# BARYON MASSES IN THE BOUND STATE APPROACH TO STRANGENESS IN THE SKYRME MODEL* 

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#### Abstract

We diagonalize exactly the $O\left(N_{c}^{0}\right)$ Hamiltonian relevant to the bound state approach to strangeness in the Skyrme model. The hyperfine splittings of strange baryons computed within this framework agree well with the experimental values.


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[^0]The Skyrme model works surprisingly well in describing non-strange baryons. Various properties of nucleons and deltas have been calculated in the $\operatorname{SU}(2)$ collective coordinate quantization of the spherically symmetric soliton [1]. They agree with the observed values to within 30 percent accuracy. Moreover, certain model independent relations, based only on rotator quantization, hold to within 3 percent [1]. An application of analogous ideas to strange baryons involves $S U(3)$ collective coordinates [2]. The effects of quark masses are then included in perturbation theory. Unfortunately, this procedure works poorly in predicting the mass splittings in the octet and decuplet of baryons [3]. In ref. [4] it was suggested that the problem is due to the relatively large mass of the strange quark. An approximation scheme was developed where the strange quark was treated as heavy rather than light, while the interaction Lagrangian was taken to be $\mathrm{SU}(3) \times \mathrm{SU}(3)$ symmetric. In this scheme the physically relevant configurations are small deformations of the $\mathrm{SU}(2)$ skyrmion into the strange directions. After expanding to second order in the strange fields, the problem reduces to the motion of kaons in the background of the hedgehog built out of non-linear excitations of the pion fields. Bound states of kaons and skyrmions are identified with the strange baryons; their quantum numbers are the same as those computed in the quark model. In ref. [4] it was stated that the numerical predictions obtained within this framework work well to $O\left(N_{c}^{0}\right)$ but fail at $O\left(1 / N_{c}\right)$ in the large $N_{c}$ expansion. In this paper we present an improved diagonalization of the Hamiltonian relevant to these bound states. We find that, after this improvement, the numerical predictions work well both to $O\left(N_{c}^{0}\right)$ and to $O\left(1 / N_{c}\right)$. It is important to stress that the physical picture formulated in ref. [4] does not need revision.

The basic defficiency of the calculations in ref. [4] involved the treatment of the Wess-Zumino term. When expanded around an SU(2) background, this term, which is in general topological, becomes an ordinary Lagrangian term which couples the baryon number current to the strangeness current. In the hedgehog background it performs the crucial function of splitting the states of positive and negative strangeness. Inclusion of the $\mathrm{W}-\mathrm{Z}$ term, which is linear in time
derivatives, makes diagonalization of the Hamiltonian somewhat unconventional. In ref. [4] the W-Z term was treated as a perturbation. Even for a small coefficient in front of the W-Z term it was not entirely clear how reliable the perturbative approach was. Also, as pointed out in [5,6], a factor error of $5 / 2$ was made in [4] in the expansion of the Wess-Zumino term in kaon fluctuations. In this paper we show that an exact treatment of the bound state approach is possible to $O\left(N_{c}^{0}\right)$. To this order, the model is described by a Lagranigan which is quadratic in fields and at most quadratic in time derivatives and is, therefore, exactly soluble. The eigenvalues of the complete $O\left(N_{c}^{0}\right)$ Hamiltonian are easy to find numerically. Perhaps the most important consequence of this improved treatment is the effect it makes on the calculation of the $O\left(1 / N_{c}\right)$ corrections to the masses of hyperons. We now find the hyperfine splittings not only to have the correct sign, but also to be very close to the experimentally observed values.

