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SUPERSYMMETRIC CHIRAL BOSONS ON GROUP MANIFOLDS *

Stefano Bellucci [1]
Department of Physics
University of California
Davis, California 95616 U.S.A.

Roger Brooks [2]
Center for Theoretical Physics,
Laboratory for Nuclear Science
and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 U.S.A.

and

Jacob Sonnenschein [3]
Stanford Linear Accelerator Center
Stanford University
Stanford, California 94305 U.S.A.

ABSTRACT

We give the WZW models, for (1,0) supersymmetric leftons and rightons. Super-Kac-Moody currents and the coupling to non-abelian, gauge superfields are derived. The locally supersymmetric theory is discussed.

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[1] On leave from INFN-Laboratori Nazionali di Frascati, C.P. 13, 00044 Frascati (Roma), Italy. Research partially supported by D.O.E. at Davis.

[2] Address after January 1, 1988: SLAC, Stanford University, Stanford, CA 94305 USA. Research partially supported by a M.I.T. Minority Postdoctoral Fellowship.

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I. Introduction

Due in part to the progress made in the compactification of superstrings, the study of $U(1)$ chiral bosons [1] has blossomed in the past year [2]. Non-abelian chiral bosons have also been studied [2b,3], although much less so than their abelian limits. Much work [1,4] has also been done on the supersymmetric extensions of the $U(1)$ theories. Studies of the supersymmetric, non-abelian theory are also present in the literature [5]. In this letter, we will present some of the results we have obtained for the (1,0) supersymmetric theories. Further details on this work will be publish elsewhere [11].

Our non-linear sigma model actions will be presented in the next section. We will also derive the self-duality constraint there. Following this, in section III, a discussion of the super Kac-Moody invariance and its associated super-currents, will be given. Section IV will describe possible couplings to non-abelian gauge superfields. The superconformal theory and the coupling to supergravity will be presented in section V. This will include the conditions for consistent propagation of the leftons (left moving) and rightons (right moving). Our notation will generally be that of our previous work [4c].

During the completion of this work we received a preprint by Gates and Siegel [10] in which some of the results obtained here are discussed.

II. Lefton And Righton Non-Linear Sigma Models

The non-linear sigma models for non-supersymmetric leftons and rightons have recently been given [2b,3]. In this section, we will present the superspace actions for the unidexterous theories. This is done by extending our previous works [4a,4c] on the $U(1)$ theories.

Following Ref. [6], we define the non-abelian group element superfield, $\mathcal{U}$, which satisfies $\mathcal{U}\mathcal{U}^{\dagger} = 1$. This matrix superfield can be expressed by $\mathcal{U} \equiv \exp[i\Phi \cdot t]$. The real superfield, $\Phi(\sigma^{\pm\pm}, \varsigma^{+})$, represents a lefton or righton and t represents the hermitian generators of the group G. Our super world-sheet has bosonic light-cone coordinates $\sigma^{\pm\pm} \equiv (\tau \pm \sigma)$ and a real Grassmann coordinate, ς^{+}. In writing the Wess-Zumino(WZ) terms we will introduce a third coordinate y with $0 \leq y \leq 1$. This will allow us to define the superfield, $\tilde{\Phi}(\sigma^{\pm\pm}, \varsigma^{+}, y)$, over the solid ball, by $\tilde{\Phi}(\sigma^{\pm\pm}, \varsigma^{+}, y = 1) = \Phi(\sigma^{\pm\pm}, \varsigma^{+})$ and $\tilde{\Phi}(\sigma^{\pm\pm}, \varsigma^{+}, y = 0) = 0$. Then the mapping is $\tilde{\mathcal{U}} \equiv \exp[i\tilde{\Phi} \cdot t]$. (Here we assume that the second homotopy group of G is trivial, $\pi_2(G) = 0$.) It will also prove useful to define $\Pi_A \equiv \mathcal{U}^{-1} D_A \mathcal{U}$ where D_A represents a supersymmetric, covariant derivative. A similar definition for $\tilde{\Pi}_A$ holds with $\mathcal{U} \rightarrow \tilde{\mathcal{U}}$ and $\tilde{\Pi}_y \equiv \tilde{\mathcal{U}}^{-1} \frac{d}{dy} \tilde{\mathcal{U}}$.

Our actions will be of the form $S(\mathcal{U}) = S_{WZW}(\mathcal{U}) + S_{\Lambda}(\mathcal{U}) = S_{\sigma}(\mathcal{U}) + nS_{WZ}(\mathcal{U}) + S_{\Lambda}(\mathcal{U})$; where $n \in (n_L, n_R)$ is an integer [6] which in general has different values for the lefton and righton sectors. Here S_{WZW} is the Wess-Zumino-Witten action, S_{σ} is the sigma model action, S_{WZ} is the WZ term, and S_{Λ} is the Siegel term which leads to the self-duality of $\mathcal{U}$.

