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THE QUESTION OF CP NONINVARIANCE— AS SEEN THROUGH THE EYES OF NEUTRAL BEAUTY*

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ABSTRACT

We present a detailed review of the expected phenomenology of CP violation in neutral B [&D] meson decays. When stating predictions as obtained from the standard model, we emphasize the basic concepts involved and give general expressions; the numbers that we quote are meant to illustrate the method and provide guidelines, not to be precise predictions.

PROLOGUE

Among the many statements that can and have been made on CP violation, three stand out since they are unassailable without being trivial:

- A breakdown of CP invariance has been directly observed in nature, namely in K_L decays.
- CP violation, despite its shy appearance on the stage of physics, represents a truly fundamental phenomenon—as it has been duly recognized from the beginning.
- We cannot claim to have developed a real understanding of this phenomenon. This —in view of the first two points—is highly unsatisfactory, if not outright embarrassing; it actually refers to two different levels:

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(a) Considerable progress has been made in "Theoretical Engineering;" we have developed a rather clear picture on the various generic ways of imbedding CP violation into a given theory.[1] But we have been unable to decide which of these mechanisms is the source, or even the dominant source, of the observed CP violation.

(b) Considering the fundamental importance of CP violation one yearns for a deeper understanding that goes beyond the question of which mechanism describes the data properly.

We do not have anything specific to say concerning point (b); however, we believe that it can hardly find a satisfactory answer if point (a) remains unanswered. Furthermore, we feel strongly that CP violation has to be found outside the decays of neutral kaons before light can be shed on the underlying source.

Beauty hadrons carry excellent promise to exhibit large observable CP asymmetries in their decays. This holds, in particular, in the Kobayashi–Maskawa (KM) ansatz[2] where it is the interplay of three quark families that makes the *phases* of the weak couplings and thus CP violation observable. Beauty decays are then the process of choice: b quarks belong to the third family, yet have to decay into members of the lower families; even the top quarks are drawn into this affair via B°–$\overline{B}^\circ$ mixing.

This argument can of course be made in a more precise way: the unitarity of the KM matrix V yields, among others, the following three relations which are evidently invariant under changes in the phase convention adopted for the quark fields

$$V(ud)V^*(td) + V(us)V^*(ts) + V(ub)V^*(tb) = 0 \ , \tag{1}$$

$$V(ub)V^*(ud) + V(cb)V^*(cd) + V(tb)V^*(td) = 0 \ , \tag{2}$$

$$V(cd)V^*(td) + V(cs)V^*(ts) + V(cb)V^*(tb) = 0 \ . \tag{3}$$

Up to small corrections of order $\sin^2\theta_c$, these equations can be rewritten in a simplified fashion:

$$V^*(td) + \lambda V^*(ts) + V(ub) \simeq 0 \ , \tag{4}$$

$$V(ub) - \lambda V(cb) + V^*(td) \simeq 0 \ , \tag{5}$$

$$-\lambda V^*(td) + V^*(ts) + V(cb) \simeq 0 \ , \tag{6}$$

where

$$\lambda \equiv \sin\theta_c \quad .$$

These equations represent triangle relations in the complex plane and—as first pointed out by Bjorken, L.–L. Chau and Jarlskog[25]—are quite accessible to geometric intuition:

(*i*) Different parametrizations of the KM matrix correspond just to different rotations of these triangles.

(*ii*) The two triangles defined by Eqs. (4) and (5) actually agree to this order in λ since obviously $V^*(ts) \simeq -V(cb)$. This triangle is shown in Fig. 1 with a shape that is "typical" as explained later.

(*iii*) The triangle defined by Eq. (6) is quite a squashed one since $|V(ts)| \simeq |V(cb)| \gg \lambda |V(td)|$.

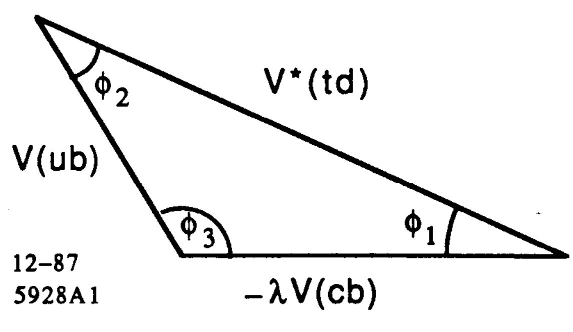

Fig. 1. Triangle depicting dominant relative phases in KM ansatz with three families.

Figure 1 contains the main observation: there are sizeable relative phases between $V(ub), V(cb)$ and $V(td)$. They can all be probed with high sensitivity in beauty decays (where $V(td)$ drives $B_d - \overline{B}_d$ mixing).

In K and D decays, on the other hand, the situation is much less favorable: $V(cs)$ contains a CP violating phase, but only at order λ^4; $V(td)$ is quite crucial for ϵ_K and ϵ', yet its numerical impact is greatly reduced by the smallness of $V(td)V^*(ts)$; furthermore, the dynamical "accident" (as far as CP violation is concerned) of the $\Delta I = 1/2$ rule reduces CP asymmetries like ϵ' by an additional order of magnitude.

For more detailed considerations it is still useful to employ an explicit form of the KM matrix

$$\begin{array}{cccc} & d & s & b \\ V_{KM} = & \left[\begin{array}{ccc} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3\left(\rho - i\eta\left\{1-\frac{1}{2}\lambda^2\right\}\right) \\ -\lambda & 1-\frac{1}{2}\lambda^2 - i\eta A^2\lambda^4 & A\lambda^2(1+i\eta\lambda^2) \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{array}\right] & \begin{array}{c} u \\ c \\ t \end{array} \end{array}, \tag{7}$$

where we have used the Wolfenstein expansion up to order $\lambda^4(\lambda^6)$ for the real (imaginary) parts of the charged current couplings.

Meaningful bounds now exist on all these parameters; one typically finds

$$A = 1.1 \pm 0.2 \quad \text{from} \quad \tau_B \ , \tag{8}$$

$$0.08 \lesssim \rho^2 + \eta^2 \lesssim 1.0 \quad \text{from} \quad B \to p\bar{p}\pi(\pi) \quad , \quad l\nu \quad \text{noncharm} \ , \tag{9}$$

$$\rho < 0 \ \text{from} \ B_d - \overline{B}_d \ \text{mixing} \quad . \tag{10}$$

To derive more specific numbers one has to proceed with considerable care and caution. For at present no precise scheme exists for describing weak decays that has been derived from first principles; instead one is limited to employing various phenomenological prescriptions whose reliability is not well established. Furthermore they introduce systematic *correlations* among the numerical values for the KM parameters as they are inferred from the data. Accordingly, one has to apply these schemes consistently.

