couplings could in turn lead more directly to a realistic mass matrix.

We have reported in this paper on a first attempt at model-building with the fermionic formulation of four-dimensional strings [8,9]. We arrived at a supersymmetric flipped SU(5) GUT [12] which is distinguished by its natural doublet-triplet mass-splitting mechanism, its see-saw neutrino mass matrix and the absence of any adjoint (or higher) Higgs representations. We have found a fermionic string theory which almost reproduces the desired model, modulo questions about the charge $+2/3$ quark masses, the conjugate neutrino masses, U(1) anomalies and the scale of supersymmetry breaking. As we have argued above, some of these difficulties may resolve each other. One possible line of future research would be to tune up the model presented here. Another would be to look for other phenomenologically appealing models in the fermionic formulation of four-dimensional strings. However, even if a very attractive string theory were found, there would still be the problem of divining how and why Nature chose that particular string theory.

Acknowledgements

The work of D.V.N. was supported in part by Department of Energy grant DE–AC02–76ER0081 and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.

References

[1] M. B. Green and J. H. Schwarz, Phys. Lett. **149B** (1984) 117; D J. Gross, J. A. Harvey, E. Martinec and R. Rohm, Nucl. Phys. **B256**, 468 (1985) and **B267** (1986) 75.

[2] P. Candelas, G. T. Horowitz, A. Strominger and E. Witten, Nucl. Phys. **B258** (1985) 46.

[3] E. Witten, Nucl. Phys. **B268** (1986) 79.

[4] L. Dixon, J. A. Harvey, C. Vafa and E. Witten, Nucl. Phys. **B261** (1985) 678 and **B274** (1986) 285.

[5] K. Narain, Phys. Lett. **169B** (1986) 41; W. Lerche, D. Lüst and A. N. Schellekens, Nucl. Phys. **B287**, (1987) 477.

[6] I. Antoniadis, C. Bachas, C. Kounnas and P. Windey, Phys. Lett. **171B** (1986) 51.

[7] H. Kawai, D. Lewellen and S.-H.H. Tye, Phys. Rev. Lett. **57** (1986) 1832 and Nucl. Phys. **B288** (1987) 1.

[8] I. Antoniadis, C. Bachas and C. Kounnas, Nucl. Phys. **B289** (1987) 87.

[9] I. Antoniadis and C. Bachas, Nucl. Phys. **B298** (1988) 586.

[10] For a review and references, see J. Ellis–CERN preprint TH.4840/87 (1987).

[11] L. Ibáñez, H.–P. Nilles and F. Quevedo, Phys. Lett. **187B** (1987) 25 and **192B** (1987) 332; L. Ibáñez, J. E. Kim, H. -P. Nilles and F. Quevedo, Phys. Lett. **191B** (1987) 282.

[12] I. Antoniadis, J. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. **194B** (1987) 231.

[13] B. A. Campbell, J. Ellis, J. S. Hagelin, D. V. Nanopoulos and R. Ticciati, Phys. Lett. **198B** (1987) 200.

[14] B. A. Campbell, J. Ellis, J. S. Hagelin, D. V. Nanopoulos and K. A. Olive, Phys. Lett. **197B** (1987) 355.

[15] I. Antoniadis, C. Bachas and C. Kounnas, CERN preprint TH.4863/87 (1987).

[16] I. Antoniadis, J. Ellis, E. Floratos, D. V. Nanopoulos and T. Tomaras, Phys. Lett. **191B** (1987) 96, S. Ferrara, L. Girardello, C. Kounnas and M. Porrati, Phys. Lett. **192B** (1987) 368 and **194B** (1987) 358.

[17] C. Kounnas, A. B. Lahanas, D. V. Nanopoulos and M. Quirós, Nucl. Phys. **B236** (1984) 438.

[18] R. Barbieri, S. Ferrara and D. V. Nanopoulos, Zeit. für Phys. **C13**, 267 (1982) and Phys. Lett. **116B** (1982) 16.

[19] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. **B289** (1987) 589.