In the (1,0) theory, both the leftons and rightons will have the S_{σ} and S_{WZ} actions defined as follows:

$$S_{\sigma}(\mathcal{U}) = -i\tfrac{1}{4l^2}\int d^2\sigma d\varsigma^{-} \; Tr[\Pi_{+}\Pi_{--}] \ , \tag{2.1a}$$

$$S_{WZ}(\mathcal{U}) = -i\tfrac{1}{16\pi}\int d^2\sigma d\varsigma^{-} \int_0^1 dy \; Tr[\tilde{\Pi}_y(D_{+}\tilde{\Pi}_{--} - \partial_{--}\tilde{\Pi}_{+})] \ . \tag{2.1b}$$

For leftons, we employ the notation $\mathcal{U} \rightarrow \mathcal{U}_L$ and $\tilde{\mathcal{U}} \rightarrow \tilde{\mathcal{U}}_L$. The rightons will be adorned with a "R" label.

The component form of this action at the special coupling $(\frac{1}{l^2} - \frac{n}{4\pi}) = 0$ has the form [5]

$$S_{WZW}(\mathcal{U}) = S_{WZW}(u) - i\frac{n}{8\pi}\int d^2\sigma Tr[\beta_{+} \cdot \partial_{--}\beta_{+}] \tag{2.2a}$$

where u is a group element of G and β_+ is a Weyl-Majorana fermion in the adjoint representation of G. It is defined via:

$$\mathcal{U}| \equiv u \ , \quad D_+\mathcal{U}| \equiv iu\beta_+ \ . \tag{2.2b}$$

The variation of the actions (2.1a) and (2.1b) with respect to $\mathcal{U}$ are the following:

$$\delta S_\sigma(\mathcal{U}) = i\tfrac{1}{4l^2}\int d^2\sigma d\varsigma^- \ Tr[\mathcal{U}^{-1}\delta\mathcal{U}(\partial_{--}\Pi_+ + D_+\Pi_{--})] \ , \tag{2.3a}$$

$$\delta S_{WZ}(\mathcal{U}) = \frac{i}{16\pi}\int d^2\sigma d\varsigma^- \ Tr[\mathcal{U}^{-1}\delta\mathcal{U}(\partial_{--}\Pi_+ - D_+\Pi_{--})] \ , \tag{2.3b}$$

For the special coupling $(\frac{1}{l^2} - \frac{n}{4\pi}) = 0$ the variation of the WZW action takes the form

$$\delta S_{WZW}(\mathcal{U}) = i\frac{n}{8\pi}\int d^2\sigma d\varsigma^- \ Tr[\mathcal{U}^{-1}\delta\mathcal{U}\partial_{--}\Pi_+] \ , \tag{2.3c}$$

II.1 Non-abelian (1,0) Leftons:

As a generalization of the abelian leftons superfields, $\Phi^{\hat{\alpha}}$, which *a priori* obey $\partial_{--}\Phi^{\hat{\alpha}} = 0$, *i.e.* $\Phi^{\hat{\alpha}} = \Phi^{\hat{\alpha}}(\sigma^{++}, \varsigma^+)$, we define a lefton matrix superfield, $\mathcal{U}_L$, by $\partial_{--}\mathcal{U}_L = 0$. These group element leftons can be constructed from the former leftons via $\mathcal{U}_L = \exp[i\Phi \cdot t]$. The chiral nature of the matrix superfields $\mathcal{U}_L$ is achieved by adding to the $S_{WZW}(\mathcal{U}_L)$ the following lagrange multiplier $(\Lambda_+{}^{--})$ or Siegel term

$$S_\Lambda(\mathcal{U}_L) = -i\tfrac{1}{4l^2}\int d^2\sigma d\varsigma^- \Lambda_+{}^{--} \ Tr[\Pi_{--}{}^{(L)}\Pi_{--}{}^{(L)}] \ . \tag{2.5}$$

The variation of this term with respect to $\delta\mathcal{U}_L$ is given by

$$\delta S_\Lambda(\mathcal{U}_L) = i\tfrac{1}{2l^2}\int d^2\sigma d\varsigma^- \ Tr[\mathcal{U}_L^{-1}\delta\mathcal{U}_L\partial_{--}(\Lambda_+{}^{--}\Pi_{--}{}^{(L)})] \ . \tag{2.6}$$

Using the variations (2.3) and (2.6) it is straightforward to verify that both $S_{WZ}(\mathcal{U}_L)$ and $S_\sigma(\mathcal{U}_L) + S_\Lambda(\mathcal{U}_L)$ are invariant under the Siegel transformations

$$\begin{aligned} \delta_\Upsilon\mathcal{U}_L &= \Upsilon^{--}\partial_{--}\mathcal{U}_L \ , \\ \delta_\Upsilon\Lambda_+{}^{--} &= -D_+\Upsilon^{--} + \Upsilon^{--}\overleftrightarrow{\partial}_{--}\Lambda_+{}^{--} \ . \end{aligned} \tag{2.7}$$

