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TOP DECAYS WHEN $m_t \approx M_W + m_b$*

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ABSTRACT

We discuss the special situation for decays of the top quark that occurs when $m_t \approx M_W + m_b$ and how it affects exclusive channels, $\Gamma(t \to b+W)/\Gamma(t \to s+W)$, and the relative proportion of longitudinally and transversely polarized W's in t decay.

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1. Introduction

The t quark mass is constrained to be above 25 GeV from TRISTAN,[1] above 44 GeV from UA1,[2] and above about 50 GeV from theoretical considerations[3] based on the ARGUS result[4] for $B - \bar{B}$ mixing. In fact, the $B - \bar{B}$ mixing results, interpreted within the standard model, would have one entertain t quark masses in the vicinity of 100 GeV. Even for values of $M_t \approx 50$ GeV, the finite mass of the W results in a $\approx 25\%$ increase in the t decay width over the value calculated with the point (infinite M_W) Fermi interaction; for $M_t \approx 100$ GeV we have decay into a "real" W resonance and the width is proportional to G_F rather than G_F^2.

In this paper we consider in some detail the transition region between the production of "virtual" and "real" W's in t decays, *i.e.*, values of $m_t \approx M_W + m_b$. The absolute width for a t quark with a mass in the range has been considered previously,[5] usually as a special case of a generic heavy quark decaying to a "real" W.

We repeat some of this analysis in the next Section, and then we examine some particular properties of the region where $m_t \approx M_W + m_b$, noting especially how the possibility of a sharp transition or threshold is smeared out by the finite width of the W. In Section III, we consider the decay rate for $t \rightarrow s + W$ compared to that for $t \rightarrow b + W$. The first process is suppressed relative to the second by the ratio of Kobayashi–Maskawa matrix elements squared, $|V_{ts}|^2/|V_{tb}|^2$, which is known[6] to be $\approx 1/500$. There is a region, however, where the first process is above threshold for production of a real W, while the second is below threshold. The question of whether this can compensate for the Kobayashi–Maskawa suppression is answered (negatively) in Section III.

In Section IV we consider the possibility that the hadronic final state recoiling against the W and containing a b quark will be dominated by a very few hadronic states, rather than be a sum of many states in the form of a jet. We calculate the specific matrix elements in this case in the quark model—one of the few cases in which the nonrelativistic quark model may really be well-justified *a priori*.

This ties into Section V, where we examine the relative population of longitudinal and transverse W's as we move through the threshold region. The ratio of decay widths involving longitudinal and transverse W's varies fairly rapidly near the threshold and we show how the associated lepton or quark jet angular distribution in the W decay can be used to measure this quantity and help determine the t quark mass to a few GeV.

2. The Decay Rate for $t \rightarrow b\, e^+\nu_e$

Consider the semileptonic decay of t to b. The tree-level width, for any value of m_t, is given by

$$\Gamma(t \rightarrow b\, e^+\nu) =$$

$$\frac{G_F^2 m_t^5}{24\pi^3} \int\limits_0^{(m_t-m_b)^2} dQ^2 \frac{M_W^4 \,|\vec{Q}|}{(Q^2 - M_W^2)^2 + M_W^2\Gamma_W^2}\left[2|\vec{Q}|^2 + 3\, Q^2(1 - \frac{Q_0}{m_t})\right] , \tag{1}$$

where Γ_W is the total width of the W and the integration variable Q^2 is the square of the four-momentum which it carries, with the associated quantities $Q_0 = (m_t^2 + Q^2 - m_b^2/2m_t)$ and $|\vec{Q}|^2 = Q_0^2 - Q^2$. In general, the right-hand side of Eq. (1) should contain the square of the relevant Kobayashi–Maskawa matrix element, $|V_{tb}|^2$, which in the case of three generations is one to high accuracy.