Let us review the basic formalism of the bound state approach to strangeness. We start with the Skyrme model Lagrangian [7]

$$
\begin{equation*}
\mathcal{L}_{S k}=\frac{F_{\pi}^{2}}{16} \operatorname{tr}\left(\partial_{\mu} U^{\dagger} \partial^{\mu} U\right)+\frac{1}{32 e^{2}} \operatorname{tr}\left[\partial_{\mu} U U^{\dagger}, \partial_{\nu} U U^{\dagger}\right]^{2}+\operatorname{tr} M\left(U+U^{\dagger}-2\right) \tag{1}
\end{equation*}
$$

where $U(\vec{x}, t)$ takes values in $\mathrm{SU}(3) . M$ is the diagonal matrix whose entries are proportional to the $u, d$ and $s$ quark masses. For simplicity, we set $m_{u}=m_{d}=0$ but consider the strange quark mass to be large. The relevant configurations of unit baryon number are then the small fluctuations into strange directions about the pionic soliton solution. To identify the strange fields correctly, we make the ansatz [4]

$$
\begin{equation*}
U=\sqrt{U}_{\pi} U_{K} \sqrt{U}_{\pi} \tag{2}
\end{equation*}
$$

where $U_{\pi}=\exp \left(i \frac{2}{F_{\pi}} \lambda_{j} \pi^{j}\right)$ and $U_{K}=\exp \left(i \frac{2}{F_{\pi}} \lambda_{a} K^{a}\right)$ with $j$ running from 1 to 3 and $a$ running from 4 to 7. $\lambda$ are the generators of $\mathrm{SU}(3)$ normalized to
$\operatorname{Tr}\left(\lambda_{k} \lambda_{l}\right)=2 \delta_{k l}$. After expanding (1) to second order in $K$ we get

$$
\begin{equation*}
\mathcal{L}=\mathcal{L}_{S k}\left(U_{\pi}\right)+\left(D_{\mu} K\right)^{\dagger} D^{\mu} K-m_{K}^{2} K^{\dagger} K+\ldots \tag{3}
\end{equation*}
$$

where the ellipsis represents a lengthy expression [4] depending on

$$
\begin{equation*}
A_{\mu}=\frac{1}{2}\left(\sqrt{U}_{\pi}^{\dagger} \partial_{\mu} \sqrt{U}_{\pi}-\sqrt{U}_{\pi} \partial_{\mu} \sqrt{U}_{\pi}^{\dagger}\right) \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
D_{\mu} K=\partial_{\mu} K+\frac{1}{2}\left(\sqrt{U}_{\pi}^{\dagger} \partial_{\mu} \sqrt{U}_{\pi}+\sqrt{U}_{\pi} \partial_{\mu} \sqrt{U}_{\pi}^{\dagger}\right) K \tag{5}
\end{equation*}
$$

$K$ is the standard complex isodoublet:

$$
\begin{equation*}
K=\frac{1}{\sqrt{2}}\binom{K_{4}-i K_{5}}{K_{6}-i K_{7}}=\binom{K^{+}}{K^{0}} \tag{6}
\end{equation*}
$$

The Lagrangian (3) has $\mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry, with the axial subgroup realized non-linearly [8].

Witten has emphasized that the model is not complete without the WessZumino term. He discovered that, in the rotator quantization, this term requires the soliton to be a fermion for an odd $N_{c}$ and a boson for an even $N_{c}$ [9]. In general, it can only be written as an action term

$$
\begin{equation*}
S_{W Z}=-\frac{i N_{c}}{240 \pi^{2}} \int d^{5} x \epsilon^{\mu \nu \alpha \beta \gamma} \operatorname{tr}\left(U^{\dagger} \partial_{\mu} U U^{\dagger} \partial_{\nu} U U^{\dagger} \partial_{\alpha} U U^{\dagger} \partial_{\beta} U U^{\dagger} \partial_{\gamma} U\right) \tag{7}
\end{equation*}
$$

Expanding this to second order in $K$ converts it into an ordinary Lagrangian term

$$
\begin{equation*}
\mathcal{L}_{W Z}=\frac{i N_{c}}{F_{\pi}^{2}} B^{\mu}\left(K^{\dagger} D_{\mu} K-\left(D_{\mu} K\right)^{\dagger} K\right) \tag{8}
\end{equation*}
$$

where $B^{\mu}$ is the baryon number current of the $\operatorname{SU}(2)$ soliton configuration. Terms of this kind describe the interaction of a charged field with a vector potential.