In addition the action for the non-abelian leftons is invariant under the global super-chiral transformations:

$$\mathcal{U} \to A\mathcal{U} \ , \qquad \mathcal{U} \to \mathcal{U}B^{-1} \ . \tag{2.8}$$

Where A and B are constant matrices and are elements of G. The associated chiral super-currents are

$$\begin{aligned} J^{(L)}_{+(l)} &= i(\frac{1}{4l^2} + \frac{n_L}{16\pi})\Pi_+{}^{(L)} + i\frac{1}{2l^2}\Lambda_+{}^{--}\Pi_{--}{}^{(L)} \ , \\ J^{(L)}_{--(l)} &= i(\frac{1}{4l^2} - \frac{n_L}{16\pi})\Pi_{--}{}^{(L)} \ , \end{aligned} \tag{2.9a}$$

$$\begin{aligned} J^{(L)}_{+(r)} &= -i(\frac{1}{4l^2} - \frac{n_L}{16\pi})D_+\mathcal{U}_L\mathcal{U}_L{}^{-1} - i\frac{1}{2l^2}\Lambda_+{}^{--}\partial_{--}\mathcal{U}_L\mathcal{U}_L{}^{-1} \ , \\ J^{(L)}_{--(r)} &= -i(\frac{1}{4l^2} + \frac{n_L}{16\pi})\partial_{--}\mathcal{U}_L\mathcal{U}_L{}^{-1} \ , \end{aligned} \tag{2.9b}$$

where the right current is associated with the first transformation in eqn. (2.8).

Using the expressions for the variation of the various terms of the action with respect to $\mathcal{U}_L$, we get the following equation of motion

$$\begin{aligned} &(\frac{1}{l^2} - \frac{n_L}{4\pi})D_+(\mathcal{U}_L{}^{-1}\partial_{--}\mathcal{U}_L) + (\frac{1}{l^2} + \frac{n_L}{4\pi})\partial_{--}(\mathcal{U}_L{}^{-1}D_+\mathcal{U}_L) \\ &+ \frac{2}{l^2}\partial_{--}(\Lambda_+{}^{--}\mathcal{U}_L{}^{-1}\partial_{--}\mathcal{U}_L) = 0 \ , \end{aligned} \tag{2.10a}$$

The second equation of motion associated with the variation of $\Lambda_+{}^{--}$ is

$$(\mathcal{U}_L{}^{-1}\partial_{--}\mathcal{U}_L)^2 = 0 \ , \tag{2.10b}$$

This equation of motion, as will be clarified below, is potentially anomalous. To solve these equations we take

$$\partial_{--}\mathcal{U}_L = 0 \ . \tag{2.10c}$$

From the equation of motion (2.10a) we get the current conservation laws:

$$D_+J^{(L)}_{--\binom{l}{r}} + \partial_{--}J^{(L)}_{+\binom{l}{r}} = 0 \ . \tag{2.11}$$

The vector and axial supercurrents $J_{\binom{v}{a}} = [J_{(l)} \pm J_{(r)}]$ are obviously also conserved. Note, however, that now unlike the abelian case [3,4c] the vector current is not topologically conserved as the equation of motion (2.10a) must be applied.

II.2 Non-abelian (1,0) Rightons:

Just as for the leftons we now generalize the abelian rightons. The latter are superfields that *a priori* obey $\bar{D}_+\Phi^{\hat{a}} = \dot{0}$. We define a righton matrix superfield $\mathcal{U}_R$ by $D_+\mathcal{U}_R = 0$. This means that the component plus-spinor, β_+ defined in eqn. (2.2b), vanishes. The lagrange multiplier ($\Lambda_{--}{}^{++}$) term takes now the form

$$S_\Lambda(\mathcal{U}_R) = -i\int d^2\sigma d\varsigma^- \Lambda_{--}{}^{++} Tr[\tfrac{1}{4l^2}\Pi_+{}^{(R)}\Pi_{++}{}^{(R)} + i\tfrac{n_R}{48\pi}\Pi_+{}^{(R)}\Pi_+{}^{(R)}\Pi_+{}^{(R)}] \ . \tag{2.12}$$

Notice that the second term is needed to compensate for the Siegel transformation of the WZ term. The variation of this Siegel term with respect to $\mathcal{U}_R$ is

$$\delta S_\Lambda(\mathcal{U}_R) = i\int d^2\sigma d\varsigma^- \ Tr[\mathcal{U}_R^{-1}\delta\mathcal{U}_R\{\tfrac{1}{2l^2}[D_+(\Lambda_{--}{}^{++}\Pi_{++}^{(R)}) + \partial_{++}(\Lambda_{--}{}^{++}\Pi_{+}^{(R)})] \\ + i\tfrac{n_R}{16\pi}D_+(\Lambda_{--}{}^{++}\Pi_+^{(R)}\Pi_+^{(R)})\}] \tag{2.13}$$