In the limit that $m_t \leq M_W$, the momentum dependence of the W propagator can be neglected and the expression simplifies to

$$
\begin{aligned}
\Gamma(t \to b\ e^+\nu) =& \frac{G_F^2 m_t^5}{24\pi^3} \int\limits_0^{(m_t-m_b)^2} dQ^2\ |\vec{Q}|\Big[2|\vec{Q}|^2 + 3\ Q^2(1 - Q_0/m_t)\Big] \\
=& \frac{G_F^2 m_t^5}{6\pi^3} \int\limits_0^{(m_t-m_b)^2} dQ^2\ |\vec{Q}|^3 \\
=& \frac{G_F^2 m_t^5}{192\pi^3}\Big[1 - 8\Delta^2 + 8\Delta^6 - \Delta^8 - 24\Delta^4\ ln\Delta\Big]\ ,
\end{aligned}
\tag{2}
$$

where $\Delta = m_b/m_t$.

In the other limit, where $m_t >> M_W$, we may integrate over the Breit–Wigner for producing a "real" W, and using

$$
\Gamma(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\pi\sqrt{2}}\ , \tag{3}
$$

rewrite Eq. (1) as

$$
\Gamma(t \to b + W \to b\ e^+\nu_e) = B(W \to e\nu) \cdot \frac{G_F|\vec{Q}|}{2\pi\sqrt{2}}\Big[2|\vec{Q}|^2 + 3M_W^2(1 - \frac{Q_0}{m_t})\Big]\ , \tag{4}
$$

where now $Q^2 = M_W^2$ so that $Q_0 = (m_t^2 + M_W^2 - m_b^2/2m_t)$ and $|\vec{Q}|^2 = Q_0^2 - M_W^2$. For very large values of m_t, the width in Eq. (4) behaves as $B(W \to e\nu) \cdot G_F m_t^3/8\pi\sqrt{2}$, to be contrasted with Eq. (2).

The finite width of the W determines the behavior of the rate as we cross the threshold for producing a real W. Once we are several full widths of the W above threshold, the much larger width given in Eq. (4) for producing a "real" W dominates the total t decay rate. This is seen in Figure 1, where the $t \to be^+\nu_e$ decay rate is plotted *versus* m_t. The dashed curve is the result in Eq. (4) which would hold for production of a real, infinitely narrow W, while the solid curve gives the result of integrating Eq. (1) numerically.[7] For smaller values of m_t the width is less than $G_F^2 m_t^5/192\pi^3$ because of the finite value of m_b [here taken to be 5 GeV, see Eq. (2)], but then is enhanced by the W propagator as m_t increases. The exact result quickly matches that for an infinitely narrow W once we are several W widths above threshold. The finite W width simply provides a smooth interpolation as the decay rate jumps by over an order of magnitude in crossing the threshold.

3. Ratio of $t \to b$ to $t \to s$

Ordinarily the weak transition $t \to s$ is suppressed relative to $t \to b$ by the ratio of the relevant Kobayashi–Maskawa matrix elements squared,[6] $|V_{ts}|^2/|V_{tb}|^2 \approx 1/500$. However, we have seen that $\Gamma(t \to b\, e^+\nu_e)$ increases sharply as m_t crosses the W threshold, changing from being proportional to G_F^2 to being proportional to G_F. Thus we expect $\Gamma(t \to se^+\nu_e)$ to be enhanced relative to $\Gamma(t \to be^+\nu_e)$ when m_t lies between the two thresholds: $M_W + m_s < m_t < M_W + m_b$. The question is whether the threshold enhancement "wins" over the Kobayashi–Maskawa suppression.

