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$(g-2)$ OF THE MUON FROM COMPOSITENESS IN THE MODEL OF ABBOTT AND FARHI*

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ABSTRACT

We use a simple model to estimate the contribution to $(g-2)$ for the muon in the composite model of Abbott and Farhi. The dichotomy between the left- and right-handed muon in this model allows us to fix the value of an unknown coupling constant. We investigate several scenarios for possible constituent masses.

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As an alternative to the highly successful standard model of electroweak interactions, Abbott and Farhi have proposed an $SU(2)_L \times U(1)$ theory[1] which can successfully predict the correct low energy phenomenology, but has the feature that the $SU(2)_L$ gauge group is strongly confining at the weak scale $G_F^{-1/2}$.[2] In this model the observed weak interactions are simply the residual forces between composite SU(2) singlets, which are identified as the left-handed quarks and leptons, $W^{\pm}$, Z°, and Higgs boson we are familiar with. The underlying $SU(2)_L$ symmetry is unbroken in this scenario. An unusual feature of this model is that the right-handed quarks and leptons are elementary.

The standard left-handed fermions in the Abbott–Farhi model are bound states of a fermion and scalar, whereas the Higgs and spin-1 bosons are scalar-scalar bound states. The composite nature of these particles will obviously modify the form factors for scattering experiments involving them.[3] The values of the gyromagnetic ratios for the leptons so far agree extremely well with the expected QED result once QCD corrections are made, which places stringent limits on the possible contributions due to compositeness.[4] In this note we discuss the contribution to $(g-2)$ of the muon in this model from the preonic fermion and scalar constituents. We first note that the Abbott–Farhi model satisfies the 't Hooft anomaly matching conditions[5] by which massless bound states may arise in a confining theory. (The small but finite masses of the physical quarks and leptons are generated by perturbations such as the electromagnetic interaction.) The clearest manifestation of this mechanism is in the case of massless fermionic subconstituents in the theory. Later we will investigate the consequences of relaxing this condition.

We write an effective coupling between the muon and these constituents (f = fermion, ϕ = scalar), as

$$\mathcal{L}_{\text{eff}} = g_0 \, \overline{\psi} \, L \, \gamma_\mu \, \phi + \text{h.c.}$$

where $L = \frac{1}{2}(1-\gamma_5)$ and g_0 is an effective coupling. This is the simplest form allowed by Lorentz invariance. In principle, the calculation would require an understanding of the composite lepton wavefunction, but we may obtain an estimate using this simple representation. The compositeness is built into our approach via a Pauli–Villars subtraction in which the confinement scale is explicitly introduced. Other approaches to this problem which could be used are dispersion relations,[6] or the Drell–Hearn–Gerasimov sum rules, along the lines of the calculation of Alterelli et al. (See Reference 4 for a discussion of this point.) Our model can also be used to calculate the lepton $g-2$ using the DHG approach with the same results.

The relevant diagrams are show in Fig. 1, where (a) and (b) involve the scattering of a photon from a composite left-handed muon and (c) shows the analogous process for a right-handed muon. Note that Fig. 1(a) will produce a contribution to the anomalous moment even though the incoming and outgoing muons are both left-handed. The point is "left-handed" here refers to the chirality state $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$, which is not the same as the helicity state in the case of a massive particle such as the muon. If such a chiral protection was not present, we would expect the anomalous moment to go like m_μ/Λ.[4] As we shall see, the contribution in this model is quadratic in this quantity. Using the Pauli–Villars regularization to ensure gauge invariance, we obtain for Figs. 1(a) and (b) the amplitude

$$M_\mu = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{1}{4m_\mu}[\not{q},\gamma_\mu]F_2(q^2)\right]u(p) + \gamma_5 \text{ parts}$$

where

$$F_1(0) = \frac{(e_f+e_s)g_0^2}{32\pi^2}\int_0^1 x dx\, ln\left(\frac{\Lambda^2 x + m_f^2(1-x) - m_\mu^2 x(1-x)}{\lambda^2 x + m_f^2(1-x) - m_\mu^2 x(1-x)}\right)$$

and

$$F_2(0) = m_\mu^2\frac{(e_f+e_s)g_0^2}{16\pi^2}\left\{\int_0^1 \frac{x^2(1-x)dx}{\lambda^2 x + m_f^2(1-x) - m_\mu^2 x(1-x)} - (\lambda^2\to\Lambda^2)\right\} .$$