The full action, $S(\mathcal{U}_R)$ is invariant under the Siegel transformations

$$\begin{aligned} \delta_\Upsilon\mathcal{U}_R &= \Upsilon^{++}\partial_{++}\mathcal{U}_R - i\tfrac{1}{2}D_+\Upsilon^{++}D_+\mathcal{U}_R \ , \\ \delta_\Upsilon\Lambda_{--}{}^{++} &= -\partial_{--}\Upsilon^{++} + \Upsilon^{++}\partial_{++}\Lambda_{--}{}^{++} - i\tfrac{1}{2}D_+\Upsilon^{++}D_+\Lambda_{--}{}^{++} \ . \end{aligned} \tag{2.14}$$

Here again the action is invariant under the chiral transformation of (2.8). The associated chiral currents are now

$$\begin{aligned} J_{+(r)}^{(R)} &= -i(\frac{1}{4l^2} - \frac{n_R}{16\pi})D_+\mathcal{U}_R\mathcal{U}_R{}^{-1} \ , \\ J_{--(r)}^{(R)} &= -i(\frac{1}{4l^2} + \frac{n_R}{16\pi})\partial_{--}\mathcal{U}_R\mathcal{U}_R{}^{-1} + (\frac{1}{4l^2} - \frac{n_R}{16\pi})\Lambda_{--}{}^{++}D_+\mathcal{U}_R D_+\mathcal{U}_R^{-1} \\ &\quad - i\frac{1}{2l^2}[\Lambda_{--}{}^{++}\partial_{++}\mathcal{U}_R\mathcal{U}_R{}^{-1} - \frac{i}{2}D_+\Lambda_{--}{}^{++}D_+\mathcal{U}_R\mathcal{U}_R{}^{-1}] \ , \\ J_{+(l)}^{(R)} &= i(\frac{1}{4l^2} + \frac{n_R}{16\pi})\mathcal{U}_R{}^{-1}D_+\mathcal{U}_R \ , \\ J_{--(l)}^{(R)} &= i(\frac{1}{4l^2} - \frac{n_R}{16\pi})\mathcal{U}_R{}^{-1}\partial_{--}\mathcal{U}_R - (\frac{1}{4l^2} + \frac{n_R}{16\pi})\Lambda_{--}{}^{++}\Pi_+^{(R)}\Pi_+^{(R)} \\ &\quad + \frac{i}{2l^2}[\Lambda_{--}{}^{++}\mathcal{U}_R{}^{-1}\partial_{++}\mathcal{U}_R - i\tfrac{1}{2}D_+\Lambda_{--}{}^{++}\mathcal{U}_R{}^{-1}D_+\mathcal{U}_R] \ , \end{aligned} \tag{2.15}$$

From the variation of the action (2.13) plus (2.3), with respect to $\mathcal{U}_R$, we find the equation of motion associated with the action $S(\mathcal{U}_R) = S_\sigma(\mathcal{U}_R) + n_R S_{WZ}(\mathcal{U}_R) + S_\Lambda(\mathcal{U}_R)$,

$$\begin{aligned}
&(\frac{1}{l^2} - \frac{n_R}{4\pi})D_+(\mathcal{U}_R{}^{-1}\partial_{--}\mathcal{U}_R) \quad + \quad (\frac{1}{l^2} + \frac{n_R}{4\pi})\partial_{--}(\mathcal{U}_R{}^{-1}D_+\mathcal{U}_R) \\
&+ \quad \frac{2}{l^2}D_+[\Lambda_{--}{}^{++}\mathcal{U}_R{}^{-1}\partial_{++}\mathcal{U}_R \quad - \quad i\tfrac{1}{2}(D_+\Lambda_{--}{}^{++})\mathcal{U}_R{}^{-1}D_+\mathcal{U}_R] \\
&+ \quad i(\frac{1}{4l^2} + \frac{n_R}{16\pi})D_+[\Lambda_{--}{}^{++}\Pi_+^{(R)}\Pi_+^{(R)}] \quad = \quad 0
\end{aligned} \tag{2.16a}$$

Similarly, the $\Lambda_{--}{}^{++}$ variation, which again is potentially anomalous, leads to

$$\frac{1}{4l^2}(\mathcal{U}_R{}^{-1}D_+\mathcal{U}_R)(\mathcal{U}_R{}^{-1}\partial_{++}\mathcal{U}_R) \quad + \quad i\frac{n_R}{48\pi}(\mathcal{U}_R{}^{-1}D_+\mathcal{U}_R)^3 \quad = 0 \ . \tag{2.16b}$$

To solve these equations we take

$$D_+\mathcal{U}_R \quad = \quad 0 \ . \tag{2.16c}$$

In analogy with the leftons, the chiral super-currents of the rightons here too, are conserved due to eqn. (2.16a).