In our case, the charge is strangeness, and the role of the vector potential is played by the baryon current. The analogy is incomplete because the bound state Lagrangian has no term $\sim B_{\mu} B^{\mu} K^{\dagger} K$. Clearly, the interaction (8) distinguishes between positive and negative strangeness and therefore plays a crucial role in our approach.

We have reduced the problem to the motion of kaons in the classical background of the $\mathrm{SU}(2)$ soliton. The Lagrangian is quadratic in the quantized kaon fields. In ref. [4] it was shown that kaons and skyrmions can form bound states. Our goal is to calculate their masses in the large $N_{c}$ expansion. In the semiclassical approximation solitons rotate slowly, with velocities of order $1 / N_{c}$. Therefore, to find the kaon energy levels to $O\left(N_{c}^{0}\right)$ it is sufficient to treat $U_{\pi}$ as a static background hedgehog

$$
\begin{equation*}
U_{0}(\vec{r})=\exp (i \vec{\tau} \cdot \hat{r} F(r)) \tag{9}
\end{equation*}
$$

where $F(r)$ is the profile function obtained in ref. [1]. Since the background is symmetric under combined spatial and isospin rotations $\vec{T}=\vec{I}+\vec{L}$, we can write the kaon eigenmodes as

$$
\begin{equation*}
K(\vec{r}, t)=k(r, t) Y_{T L T_{x}} \tag{10}
\end{equation*}
$$

After substituting this into $\mathcal{L}_{S k}+\mathcal{L}_{W Z}$ we find the following effective Lagrangian for the radial field $k(r, t)$ (It is convenient to transform to dimensionless variables with the mass scale set by $e F_{\pi}$.):
$L=4 \pi \int d r r^{2}\left(f(r) \dot{k}^{\dagger} \dot{k}+i \lambda(r)\left(k^{\dagger} \dot{k}-\dot{k}^{\dagger} k\right)-h(r) \frac{d}{d r} k^{\dagger} \frac{d}{d r} k-k^{\dagger} k\left(m_{K}^{2}+V_{e f f}(r ; T, L)\right)\right.$
where $f(r), h(r)$, and $V_{e f f}(r ; T, L)$ are given in terms of $F(r)$ in the eqn. (3.1) of ref. [4] and

$$
\begin{equation*}
\lambda(r)=-\frac{N_{c} e^{2}}{2 \pi^{2} r^{2}} \sin ^{2} F F^{\prime} \tag{12}
\end{equation*}
$$

$\bar{T}$ he term linear in time derivatives originates in the W-Z term. The Lagrangian (11) is analogous to the interaction of a relativistic charged field with a back-
ground static, radial electric freld. We find that the negative strangeness particles are attracted to the origin while the positive are repelled. In ref. [4] it was shown that, in the absence of the W-Z term, the lowest bound state has quantum numbers $T=1 / 2, L=1$. As we dial the coefficient of the W-Z term up, the $s=-1$ state becomes more tightly bound to the soliton, while the $s=1$ state gets pushed out eventually into the continuum. This is the essential role of the W-Z term in our treatment of strangeness.

The Lagrangian (11) is quadratic in fields and at most quadratic in time derivatives. In general, physical systems described by such Lagrangians can be treated exactly. The observation crucial for our purposes is that the quantum energy levels of the system are given by the classical eigenfrequencies. We demonstrate how this works in our specific example. The variational equation resulting from (11) is

$$
\begin{equation*}
-f(r) \frac{d^{2}}{d t^{2}} k+2 i \lambda(r) \frac{d}{d t} k+O k=0 \tag{13}
\end{equation*}
$$

where

$$
\begin{equation*}
O=\frac{1}{r^{2}} \frac{d}{d r} h(r) r^{2} \frac{d}{d r}-m_{K}^{2}-V_{e f f}(r ; T, L) \tag{14}
\end{equation*}
$$

is a hermitian operator. Let us expand the field $k$ in terms of its eigenmodes:

$$
\begin{equation*}
k(r, t)=\sum_{n>0}\left(\tilde{k}_{n}(r) e^{i \tilde{\omega}_{n} t} b_{n}^{\dagger}+k_{n}(r) e^{-i \omega_{n} t} a_{n}\right) \tag{15}
\end{equation*}
$$

where $\omega_{n}$ and $\tilde{\omega}_{n}$ are assumed to be positive. By substituting (15) into (13) we find that the eigenmodes satisfy

$$
\begin{equation*}
\left(f(r) \omega_{n}^{2}+2 \lambda(r) \omega_{n}+O\right) k_{n}=0 \tag{16}
\end{equation*}
$$

and

$$
\begin{equation*}
\left(f(r) \tilde{\omega}_{n}^{2}-2 \lambda(r) \tilde{\omega}_{n}+O\right) \tilde{k}_{n}=0 \tag{17}
\end{equation*}
$$

Using the hermiticity of $O$ we derive the following orthogonality relations:

$$
\begin{align*}
& 4 \pi \int d r r^{2} k_{n}^{\star} k_{m}\left(f(r)\left(\omega_{n}+\omega_{m}\right)+2 \lambda(r)\right)=\delta_{n m}  \tag{18}\\
& 4 \pi \int d r r^{2} \tilde{k}_{n}^{\star} \tilde{k}_{m}\left(f(r)\left(\tilde{\omega}_{n}+\tilde{\omega}_{m}\right)-2 \lambda(r)\right)=\delta_{n m}  \tag{19}\\
& 4 \pi \int d r r^{2} k_{n}^{\star} \tilde{k}_{m}\left(f(r)\left(\omega_{n}-\tilde{\omega}_{m}\right)+2 \lambda(r)\right)=0 \tag{20}
\end{align*}
$$

Upon carrying out canonical quantization we find that the momentum conjugate to $k$ is

$$
\begin{equation*}
\pi(r, t)=f(r) \dot{k}^{\dagger}+i \lambda(r) k^{\dagger} \tag{21}
\end{equation*}
$$

Canonical commutation relations between the fields and their conjugate momenta and eqns. (18)-(20) imply that the oscillators have the usual algebra

$$
\begin{equation*}
\left[a_{n}, a_{m}^{\dagger}\right]=\delta_{n m}, \quad\left[b_{n}, b_{m}^{\dagger}\right]=\delta_{n m} \tag{22}
\end{equation*}
$$

with the rest of the commutators vanishing. In terms of the creation and annihilation operators the Hamiltonian reduces to

$$
\begin{equation*}
H=\sum_{n>0}\left(\omega_{n} a_{n}^{\dagger} a_{n}+\tilde{\omega}_{n} b_{n}^{\dagger} b_{n}\right) \tag{23}
\end{equation*}
$$

This proves that the quantum energy levels are given by the classical eigenfrequencies. The strangeness charge is

$$
\begin{equation*}
S=\sum_{n>0}\left(b_{n}^{\dagger} b_{n}-a_{n}^{\dagger} a_{n}\right) \tag{24}
\end{equation*}
$$

It follows that $a_{n}$ and $b_{n}$ annihilate the modes of strangeness -1 and 1 respectively. The bound state energy in the $s=-1$ sector can be found by solving for the lowest eigenfrequency of eqn. (16). This can easily be done numerically.

We find that in the lowest partial wave ( $T=1 / 2, L=1$ ) there is exactly one bound state with $s=-1$. We have determined its energy to be $0.23 e F_{\pi}$. This is the bound state on the basis of which we construct the $\Lambda, \Sigma$ and $\Sigma^{\star}$ baryons. We find no bound states corresponding to the exotic baryons with $s=1$. The only place where these exotics may manifest themselves is the continuous part of the spectrum [10]. This interesting application of our approach to strangeness has not yet been worked out. A notable feature of our approach is the presence of a bound state in the lowest negative parity partial wave ( $s=-1, T=1 / 2, L=0$ )[4]. This state probably corresponds to the observed $\Lambda(1405)$ which is indeed below the KN threshold. To $O\left(N_{c}^{0}\right)$ we find the energy for this state to be $0.50 e F_{\pi}$.