Here $m_f(e_f)$ is the fermion mass (charge) $\lambda(e_s)$ is the scalar mass (charge) and Λ is the regularization mass scale, which we take to be the confinement scale.

We may normalize the γ_μ part by demanding the left- and right-handed muons to have equal charges. This requires setting

$$F_1(0) = e$$

and hence we may eliminate g_0^2 from the problem. It is then a simple calculation to obtain the contribution to $(g-2)$:

$$a = \frac{F_2(0)}{F_1(0)} = \frac{m_\mu^2}{\Lambda^2} f\left(m_f^2, \lambda^2, \Lambda^2, m_\mu^2\right)$$

where $f(m_f^2, \lambda^2, \Lambda^2, m_\mu^2)$ is a dimensionless function which renders the result finite even if $m_f = \lambda = 0$. This result agrees with the calculation of Reference 4. That the contribution to $(g-2)$ in this model is $0(M_\mu^2/M_*^2)$ (with M_* a mass scale associated with the composite nature of the lepton) was anticipated in the original work of Abbott and Farhi.[1] For the electron $(g-2)$, this contribution [which will be $0(M_e^2/M_*^2)$], will lead to less stringent bounds due to the lightness of the electron, even though the electron $(g-2)$ is a more precisely measured quantity. Of course, the contribution to the magnetic moment of the tau lepton will be greater still, so possible future experiments might give more information regarding possible constituents.

We now relax the condition $m_f = \lambda = 0$ and investigate the effect on $(g-2)$. For the case $m_f = 0$, $\lambda \neq 0$, which is the most natural possibility since the scalar may itself be a fermion-antifermion bound state (or in the absence of supersymmetry is not a naturally light particle if there exists some higher mass scale analogous to the GUT scale). Also, giving a mass to the constituent fermion will explicitly break the chiral $SU(2)_L$ gauge symmetry. The result for a massless fermion is simply

$$a = \frac{1}{3}\frac{m_\mu^2}{\lambda^2}\frac{(1-\lambda^2/\Lambda^2)}{ln\Lambda/\lambda} \quad .$$

In order to keep the magnitude of the contribution to $(g-2) < 3\times 10^{-8}$, we are forced to take $\lambda > 1.1$ TeV when Λ is taken to be the weak scale $G_F^{-1/2}$. This is potentially a problem, as the decoupling of heavy particles from low energy physics would suggest that a constituent of this mass is unlikely to occur in phenomenologically viable models of composite leptons.[8] This is termed the persistent-mass condition. It has been argued, however, that the presence of scalar (or vector) constituents in a theory does not preclude the presence of chiral symmetries, which will protect the bound state from obtaining a large mass,[9] so we are not able to conclude that the model is fundamentally flawed on these grounds. In order to produce massless bound states, we must have the scalar mass compensated for by some binding energy. Since the fundamental energy scale is the confining scale in this scenario, we may reasonably expect the binding energy to be of order $\Lambda = G_F^{-1/2}$. Thus a very heavy scalar may imply some problems.

