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Measurement of the mass and lifetime differences between the heavy and light B_s eigenstates*

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I. Introduction

This report concerns measurements of the mass and lifetime differences of the B_s - $\overline{B}_s$ system. A large mass difference, $(\Delta m/\gamma)_{B_s} \gtrsim 5$, is predicted in the Kobayashi-Maskawa (KM) model with three generations of quarks.[1] This prediction follows from two factors. The first is the ARGUS collaboration report of a large B_d - $\overline{B}_d$ mixing,[2] $(\Delta m/\gamma)_{B_d} \sim 1$. The second is the unitarity of the 3 X 3 KM matrix. The prediction does not depend whether the KM model is relevant for CP violation. Thus, a much smaller $(\Delta m/\gamma)_{B_s} << 5$ would indicate new physics. An argument for an observable lifetime difference will be given. This report discusses time-dependent ways to extract the mass difference and lifetime difference of the neutral B_s system.

II. Formalism

An arbitrary neutral B-meson state

$$a\,|\,B^0> + \,b\,|\,\overline{B}^0> \tag{2.1}$$

is governed by the time-dependent Schrödinger equation

$$i\,\frac{d}{dt}\begin{pmatrix}a\\b\end{pmatrix} = H\begin{pmatrix}a\\b\end{pmatrix} \equiv \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix}a\\b\end{pmatrix} \tag{2.2}$$

Here M and Γ are 2 X 2 matrices, with $M=M^+$, $\Gamma=\Gamma^+$. CPT invariance guarantees $M_{11}=M_{22}$ and $\Gamma_{11}=\Gamma_{22}$. We assume CPT throughout to obtain the eigenstates of the mass matrix as

$$|\,B_L> = \; p\,|\,B^0> + \,q\;|\,\overline{B}^0>, \tag{2.3}$$

$$|\,B_H> = \; p\,|\,B^0> - \,q\;|\,\overline{B}^0>, \tag{2.4}$$

with eigenvalues (L="light," H="heavy")

$$\mu_{L,H} = m_{L,H} - \frac{i}{2}\,\gamma_{L,H}\,. \tag{2.5}$$

Here $m_{L,H}$ and $\gamma_{L,H}$ denote the masses and decay widths of $B_{L,H}$. Furthermore, define

$$\Delta\mu \equiv \mu_H - \mu_L \equiv \Delta m - \frac{i}{2}\,\Delta\gamma, \qquad \gamma \equiv (\gamma_L + \gamma_H)/2. \tag{2.6}$$

The neutral B meson mass difference, Δm, is related to the dispersive mass matrix part, M_{12}:

$$\Delta m \approx 2\,|M_{12}|. \tag{2.7}$$

A B_d mass difference was recently observed by the ARGUS collaboration,[2]

$$(\Delta m/\gamma)_{B_d} = 0.73 \pm 0.18. \tag{2.8}$$

The KM framework with three generations of quarks predicts a much enhanced B_s mass mixing,

$$\frac{(\Delta m/\gamma)_{B_s}}{(\Delta m/\gamma)_{B_d}} \approx \left|\frac{V_{ts}}{V_{td}}\right|^2 \sim \frac{1}{\theta^2}\,. \tag{2.9}$$

Here θ=0.22 is the sine of the Cabibbo angle. A lifetime difference in the B_s-system might also be observable.[3] Optimistic estimates of it reach the ten percent level. Qualitatively, this can be understood as follows. The lifetime difference is approximately proportional to the absorptive part, Γ_{12}.

$$\Delta\gamma \approx -2\,|\Gamma_{12}|. \tag{2.10}$$

The absorptive part, Γ_{12}, is a weighted summation over all physical[4] decay channels, f, into which both the B^o and the $\overline{B}^o$ can decay:

$$\begin{matrix} B^0 \searrow & \\ & f \\ \overline{B}^0 \nearrow & \end{matrix}\quad . \tag{2.11}$$

For the B_s-system, the leading quark transition that contributes to the absorptive part [i.e. satisfies Eq. (2.11)] is $\overline{b} \to \overline{c} + c\,\overline{s}$. This transition has a large branching ratio of order ~10%. Due to phase space considerations there will be a small number of final states with this underlying quark process $\overline{b} \to \overline{c} + c\,\overline{s}$. It is conceivable that no large cancellations will occur in the weighted summation for Γ_{12}, since the summation extends over few final states. Thus, the lifetime difference in the B_s-system might be observable.