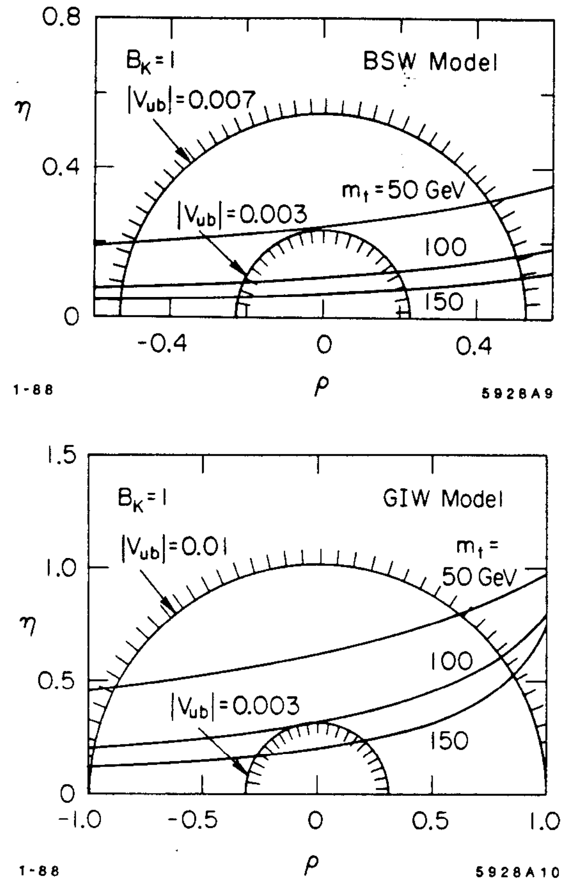

Fig. 2. (a) The allowed region in the $\eta - \rho$ plane as derived in the BSW ansatz, with $B_K = 1$. (b) Same as in (a), but using the GIW scheme.

In Fig. 2a we show the allowed range in the $\eta - \rho$ plane as obtained[26] in the BSW ansatz, Ref. 9; the upper bound $| V_{ub} | = 0.007$ is deduced from semileptonic B decays, the lower bound is suggested by BR($B \to p\bar{p}\pi(\pi)$). The constraint imposed by ϵ_K is represented by the three lines for $m_t = 50, 100, 150$ GeV when using $B_K = 1$. The ARGUS data on $B_d - \overline{B}_d$ mixing do not provide very specific constraints yet, apart from *favoring negative values for* ρ.

Figure 2b shows the results of the analogous analysis based on the GIW ansatz, Ref. 23.

From both curves we read off that $\eta = 0.2$ represents quite a reasonable value. Furthermore, UA1 data on direct production of top and ARGUS data on $B_d - \overline{B}_d$ mixing imply a lower bound on m(top)[19] while a comprehensive analysis of electroweak phenomena leads to an upper bound[20]

$$50 \text{ GeV} \lesssim m_t \lesssim 200 \text{ GeV} \quad . \tag{11}$$

Within the theoretical (and experimental) uncertainties one has to allow for at present, the KM scheme is able to reproduce the observed strength of ϵ_K and to predict $\epsilon'/\epsilon_K \sim$ few $\times 10^{-3}$.

At this point we want to emphasize another important feature of beauty physics: their theoretical treatment should be much more reliable than that of K (and D) physics since on very general grounds one expects *global* contributions from long-distance dynamics to be fairly small in beauty decays. Also, the radiative QCD corrections are smaller and in any case computed with a higher degree of confidence. Accordingly, we will see that careful studies of B°–$\overline{B}^\circ$ mixing and CP violation does much more than "just" test the Standard Model in a very sensitive way: it probes the existence of "New Physics" characterized by high-mass scales like a fourth family, right-handed currents, etc. Since this subject is outside the scope of this review we will make only brief references to it at appropriate times.

Unfortunately, it is one thing to state that among the multitude of beauty decays there are some with large CP asymmetries and an altogether quite different thing to make this statement more precise: how large an asymmetry does one really predict and in which decay mode?

It obviously is quite premature to present precise numerical estimates: even basic quantities like the lifetimes for the different beauty hadrons separately, i.e.,

$$\tau(B_d) \text{ vs. } \tau(B_u) \text{ vs. } \tau(B_s) \text{ vs. } \tau(\Lambda_b)$$

have not been measured with any accuracy. Instead we are forced to rely on theoretical estimates which state, rather conservatively, that

$$1 \lesssim \frac{\tau(B_u)}{\tau(B_d)} \lesssim 1.2 \quad , \qquad \text{etc .}$$

There are very few exclusive branching ratios known, and the errors attached to these numbers are still quite large.

Instead we have decided to dwell more on the basic methods and present expressions that are as general as appropriate. Definite numbers will at times be inserted, mainly for illustrative purposes and to show that our considerations are far from being purely academic. We want to enable the reader to insert precise numbers for various parameters whenever they become available.

The field we are going to discuss contains many elements of open-ended adventure; accordingly, we treat it like a modern drama—there are certainly many lessons in it, but we do not know when they will come out and what they will finally be. In Act I we set the stage by developing the general phenomenology of CP asymmetries in B decays; in Act II we focus more specifically on B_d decays, while B_s decays take center stage in Act III; in Act IV we address the issue of how to search for these effects in e^+e^- annihilation and in hadronic reactions before giving a summary and presenting the conclusions in Act V.

ACT I. THE PLOT: CP ASYMMETRIES IN B DECAYS

Assuming CPT invariance implies that CP violation can enter only via relative complex phases between (effective) coupling constants. These phases can be observed only if two *different* amplitudes contribute *coherently* to the same process—the asymmetry is produced by their interference. Basically, there are just two ways to realize such a scenario:

– via final state interactions, hereafter referred to as FSI

– via mixing, like ϵ_K.

The different scenarios can be distinguished also in an operational way: one compares the evolution of decay rates in proper time

$$\text{rate}\,[B(t) \to f] \propto \quad e^{-\Gamma t}\, G \quad \leftrightarrow \quad \text{rate}\,[\overline{B}(t) \to \bar{f}\,] \propto \quad e^{-\Gamma t}\, \overline{G}\,. \tag{12}$$

A difference

$$\frac{G}{\overline{G}} \neq 1$$

establishes CP violation. Such a difference can be realized in two quite distinct ways:

$$\frac{d}{dt}\,\frac{G}{\overline{G}} \equiv 0\;, \tag{13}$$

$$\frac{d}{dt}\,\frac{G}{\overline{G}} \neq 0\quad. \tag{14}$$

When the final state f is flavor specific, i.e.,

$$B(0) \begin{array}{ccc} \nearrow & f & \not\nwarrow \\ \not\searrow & \bar{f} & \swarrow \end{array} \overline{B}(0)\;, \tag{15}$$

the first situation, Eq. (13), applies. This is always (but not exclusively) the case when FSI are essential in exposing CP violation. When f (and therefore also $\bar{f}$) is common to both B and $\overline{B}$ decays, possible only for neutral B decays,

$$B^\circ \begin{array}{ccc} \nearrow & f & \nwarrow \\ \searrow & \bar{f} & \swarrow \end{array} \overline{B}^\circ\;, \tag{16}$$

the second scenario, Eq. (14), applies which involves B°–$\overline{B}^\circ$ mixing as we will see.

The decays of neutral B mesons can realize both scenarios—in contrast to the decays of charged B mesons and Λ_b baryons which are treated in a separate article.[3]

(A) General Formalism

The proper time evolution of a meson that was born as a B° or $\overline{B}^\circ$, respectively, at time $t = 0$ is given in the most general Pais–Treiman form[4] by

$$\begin{aligned}
|B^\circ(t)\rangle &= g_+(t)\ |B^\circ\rangle_\circ + \frac{q}{p}\ g_-(t)\ |\ \overline{B}^\circ\rangle_\circ\;, \\
|\overline{B}^\circ(t)\rangle &= \frac{p}{q}\ g_-(t)|B^\circ\rangle_\circ + g_+(t)\ |\overline{B}^\circ\rangle_\circ\;, \\
g_\pm(t) &= \frac{1}{2}\exp\left\{-\frac{1}{2}\Gamma_1 t\right\}\ \exp\{i m_1 t\}\ \left[1 \pm \exp\left\{-\frac{1}{2}\Delta\Gamma t\right\}\exp\{i\Delta m t\}\right]\;, \\
\Delta\Gamma &= \Gamma_2 - \Gamma_1 \quad , \quad \Delta m = m_2 - m_1 \quad , \quad \frac{q}{p} = \frac{1-\bar{\epsilon}}{1+\bar{\epsilon}}\;.
\end{aligned} \tag{17}$$

Γ_i, m_i, $i = 1, 2$ are the width and mass of the two neutral mass eigenstates B_i.