III. Super Kac-Moody Invariance

It is well known that at the special coupling $(\frac{1}{l^2} - \frac{n_L}{4\pi}) = 0$ the ordinary WZW theory [6], the chiral non-supersymmetric WZW action [2b,3], and the supersymmetric (1,1) non-chiral WZW action[5] have a fixed point where the theory is conformal and Kac-Moody invariant. In a complete analogy we will show that this property holds for our case as well. Using eqns. (2.3) and (2.6) for the variation of the leftons' action, it is straightforward to verify that at the fixed point, the action is invariant under $\mathcal{U} \to \mathcal{U}B^{-1}(\sigma^{++})$. Note however that unlike the non-chiral theory, here the second Kac-Moody invariance $\mathcal{U} \to A(\sigma^{--})\mathcal{U}$ is absent due to the Siegel term eqn. (2.12). A reversed situation appears for the rightons' action where the second super Kac-Moody transformation generates a symmetry while the first does not.

These properties can be revealed easily also from the chiral super-currents. For the leftons at the fixed point, $J^{(L)}_{--(l)} = 0$ and therefore we get the super Kac-Moody current

$$J^{(L)}_{+(l)} = i\frac{n_L}{8\pi}[\Pi_+{}^{(L)} + \Lambda_+{}^{--}\Pi_{--}{}^{(L)}] \tag{3.1}$$

which is anti-holomorphic, namely $\partial_{--}J^{(L)}_{+(l)} = 0$. The components of this super current are

$$\begin{aligned} j^{(L)}_{+(l)} &\equiv J^{(L)}_{+(l)}| = -\frac{n_L}{8\pi}[\beta_+ - i\lambda_+{}^{--}u^{-1}\partial_{--}u] \\ j^{(L)}_{++(l)} &\equiv -iD_+J^{(L)}_{+(l)}| = \frac{n_L}{8\pi}\{[\beta_+\beta_+ + i(u^{-1}\partial_{++}u + \lambda_{++}{}^{--}u^{-1}\partial_{--}u) \\ &\quad - i\lambda_+{}^{--}[(u^{-1}\partial_{--}u)\beta_+ + \partial_{--}\beta_+ - \beta_+(u^{-1}\partial_{--}u)]\} \end{aligned} \tag{3.2}$$

Expressing the super-current as $J^{(L)}_{+(l)} = J^{(L)\hat{\alpha}}_{+(l)}t^{\hat{\alpha}}$ one can check that the algebra of $j^{(L)\hat{\alpha}}_{++(l)}$ is the Kac-Moody algebra with the level equal to n_L. On the other hand, the form of the conservation law of the right current in not changed and hence there is no affine symmetry in this sector

For the righton case at the fixed point, $J^{(R)}_{+(r)} = 0$ and therefore $D_+J^{(R)}_{--(r)} = 0$ namely

$$\begin{aligned} J^{(R)}_{--(r)} &= +i\frac{n_R}{8\pi}[\mathcal{U}_R{}^{-1}\partial_{--}\mathcal{U}_R \\ &\quad + \Lambda_{--}{}^{++}\mathcal{U}_R{}^{-1}\partial_{++}\mathcal{U}_R - i\tfrac{1}{2}D_+\Lambda_{--}{}^{++}\mathcal{U}_R{}^{-1}D_+\mathcal{U}_R] \ , \end{aligned} \tag{3.3}$$

is now a super-Kac-Moody current. The component current $j^{(R)\alpha}_{--(r)}$ is holomorphic and has the Kac-Moody algebra with the level equal to n_R.

IV. Coupling To Non-Abelian Gauge Superfields

Next we want to couple the (1,0) chiral WZW actions at their fixed points to non-abelian gauge fields. We present here two possible couplings: the vector and the chiral couplings [3]. In the first approach we gauge the vector transformation $\mathcal{U} \rightarrow A\mathcal{U}A^{-1}$. This is done by introducing the gauge superfields Γ_+ and Γ_{--}, covariantizing the vector current by replacing ordinary derivatives with covariant derivatives $D_A \rightarrow D_A + i[\Gamma_A, \ \ ]$, and constructing an action whose variation with respect to the gauge superfields yields the covariantized vector current [9]. The result is the following action

$$
\begin{aligned}
S_{LNA(v)} &= S_{WZW}(\mathcal{U}_L) + S_\Lambda(\mathcal{U}_L) + S_{(v)}(\Gamma, \mathcal{U}_L) \ , \\
S_{(v)} &= i\int d^2\sigma d\varsigma^- \ Tr[\Gamma_+ J^{(L)}_{--(v)} + \Gamma_{--} J^{(L)}_{+(v)}] \\
&\quad - i\frac{n_L}{8\pi}\int d^2\sigma d\varsigma^- \ Tr[\Gamma_+ {\mathcal{U}_L}^{-1}\Gamma_{--}\mathcal{U}_L - \Gamma_+\Gamma_{--} \\
&\quad + \Lambda_+{}^{--}(\Gamma_{--}\mathcal{U}_L\Gamma_{--}{\mathcal{U}_L}^{-1} - \Gamma_{--}\Gamma_{--})] \ .
\end{aligned}
\tag{4.1}
$$