III. A mass difference measurement of the B_s

A few definitons are in order. We denote the pure B_s mesons as: B_s (composed of " $\bar{b}s$") and $\bar{B}_s$ (composed of "b $\bar{s}$"). In contrast, a time-evolved initially pure B_s meson will be denoted as $B_{s,phys}$. It is of importance not to confuse the time-evolved $B_{s,phys}$ with a pure B_s. At later times an initially pure B_s meson ($B_{s,phys}$) has a probability of being a $\bar{B}_s$. After first showing that our method is insensitive to expected lifetime differences, we will discuss how to measure Δm.

In this section we wish to discuss final states, f, such that either a pure B_s or a pure $\bar{B}_s$--but not both (B_s and $\bar{B}_s$)--decays into them:

either
$$B_s \to f \text{ and } \bar{B}_s \not\to f \tag{3.1}$$

or
$$B_s \not\to f \text{ and } \bar{B}_s \to f. \tag{3.2}$$

Without loss of generality we consider final states, f, that satisfy Eq. (3.1). One method to extract the mass difference is to measure the time-dependent rates of an initially pure B_s ($\bar{B}_s$) into f,[5]

$$B_{s,phys}(t) \to f \qquad (\bar{B}_{s,phys}(t) \to f). \tag{3.3}$$

This extraction of the mass difference is insensitive to the expected lifetime difference, which is of the order of 10% or smaller. For instance, Fig. 1 illustrates the time-dependent rate of an *initially* pure B_s that decays into f,

$$B_{s,phys}(t) \to f. \tag{3.4}$$

Recall the assumption that the pure $\bar{B}_s \not\to f$! Fig. 2 shows the time-dependent rate of an initially pure anti-meson, $\bar{B}_s$, into the same final state, f,

$$\bar{B}_{s,phys}(t) \to f. \tag{3.5}$$

In Fig. 1 (Fig. 2) a mixing of

$$(\Delta m/\gamma)_{B_s} = 5 \tag{3.6}$$

was assumed, and the time-dependent rate of $B_{s,phys}(t) \to f$ ($\overline{B}_{s,phys}(t) \to f$)
was plotted for two cases:

one with an improbably large lifetime difference of $(\Delta\gamma/\gamma)_{B_s} = -0.2$, (3.7)

the other with no lifetime difference, $(\Delta\gamma/\gamma)_{B_s} = 0$. (3.8)

Both curves in Fig. 1 (Fig. 2) coincide to the accuracy of the graph. This coincidence indicates that the time-dependent curves into final states, f, are insensitive to the predicted lifetime differences [$\lesssim$ O(10%)]. Of course, unrealistically large lifetime differences would show non-overlapping curves; see Fig. 3.

Now we will discuss the measurement of the mass difference. In Fig. 1 (Fig. 2) the horizontal distance (elapsed proper time) between extrema measures the mass difference, Δm. The inclusive decays governed by the quark transition, $\overline{b} \to \overline{c} + u\,\overline{d}$, and by $\overline{b} \to \overline{c} + l^+ \nu$ can be used [Eq. (3.1) holds]. They comprise the majority of B_s decays, making our method statistically powerful. Note, however, that this method of measuring $(\Delta m/\gamma)_{B_s}$ requires the distinction between an initially pure B_s and an initially pure $\overline{B}_s$. This is known as the tagging requirement, and reduces the number of events available for statistical evaluation. A more ambitious program might consider what can be learned from the observation of only the time-dependent rate into the inclusive final state, f, without the requirement that one know whether the final state, f, originated from an initially pure B_s or pure anti-meson $\overline{B}_s$.

IV. A lifetime difference measurement of the B_s

Having discussed a method of measuring Δm, we now wish to turn to an experiment which is sensitive to lifetime differences. Throughout this section we neglect CP violation effects in the B_s-system. We will briefly justify this assumption in the next paragraph. In the absence of CP violation, the mass eigenstates of the B_s [$(B_s)_H$ and $(B_s)_L$] have a definite CP signature. [This is analogous to the kaon system. In the absence of CP violation, the K_S decays only into CP even eigenstates, the K_L only into CP odd eigenstates.] The time-dependent rate into CP even or CP odd final states is exponential,

$$e^{-\gamma_{L,H} t} . \tag{4.1}$$

Thus, a comparison of the exponential decay rate into CP even final states with that into CP odd final states can be used to measure $\Delta\gamma$, see Fig. 4. Here no tagging is required, since CP invariance guarantees that one mass eigenstate decays only into CP even eigenstates, the other only into CP odd ones.