To examine this quantitatively we consider the ratio of the widths with the Kobayashi–Maskawa factors divided out:

$$\frac{\Gamma(t \to b\, e^+\nu_e)/|V_{tb}|^2}{\Gamma(t \to se^+\nu_e)/|V_{ts}|^2} ,$$

Either well below or well above threshold for a "real" W this ratio should be near unity. For an infinitely narrow W the denominator is strongly enhanced, but the numerator is not, when $M_W + m_s < m_t < M_W + m_b$. The ratio indeed drops dramatically near $t \to s + W$ threshold, as shown in Figure 2, for $\Gamma_W = 0.0225$ GeV (dotted curve) and even for $\Gamma_W = 0.225$ GeV (dashed curve). However, the expected W width of 2.25 GeV (solid curve) smears out the threshold effect over a mass range that is of the same order as $m_b - m_s$, and gives only a modest dip (to ≈ 0.6) in the ratio. This is hardly enough to make $t \to s$ comparable to $t \to b$.

4. Exclusive Modes

When m_t is in the present experimentally acceptable range, the rate for weak decay of the constituent t quarks within possible hadrons becomes comparable with that for electromagnetic and weak decays. Weak decays of toponium become a major fraction of, say, the $J^P = 1^-$ ground state, and even for the $T^*(t\bar{q})$ vector meson, weak decays can dominate the radiative magnetic dipole transition to its hyperfine partner, the T meson $J^P = 0^-$ ground state.[8]

In decays of heavy flavor mesons the branching ratios for typical exclusive channels scale like $(f/M_Q)^2$, where f is a meson decay constant (like f_π or f_K), of order 100 MeV, and M_Q is the mass of the heavy quark. For D mesons individual channels have branching ratios of a few percent; for B mesons they are roughly ten times smaller; and for T (or T^*) mesons they should be a hundred or more times smaller yet. It should be possible to treat T decays in terms of those of the constituent t quark, $t \to b + W^+$, with the b quark appearing in a b jet not so different from those already observed at PEP and PETRA.

There is one possible exception to these last statements, and that is when $m_t \approx m_b + M_W$, the situation under study here. In this case there is a premium on giving as much energy to the W as possible, *i.e.*, keeping as far above threshold for "real" W production as possible, and hence on keeping the invariant mass of the hadronic system containing the b quark small. Then we expect the T and T^* to decay dominantly into a few exclusive channels: a "real" W plus a B or a "real" W plus a B^*.

Furthermore, this is one place where the use of the non-relativistic quark model is *a priori* well-justified. The t quark and final W are very heavy. When $m_t \approx m_b + M_W$, the final heavy b quark is restricted to have a few GeV or less of kinetic energy if the W is to be as "real" as possible. The accompanying light quark in the T hadron is very much a spectator which simply becomes part of the final B or B^* hadron. Thus we can match up the weak current of heavy quark states,

$$\langle b(p_b,\lambda_b)|\, V^\mu - A^\mu\, |t(p_t,\lambda_t)\rangle = \bar{u}(p_b,\lambda_b)\gamma^\mu(1-\gamma^5)u(p_t,\lambda_t)\ , \tag{5}$$

sandwiched between the appropriate hadronic wavefunctions in spin and flavor space, with the matrix elements of the exclusive hadronic channels $T \to B + W$ and $T \to B^* + W$, defined in terms of the form factors[9-11]

$$\langle B(p_B)|\, V^\mu\, |T(p_T)\rangle = f_+(p_T^\mu + p_B^\mu) + f_-(p_T^\mu - p_B^\mu) \tag{6a}$$

and

$$\begin{aligned}&\langle B^*(p_B,\epsilon_B)|\, V^\mu - A^\mu\, |T(p_T)\rangle = \\ &a\epsilon_{B^*}^{\mu,\star} + b(p_T\cdot\epsilon_{B^*}^\star)(p_T+p_{B^*})^\mu + c(p_T\cdot\epsilon_{B^*}^\star)(p_T-p_{B^*})^\mu + ig\epsilon^\mu_{\alpha\beta\gamma}p_T^\alpha p_{B^*}^\beta\epsilon_{B^*}^{\gamma,\star}\ .\end{aligned} \tag{6b}$$

