SLAC–PUB–4495
December 1987
(T/E)

NON MULTA, SED MULTUM — FUTURE LESSONS FROM TWO-PRONG, TWO-BODY DECAYS OF BEAUTY*

I.I. BIGI†
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

B. STECH
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305
and
Inst. f. Theoret. Physik der Universitaet
D-6900 Heidelberg, Germany

ABSTRACT

There are four two-body, two-prong decays modes of B mesons and two for beauty baryons and they are quite rare, i.e., their branching ratios are not expected to exceed the $\mathcal{O}(10^{-4})$ level. Yet a detailed study of their relative rates with a sensitivity level of 10^{-5} can yield unique and important information on strong interactions. If the evolution of these reactions in proper time can be traced then, under favorable conditions, one can analyze $B^0 - \overline{B}^0$ mixing and CP invariance in a detailed way.

Contributed to the Workshop on High Sensitivity Beauty Physics at Fermilab, Batavia, Illinois, Nov. 11 - 14, 1987

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.
† Heisenberg Fellow.

I. Introduction

Hadronic collisions produce immense numbers of hadrons carrying beauty. Yet it represents an awesome experimental challenge to filter them out from the huge nonbeauty background. As emphasized in particular by Bjorken, it might pay off to look for two-body, two-prong final states: although these decays have very small branching ratios since they exhibit a clean and simple decay topology.

The list of two-body, two–prong decay modes of beauty hadrons is quite short:

(*i*) $B \to \pi^+ \pi^-$, K^+K^-

(*ii*) $B \to K^\pm \pi^\mp$

(*iii*) $B \to p\bar{p}$

(*iv*) $\Lambda_b \to p\pi^-$, pK^-

In this note we want to discuss the physics issues that can be addressed in dedicated studies of these decays:

(a) They will give very sensitive and unique information on $V(ub)$ and on the impact of strong interactions on nonleptonic decays of heavy flavors.

If, in addition, the evolution of the decay process in proper time can be resolved, there are two more highly fascinating topics:

(b) A detailed study of $B^0 - \overline{B}^0$ mixing can be performed where the B_d system is separated from the B_s system. In principle one can also distinguish between the relative weight of Δm and of $\Delta\Gamma$ in $B^0 - \overline{B}^0$ mixing.

(c) CP violation, both in direct decay processes and through mixing, can be studied.

The information one will gain at each step of the above program will provide essential input for the next step.

II. Discussion of Branching Ratios

To obtain benchmark numbers for the branching ratios we employ the factorization approximation as a guideline and ignore for the moment Penguin contributions and rescattering effects. One finds for $\tau_B = 1.2$ psec

$$BR\ (\overline{B}_d \to \pi^+\pi^-) \simeq BR\ (\overline{B}_s \to K^+\pi^-) \simeq 2 \times 10^{-3} \left|\frac{V(ub)}{V(cb)}\right|^2 \tag{1}$$

$$\begin{aligned} BR\ (\overline{B}_d \to K^-\pi^+) \simeq BR\ (\overline{B}_s \to K^+K^-) &\simeq \left(\frac{f_K}{f_\pi}\right)^2 tg^2\theta_c BR\ (\overline{B}_d \to \pi^+\pi^-) \\ &\sim 1.6 \times 10^{-4} \left|\frac{V(ub)}{V(cb)}\right|^2 \end{aligned} \tag{2}$$

$$BR\ (\overline{B}_d \to p\bar{p}) \lesssim few \times 10^{-4} \left|\frac{V(ub)}{V(cb)}\right|^2 \tag{3}$$

$$BR\ (\Lambda_b \to p\pi^-) \simeq 3 \times 10^{-3} \left| \frac{V(ub)}{V(cb)} \right|^2 \tag{4}$$

$$\begin{aligned} BR\ (\Lambda_b \to pK^-) &\simeq \left(\frac{f_K}{f_\pi} \right)^2 tg^2\theta_c BR\ (\Lambda_b \to p\pi^-) \\ &\sim 2.3 \times 10^{-4} \left| \frac{V(ub)}{V(cb)} \right|^2 \end{aligned} \tag{5}$$

The numerical branching ratios in Eqs. (1), (2), (4), and (5) have been estimated in the usual fashion taking the wave function overlaps for the current matrix elements from Ref. 1. Varying the model introduces an uncertainty of roughly a factor of two. The estimate on B decays into $p\bar{p}$ pairs has been obtained by considering the intermediate production of diquark states[2] and is more doubtful.