In collective coordinate quantization strange baryons acquire definite spin and isospin quantum numbers through a slow rotation of the soliton together with the bound meson:

$$
\begin{equation*}
U_{0}(\vec{r}) \rightarrow A(t) U_{0}(\vec{r}) A^{-1}(t), \quad K(\vec{r}, t) \rightarrow A(t) K(\vec{r}, t) \tag{25}
\end{equation*}
$$

In ref. [4] it was shown that a meson in a bound state orbital behaves effectively as an object of isospin zero and spin equal to the quantum number $T$ of the orbital. Therefore, a meson bound in the $T=1 / 2$ channel acquires the quantum numbers of a strange quark. The $S U(2)$ rotator can be quantized either integrally or halfintegrally. An important constraint is that, for a fixed $N_{c}$, every time we increase the occupation number of the bound state orbital, the quantization rule for the rotator must change to insure that the composite system remains fermionic (for $N_{c}$ odd) or bosonic (for $N_{c}$ even). Although from the point of view of the $\mathrm{SU}(2)$ rotator this quantization rule seems arbitrary, it follows from the existence of strange directions and the presence of the $\mathrm{W}-\mathrm{Z}$ term in $\mathrm{SU}(3)$. In particular, this rule guarantees that the quantum numbers obtained in the bound state approach for a heavy strange quark match the quantum numbers obtained in the $\mathrm{SU}(3)$ collective coordinate approach for a light strange quark [11].

We may now determine the $O\left(1 / N_{c}\right)$ corrections to the spectrum. Using the quark model language, they are the hyperfine splittings between the masses of
$\Lambda, \Sigma$ and $\Sigma^{\star}$. The relevant terms in the Lagrangian (3) depend on the rotator velocities $A^{-1} \dot{A}=i \dot{\alpha}_{a} \tau_{a}$ which are of order $1 / N_{c}$ :

$$
\begin{equation*}
\delta L=2 \Omega\left(\dot{\alpha}_{a}\right)^{2}+\dot{\alpha}_{a} \int d^{3} x \chi_{a} \tag{26}
\end{equation*}
$$

where

$$
\begin{gather*}
\chi_{a}=i f \dot{K}^{\dagger} M_{a} K+\lambda K^{\dagger} M_{a} K-2 i \lambda \epsilon_{i a c} \vec{r}_{c} K^{\dagger} D_{i} K+ \\
2 i \dot{K}^{\dagger} D_{i} K t r\left(A_{i} P_{a}\right)-6 i \dot{K}^{\dagger}\left[P_{a}, A_{i}\right] D_{i} K+c . c,  \tag{27}\\
M_{a}=\frac{1}{2}\left(\sqrt[U]{0}_{0}^{\dagger} \tau_{a} \sqrt{U}_{0}+\sqrt{U}_{0} \tau_{a} \sqrt{U_{0}^{\dagger}}\right),  \tag{28}\\
P_{a}=\frac{1}{2}\left(\sqrt{U}_{0}^{\dagger} \tau_{a} \sqrt{U}_{0}-\sqrt{U}_{0} \tau_{a} \sqrt{U}_{0}^{\dagger}\right) \tag{29}
\end{gather*}
$$

and $A_{i}$ is defined in (4). The resulting Hamiltonian is [4]

$$
\begin{equation*}
\frac{1}{2 \Omega}\left(\vec{J}_{r}^{2}-\vec{J}_{r} \cdot \int d^{3} x \vec{\chi}+\frac{1}{4}\left(\int d^{3} x \vec{\chi}\right)^{2}\right) \tag{30}
\end{equation*}
$$

where $\vec{J}_{r}$ is the angular momentum of the rotator. We are going to evaluate the mass splittings generated by (30) to first order in perturbation theory. Our calculations show that

$$
\begin{equation*}
\int d^{3} x \chi_{a}=-c a_{1 \mu}^{\dagger} \tau_{a}^{\mu \nu} a_{1 \nu}+\ldots \tag{31}
\end{equation*}
$$