to be added to the lefton action (2.2) and (2.5). To get the corresponding action for the rightons, one has to replace (in (4.1)) the indices L with R and add, instead of the last line, the following expression: $\frac{n_R}{8\pi}[\Lambda_{--}{}^{++}(\mathcal{U}_R^{-1}\Gamma_+\mathcal{U}_R - \Gamma_+)\Gamma_{++} + \frac{i}{2}(D_+\Lambda_{--}{}^{++})\mathcal{U}_R^{-1}\Gamma_+\mathcal{U}_R\Gamma_+ - i\Lambda_{--}{}^{++}\mathcal{U}_R^{-1}D_+\mathcal{U}_R\Gamma_+\Gamma_+]$. The S_Λ action for the rightons is given in eqn. (2.12).

It follows from the equations of motion derived from these actions, that the vector super current is still conserved. However, the axial current gets an anomalous divergence

$$
D_+ J^{\binom{L}{R}}_{--(a)} + \partial_{--} J^{\binom{L}{R}}_{+(a)} = +n_{\binom{L}{R}}\frac{1}{4\pi}W_- \ , \tag{4.2}
$$

where $W_- = D_+\Gamma_{--} - \partial_{--}\Gamma_+ + i[\Gamma_+, \Gamma_{--}]$. This result is just the supersymmetric generalization of the bosonized version of the non-abelian anomaly of chiral fermions [3].

In the chiral coupling, the left and right super-currents are coupled to non-abelian gauge superfields for the lefton and righton cases respectively. The corresponding actions can be derived by supersymmetrization of the action given in Ref. [3]. The resulting interaction terms are:

$$\begin{aligned} S_{(l)} &= i\int d^2\sigma d\varsigma^- \ Tr[\Gamma_+ J^{(L)}_{--(l)} \ + \ \Gamma_{--} J^{(L)}_{+(l)}] \\ &\quad + i\frac{n_L}{16\pi}\int d^2\sigma d\varsigma^- \ Tr[\Gamma_+\Gamma_{--} \ + \ \Lambda_+{}^{--}\Gamma_{--}\Gamma_{--}] \end{aligned} \tag{4.3}$$

for the leftons. Replacing the L indices with R, l with r and changing the last term $+\frac{n_R}{16\pi}\int d^2\sigma d\varsigma^- \ Tr[\Gamma_+\Gamma_{--} \ + \ \Lambda_{--}{}^{++}(\Gamma_+\Gamma_{++} + \Gamma_+^3)]$, leads to the interaction term in the righton's model. Similar to eqn. (4.2) the divergence of the chiral super currents are anomalous. The expresion one gets is just the supersymmetric gengeralization of the results given in [3].

V. Coupling To Supergravity

Just as the D=2 superconformal theory is anomalous, the super-Siegel symmetry is also potentially anomalous [2,3,4]. For the $U(1)$ theory the anomaly is removed by one of two methods. The first requires the stringy prescription of a multiplet of chiral bosons. The second requires the addition of a Liouville term to the action. The removal of the anomaly fixes the "coupling" constant of the latter term just as it singles out a value for the number of leftons or rightons.

It is known [2b] that for the non-supersymmetric theory, the Polyakov procedure of adding a Liouville, tree-level, anomalous counter-term to the sigma model action, is not well defined. Although it is possible to supersymmetrize this Liouville term, it suffers from the same pathology. It is the trace of a Lie-algebra valued construct and thus vanishes. So it appears that the $O(N)$ model must quantum mechanically conserve Siegel symmetry by some other means. However, two points concerning the Liouville term are worth mentioning. Firstly, if the group element contains at least one $U(1)$ factor [3], then the trace no longer vanishes. Of course this is defeated if the group, G, is taken to be semi-simple. Secondly, if one allows the lagrange multiplier to be Lie-algebra valued, then it must be included in the trace which is then non-trivial. Such a matrix superfield would be the $O(N)$ generalization of the family of $U(1)$ lagrange multipliers given in Ref. [4a].

Coupling self-dual superfields to supergravity requires more than the naive procedure of covariantizing the action leading to the classical conservation of two symmetries, supergravity and Siegel. One is not guaranteed consistent removal of the pure and mixed anomalies. Less covariantizing must be done. Indeed, the super-Siegel transformation laws are those of a truncated supergravity theory coupled to matter [1]. Sensible coupling of supergravity to leftons and rightons is achieved by treating the supergravity schizophrenically, when realized on these superfields [4c].