The decay rate $\overline{B}_d \to K^+K^-$ is very tiny in the factorization ansatz: it requires annihilation and recreation of $q\bar{q}$ pairs. The modes $\overline{B}_s \to \pi^+\pi^-, p\bar{p}$ are, in addition, Cabibbo–suppressed.

There is no general argument why final state interactions like rescattering and annihilation processes will necessarily contribute only small amounts to a specific beauty decay mode – in particular when the factorizable amplitude by itself is small. The question of their relative weight is one of detailed *dynamics* which has to be addressed on a case-by-case basis.

A typical channel mixing process is given by

$$\overline{B}_d \to \rho^+\rho^- \Rightarrow K^+K^- \quad . \tag{6}$$

From our experience with $D_s^+ \to \rho^0\pi^+$ vs. $D_s^+ \to \phi\pi^+$, i.e.,[3]

$$\frac{BR(D_s^+ \to \rho^0\pi^+)}{BR(D_s^+ \to \phi\pi^+)} < 0.09 \qquad (90\%\ \text{C.L.}) \quad .$$

We infer a tiny rate for process (6) and therefore

$$BR\ (B_d \to K^+K^-) \ll\ BR\ (B_d \to \pi^+\pi^-) \quad . \tag{7}$$

Similarly

$$BR\ (\overline{B}_s \to \pi^+\pi^-) \ll\ BR\ (\overline{B}_s \to K^+K^-) \tag{8}$$

$$BR\ (\overline{B}_s \to p\bar{p}) \ll BR\ (\overline{B}_d \to p\bar{p}) \tag{9}$$

where relation (9) is further strengthened by the Cabibbo suppression in $B_s \to p\bar{p}$.

The relations Eqs. (7)–(9) should be checked experimentally. Finding the expected suppression would clearly confirm our belief that we have developed an at least semiquantitative understanding of energetic two-body decays. The result would tell us to what extent annihilation is important in these decays. If however no significant suppression were found, i.e., if $BR\ (B_d \to K^+K^-) \gtrsim 1/3\ BR\ (B_d \to \pi^+\pi^-)$, etc. then we would have to admit that essential parts of nonleptonic beauty decays are not understood.

Final state interactions can, however, drastically affect the decay rates of other channels: Penguin transitions and rescattering processes may give significant, if not even dominant contributions to $\overline{B}_s \to K^+K^-, \overline{B}_d \to K^-\pi^+, \Lambda_b \to pK^-$. For the factorizable contributions to these transition rates are proportional to $|V(ub)|^2\ sin^2\theta_c$; if channel mixing is effective then these final states can be fed from intermediate states whose occurrence is controlled by $|V(cb)|^2 \cos^2\theta_c$; for instance

$$\overline{B}_s \to D_s^{(*)}\overline{D}_s^{(*)} \Rightarrow K^+K^- \tag{10}$$

$$\overline{B}_d \to \overline{D}_s^{(*)}D^{(*)} \Rightarrow K^-\pi^+ \tag{11}$$

$$\Lambda_b \to \Lambda_c\overline{D}_s^{(*)} \Rightarrow pK^- \quad . \tag{12}$$

The huge KM enhancement factor

$$\frac{|V(cb)|^2}{|V(ub)|^2\ tg^2\theta_c} \sim 500$$

can make the reactions Eqs. (10)–(12) competitive even for tiny rescattering rates! To be more specific, one expects[1] for the processes $\overline{B}_s \to \overline{D}_s^{(*)}D_s^{(*)}, \overline{B}_d \to \overline{D}_s^{(*)}D^{(*)}, \Lambda_b \to \Lambda_c\overline{D}_s^{(*)}$ branching ratios of a few percent. There is actually some preliminary experimental support for the estimate on $BR\ (\overline{B}_d \to \overline{D}_s^*D^*)$. Therefore with a *rescattering probability as small as 1/2%* one obtains branching ratios of order 10^{-4} for $\overline{B}_s \to K^+K^-, \overline{B}_d \to K^-\pi^+, \Lambda_b \to pK^-$, i.e., much higher than the expectation given in Eqs. (2) and (5).