The results from the quark level calculation are:

$$\begin{aligned}\mathcal{M}(T\to B+W_L) &= \sqrt{\frac{2m_t}{E_b+m_b}}\frac{|\vec{p}\,|}{M_W}(m_t+m_b)\\ \mathcal{M}(T\to B^*+W_T) &= \sqrt{\frac{2m_t}{E_b+m_b}}(E_b+m_b+\lambda_W|\vec{p}\,|)\\ \mathcal{M}(T\to B^*+W_L) &= \sqrt{2m_t(E_b+m_b)}\frac{(m_t-m_b)}{M_W}\ ,\end{aligned} \tag{7}$$

where W_L and W_T refer to longitudinally and transversely polarized W's, respectively, and $\vec{p} = -\vec{Q}_W$ is the final three-momentum of the b. Correspondingly, at the hadron level,

$$\begin{aligned}\mathcal{M}(T\to B+W_L) &= 2\frac{m_T}{M_W}|\vec{p}\,|f_+\\ \mathcal{M}(T\to B^*+W_T) &= (a+\lambda_W g m_t|\vec{p}\,|)\\ \mathcal{M}(T\to B^*+W_L) &= \frac{1}{M_W m_{B^*}}\left[a(|\vec{p}\,|^2+E_{B^*}E_W)+2bm_T^2|\vec{p}\,|^2\right]\ .\end{aligned} \tag{8}$$

Identifying $m_t = m_T$ and $m_b = m_B = m_{B^*}$ we get, by comparing Eqs. (7) and (8),

$$
\begin{aligned}
f_+ &= \frac{m_T + m_B}{\sqrt{2m_T(E_B + m_B)}} \\
a &= \sqrt{2m_T(E_B + m_B)} \\
g &= \frac{2}{\sqrt{2m_T(E_B + m_B)}} \\
b &= \sqrt{\frac{E_B + m_B}{2m_T}}\frac{(m_B - E_B)}{|\vec{p}|^2} \ .
\end{aligned}
\tag{9}
$$

In the limit $|\vec{p}| \to 0$ the form factors reduce to

$$
\begin{aligned}
f_+ &= \frac{m_T + m_B}{2\sqrt{m_T m_B}} \\
a &= 2\sqrt{m_T m_b} \\
g &= \frac{1}{\sqrt{m_T m_B}} \\
b &= \frac{-1}{2\sqrt{m_T m_B}} \ .
\end{aligned}
\tag{10}
$$

These results agree in the appropriate limit with previous results[9,10,11] . The form factors f_+, a, and g all have straightforward limits as $|\vec{p}| \to 0$, while that for b can be subtle, as explicitly seen in Eq. (9). It is more sensitive to bound quarks being off the mass-shell.[10] Our result agrees with that of Ref. 11 with the appropriate change of flavors.

5. W Polarization in t Decay

Within the scenario of discovery of the top quark at a hadron collider, it would be useful to have several handles on the value of m_t. An indirect method would be to measure a quantity in top decays which depends strongly on the top mass. For m_t in the vicinity of $M_W + m_b$, we now show that such a quantity is the ratio of the production of longitudinal W's to that of transverse W's in top decay.

The decay widths into longitudinal and transverse W's are defined by decomposing the numerator of the W propagator as

$$g_{\mu\nu} - Q_\mu Q_\nu / M_W^2 = \sum_\lambda \epsilon_\mu(\lambda)\epsilon_\nu^\star(\lambda) = \epsilon_\mu^{(+)}\epsilon_\nu^{(+)\star} + \epsilon_\mu^{(0)}\epsilon_\nu^{(0)\star} + \epsilon_\mu^{(-)}\epsilon_\nu^{(-)\star} \ , \tag{11}$$