For the decay rate into a final state f and its CP conjugate $\overline{f}$ one finds accordingly as a function of proper time t

$$\begin{aligned} \text{rate}[B^\circ(t) \to f] \propto\ & \frac{1}{2} e^{-\Gamma_1 t}\, G\ , \\ G = a + b e^{-\Delta\Gamma t} &+ c e^{-1/2\Delta\Gamma t} \cos \Delta m t + d e^{-1/2\Delta\Gamma t} \sin \Delta m t\ , \end{aligned} \tag{18}$$

$$\begin{aligned} \text{rate}[\overline{B}^\circ(t) \to \overline{f}\,] \propto\ & \frac{1}{2} e^{-\Gamma_1 t}\, \overline{G}\ , \\ \overline{G} = \overline{a} + \overline{b} e^{-\Delta\Gamma t} &+ \overline{c} e^{-1/2\Delta\Gamma t} \cos \Delta m t + \overline{d}\, e^{-1/2\Delta\Gamma t} \sin \Delta m t\ , \end{aligned} \tag{19}$$

where

$$\begin{aligned}
\overset{(-)}{A}\,(\overset{(-)}{f}\,) &= \langle \overset{(-)}{f} | \mathcal{L}\ (\Delta B = 1)\ | \overset{(-)}{B}{}^\circ \rangle_\circ\ , \\
\overline{\rho}(f) &= \frac{\overline{A}(f)}{A(f)} \quad ; \quad \rho(\overline{f}) = \frac{A(\overline{f})}{\overline{A}(\overline{f})}\ , \\
a &= |A(f)|^2 \left\{ \frac{1}{2} \left[1 + |\frac{q}{p} \overline{\rho}(f)|^2 \right] + Re \left[\frac{q}{p}\, \overline{\rho}(f) \right] \right\}\ , \\
b &= |A(f)|^2 \left\{ \frac{1}{2} \left[1 + |\frac{q}{p} \overline{\rho}(f)|^2 \right] - \text{Re} \left[\frac{q}{p}\, \overline{\rho}(f) \right] \right\}\ , \\
c &= |A(f)|^2 \left\{ 1 - \left| \frac{q}{p} \right|^2 |\overline{\rho}(f)|^2 \right\}\ , \\
d &= 2|A(f)|^2\ \text{Im} \left[\frac{q}{p}\, \overline{\rho}(f) \right]\ , \\
\overline{a} &= |\overline{A}\ (\overline{f})|^2 \left\{ \frac{1}{2} (1 + |\frac{p}{q} \rho(\overline{f})|^2) + \text{Re} \left[\frac{p}{q}\, \rho(\overline{f}) \right] \right\}\ , \\
\overline{b} &= |\overline{A}(\overline{f})\ |^2 \left\{ \frac{1}{2} (1 + |\frac{p}{q} \rho(\overline{f})|^2) - \text{Re} \left[\frac{p}{q}\, \rho(\overline{f}) \right] \right\}\ , \\
\overline{c} &= |\overline{A}(\overline{f})|^2 \left\{ 1 - \left| \frac{p}{q} \right|^2 |\rho(\overline{f})|^2 \right\}\ , \\
\overline{d} &= 2|\overline{A}(\overline{f})|^2\ \text{Im} \left[\frac{p}{q} \rho\ (\overline{f}) \right]\quad .
\end{aligned} \tag{20}$$

CP invariance is clearly violated if (for $\Delta\Gamma, \Delta\text{m} \neq 0$),

$$a \neq \overline{a} \text{ or } b \neq \overline{b} \text{ or } c \neq \overline{c} \text{ or } d \neq \overline{d}\quad . \tag{21}$$

Equations (18)–(20) describe the most general proper time evolution for the decay of a neutral B meson. Staring at the most general case of a problem is rarely illuminating; instead we will use Eqs. (18)–(20) as master equations for the subsequent discussion where we consider complementary special cases.

We have already introduced two basic categories of final states f which will exhibit quite a different phenomenology, namely "flavor specific" decay modes where

$$A(f) \cdot \overline{A}(f) = 0 = A(\bar{f}) \cdot \overline{A}(\bar{f}) \ , \tag{22}$$

and those which are common to direct B° as well as $\overline{B}^\circ$ decays:

$$A(f) \cdot \overline{A}(f) \neq 0 \neq A(\bar{f}) \cdot \overline{A}(\bar{f}) \quad . \tag{23}$$

Four examples—two each from these two categories—will help to illustrate the general scenario.

(i) Flavor Specific Decays

Semileptonic decays represent the most convenient (though not only) example of flavor specific decays. In the Standard Model b quarks can decay directly into negatively charged leptons only:

$$b \begin{array}{l} \nearrow \ l^- \ \bar{\nu} \ q \\ \not\searrow \ l^+ \ \nu \ q \end{array} \quad . \tag{24}$$

With the convention

$$B^\circ = (\bar{b}q) \quad , \quad q = d, s \quad , \tag{25}$$

one can express Eq. (24) as a $\Delta B = \Delta Q_l$ rule

$$\begin{array}{c} B^\circ \rightarrow \ell^+ \ \nu X \not\leftarrow \overline{B}^\circ \ , \\ B^\circ \not\rightarrow \ell^- \ \nu X \leftarrow \overline{B}^\circ \ . \end{array} \tag{26}$$

Accordingly,

$$A(l^- \bar{\nu} \ X) = \overline{A}(l^+ \nu \ X) = 0 \quad . \tag{27}$$

Example 1: $B^\circ(t) \to l^+\nu\, X$ vs. $\overline{B}^\circ(t) \to l^-\nu\, X$

Equation (27) is re-expressed in the notation of Eq. (20)

$$\begin{aligned} &a = b = \frac{1}{2}\, c = \frac{1}{2}|A(l^+\nu\, X)|^2 \ , \\ &\bar{a} = \bar{b} = \frac{1}{2}\, \bar{c} = \frac{1}{2}|\overline{A}(l^-\bar{\nu}\, X)|^2 \ , \\ &d = \bar{d} = 0 \ , \end{aligned}$$

and therefore [see Eqs. (18) and (19)]

$$G = |A(l^+\nu\, X)|^2 \left\{ \frac{1}{2}\left(1 + e^{-\Delta\Gamma t}\right) + e^{-1/2\Delta\Gamma t}\cos\Delta m t \right\} \ , \tag{28}$$

$$\overline{G} = |\overline{A}(l^-\bar{\nu}\, X)|^2 \left\{ \frac{1}{2}\left(1 + e^{-\Delta\Gamma t}\right) + e^{-1/2\Delta\Gamma t}\cos\Delta m t \right\} \ , \tag{29}$$

$$\frac{d}{dt}\frac{G}{\overline{G}} = \frac{d}{dt}\frac{|A(l^+\nu\, X)|^2}{|\overline{A}(l^-\bar{\nu}\, X)|^2} \equiv 0 \quad . \tag{30}$$

A CP asymmetry can then exist only if

$$|A(l^+\nu\, X)| \neq |\overline{A}(l^-\bar{\nu}\, X)| \quad , \tag{31}$$

i.e., if there is CP violation in $\Delta B = 1$ transitions. This is referred to as *direct CP violation.* Since we will return to it in a more detailed way we make only a brief remark in passing: it is highly unlikely for Eq. (31) to be realized in nature, nonleptonic decay modes offer much better prospects; nevertheless this example is useful for illustrating the general phenomenology.