Penguin type graphs actually produce two terms: one contains top quarks in the internal loop—clearly a short distance contribution; the other one has charm quarks and involves more than pure short distance dynamics since $m(B)$ does not lie well above the $D\overline{D}$ threshold. An important part of the long-range contribution is given by the channel mixing processes described above.

To summarize our discussion so far:

— The results based on factorization are presented in Eqs. (1)–(5).

— A significant violation of the relations stated in Eqs. (7)–(9) is not to be expected. If observed, it would overturn our picture of energetic two-body decays.

— The branching ratios for the modes $\overline{B}_s \to K^+K^-, \overline{B}_d \to K^-\pi^+, \Lambda_b \to pK^-$, on the other hand, can be strongly enhanced by final state interactions. Deviations from the results of factorization are here indeed expected and the branching ratios will yield important information on the relative importance of interfering processes. It generates a "Scenario of acceptable dissent" from factorization.

A final qualitative note may help to illustrate some of the issues involved in final state interactions. An important part of the effect of final state interactions is given by on-mass-shell rescattering processes: the bare decay amplitudes $A^0(\overline{B} \to f)$ are modified according to

$$A(\overline{B} \to f) = \sum_{f'} (S^{1/2})_{ff'} A^0(\overline{B} \to f') \tag{13}$$

where $S^{1/2}$ is the square root of the ***strong*** interaction S matrix of definite isospin. Note also that we deal here with $s-$ and $p-$wave decays only, i.e., rescattering has to be treated like central collisions.

Using unitarity and time reversal invariance one can express $S^{1/2}$ in terms of the S matrix; there are actually two equivalent ways of doing that:

$$S^{1/2} = \frac{1}{\sqrt{2}}(1 + ReS)^{-1/2}(1 + S) \tag{14}$$

$$S^{1/2} = \frac{1+i}{2}(1 + ImS)^{-1/2}(1 - iS) \quad . \tag{15}$$

The effect of rescattering for example in the elastic channel $f \to f$ (like $2\pi \to 2\pi$) can thus be expressed as follows:

$$(S^{1/2})_{ff} = \frac{1}{\sqrt{2}} \langle f \left|(1 + ReS)^{-1/2}\right| f\rangle \; (1 + \eta_f e^{2i\delta_f}) + \frac{i}{\sqrt{2}} \sum_{f' \neq f} \langle f \left|(1 + ReS)^{-1/2}\right| f'\rangle T_{f'f} \tag{16}$$

$$(S^{1/2})_{ff} = \frac{1+i}{2}(\langle f \left|(1 + ImS)^{-1/2}\right| f\rangle \; (1 - i\eta_f e^{2i\delta_f}) + \sum_{f' \neq f} \langle f \left|(1 + ImS)^{-1/2}\right| f'\rangle T_{f'f}) \tag{17}$$

Here we have used the usual notation:

$$S = 1 + iT, \;\; S_{ff} = \eta_f e^{2i\delta_f} \quad . \tag{18}$$

From Eqs. (16) and (17) one reads off that even total absorption in the final state, i.e., $\eta_f = 0$, does not necessarily and not even likely produce a strong suppression of $S^{1/2}$; it might get reduced by a factor of $1/\sqrt{2}$ only.

Comparing Eqs. (16) and (17) shows that the inelastic transition amplitudes $T_{f'f}$ will in general contain sizeable complex phases. For otherwise the two expressions (16) and (17) are equivalent only under very special circumstances since the phase structure of the first term in each equation is quite different.

All of this is only meant to illustrate two things: firstly, final state interactions will in general introduce phase shifts; secondly, no specific prediction can be made on it.