where we have explicitly shown the crucial term which determines the bound state splittings. $a_{1 \mu}$ are the annihilation operators for the meson in the lowest orbital ( $T=1 / 2, L=1$ ), with the Greek index labeling the two possible values of $T_{z}$, the effective angular momentum. After a rather involved calculation we find that

$$
\begin{equation*}
c=1-\frac{\int d r k^{\star} k \omega\left(\frac{4}{3} f r^{2} \cos ^{2} \frac{F}{2}-2\left(\frac{d}{d r}\left(r^{2} \frac{d F}{d r} \sin F\right)-\frac{4}{3} \sin ^{2} F \cos ^{2} \frac{F}{2}\right)\right)}{\int d r r^{2} k^{\star} k(f \omega+\lambda)} \tag{32}
\end{equation*}
$$

where $\omega$ and $k(r)$ are the bound state eigenvalue and eigenfunction. This formula differs from the corresponding formula in ref. [4] because we have now treated the bound state dynamics exactly to $O\left(N_{c}^{0}\right)$.

The last term in (30) is quartic in creation and annihilation operators. Therefore, its bound state matrix element suffers from operator ordering ambiguities. However, in some sense this term is the least important because its contribution does not depend on the spin and isospin quantum numbers. It provides the same shift for all baryon states of strangeness -1 . Using the fact that $\frac{1}{2} a^{\dagger} \vec{\tau} a$ is the effective angular momentum of the bound meson we derive our mass formula for baryons with $s=-1$ :

$$
\begin{equation*}
M=M_{c l}+\omega+\frac{1}{2 \Omega}(c J(J+1)+(1-c) I(I+1))+\delta \tag{33}
\end{equation*}
$$

where $\delta$ is some common shift of order $1 / N_{c}$ which depends on the operator ordering prescription. Numerically, we find $c=.60$. We can determine the experimental value for $c$ by noting that

$$
\begin{equation*}
M_{\Sigma}-M_{\Lambda}=\frac{1-c}{\Omega} \tag{34}
\end{equation*}
$$

and

$$
\begin{equation*}
M_{\Sigma^{\star}}-M_{\Sigma}=\frac{3 c}{2 \Omega} \tag{35}
\end{equation*}
$$

After taking the ratio of the above equations we find $c_{\exp }=.62$, in excellent agreement with our calculation.

It is important to note that the quark model predicts a similar structure for the hyperfine splittings based on the Hamiltonian term

$$
\begin{equation*}
\delta H=\frac{1}{\Omega} \sum_{k<l} c_{k l} \vec{j}_{k} \cdot \vec{j}_{l} \tag{36}
\end{equation*}
$$

where $\vec{j}_{k}$ is the angular momentum of the k -th quark. Hyperon splittings identical to eqns. (34) and (35) are obtained in the quark model if $c_{k l}=1$ when $k$ and $l$ refer to light quarks and $c_{k l}=c$ when either $k$ or $l$ refers to a heavy quark. It is instructive to make a comparison of the values of $c$ in the bound state approach
to those in the quark model as we vary the heavy meson mass. In the $\mathrm{SU}(3)$ limit $m_{K} \rightarrow 0$ and the bound state becomes a zero mode ( $\omega=0$ ). In this limit eqn. (32) predicts $c=1$. The quark model makes the same prediction since all the quarks are massless and interact with equal strengths. As we dial the mass of the heavy quark up, the variation of $c$ can be estimated in the quark model by a one gluon exchange calculation [12]. One finds that $c$ is a monotonically decreasing function of the heavy quark mass which falls off to zero in the infinite mass limit. Our calculations with eqn. (32) indicate that, as we increase $m_{K}$, the same qualitative behaviour occurs in the bound state approach to heavy flavors. In fact, $c$ remains positive for $m_{K}$ as large as the charmed meson mass, which is well outside the region of validity for our approximations. To summarize, the behavior of hyperfine splittings we obtain is in excellent agreement with the quark model predictions for a wide range of the heavy quark masses.