where the superscripts give the helicity of the W, whether virtual or real. In calculating the t decay rate in Eq. (1), we define $\Gamma_L = \Gamma^{(0)}$, originating from W's with helicity zero, and $\Gamma_T = \Gamma^{(+)} + \Gamma^{(-)}$, originating from W's with helicity ± 1. Separating in this way the portions of Eq. (1) that originated from longitudinal and transverse W's, we find

$$\Gamma_L = \frac{G_F^2 m_t^5}{24\pi^3} \int\limits_0^{(m_t-m_b)^2} dQ^2 \frac{M_W^4 \ |\vec{Q}\,|}{(Q^2 - M_W^2)^2 + M_W^2\Gamma_W^2} \left[2|\vec{Q}\,|^2 + Q^2(1 - \frac{Q_0}{m_t})\right] \ , \tag{12a}$$

$$\Gamma_T = \frac{G_F^2 m_t^5}{24\pi^3} \int\limits_0^{(m_t-m_b)^2} dQ^2 \frac{M_W^4 \ |\vec{Q}\,|}{(Q^2 - M_W^2)^2 + M_W^2\Gamma_W^2} \left[2\ Q^2(1 - \frac{Q_0}{m_t})\right] \ . \tag{12b}$$

In the case $m_t << M_W$ the integrals can be done with the result

$$\frac{\Gamma_L}{\Gamma_T} = 2 \ . \tag{13}$$

Sufficiently far above the W threshold we need only calculate the relative production of longitudinal and transverse real W's:

$$\frac{\Gamma_L}{\Gamma_T} = \frac{1}{2} + \frac{m_t|\overrightarrow{Q}_W|^2}{E_b M_W^2} \; . \tag{14}$$

Precisely at threshold, $\Gamma_L/\Gamma_T = 1/2$. The value of Γ_L/Γ_T near the threshold is shown in Figure 3 for $\Gamma_W = 0.0225$ GeV (dotted curve), 0.225 GeV (dashed curve), and the expected 2.25 GeV (solid curve). In this case we see that even for the expected value of Γ_W the ratio varies rapidly with m_t, especially just below the threshold.

The ratio of longitudinal to transverse W's is reflected in the angular distribution of the electrons[12] from its decay. With the final b quark direction as a polar axis,

$$\frac{d\Gamma}{d\cos\theta} = 1 + \alpha \; \cos^2\theta \; , \tag{14}$$

where

$$\alpha = \frac{\Gamma_T - \Gamma_L}{\Gamma_T + \Gamma_L} \; . \tag{15}$$

Thus a measurement of α gives a value for Γ_L/Γ_T and indirectly a value for m_t. In particular, α becomes positive only a few GeV below the threshold, and this may provide a useful lower bound on m_t,

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7. We take the total width of the W to be $\Gamma_W = 2.25$ GeV $= 9\Gamma(W \to e\nu_e)$, accounting for three generations of leptons and only two of quarks, since here we are only interested in the situation where $W^+ \to t\bar{b}$ is kinematically forbidden or at least very highly suppressed.