II. $B^0 - \overline{B}^0$ Mixing

We assume here that no flavor tagging of the beauty hadron that was produced in conjunction with the neutral B can be performed. *A priori* we do therefore not know whether the decaying B was born as a B^0 or a $\overline{B}^0$. $B^0 - \overline{B}^0$ mixing can then be studied only under rather favorable circumstances. Throughout this section we assume CP variance to hold.

(a) $\Delta\Gamma$:

$\pi^+\pi^-$, K^+K^- are even CP eigenstates. Thus

$$\text{rate}\left(B^0(t) \to \pi^+\pi^- \ , \ K^+K^-\right) \alpha \ e^{-\Gamma_+ t} \tag{20}$$

where Γ_+ denotes the width of the even CP eigenstate among the two mass eigenstates of B^0, $\overline{B}^0$, be they B_d or B_s. In $B^0 \to p\bar{p}$ we have a combination of even and odd CP eigenstates. The time evolution should then exhibit two exponentials:

$$\text{rate}\ \left(B^o(t) \to p\bar{p}\right) \alpha\ f_1 e^{-\Gamma_+ t} + f_2 e^{-\Gamma_- t} \tag{21}$$

where f_1 and f_2 denote the weight of the s-wave and p-wave configuration of the $p\bar{p}$ final state. Thus

$$\frac{\text{rate}\left(B^0(t) \to p\bar{p}\right)}{\text{rate}\left(B^0(t) \to \pi^+\pi^-\right)}\ \alpha\ \ 1 + \ \frac{f_2}{f_1}\ e^{\Delta\Gamma t} \ \ , \ \Delta\Gamma = \Gamma_+ - \Gamma_- \quad .$$

These processes do not yield information on Δm, as long as CP invariance holds. For measuring Δm one has to turn to final states that are *not* CP eigenstates.

(b) Δm :

$$\text{rate}\left(\overline{B}_d \to K^-\pi^+\right) \alpha\ e^{-\Gamma_d t} \left(\frac{1}{2}\left(1 + e^{-\Delta\Gamma_d t}\right) + e^{-\frac{1}{2}\Delta\Gamma_d t} \cos\Delta m_d t\right) \tag{22}$$

$$\text{rate}\left(B_d \to K^-\pi^+\right) \alpha\ e^{-\Gamma_d t} \left(\frac{1}{2}\left(1 + e^{-\Delta\Gamma_d t}\right) - e^{-\frac{1}{2}\Delta\Gamma_d t} \cos\Delta m_d t\right) \tag{23}$$

$$\text{rate}\left(\overline{B}_s \to K^-\pi^+\right) \alpha\ e^{-\Gamma_s t} \left(\frac{1}{2}\left(1 + e^{-\Delta\Gamma_s t}\right) - e^{-\frac{1}{2}\Delta\Gamma_s t} \cos\Delta m_s t\right) \tag{24}$$

$$\text{rate}\ \left(B_s \to K^-\pi^+\right) \alpha\ e^{-\Gamma_s t} \left(\frac{1}{2}\left(1 + e^{-\Delta\Gamma_s t}\right) + e^{-\frac{1}{2}\Delta\Gamma_s t} \cos\Delta m_s t\right) \quad . \tag{25}$$

Therefore

$$\text{rate}\left(B_d^{neut} \to K^-\pi^+\right) \alpha\ e^{-\Gamma_d t} \left(\frac{1}{2}\left(1 + e^{-\Delta\Gamma_d t}\right) + \frac{\overline{N} - N}{\overline{N} + N} \cos\Delta m_d t\right) \tag{26}$$

$$\text{rate}\left(B_s^{neut} \to K^-\pi^+\right) \alpha\ e^{-\Gamma_s t}\left(\frac{1}{2}\left(1+e^{-\Delta\Gamma_s t}\right) - \frac{\overline{N}-N}{\overline{N}+N}\cos\Delta m_s t\right) \quad . \tag{27}$$

$$N\left[\,\overline{N}\,\right] = \text{number of } B^0\left[\overline{B}^0\right] \text{ produced} \quad .$$

Here we encounter the first situation where we have to hope for some luck: unless the production process is such that $N \neq \overline{N}$ holds within the acceptance region we cannot measure Δm without flavor tag.