There is another interesting comparison to be made. We would like to compare the bound state approach with the rigid rotator approach of Yabu and Ando [13]. These authors consider rigid rotation of the hedgehog in the presence of a kaon mass term. Although this approach ignores soliton deformation as it fluctuates into strange directions, it can give us some qualitative idea of the mass splittings. In the heavy kaon limit Yabu and Ando find a mass formula analogous to ours with the splitting coefficient $c$ given by

$$
\begin{equation*}
c_{Y A}=1-\frac{\Omega}{2 \Phi}+O(1 / \omega) \tag{37}
\end{equation*}
$$

where

$$
\begin{equation*}
\Omega=\frac{2 \pi}{3} \int_{0}^{\infty} d r \sin ^{2} F\left(r^{2}+4 \sin ^{2} F+4 r^{2}\left(F^{\prime}\right)^{2}\right) \tag{38}
\end{equation*}
$$

and
$-\quad \Phi=\pi \int_{0}^{\infty} d r \sin ^{2} \frac{F}{2}\left(r^{2}+2 \sin ^{2} F+r^{2}\left(F^{\prime}\right)^{2}\right)$
are the moments of inertia from $\mathrm{SU}(3)$ collective coordinate quantization and $\omega$
is the lowest eigenfrequency. We can derive a similar result in our approach. The crucial step is to show that a small rigid rotation of the hedgehog in a strange direction generates a kaon configuration in the lowest partial wave with the radial function given by $k(r) \sim \sin \left(\frac{F}{2}\right)$. Substituting this radial function into (32) we find that in the rigid rotator approximation

$$
\begin{equation*}
c=1-\frac{\Omega}{2 \Phi+N_{c} e^{2} / 4 \omega} . \tag{40}
\end{equation*}
$$

Therefore, to the order that Yabu and Ando retained, their formula agrees with (40). However, the extra term in the denominator of (40) arises naturally from the Wess-Zumino term and should not be discarded. It is of the same order in the $1 / N_{c}$ expansion as the rest of the terms in (40) and is numerically significant. If it were ignored, $c$ would have been found negative, in drastic disagreement with the quark model and experiment. The above discussion demonstrates quite clearly the importance of the W-Z term in our approach. Further details on the relationship between the bound state and rigid rotator approaches are in preparation [14].

To compare our calculations with the baryon masses, we set $e=5.45, F_{\pi}=$ 129 MeV . These values are chosen to fit the nucleon and delta masses exactly [1]. In fitting the masses of strange baryons we treat $\delta$ as an adjustable parameter. We find $M_{c l}=864 \mathrm{MeV}$ and $\omega=161 \mathrm{MeV}$. Our $\omega$ can be identified with the difference between the strange quark and the light quark constituent masses, calculated to $O\left(N_{c}^{0}\right)$. Let us choose $\delta=90 \mathrm{MeV}$ to fit the $\Lambda$ mass to the observed value of 1115 MeV . Our mass formula then predicts $M_{\Sigma}=1193 \mathrm{MeV}$ and $M_{\Sigma^{*}}=1370 \mathrm{MeV}$. The observed values are 1193 MeV and 1382 MeV respectively. Therefore, the masses of three baryons of strangeness - 1 can be fitted almost exactly with one adjustable parameter $\delta$.

In conclusion, we would like to emphasize that the bound state approach works very well in describing the masses of strange baryons. This opens possibilities for further research. For instance, our approach can be used to investigate
further properties of strange baryons, such as magnetic moments and strangeness distributions. In fact, it has already been applied to calculations of the effects of strangeness on the size [15] and shape [16] of baryons. It has also been used in a model where the solitons are stabilized by vector mesons [6]. As stated in refs. [4] and [17], the bound state approach appears to be particularly useful for studying the formation of kaon condensates in dense nuclear matter. A method for treating dense skyrmion matter was proposed in ref. [18]. Some work on kaon condensation in the Skyrme model has already been performed [19].

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