Example 2: $B^\circ(t) \to l^-\nu\, X$ vs. $\overline{B}^\circ(t) \to l^+\nu\, X$

New features appear here since these decays can occur only due to $B^\circ - \overline{B}^\circ$ mixing. Equation (27) is now re-expressed as

$$a = b = -1/2\, c = 1/2\, \left|\frac{q}{p}\right|^2 \left|\overline{A}(l^-\nu\, X)\right|^2 \ , \tag{32}$$

$$\bar{a} = \bar{b} = -\frac{1}{2}\,\bar{c} = \frac{1}{2}\left|\frac{p}{q}\right|^2 \left|A(l^+\nu\; X)\right|^2 \;, \tag{33}$$

$$d = \bar{d} = 0 \;, \tag{34}$$

and therefore

$$G = \left|\frac{q}{p}\right|^2 |\overline{A}(l^-\nu\; X)|^2 \left\{\frac{1}{2}\left(1 + e^{-\Delta\Gamma t}\right) - e^{-1/2\,\Delta\Gamma t}\cos\Delta m t\right\} \;, \tag{35}$$

$$\overline{G} = \left|\frac{p}{q}\right|^2 |\overline{A}(l^+\nu\; X)|^2 \left\{\frac{1}{2}\left(1 + e^{-\Delta\Gamma t}\right) - e^{-1/2\,\Delta\Gamma t}\cos\Delta m t\right\} \;, \tag{36}$$

$$\frac{d}{dt}\,\frac{G}{\overline{G}} = \frac{d}{dt}\left|\frac{q}{p}\right|^4 \frac{|\overline{A}(l^-\nu\; X)|^2}{|A(l^+\nu\; X)|^2} = 0 \quad . \tag{37}$$

There is a CP asymmetry even if the $\Delta B = 1$ amplitudes are, as expected, identical, namely when

$$\left|\frac{q}{p}\right|^2 \neq 1 \quad . \tag{38}$$

As described in more detail later, $|q/p|^2 \neq 1$ describes one aspect of CP violation in B°–$\overline{B}^\circ$ mixing, i.e., in $\Delta B = 2$ transitions. We will also see that the prospects for ever observing such an effect are quite discouraging.

(ii) Flavor Nonspecific Decays

There are quite a few nonleptonic channels that are common to B° and $\overline{B}^\circ$ decays. Among them is a special class of such final states f, namely CP eigenstates

$$CP|f_\pm\rangle = \pm|f_\pm\rangle \quad . \tag{39}$$

Their special role is exhibited in the following *theorem:*

Let $B_{\rm neut}$ denote any combination of B° and $\overline{B}^\circ$ mesons and f a final state of definite CP parity. CP violation is established by finding the proper time evolution for the rate $B_{\rm neut} \rightarrow f$ to be different from a *single, pure* exponential, i.e.,

$$\frac{d}{dt}e^{\Gamma t}\ \text{rate}\ (B_{\rm neut}(t) \rightarrow f) \neq 0 \;. \tag{40}$$

for all Γ.

The proof is completely elementary. Assume CP to be conserved; then the mass eigenstates are CP eigenstates as well: $CP|B_\pm\rangle = \pm|B_\pm\rangle$; furthermore $B_+ \to f_+, B_- \not\to f_+$ for $CP|f_+\rangle = |f_+\rangle$. Therefore

$$\text{rate}[B_{\text{neut}}\ (t) \to f_+] = \frac{N_+}{N_+ + N_-}\ \text{rate}\ (B_+(t) \to f_+) = e^{-\Gamma t}\ \text{const}\ .$$

where $N_+[N_-]$ denotes the original number of $B_+[B_-]$ mesons in the B_{neut} beam, Q.E.D.

This can be seen also explicitly by applying Eq. (17): if CP is conserved, one can set $q/p = 1$ without loss of generality; then one writes down

$$|B^0(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-\frac{1}{2}\Gamma_1 t}e^{im_1 t}|B_+\rangle + e^{-\frac{1}{2}\Gamma_2 t}e^{im_2 t}|B_-\rangle\right)\ .$$

Here we have used the definition

$$CP|B^0\rangle \equiv |\overline{B}^0\rangle\ , \tag{41}$$

which leads to

$$|B_\pm\rangle = \frac{1}{\sqrt{2}}\left(|B^0\rangle \pm |\overline{B}^0\rangle\right)\ .$$

Example 3: $B^\circ \to \psi K_S$ vs. $\overline{B}^\circ \to \psi K_S$

The final state ψK_S besides being an odd CP eigenstate is described by a single isospin amplitude. Therefore as explained later in detail

$$|\overline{\rho}(\psi K_S)| = \left|\frac{\overline{A}(\psi K_S)}{A(\psi K_S)}\right| = 1\ , \tag{42}$$

$$\begin{aligned} G = |A(\psi K_S)|^2 \Bigg\{ & 1 + \text{Re}\left[\frac{q}{p}\,\overline{\rho}(\psi K_S)\right] + e^{-\Delta\Gamma t}\left(1 - \text{Re}\left[\frac{q}{p}\,\overline{\rho}\ (\psi K_S)\right]\right)\ , \\ & + \left(1 - \left|\frac{q}{p}\right|^2\right) e^{-1/2\Delta\Gamma t}\cos\Delta m t + 2\ \text{Im}\left[\frac{q}{p}\,\overline{\rho}\ (\psi K_S)\right] e^{-1/2\Delta\Gamma t}\ \sin\Delta m t \Bigg\}\ , \end{aligned} \tag{43}$$

$$\begin{aligned} \overline{G} = |A(\psi K_S)|^2 \Bigg\{ & 1 + \text{Re}\left[\frac{p}{q}\,\overline{\rho}\ (\psi K_S)^{-1}\right] + e^{-\Delta\Gamma t}\left(1 - \text{Re}\left[\frac{p}{q}\,\overline{\rho}\ (\psi K_S)^{-1}\right]\right)\ , \\ & + \left(1 - \left|\frac{p}{q}\right|^2\right) e^{-\frac{1}{2}\Delta\Gamma t}\cos\Delta m t + 2\ \text{Im}\left[\frac{p}{q}\,\overline{\rho}\ (\psi K_S)^{-1}\right]\ e^{-\frac{1}{2}\Delta\Gamma t}\sin\Delta m t \Bigg\}\ . \end{aligned} \tag{44}$$

As discussed in Acts II and III one expects that $|q/p| \simeq 1$; $q/p\,\overline{\rho}$ is then described by a unit vector in the complex plane:

$$\frac{q}{p}\,\overline{\rho} \equiv e^{i\phi} \quad .$$

Accordingly, $\mathrm{Re}\,(q/p)\,\overline{\rho} = \mathrm{Re}\left[(p/q)\,\overline{\rho}^{-1}\right]$; $\mathrm{Im}\left[(q/p)\,\overline{\rho}\right] = -\mathrm{Im}\left[(p/q)\,\overline{\rho}^{-1}\right]$ and

$$\begin{aligned} G = |A(\psi K_S)|^2 \Big\{ (1+e^{-\Delta\Gamma t}) + (1-e^{-\Delta\Gamma t})\ \mathrm{Re}\left[\frac{q}{p}\,\overline{\rho}\,(\psi K_S)\right] \ , \\ + 2\ \mathrm{Im}\left[\frac{q}{p}\,\overline{\rho}\,(\psi K_S)\right] e^{-1/2\Delta\Gamma t}\ \sin\Delta m t \Big\} \ , \end{aligned} \tag{45}$$

$$\begin{aligned} \overline{G} = |A(\psi K_S)|^2 \Big\{ (1+e^{-\Delta\Gamma t}) + (1-e^{-\Delta\Gamma t})\ \mathrm{Re}\left[\frac{q}{p}\,\overline{\rho}\,(\psi K_S)\right] \ , \\ - 2\ \mathrm{Im}\left[\frac{q}{p}\,\overline{\rho}\,(\psi K_S)\right] e^{-1/2\Delta\Gamma t}\ \sin\Delta m t \Big\} \ . \end{aligned} \tag{46}$$