Hadronic collisions will produce $N \neq \overline{N}$ for certain kinematical regimes; yet at present it is quite impossible to predict the size of $N - \overline{N}$ and the best kinematical regime. This suggests the following procedure: one studies the time evolution of $B \to K^{\mp}\pi^{\pm}$ very carefully and searches for a $\cos\Delta mt$ term. If found, one extracts two pieces of information:

(*i*) Δm which determines mixing;

(*ii*) $\frac{\overline{N}-N}{\overline{N}+N}$ calibrating the production asymmetry. This will be very important when searching for CP asymmetries as discussed next.

III. CP Asymmetries

CP violation is established most directly by observing a difference between CP conjugate decay rates. When comparing

$$\text{rate}\left(B(t) \to f\right) = e^{-\Gamma t} G \leftrightarrow \text{rate}\left(\overline{B}(t) \to \bar{f}\right) = e^{-\Gamma t}\overline{G} \tag{28}$$

one can encounter two basic types of differences,

$$\frac{G}{\overline{G}} \neq 1 \quad , \quad \frac{d}{dt}\left(\frac{G}{\overline{G}}\right) = 0 \tag{29}$$

or

$$\frac{G}{\overline{G}} \neq 1 \quad , \quad \frac{d}{dt}\left(\frac{G}{\overline{G}}\right) \neq 0 \quad . \tag{30}$$

The first scenario appears when a CP asymmetry arises in conjunction with non-zero phases from final state interactions, the second when $B^0 - \overline{B}^0$ mixing is involved.[4]

Direct CP violation can be searched for in $\overline{B} \to K^-\pi^+$ vs. $B \to K^+\pi^-$ since the final states are flavor-specific (within the Standard Model). The presence of mixing at first sight complicates the situation:

$$\frac{\text{number of}(K^-\pi^+)_B}{\text{number of}(K^+\pi^-)_B} = \frac{1+\frac{\overline{N}-N}{\overline{N}+N}f(t)}{1-\frac{\overline{N}-N}{\overline{N}+N}f(t)} \; \frac{|\,\text{Ampl}(\overline{B} \to K^-\pi^+)\,|^2}{|\,\text{Ampl}(B \to K^+\pi^-)\,|^2} \tag{31}$$

$$f(t) = \frac{2e^{-\frac{1}{2}\Delta\Gamma t}}{1+e^{-\Delta\Gamma t}}\cos\Delta mt \;\; \simeq \;\; \cos\Delta mt$$

for $\Delta\Gamma \simeq 0$. CP violation is established if the second factor in Eq. (31) is found to differ from unity. Thus we realize that $B^0 - \overline{B^0}$ mixing is actually far from being a nuisance;

by the rather special dependence on proper time it introduces, one can extract the first factor in Eq. (31) quite independently of any CP analysis. This allows to determine the ratio $BR(\overline{B} \to K^-\pi^+)/BR(B \to K^+\pi^-)$ directly. What are the prospects for $BR(\overline{B} \to K^-\pi^+)$ to differ from $BR(B \to K^+\pi^-)$?

For $\overline{B}_d \to K^-\pi^+$ vs. $B_d \to K^+\pi^-$ they are quite good:

- KM parameters with a large phase enter (like $V(ub)$ in the Wolfenstein representation).
- As discussed in Sec. II final state interactions are presumably quite virulent in this rare mode.

Therefore CP asymmetries of around 10% could emerge here.

The prospects are less promising for $\overline{B}_s \to K^+\pi^-$ vs. $B_s \to K^-\pi^+$, since rescattering and Penquin graphs are Cabibbo-suppressed.