The schizophrenic general-supercoordinate transformation laws (with super-parameter $K^{\pm\pm}$) for the lefton and righton group elements are as given in Eqns. (2.7) and (2.14) but with $\Upsilon^{\pm\pm} = -K^{\pm\pm}$ and $R \leftrightarrow L$ [4c]. We minimally covariantize with respect to these transformations. The realization of the (1,0) supergravity algebra on the $\mathcal{U}_L$ and $\mathcal{U}_R$ is exactly as given in Ref. [4c] for the abelian leftons and rightons. The locally supersymmetric and super-dilatation invariant action is then the sum $S(\mathcal{U}_L, \mathcal{U}_R) = S_\sigma(\mathcal{U}_L, \mathcal{U}_R) + S_{WZ}(\mathcal{U}_L, \mathcal{U}_R)$ where $S_\sigma(\mathcal{U}_L, \mathcal{U}_R) = S_\sigma(\mathcal{U}_L) + S_\sigma(\mathcal{U}_R)$ and $S_{WZ}(\mathcal{U}_L, \mathcal{U}_R) = n_L S_{WZ}(\mathcal{U}_L) + n_R S_{WZ}(\mathcal{U}_R)$ with the density E^{-1} inserted into the measure and $D_A \to \nabla_A$. (Our notation is explained in Ref. [4c].) In our discussion we will assume that the group $G = G_S \times G_A$, where G_A is a product of some number, $N_A = N_{AL} + N_{AR}$, of abelian factors from the lefton and righton sectors. With this we take G_S to be simple and compact, as in Ref. [2b]. Furthermore, we include D space-time coordinates. Then our locally supersymmetric and super-dilatation invariant action is

$$
\begin{aligned}
S_{CLR} \;=\; & -i\tfrac{1}{4l^2}\int d^2\sigma d\varsigma^- E^{-1}[\nabla_+\hat{\Phi}^{\hat{A}}\nabla_{--}\hat{\Phi}^{\hat{B}}]\eta_{\hat{A}\hat{B}} \\
& -i\tfrac{1}{4l^2}\int d^2\sigma d\varsigma^- E^{-1}\ Tr[\aleph_+{}^{(L)}\aleph_{--}{}^{(L)} \;+\; \aleph_+{}^{(R)}\aleph_{--}{}^{(R)}] \\
& -i\tfrac{1}{16\pi}\int d^2\sigma d\varsigma^- E^{-1}\int_0^1 dy\ Tr[n_L\tilde{\Pi}_y{}^{(L)}(\nabla_+\tilde{\aleph}_{--}{}^{(L)} \;-\; \nabla_{--}\tilde{\aleph}_+{}^{(L)}) \\
& \qquad + n_R\tilde{\Pi}_y{}^{(R)}(\nabla_+\tilde{\aleph}_{--}{}^{(R)} \;-\; \nabla_{--}\tilde{\aleph}_+{}^{(R)})]\ ,
\end{aligned} \tag{5.1}
$$

where now $\aleph_A \equiv \mathcal{U}^{-1}\nabla_A\mathcal{U}$ and the internal $\hat{\Phi}$'s are $U(1)$ valued. A discussion of the first action (for the latter superfields) was given in Ref. [4c]. The super-dilatation (with S as its local super-parameter) transformations under which the simple action is invariant, are

$$
\delta_S\aleph_+ \;=\; \tfrac{1}{2}S\aleph_+\ , \qquad \delta_S\aleph_{\pm\pm} \;=\; S\aleph_{\pm\pm}\ , \qquad \delta_S\Pi_y \;=\; 0\ , \qquad \delta_S E \;=\; \tfrac{3}{2}SE\ , \tag{5.2}
$$

with similar transformations for the $\tilde{\aleph}_A$. By construction, the abelian and space-time coordinate actions are super-Weyl invariant. For the superstring, $l^2 = -\pi\alpha'$.