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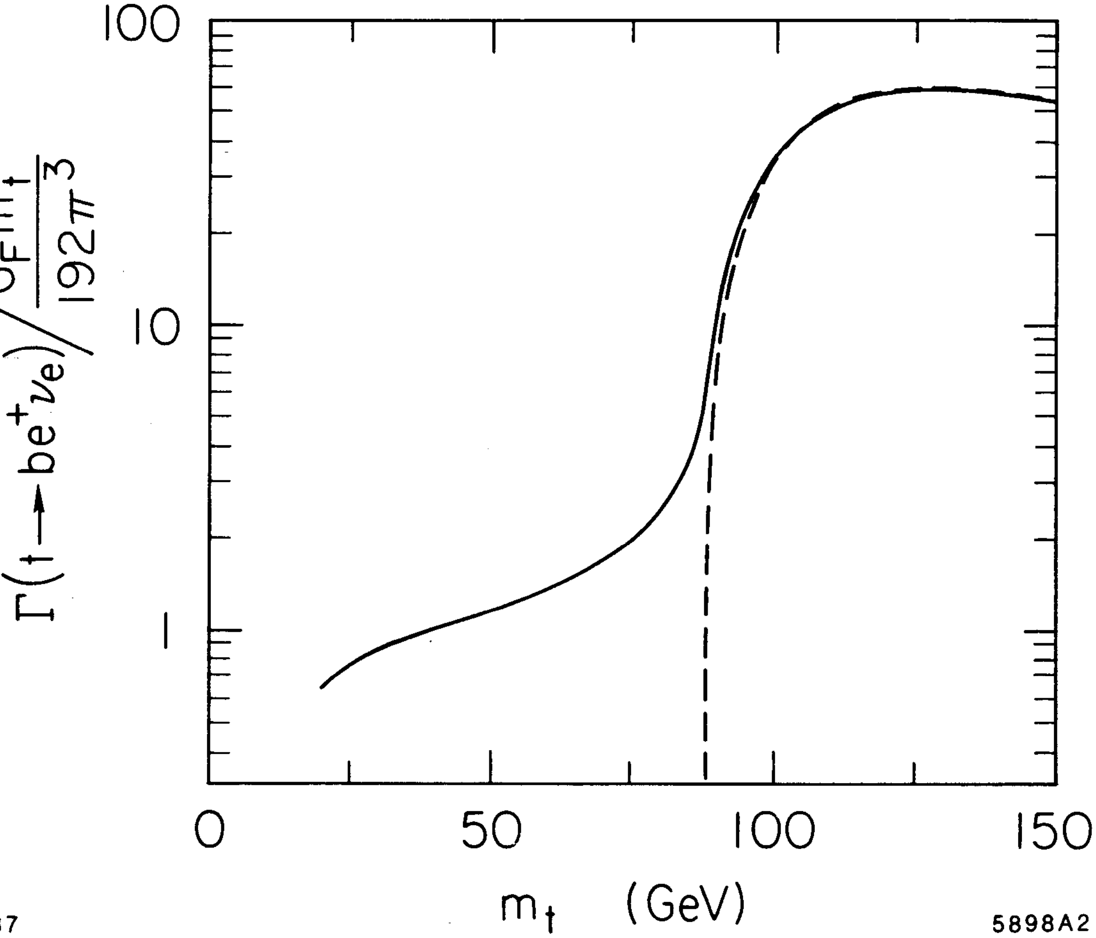
12. The direction of muons, taus or quark jets in other decays of the W will of course serve the same purpose.

FIGURE CAPTIONS

1. $\Gamma(t \to b\ e^+\nu_e)/(G_F^2 m_t^5/192\pi^3)$ as a function of m_t from the full expression in Eq. (1) for $M_W = 83$ GeV, $\Gamma_W = 2.25$ GeV and $m_b = 5$ GeV (solid curve), and from Eq. (4) for decay into a real, infinitely narrow W (dashed curve).