If finally, as expected, $\Delta\Gamma \ll \Gamma = \frac{1}{2}(\Gamma_1 + \Gamma_2)$, very simple expressions emerge:

$$G = 2|A(\psi K_S)|^2\ (1 + AS\ \sin\Delta m t)\ , \tag{47}$$

$$\overline{G} = 2|A(\psi K_S)|^2\ (1 - AS\ \sin\Delta m t)\ , \tag{48}$$

$$AS = \mathrm{Im}\left[\frac{q}{p}\,\overline{\rho}\,(\psi K_S)\right] \quad . \tag{49}$$

Comparing Eqs. (47) and (48) also shows that this CP asymmetry can be observed only if B° decays can be distinguished from $\overline{B}^\circ$ decays at least partially (unless one is able to detect $e^{-\Delta\Gamma t}$ effects). This exemplifies the general need for "flavor tagging" in these studies—a topic to be addressed in Act IV.

For final states that are *not* CP eigenstates one typically finds $|\overline{\rho}(f)|^2 \gg 1$ or $\ll 1$. For instance,

$$|\overline{\rho}\ (D^-\pi^+)|^2 \sim \mathcal{O}\left(\left|\frac{V(ub)V(cd)}{V(cb)V(ud)}\right|^2\right) \ll 1\ , \tag{50}$$

$$|\overline{\rho}(D^+\pi^-)|^2 \sim \mathcal{O}\left(\left|\frac{V(cb)V(ud)}{V(ub)V(cd)}\right|^2\right) \gg 1\quad . \tag{51}$$

However, an inspection of Eq. (20) shows that a CP asymmetry is greatly damped in both cases. The optimal value is actually $|\overline{\rho}(f)|^2 \sim 1$, which is intuitively clear since one searches for an interference between the two amplitudes $A(f)$ and $\overline{A}(f)$ and maximal interference is attained for $|A(f)| = |\overline{A}(f)|$.

As already indicated, and we will discuss it in more detail later on, $|\overline{\rho}(f)| = 1$ is a natural value when f is a CP eigenstate. However such a scenario can be reached also for other final states, though at the cost of quite a small branching ratio.

Example 4: $B^\circ \rightarrow \overline{D}^\circ\ K_S$ vs. $\overline{B}^\circ \rightarrow D^\circ\ K_S$

Hadronic complications arise here that will be addressed later. Very roughly one finds

$$\begin{aligned} |\overline{\rho}(\overline{D}^\circ\ K_S)| &\sim \mathcal{O}\left(\left|\frac{V(ub)V(cs)}{V(cb)V(us)}\right|\right)\ , \\ &\sim \mathcal{O}\left(\sqrt{\rho^2+\eta^2}\right) \sim \mathcal{O}(1) \sim |\rho(D^\circ\ K_S)|\ , \end{aligned} \tag{52}$$

and thus

$$\begin{aligned} G &\sim |A(\overline{D}^\circ\ K_S)|^2\ \left\{1+\ \mathrm{Im}\left[\frac{q}{p}\overline{\rho}\ (\overline{D}^\circ\ K_S)\right]\sin\Delta mt\right\}\ , \\ \overline{G} &\sim |\overline{A}(D^\circ\ K_S)|^2 \left\{1+\mathrm{Im}\left[\frac{p}{q}\ \rho\ (D^\circ\ K_S)\right]\sin\Delta mt\right\}\ , \\ \mathrm{Im}\left[\frac{p}{q}\ \rho\ (D^\circ\ K_S)\right] &\sim -\ \mathrm{Im}\left[\frac{q}{p}\ \overline{\rho}\ (\overline{D}^\circ\ K_S)\right]\ . \end{aligned} \tag{53}$$

Two remarks on time-integrated quantities will conclude this subsection:

- Since the CP asymmetries in flavor specific decays are ***independent*** of (proper) time, one can integrate over all decay times with impunity, i.e., without diluting a possibly existing asymmetry.
- CP asymmetries in flavor nonspecific decays exhibit a very special time dependence involving terms like $e^{-\Gamma t}\sin\Delta mt$. Integrating over all decay times yields

$$\Gamma\int_0^\infty dte^{-\Gamma t}\sin\Delta mt = \frac{x}{1+x^2} \quad , \qquad x = \frac{\Delta m}{\Gamma} \quad . \tag{54}$$

Obviously mixing has to occur, i.e., $x \neq 0$, for this effect to become observable. Yet too much mixing, i.e., $x \gg 1$ leads to a suppression $\sim 1/x$: this is easily understood by remembering that in that case one sums over B° as well as $\overline{B}^\circ$ decays which necessarily leads to a suppression of the asymmetry.

So far our discussion has been completely phenomenological; in the next two subsections we discuss how the various CP asymmetries can be classified in a more systematic way.

(B) CP Asymmetries and Final State Interactions

When two different amplitudes contribute to the decay of a beauty hadron B into a final state f, one writes down for the amplitude

$$A = \langle f|\mathcal{L}(\Delta B = 1)|B\rangle = \langle f|\mathcal{L}_1|B\rangle + \langle f|\mathcal{L}_2|B\rangle = g_1 M_1 e^{i\alpha_1} + g_2 M_2 e^{i\alpha_2} \; . \tag{55}$$

M_1, M_2 denote the matrix elements for the weak transition operators $\mathcal{L}_1, \mathcal{L}_2$ with the KM parameters g_1, g_2 and the strong (or electromagnetic) phase shifts α_1, α_2 factored out. The amplitude for the CP conjugate decay $\overline{B} \to \bar{f}$ then reads:

$$\overline{A} = \langle \bar{f}|\mathcal{L}(\Delta B = 1)|\overline{B}\rangle = g_1^* M_1 e^{i\alpha_1} + g_2^* M_2 e^{i\alpha_2} \; , \tag{56}$$

where it is the ***CP invariance of the strong forces*** that fixes the phase shifts in Eq. (55) following the usual prescription of field theory.

Comparing Eqs. (55) and (56), one obtains

$$\Gamma(B \to f) - \Gamma(\overline{B} \to \bar{f}) \propto \text{ Im } g_1^* g_2 \sin(\alpha_1 - \alpha_2) M_1 M_2 \quad . \tag{57}$$

Such a CP asymmetry is usually called a ***direct CP violation*** since it involves only decay, i.e., $\Delta B = 1$ processes.

Two conditions have thus to be met simultaneously for such an asymmetry to show up (in the absence of $B^\circ - \overline{B}^\circ$ mixing):

(*i*) There has to be a relative complex phase between the two weak couplings g_1 and g_2. Within the Standard Model this means that the branching ratios for such modes are suppressed by small, or even tiny, KM parameters, as will be illustrated by the examples given later. It is not necessarily true anymore in the presence of substantial contributions due to *New Physics*.

(*ii*) Nontrivial phase shifts $\alpha_1 \neq \alpha_2$ have to be generated by the strong forces. In principle, this could be done by the electromagnetic forces as well; in practice, however, such effects are far too small.