We now wish to show that CP violation can be neglected in a first approximation. The KM favored quark transition $\bar{b} \rightarrow \bar{c} + c\,\bar{s}$ is responsible for a major fraction of B_s decays into CP eigenstates. The three generation KM model predicts a tiny CP violation in the B_s decays which are governed by this quark transiton, $\bar{b} \rightarrow \bar{c} + c\,\bar{s}$. The preceding explains the neglect of CP violation in this section. A careful study, however, could consider the complications that CP violation introduces.

Acknowledgements

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References

[1]See (e.g.): H. Harari, SLAC preprint, SLAC-PUB-4327, May 1987; J. Ellis, J. S. Hagelin and S. Rudaz, Phys. Lett. **192B**, 201 (1987); I. Dunietz, Ph. D. Thesis, Enrico Fermi Institute report, EFI 87-86, Oct. 1987, submitted to Annals of Physics.

[2]H. Albrecht et al., Phys. Lett. **192B**, 245 (1987).

[3]Dan-di Wu, LBL-preprint, July 1985, LBL-19982 (unpublished); A. Datta, E. A. Paschos and U. Türke, Dortmund University report, DO-TH 87/10, June 1987; Dunietz, Ref. 1.

[4] In contrast to virtual decay channels, which contribute to the dispersive part.

[5]Time-dependent formulae without approximations can be found in I. Dunietz and J. L. Rosner, Phys. Rev. **D34**, 1404 (1986).