If we now take $m_f \neq 0$ as well, we may investigate whether we can satisfy the experimental bound for moderate values of the fermion mass, keeping in mind that for large values of the fermion mass the underlying chiral symmetry is clearly broken, and it may be difficult to keep the bound states light. Due to the chiral nature of the gauge symmetry in this model, it is an unattractive step to allow $m_f \neq 0$, which may even result in deconfinement (due to the present of right-handed preons which will not feel the $SU(2)_L$ force), but we present the results here anyway since they will be qualitatively applicable to any model with a scalar-fermion bound state. For small values of m_f, the approximate chiral symmetry will protect the lepton mass to some extent, as the bound state is allowed to be massless in the limit of massless constituents, and we would expect some sort of continuity argument to apply.[10]

If we take the confinement scale to be $G_F^{-1/2}$, which is the canonical scale for this model, we find that it is not possible to keep the contribution from compositeness within the experimental limits ($|a_\mu| < 3 \times 10^{-8}$) with m_f and $\lambda < \Lambda$. We are therefore forced to take constituent masses greater than the confinement scale. We will allow the scalar to be the more massive constituent for the reasons presented above. We show in Fig. 2 that it is possible to keep $m_f < \Lambda$ and satisfy the experimental bounds on a_μ by taking larger values of λ. The three curves are labeled with the values of m_f taken. It is problematical whether the masses required are viable, since for small ($\ll \Lambda$) values of m_f we are led to values of λ almost as high as before, whereas values of $m_f \simeq \Lambda$ represent an unacceptably large breaking of the $SU(2)_L$ gauge symmetry.

If we take a confinement scale higher than the weak scale, we might expect the situation to be somewhat better regarding the hierarchy of mass scales within the model. In Fig. 3 we present the contribution to $(g-2)$ of the muon for a confinement scale of 500 GeV for various values of the constituent masses. It is clear that in this instance we can keep both the scalar and fermion masses below the confinement scale and still remain within the current experimental limit, though we are still facing problems regarding decoupling and the persistent-mass condition. For higher confinement scales still, we clearly have less trouble satisfying this bound. These general considerations will hold for any model in which fermions are fermion-scalar bound states.

Note that these considerations may be modified by the contributions of particles other than those appearing in the standard model. For example, there is the possibility of a series of $(\phi - \phi)$ bound states of spin 2,3,... (the possibility of a spin-2 particle appearing from string considerations has also been considered),[12] or we might have contributions from excited states of standard model particles. The contribution from a spin-2 particle of mass 100 GeV has been calculated to be $\simeq 1.5 \times 10^{-8}$.[13] We have seen, however, that achieving phenomenologically valid values of $(g-2)$ in the Abbott–Farhi model is not easily done on the grounds of compositeness alone. In any case, it is difficult to see how the contribution to $(g-2)$ can be made much less than 10^{-8}. For comparison, the standard model weak corrections are $\simeq 10^{-9}$. We note that a new experiment has been approved at BNL, which should provide new data and may distinguish between the competing models.

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FIGURE CAPTIONS

Figure 1. The diagrams relevant to the calculation of $(g-2)$ for the muon in the Abbott–Farhi model. The muon in diagrams (a) and (b) is the left-handed composite, while that in diagram (c) is the right-handed elementary one.

Figure 2. The curves on this graph show the contribution to $(g-2)$ from compositeness as a function of the scalar mass λ for various values of the fermion mass m_f (0, 100, 100 and 250 GeV for curves A, B, C and D, respectively). The confinement scale for these curves is 300 GeV. Also shown is the current experimental limit on this quantity. The limit for $m_f = 0$ is $\lambda > 1.1$ TeV.

Figure 3. The contribution from compositeness for a confinement scale of 500 GeV. The bound on λ is less strict than in the previous figure for all values of the fermion mass.