These two requirements make it obvious that only nonleptonic beauty decays have a realistic chance to exhibit direct CP violation, but not the semileptonic decays we had used previously as example.

Satisfying condition (ii) is not expected to pose a severe restriction in principle. Invoking the concept of duality, one can argue quite forcefully that *inclusive* decay rates of hadrons as heavy as B mesons should hardly be affected by final state interactions (= FSI) since a summation over many channels is involved. Yet the situation is quite different for *exclusive* decays, as can be illustrated by the following qualitative observations.

An important part of the effect of FSI is given by on-mass-shell rescattering processes which modify the bare decay amplitude $A^\circ(B \to f)$ as follows:

$$A(B \to f) = \sum_{f'} \left(S^{\frac{1}{2}}\right)_{ff'} A^\circ(B \to f') \ , \tag{58}$$

where $S^{1/2}$ is the square root of the *strong* interaction S matrix of definite isospin and f' denotes the intermediate states. This will introduce absorption, phase shifts and channel mixing. There is no reason why these effects will generally be small in B decays.

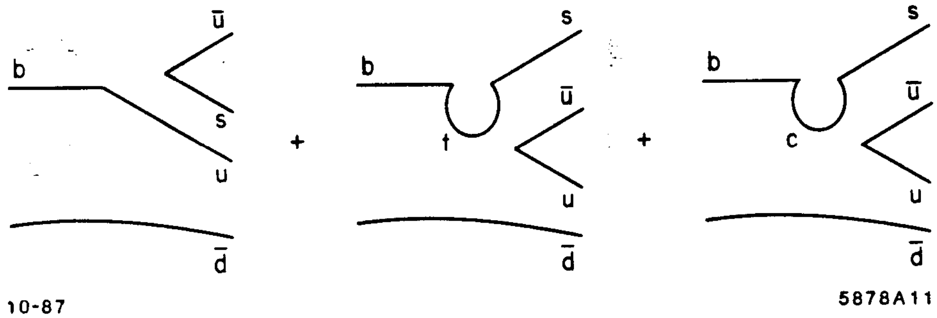

Fig. 3. Quark diagrams for $(b\bar{q}) \rightarrow s\bar{u}u\bar{q}$ decays.

The qualitative and sketchy nature of this discussion[5] points to the real problem that condition (ii) poses: while rescattering, etc., is bound to occur, there exists at present no scheme whatsoever for dealing with FSI in a numerically reliable fashion.

An interesting scenario[6] in our judgment is provided by contributions from the Penguin operator which is shown by the last two diagrams in Fig. 3:

$$0_P = \frac{G_f}{\sqrt{2}} \left[\bar{s}\, \gamma_\mu (1 - \gamma_5) \frac{t^a}{2}\, b \right] \left(\sum_q \bar{q}\, \gamma_\mu \frac{t^a}{2}\, q \right) , \tag{59}$$

with t^a denoting the SU(3) color matrices. This operator emerges when $\mathcal{L}(\Delta B = 1)$ is renormalized from the scale M_W down to m_b:

$$\mathcal{L}_{eff} \left(\Delta B = 1 \, , \, m_b^2 \right) = c_P \, 0_P + \ldots , \tag{60}$$

where the dots denote the usual current–current operators. The coefficient c_P contains, of course, the KM parameters, namely

$$c_P = V(q\,b) V^*(qs)\, \tilde{c}_q \quad , \tag{61}$$

with $q = c, t$. For $q = u$ one encounters a strong Cabibbo suppression $\sim \lambda^2$; Penguin transitions $b \rightarrow d$ instead of $b \rightarrow s$, as in Eq. 61, are reduced by λ.

The $\widetilde{c}_q$ in Eq. (61) reflects the renormalization process; rough estimate yields

$$\widetilde{c}_q \sim \frac{\alpha_s(m_b^2)}{3\pi} \log \frac{M_W^2}{m_b^2} \sim 0.07 \quad . \tag{62}$$

The Penguin transitions $b\bar{q} \rightarrow s\,\bar{q}_1 q_1 \bar{q}$ produce charmless final states; for the *inclusive* width one estimates

$$\Gamma_P(B \rightarrow \text{noncharm}) \sim |c_P|^2 \, \frac{G_f^2 m_b^5}{192\pi^3} \, \frac{N_c^2 - 1}{8N_c} N_f \quad , \tag{63}$$

where the color, spin and flavor structure of 0_P is taken into account; $N_c[N_f]$ denotes the numbers of colors [flavors]. The resulting branching ratio is of order 10^{-3}. More relevant for our discussion is the observation that the Penguin transitions can interfere with the KM-suppressed tree level spectator process $b\bar{q} \rightarrow s\bar{u}u\bar{q}$, as shown in Fig. 3. Its branching ratio is given roughly by $\sim 2 \cdot (|V(ub)|^2/|V(cb)|^2)\,|V(us)|^2 \sim \mathcal{O}(\text{few } \times 10^{-3})$, i.e., a quite comparable rate. For the total amplitude one can then write down schematically

$$\begin{aligned} A\left(\overline{B}_q \rightarrow [S=-1]\right) = V(ub)V^*(us)M_{\text{spect}} + V(tb)V^*(ts)M_P^{(t)} \\ + V(cb)V^*(cs)M_P^{(c)} e^{i\alpha_c} \end{aligned} \quad , \tag{64}$$

where the matrix elements M have been chosen real as in Eq. (55). For illustrative purposes only, we have made a somewhat artificial separation between $M_p^{(t)}$ and $M_p^{(c)}$ where $M_p^{(q)}$ denotes the Penguin transition with *internal* quark q.

Both requirements for observable CP asymmetries are apparently fulfilled:

ad(i) The relevant KM parameters exhibit large complex phases, in particular for $V(ub)$, see Eq. (7). This is not surprising since all three families contribute to these decays.

ad(ii) The amplitude for the tree-level Spectator process is real. The (nonlocal) Penguin operator with internal charm lines can produce the required FSI: since $2m_c < m_b$, they can be on-shell, thus producing an imaginary part.

Yet big uncertainties emerge when one attempts to transfer these qualitative statements into quantitative ones. At present, we have to content ourselves with rough order of magnitude estimates like those expressed in Eq. (62).

As far as FSI or α_c in Eq. (64) are concerned one can invoke the usual simple substitution

$$\log \frac{M_W^2}{m_b^2} \to \log \frac{M_W^2}{M_b^2} + i\pi \quad , \tag{65}$$

to obtain a ballpark estimate. However one has to keep in mind that this procedure ignores the usual soft FSI — a point we will return to in Act II.

All these considerations lacking as they might be in precision apply to *inclusive* decays only. New elements and questions have to be considered when one is dealing with *exclusive* modes. A more detailed discussion will be given in Act II. Suffice to say at this point that one expects quite generally and confidently that some representatives of this class of CP asymmetries will exceed the 10^{-3} level in a significant manner.

(C) CP Asymmetries and B°–$\overline{B}^\circ$ Mixing

Proper care has to be used in this discussion, mainly because q/p as defined by Eq. (17) is *not* an observable by itself since it is not invariant under the transformation

$$|\overline{B}^\circ\rangle_\circ \to e^{i\alpha}|\,\overline{B}^\circ\rangle_\circ \quad . \tag{66}$$

Equation (66) describes just a change in the *phase convention* for $\overline{B}^\circ$ which is arbitrary. For the *flavor* eigenstates B° and $\overline{B}^\circ$ are *defined* by the strong interactions only, which leaves the relative phase between B° and $\overline{B}^\circ$ undetermined:

$$|\overline{B}^0\rangle \equiv e^{-i\alpha} CP|B^0\rangle \ .$$

In Eq. (41) we had set $\alpha = 0$ for convenience.