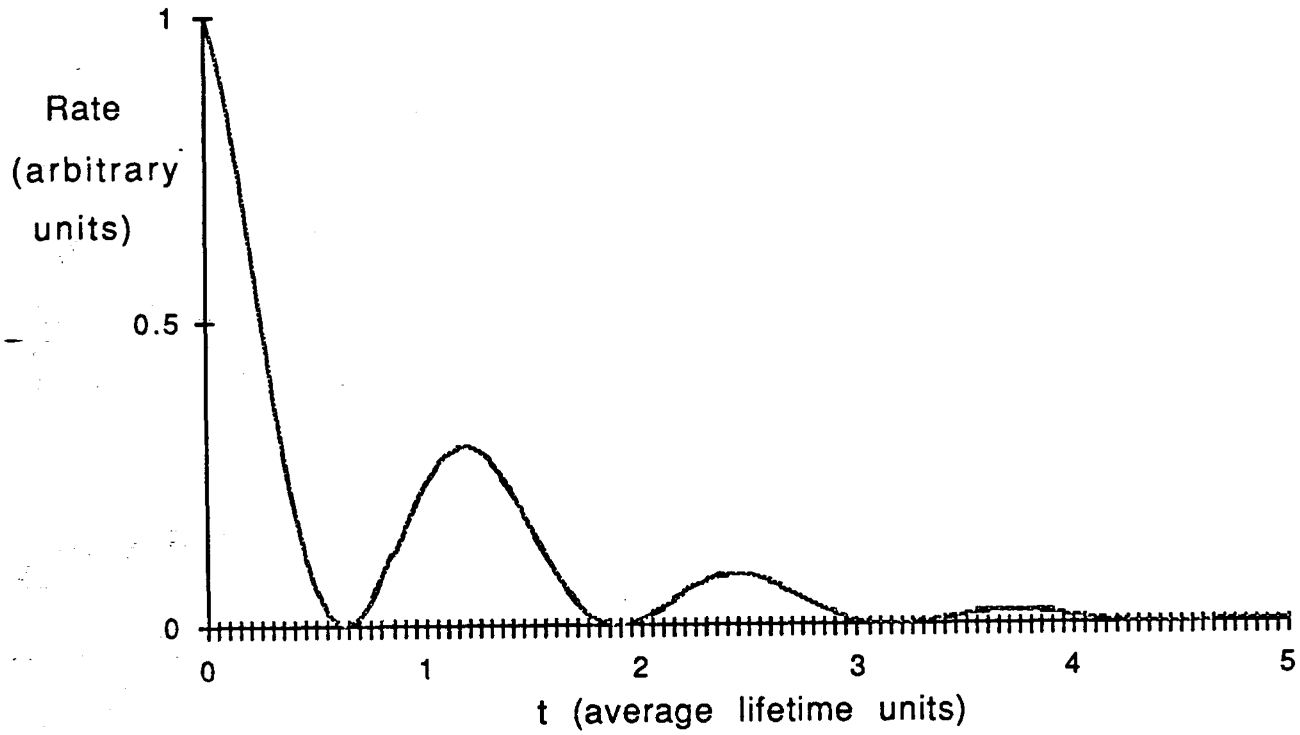
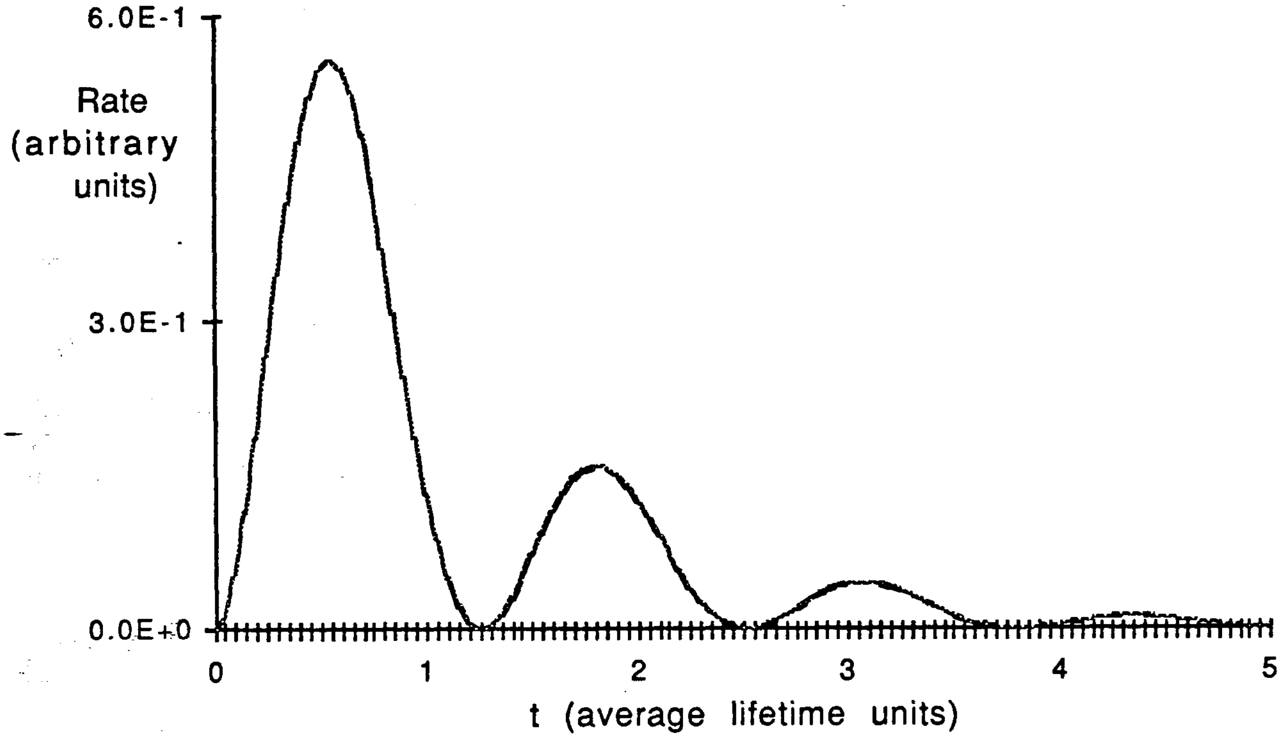


FIGURE 1



6.0E-1
Rate
(arbitrary
units)
3.0E-1
0.0E+0
0
1
2
3
4
5
t (average lifetime units)