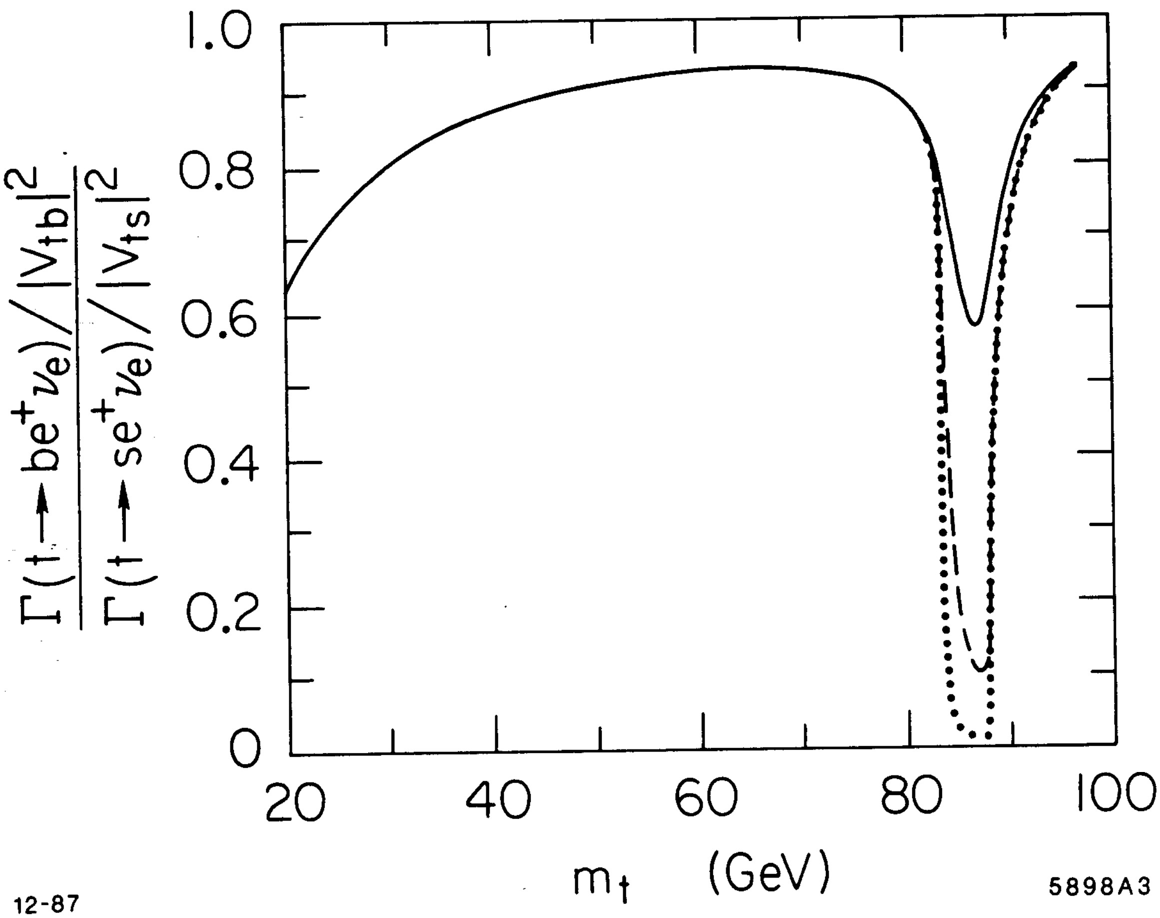
2. The ratio of decay rates with Kobayashi–Maskawa factors taken out, $\left(\Gamma(t \to b\ e^+\nu_e)/|V_{tb}|^2\right)/\ \left(\Gamma(t \to s e^+\nu_e)/|V_{ts}|^2\right)$ with $m_b = 5$ GeV and $m_s = 0.5$ GeV and Γ_W equal to ficticious values of 0.0225 GeV (dotted curve) and 0.225 GeV (dashed curve), and the expected 2.25 GeV (solid curve).

3. The ratio Γ_L/Γ_T of $t \to b + W \to b\ e^+\nu_e$ decay widths into longitudinal compared to transverse W's as a function of m_t for Γ_W equal to ficticious values of 0.0225 GeV (dotted curve) and 0.225 GeV (dashed curve), and the expected 2.25 GeV (solid curve).