Diagonalizing the $B^\circ - \overline{B}^\circ$ mass matrix yields

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \quad , \quad M_{12} - \frac{i}{2}\Gamma_{12} = \langle B^\circ|\mathcal{L}(\Delta B = 2)|\overline{B}^\circ\rangle \quad . \tag{67}$$

The transformation (66) then leads to

$$(M_{12}, \Gamma_{12}) \to e^{i\alpha}(M_{12}, \Gamma_{12}) \, , \qquad \frac{q}{p} \to e^{-i\alpha}\,\frac{q}{p} \quad . \tag{68}$$

Nevertheless, q/p has physical content:

(*i*) Its modulus is obviously invariant under (66). Rewriting it as follows

$$\left|\frac{q}{p}\right|^4 = 1 + \frac{2|\Gamma_{12}|^2 \mathrm{Im}\frac{M_{12}}{\Gamma_{12}}}{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 - |\Gamma_{12}|^2 \mathrm{Im}\frac{M_{12}}{\Gamma_{12}}}$$

$$= 1 + \frac{2|\frac{\Gamma_{12}}{M_{12}}| \sin\phi(\Delta B = 2)}{1 + \frac{1}{4}|\frac{\Gamma_{12}}{M_{12}}|^2 - |\frac{\Gamma_{12}}{M_{12}}|\sin\phi(\Delta B = 2)} \tag{69}$$

$$\phi(\Delta B = 2) = \arg\frac{M_{12}}{\Gamma_{12}} \quad ,$$

one realizes two things:

- $\phi(\Delta B = 2)$ measures the strength of CP violation in $B^\circ - \overline{B}^\circ$ mixing in an unambiguous way.

- $|q/p|$, however, provides a poor handle on it since one expects quite generally and independently of the strength of CP violation (as discussed in Act II):

$$|\frac{\Gamma_{12}}{M_{12}}| \ll 1$$

  and thus

$$\left|\frac{q}{p}\right| \simeq 1 + \frac{1}{2}\left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi(\Delta B = 2) \quad . \tag{70}$$

In the K°–$\overline{K}^\circ$ system, on the other hand, $|\frac{\Gamma_{12}}{M_{12}}| \simeq 1$ is known to hold—yet this is due to accidental reasons: K mesons are still quite light relative to most hadrons; $K_S[K_L]$ decays are therefore dominated by two [four] channels. It is this paucity of available final states that generates $\Gamma_S \gg \Gamma_L$—a result that is further enhanced by the proximity of the three pion threshold to the kaon mass.

(*ii*) Equation (20) shows that the physical content of q/p is not completely exhausted by the relation (69): Im $(q/p)\,\bar{\rho}(f)$ denotes another observable characterizing CP violation.[7] For

$$\bar{\rho}(f) = \frac{\langle f|\mathcal{L}(\Delta B = 1)|\overline{B^\circ}\rangle}{\langle f|\mathcal{L}(\Delta B = 1)|B^\circ\rangle} \rightarrow e^{i\alpha}\bar{\rho}(f) \tag{71}$$

under (66) and thus [see (68)]

$$\frac{q}{p}\bar{\rho}(f) \rightarrow \frac{q}{p}\bar{\rho}(f) \quad . \tag{72}$$

Accordingly, one can define a new phase

$$\Phi_{CPV}(f) \equiv \arg\left[\frac{q}{p}\bar{\rho}(f)\right] \quad . \tag{73}$$

It reflects the combined effect of $\Delta B = 2$–q/p –and $\Delta B = 1$–$\bar{\rho}(f)$–transitions. As long as this quantity is studied in a *single* decay mode only, one cannot disentangle the two effects in a meaningful way—see Eqs. (68) and (71). This can be achieved only if (at least) two different decay modes f_1 and f_2 are examined: if

$$\Phi_{CPV}(f_1) \neq \Phi_{CPV}(f_2) = \dots \, , \tag{74}$$

is found, then one has discovered not "just" CP violation—direct CP violation has been established as well:

$$\arg\frac{\bar{\rho}(f_1)}{\bar{\rho}(f_2)} = \Phi_{CPV}(f_1) - \Phi_{CPV}(f_2) \equiv \phi(\Delta B = 1,\, f_1 - f_2) \quad . \tag{75}$$

If, on the other hand, a universal phase is found for different classes of decay modes

$$\Phi_{CPV}(f_1) = \Phi_{CPV}(f_2) = \dots \, , \tag{76}$$

then one has uncovered a *superweak* scenario of CP violation. For in that case there exists a phase convention such that all $\bar{\rho}(f_i)$ are real.

(D) On the Sign of CP Asymmetries

The asymmetries produced by $\sin(\Delta m t)\mathrm{Im}\left[(q/p)\,\bar{\rho}(f)\right]$ are characterized by their magnitude and their sign. Both will be discussed in detail later on; here we want to make qualitative remarks only:

(*i*) Neither the size nor the sign of Δm has an intrinsic connection with CP violation. Δm can be measured in $B^\circ - \overline{B}^\circ$ mixing studies; however, it enters via a $\cos\Delta m t$ function—thus its sign cannot be measured there (without actually observing CP violation). Regeneration experiments that allowed to determine $\Delta m_K = m(K_L) - m(K_S) > 0$ seem not feasible here.

The question on the sign of $\Delta m = m_2 - m_1$ actually goes deeper: the subscripts 1 and 2 are just labels at this point without physical meaning (apart from $m_2 \neq m_1$). The proper procedure to follow is like in the K° case:

- define the mass eigenstate B_2 such that $\Gamma_1 > \Gamma_2$;
- check whether $m_2 > m_1$, or $m_2 < m_1$.

For practical reasons, however, such a program is unlikely to be executed here:

– it is doubtful that one will be able to extract $\Delta\Gamma$ from the data;

– as outlined above, one will not be able to measure the sign of Δm.

(*ii*) The *relative* sign between the CP asymmetry in two different decay modes is determined by $\bar{\rho}(f)$ and does not depend on Δm or q/p. When f is a CP eigenstate, then its CP parity determines the sign of $\bar{\rho}(f)$:

$$\bar{\rho}(f) = \frac{\langle f_\pm|\mathcal{L}(\Delta B = 1)|\overline{B}^\circ\rangle}{\langle f_\pm|\mathcal{L}(\Delta B = 1)|B^\circ\rangle} = \pm e^{-i\alpha}\,\frac{\langle f_\pm|\mathcal{L}^{CP}|B^\circ\rangle}{\langle f_\pm|\mathcal{L}|B^\circ\rangle}\ , \tag{77}$$

with $\mathcal{L}^{CP}$ denoting the CP transformed version of $\mathcal{L}$ and α the arbitrary phase in defining $\overline{B}^\circ$. In Eq. (41) we had used the convention $\alpha = 0$. Therefore (if both $B \to f_+$ and $B \to f_-$ proceed via the same quark decay),

$$\frac{\bar{\rho}(f_-)}{\bar{\rho}(f_+)} < 0\quad . \tag{78}$$

A similar situation holds also when f is not a CP eigenstate. For a positive asymmetry in $\overline{B}^\circ \to D^+\pi^-$ vs $B^\circ \to D^-\pi^+$ translates into a negative asymmetry in $\overline{B}^{\circ} \to D^+\pi^-\pi^\circ$ vs. $B^\circ \to D^-\pi^+\pi^\circ$ (at least for soft π^0) since

$$CP|\pi^\circ\rangle = -|\pi^\circ\rangle \ , \tag{79}$$

$$\begin{aligned} \langle D^+\pi^-|\mathcal{L}|\overline{B}^{\circ}\rangle &= e^{-i\alpha}\langle D^-\pi^+|\mathcal{L}^{CP}|B^\circ\rangle \ , \\ \langle D^+\pi^-\pi^\circ|\mathcal{L}|\overline{B}^{\circ}\rangle &= -e^{-i\alpha}\langle D^-\pi^+\pi^\circ|\mathcal{L}^{CP}|B^\circ\rangle \quad . \end{aligned} \tag{80}$$

Equivalently, one finds

$$\langle D^+\rho^-|\mathcal{L}|\overline{B}^{\circ}\rangle = -e^{-i\alpha}\langle D^-\rho^+|\mathcal{L}^{CP}|B^\circ\rangle \ ,$$

since D and ρ have to form a P wave configuration.