FIGURE 2

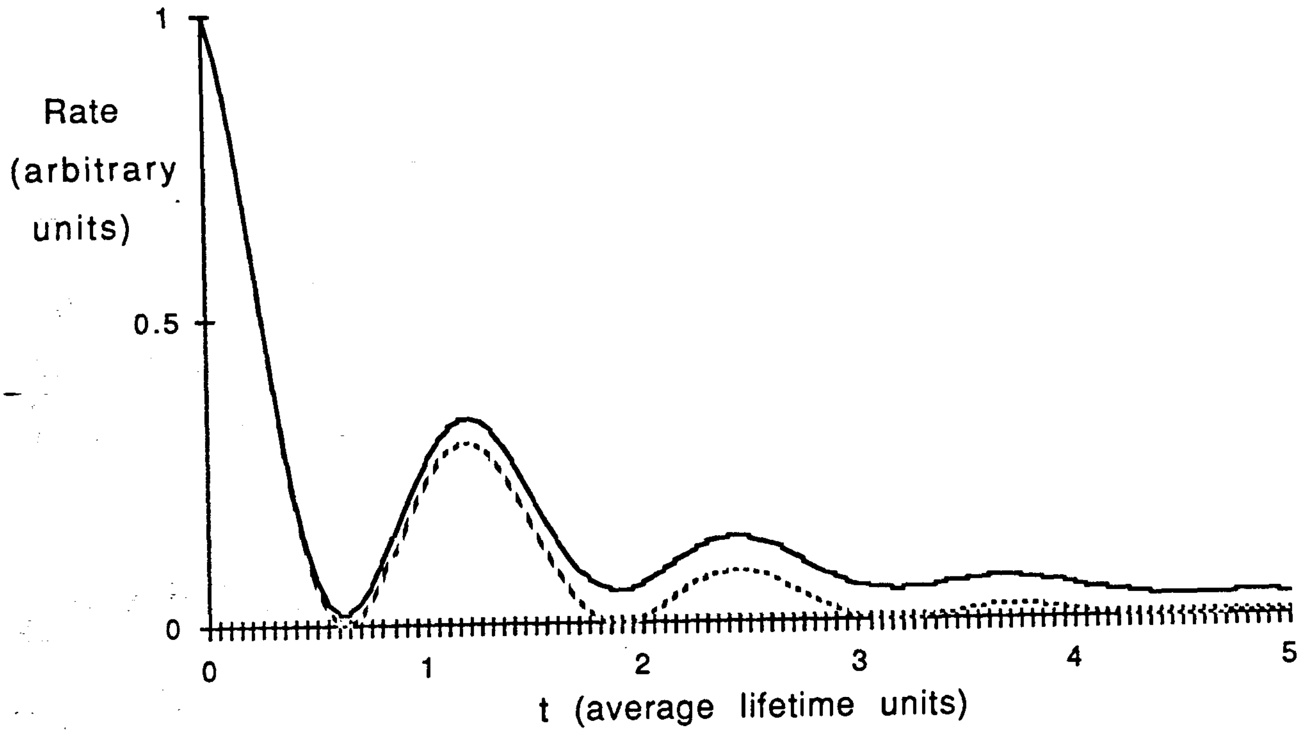


FIGURE 3

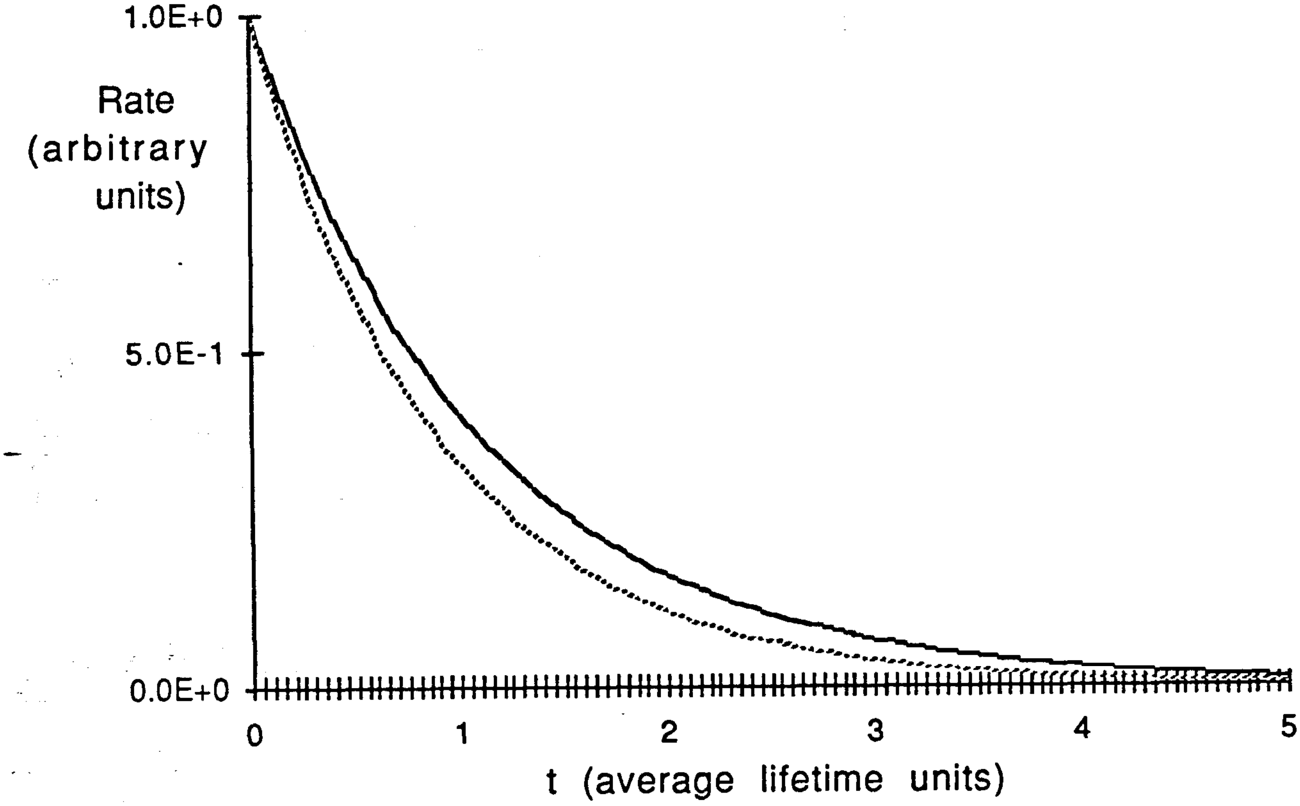

1.0E+0
Rate
(arbitrary
units)
5.0E-1
0.0E+0
0
1
2
3
4
5
t (average lifetime units)

FIGURE 4

Figure Captions:

Fig. 1. The time-dependent inclusive decay rate of an initially pure B_s into final states f that satisfy Eq. (3.1). Here $(\Delta m/\gamma)_{B_s} = 5$ was used.

Solid curve: $B_{s,phys}(t) \rightarrow f$ with a lifetime difference of $(\Delta\gamma/\gamma)_{B_s} = -0.2$.

Dashed curve: $B_{s,phys}(t) \rightarrow f$ with no lifetime difference, $(\Delta\gamma/\gamma)_{B_s} = 0$.

Fig. 2. The time-dependent inclusive decay rate of an initially pure $\overline{B}_s$ into final states f that satisfy Eq. (3.1). Here $(\Delta m/\gamma)_{B_s} = 5$ was used.

Solid curve: $\overline{B}_{s,phys}(t) \rightarrow f$ with a lifetime difference of $(\Delta\gamma/\gamma)_{B_s} = -0.2$.

Dashed curve: $\overline{B}_{s,phys}(t) \rightarrow f$ with no lifetime difference, $(\Delta\gamma/\gamma)_{B_s} = 0$.

Fig. 3. The time-dependent inclusive decay rate of an initially pure B_s into final states f that satisfy Eq. (3.1). Here $(\Delta m/\gamma)_{B_s} = 5$ was used.

Solid curve: $B_{s,phys}(t) \rightarrow f$ with an <u>exaggerated</u> lifetime difference of $(\Delta\gamma/\gamma)_{B_s} = -1.2$.

Dashed curve: $B_{s,phys}(t) \rightarrow f$ with no lifetime difference, $(\Delta\gamma/\gamma)_{B_s} = 0$.

Fig. 4. Exponential inclusive decay rate into CP eigenstates, F and F'. The CP signatures of F and F' are reversed. Here $(\Delta\gamma/\gamma)_{B_s} = -0.2$ was assumed.

Solid curve: $(B_s)_H(t) \rightarrow$ F.

Dashed curve: $(B_s)_L(t) \rightarrow$ F'.