One can search for CP asymmetries also in the decays of beauty baryons:

$$\frac{\text{number of } (\Lambda_b \to p\pi^-[K^-])}{\text{number of } (\overline{\Lambda}_b \to \bar{p}\pi^+[K^+])} = \frac{N(\Lambda_b)}{\overline{N}(\overline{\Lambda}_b)} \frac{BR(\Lambda_b \to p\pi^-[K^-])}{BR(\overline{\Lambda}_b \to \bar{p}\pi^+[K^+])} \quad . \tag{32}$$

Forming the ratio of ratios (32) leads to

$$\frac{N(\Lambda_b \to p\pi^-)\overline{N}(\overline{\Lambda}_b \to \bar{p}K^+)}{N(\overline{\Lambda}_b \to \bar{p}\pi^+)\overline{N}(\Lambda_b \to pK^-)} = \frac{BR(\Lambda_b \to p\pi^-)}{BR(\overline{\Lambda}_b \to \bar{p}\pi^+)} \frac{BR(\overline{\Lambda}_b \to \bar{p}K^+)}{BR(\Lambda_b \to pK^-)} \quad . \tag{33}$$

If this double ratio were found to be unity, we would not have learned anything about CP invariance. Yet again, the FSI are expected to be much more important in $\Lambda_b \to pK^-$ than in $\Lambda_b \to p\pi^-$. Therefore it is not unreasonable to entertain the idea that $\Lambda_b \to pK^-$ decays exhibit CP asymmetries of up to 10% while $BR(\Lambda_b \to p\pi^-) = BR(\overline{\Lambda}_b \to \bar{p}\pi^+)$ to a very good approximation.

CP violation can be made observable also via $B^0 - \overline{B}^0$ mixing. For the simplest case $B^0 \to \pi^+\pi^-$ one can write down the general expression

$$\text{rate } \left(B_{neut}(t) \to \pi^+\pi^-\right) \;\alpha\, e^{-\Gamma t} A \left(1 + \frac{B}{A} e^{\Delta\Gamma t} + \frac{C}{A} e^{-\frac{\Delta}{2}\Gamma t} \sin\Delta m t\right) \tag{34}$$

with

$$A = 1 + Re\,\frac{q}{p}\bar{\rho}_f \;,\; B = 1 - Re\frac{q}{p}\bar{\rho}_f \;,\; C = \frac{2(N - \bar{N})}{N + \overline{N}} Im\frac{q}{p}\bar{\rho}_f$$
$$\frac{q}{p} = \frac{1-\epsilon}{1+\epsilon} \;,\; \bar{\rho}_f = \frac{\text{Ampl}(\overline{B}^0 \to \pi^+\pi^-)}{\text{Ampl}(B^0 \to \pi^+\pi^-)} \quad . \tag{35}$$

CP invariance is violated if $C \neq O$; or for $\Delta\Gamma \neq O$ if $B \neq O$. Unless $\Delta\Gamma$ is sufficiently large, our only hope to observe this kind of CP violation rests on the measurement of C and thereby on the occurrence of a production asymmetry, i.e., $N \neq \overline{N}$. As discussed in Sec. III, $N - \overline{N}$ can, at least in principle, be extracted from $B^0 - \overline{B}^0$ mixing studies.

IV. Summary

Two-body, two-prong decay modes of beauty have to appear on the 10^{-5} level in branching ratio, yet branching ratios of up to 10^{-4} are quite conceivable for some modes. The relative weight of the various decay modes will teach us important lessons on strong interactions, in particular on the relevance of final state interactions.

A high premium has to be placed on the ability to trace the proper time evolution of these beauty decays. If production asymmetries occur, then one can perform very detailed studies of $B^0 - \overline{B}^0$ mixing and CP invariance.

Acknowledgments

We benefitted from useful discussions with A.I. Sanda. One of us (B. S.) gratefully acknowledges financial support from the German Ministery of Research and Technology.

References

(1) M.Bauer, B.Stech and M.Wirbel, Z.Phys.C34(1987)103.

(2) I.I.Bigi, Phys.Lett.106B(1981)510; B.Stech, Invited talk given at the San Miniato Topical Seminar on Heavy Flavours, San Miniato, 1987, preprint HD-THEP-87-18.

(3) M.Witherell, Invited talk given at the International Symposium on the Production and Decay of Heavy Flavours, Stanford, California, Sept. 1-5, 1987.

(4) For a comprehensive review see: I.I.Bigi, V.A.Khoze, N.G.Uraltsev and A.I.Sanda, SLAC-PUB-4476, to appear in "CP Violation", ed. C.Jarlskog, World Scientific, Singapore