Eqn. (5.1) reduces to

$$
\begin{aligned}
S_{CLR} \;&=\; S_C \;+\; S_{AL} \;+\; S_{AR} \;+\; S_{SL} \;+\; S_{SR}\ , \\
S_C \;&=\; -i\tfrac{1}{4l^2}\int d^2\sigma d\varsigma^- E^{-1}[\nabla_+X^{\underline{a}}\nabla_{--}X_{\underline{a}}]\ , \\
S_{AL} \;&=\; -i\tfrac{1}{4l^2}\int d^2\sigma d\varsigma^-[D_+\hat{\Phi}_L\cdot\partial_{--}\hat{\Phi}_L \;+\; H_{--}{}^{++}D_+\hat{\Phi}_L\cdot\partial_{++}\hat{\Phi}_L]\ , \\
S_{AR} \;&=\; -i\tfrac{1}{4l^2}\int d^2\sigma d\varsigma^-[D_+\hat{\Phi}_R\cdot\partial_{--}\hat{\Phi}_R \;+\; H_+{}^{--}\partial_{--}\hat{\Phi}_R\cdot\partial_{--}\hat{\Phi}_R]\ , \\
S_{SL} \;&=\; -i\tfrac{1}{4l^2}\int d^2\sigma d\varsigma^-\ Tr[\Pi_+{}^{(L)}\Pi_{--}{}^{(L)} \;+\; H_{--}{}^{++}\Pi_+{}^{(L)}\Pi_{++}{}^{(L)}] \\
&\quad -i\tfrac{n_L}{16\pi}\int d^2\sigma d\varsigma^-\int_0^1 dy\ Tr[\tilde{\Pi}_y{}^{(L)}(\tilde{\Pi}_{--}{}^{(L)}\tilde{\Pi}_+{}^{(L)} \;-\; \tilde{\Pi}_+{}^{(L)}\tilde{\Pi}_{--}{}^{(L)})] \\
&\quad + \tfrac{n_L}{48\pi}\int d^2\sigma d\varsigma^- H_{--}{}^{++}Tr[\Pi_+{}^{(L)}\Pi_+{}^{(L)}\Pi_+{}^{(L)}]\ ,
\end{aligned} \tag{5.3}
$$

$$S_{SR} = -i\tfrac{1}{4l^2}\int d^2\sigma d\varsigma^- \;\; Tr[\Pi_+{}^{(R)}\Pi_{--}{}^{(R)} \;+\; H_+{}^{--}\Pi_{--}{}^{(R)}\Pi_{--}{}^{(R)}]$$
$$-i\tfrac{n_R}{16\pi}\int d^2\sigma d\varsigma^- \int_0^1 dy \;\; Tr[\tilde{\Pi}_y{}^{(R)}(\tilde{\Pi}_{--}{}^{(R)}\tilde{\Pi}_+{}^{(R)} - \tilde{\Pi}_+{}^{(R)}\tilde{\Pi}_{--}{}^{(R)})] \;\; .$$

This is an exact (to full non-linear order) result. As explained in our earlier work [4c], there is a reversal, $R \leftrightarrow L$, in the coupling to the gauge superfields. Supergravity only couples to the "physical movers" in the superfields. Now, the $\mathcal{U}_R$'s are coupled to $H_+{}^{--}$ exactly as in Eqns. (2.1) and (2.5).

We now inquire as what other conditions are required for consistent quantum superconformal coupling at the fixed point, *i.e.* we look for anomalies. The left and right energy-momentum tensor elements may be read off from Eqn. (5.3). They are

$$\mathbf{T}_{+,++}{}^{(L)} = D_+X\cdot\partial_{++}X \;+\; D_+\hat{\Phi}_L\cdot\partial_{++}\hat{\Phi}_L \;+\; Tr[\Pi_+{}^{(L)}\Pi_{++}{}^{(L)}]$$
$$+\,i\tfrac{1}{3}Tr[\Pi_+{}^{(L)}\Pi_+{}^{(L)}\Pi_+{}^{(L)}] \;\; ,$$
$$\mathbf{T}_{--,--}{}^{(R)} = \partial_{--}X\cdot\partial_{--}X \;+\; \partial_{--}\hat{\Phi}_R\cdot\partial_{--}\hat{\Phi}_R \;+\; Tr[\Pi_{--}{}^{(R)}\Pi_{--}{}^{(R)}] \;\; , \qquad (5.4)$$

for the fixed point $n_R = n_L = n = \frac{4\pi}{l^2}$. As is well known [7], the traced terms may be written in terms of the super-Kac-Moody currents. Furthermore, the resulting expression must be suitably normalized in order to obtain the Virasoro algebra or correct OPE's. The normalization factor is $\frac{1}{\kappa}$, where $\kappa = c_V + n$. The constant, c_V, is the second Casimir of the adjoint representation defined by $f^{acd}f^{bcd} \equiv c_V\delta^{ab}$. With $d_G \equiv \dim\; G$, generically and $G \equiv G_{SL}\times G_{SR}$ denoting the factoring of the group G into simple groups, the critical dimension formulae are [7,8]

$$D \;+\; N_{AL} \;+\; \frac{2}{3}\frac{d_{G_{SL}}}{1+\frac{c_{VL}}{n}} \;+\; \frac{1}{3}d_{G_{SL}} \;=\; 10 \;\; ,$$
$$D \;+\; N_{AR} \;+\; \frac{d_{G_{SR}}}{1+\frac{c_{VR}}{n}} \;=\; 26 \;\; . \qquad (5.5)$$

The first equation is from the lefton (supersymmetric) sector and the second is from the righton (non-supersymmetric) sector. For semi-simple groups, the d_G

terms become sums over each factor in the group. Solutions of eqn. (5.6) for $N_{AL} = N_{AR} = 0$ have been given in Ref. [8].

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