100
10
1
0
50
100
150
$\Gamma(t \to b e^+ \nu_e) \Big/ \frac{G_F^2 m_t^5}{192\pi^3}$
m_t (GeV)
12-87
5898A2

Fig. 1



$\frac{\Gamma(t \rightarrow be^+\nu_e)/|V_{tb}|^2}{\Gamma(t \rightarrow se^+\nu_e)/|V_{ts}|^2}$
1.0
0.8
0.6
0.4
0.2
0
20
40
60
80
100
m_t (GeV)
12-87
5898A3

Fig. 2

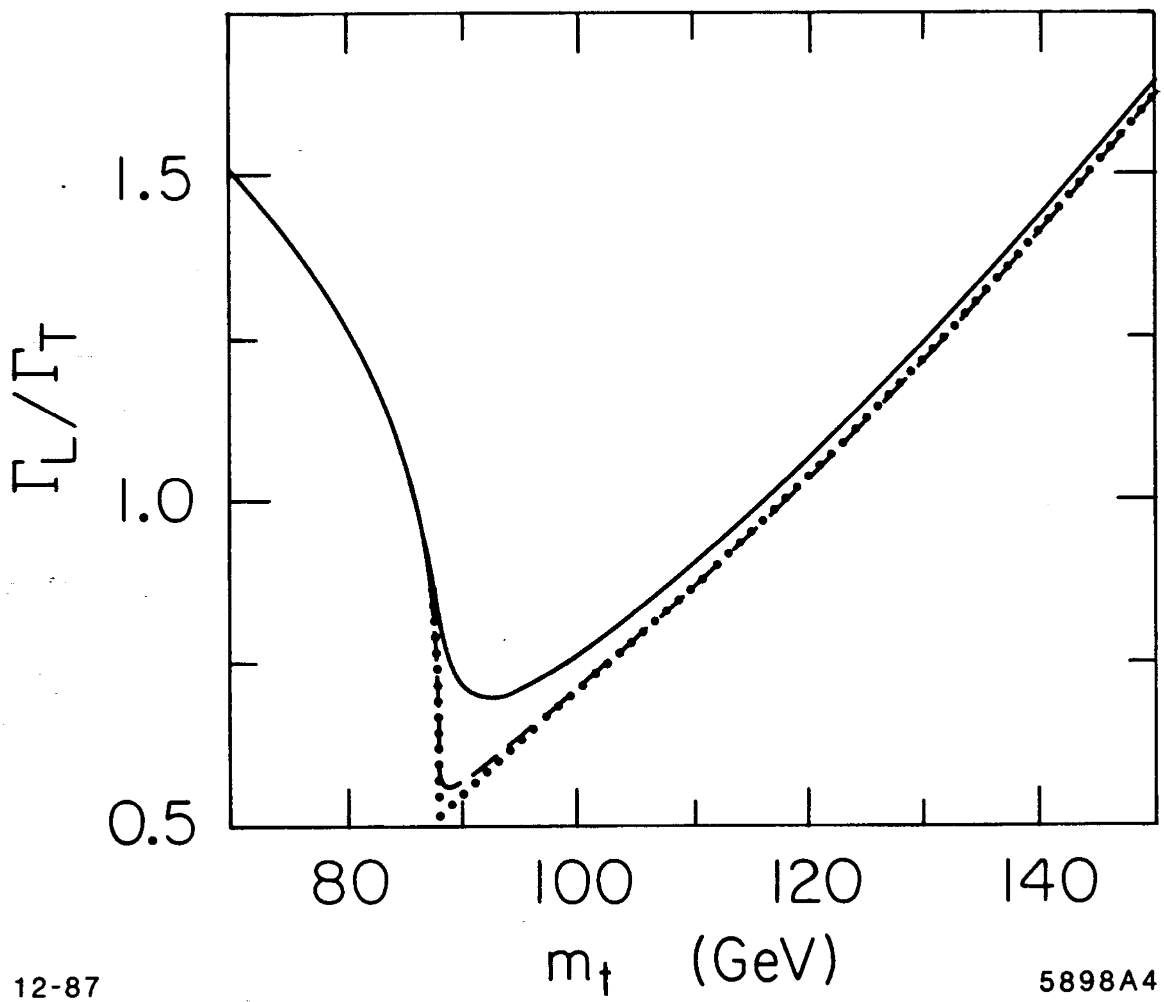


Fig. 3