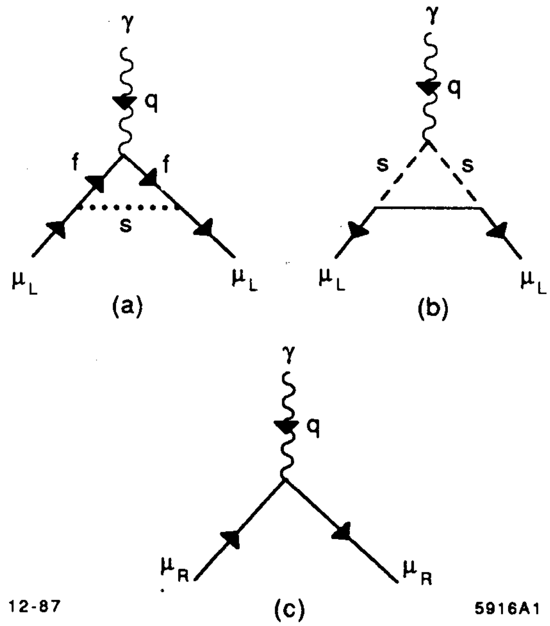
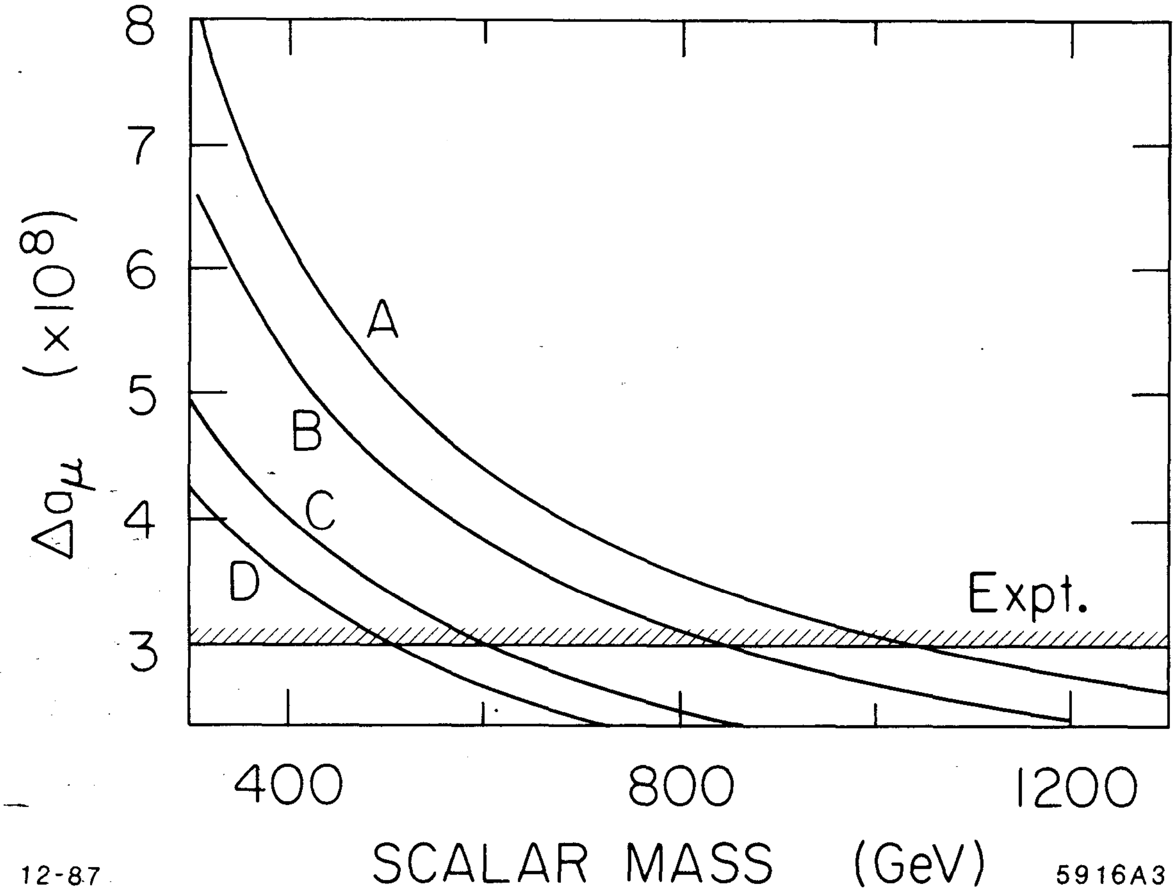
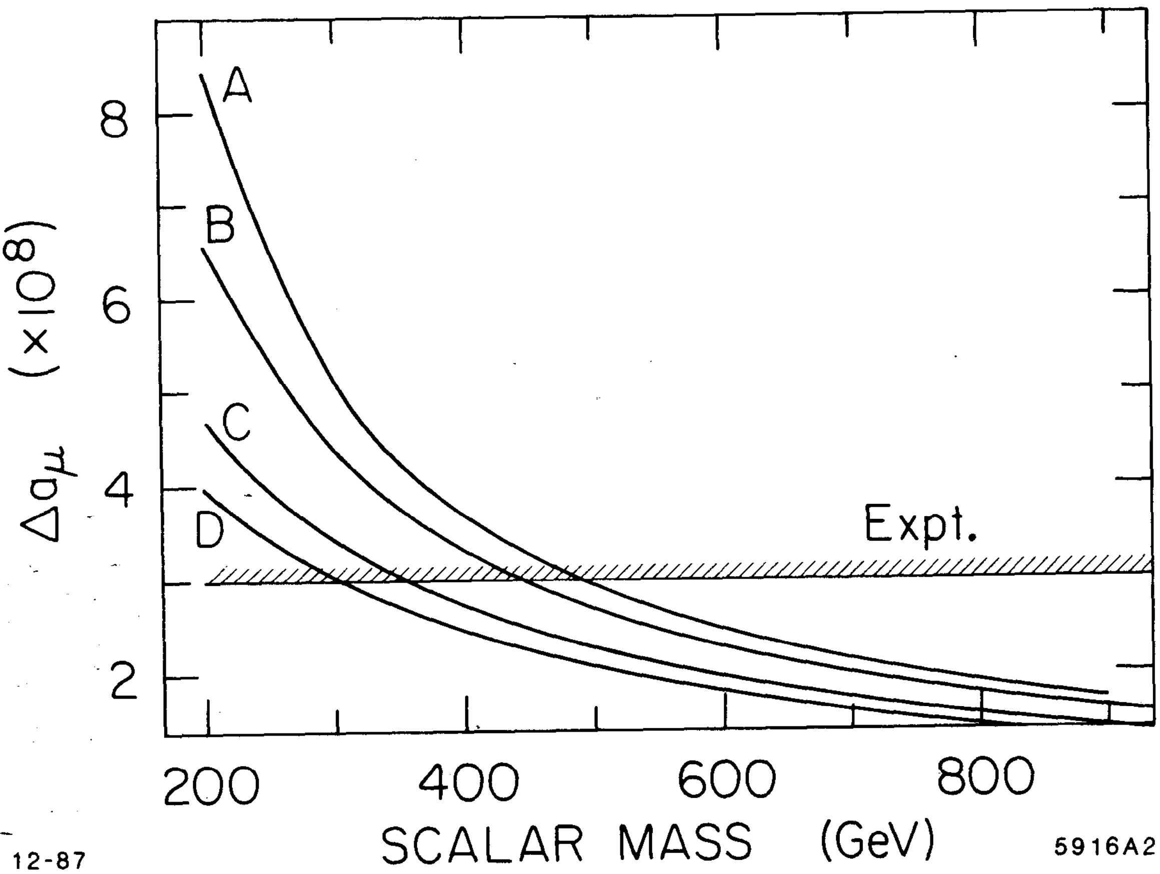


Fig. 1



8
7
6
5
4
3
Δa_μ $(\times 10^8)$
A
B
C
D
Expt.
400
800
1200
SCALAR MASS (GeV)
12-87
5916A3

Fig. 2



A
B
C
D
Expt.
8
6
4
2
Δa_μ ($\times 10^8$)
200
400
600
800
SCALAR MASS (GeV)
12-87
5916A2

Fig. 3