Synopsis of Act I

The decays of neutral beauty mesons allow to study the full range of possible CP asymmetries:

(*i*) CP asymmetries that require the intervention of nontrivial final state interactions: there are rare nonleptonic decay modes which could—under favorable conditions—exhibit CP asymmetries of up to 10%.

(*ii*) CP asymmetries purely in $\Delta B = 2$ transitions as could emerge in semileptonic B° decays. Their size is determined by the (supposedly tiny) deviation of $|q/p|$ from unity.

(*iii*) CP asymmetries in such nonleptonic channels that are common to B° and to $\overline{B}^{\circ}$ decays; CP eigenstates provide particularly simple examples of this category. The intrinsic strength of CP violation is expressed in terms of $\mathrm{Im}\,(q/p\,\bar{\rho}(f))$ and thus combines the effects of both $\Delta B = 1$ and $\Delta B = 2$ transitions. Huge CP asymmetries of around 5 to 30% are expected. The decay rate evolution in proper time would provide striking signatures; on the other hand, the need for flavor tagging is almost inescapable.

ACT II. THE LIKELY HERO: B_d AND ITS DECAYS

The discussion given above was rather general and qualitative in nature. We are going to make it more specific by concentrating first on B_d decays.

(A) CP Asymmetries and B_d–$\overline{B}_d$ Mixing

Before a real experiment can be designed, one has to address the following questions:

(i) Which are the promising decay modes and what branching ratios do they command?

(ii) How large a deviation from CP invariance does one expect?

One would like to give definitive, quantitative answers to all of these questions. This is, however, not possible at present—a sad, yet hardly surprising realization considering that beauty physics has not left its adolescent phase yet. To cite but one example: only a few fistful of B decays have been reconstructed. What we can and will do instead is

– to present semiquantitative scenarios and

– to state how they can be refined in due course.

There are five types of parameters that enter into an analysis of $B^\circ, \overline{B}^\circ \to f, \bar{f}$:

(i) $|q/p|$;

(ii) $\mathrm{Im}\,(q/p\ \bar{\rho}_f)$.

These two quantities are intrinsically connected with CP violation.

(iii) $\Delta m/\Gamma$;

(iv) $\Delta\Gamma/2\Gamma$.

These two quantities describe the strength of $B^\circ - \overline{B}^\circ$ mixing; as such, they are intrinsically independent of CP violation, yet are often essential in making CP asymmetries observable.

(v) The branching ratios $\mathrm{BR}(B \to f)$, etc.

1) Estimates on B°–$\overline{B}^\circ$ Mixing

(a) $\Delta\Gamma$:

Employing quark box diagrams (see Figs. 4 and 5) one obtains[11]

$$\left|\frac{\Delta\Gamma}{\Delta m}\right| \simeq \frac{3\pi}{2}\,\frac{m_b^2}{m_t^2}\,\frac{1}{E'\left(\dfrac{m_t^2}{M_W^2}\right)} \quad , \qquad (81)$$

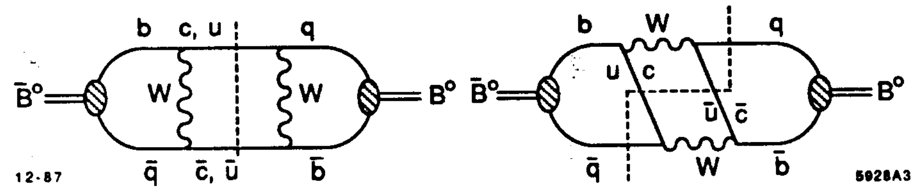

Fig. 4. Quark box diagrams for $\Delta\Gamma$.

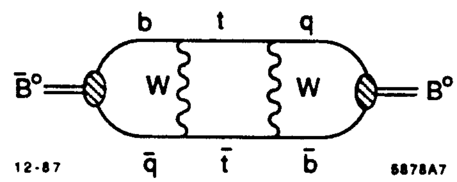

Fig. 5. Quark box diagrams for Δm.

where E' is a smooth and slowly decreasing function of m_t^2/M_W^2 which reflects the contraction of the W boson lines[12]

$$E'(x) = 1 - \frac{3}{4}\frac{x+x^2}{(1-x)^2} - \frac{3}{2}\frac{x^2}{(1-x)^3}\, log\, x\,, \qquad (82)$$

$$E'(0) = 1 \quad , E'(1) = \frac{3}{4} \quad , E'(\infty) = \frac{1}{4} \quad .$$

For $m_t \simeq 50$ GeV, one concludes from Eq. (81)

$$\left|\frac{\Delta\Gamma}{\Delta m}\right| \sim 0.05 \quad , \qquad (83)$$

which is a small number.

Another quite different and actually complementary argument can be given that arrives at roughly the same conclusion: There are quite a few common decay channels of B_d and $\overline{B}_d$ mesons; they can produce a nonvanishing $\Delta\Gamma$. However, they are at least Cabibbo-suppressed—like like $B_d, \overline{B}_d \to D\overline{D}$—or contribute with alternating signs to $\Delta\Gamma = \Gamma_2 - \Gamma_1$. For individual channels, one guestimates contributions of order 10^{-3} at most and thus arrives at a rather conservative bound

$$\frac{\Delta\Gamma}{\Gamma} \lesssim \mathcal{O}(1\%) \quad . \qquad (84)$$

(b) Δm:

Since $\Delta\Gamma \ll \Delta m$, the mixing rate is determined by Δm. For the *time-integrated* B_d decay rate into "wrong-sign" leptons, one can then write down

$$r = \frac{\Gamma(B_d \to \ell^- x)}{\Gamma(B_d \to \ell^+ x)} \simeq \frac{x^2}{2+x^2} \quad , \qquad x = \frac{\Delta m}{\Gamma} \quad . \tag{85}$$

From the ARGUS data[10] on like-sign di-lepton events in $\Upsilon(4S) \to B\overline{B}$, one infers $x = 0.73 \pm 0.18$. For our numerical estimates on CP asymmetries we will use

$$x = \frac{\Delta m}{\Gamma}(B_d) \simeq 0.5 - 1.0 \quad . \tag{86}$$

Future experiments should allow us to determine x with much better accuracy. This value for the ratio between the mixing rate and the decay rate is (pleasantly) large. Nevertheless, the Standard Model can (at our present level of understanding) accommodate Eq. (86) as long as top quarks are sufficiently heavy, i.e.,[19]

$$m_t \gtrsim 50 \text{ GeV} \quad , \tag{87}$$

another numerical input into our analysis. It is of relevance for later on to note that $\Delta m(B_d)$ is a steeply increasing function of the top mass: to first approximation $\Delta m(B_d) \propto (m_t/M_W)^2$.

2) Estimates on the Intrinsic Strength of CP Violation

(a) $|q/p|$:

With $|\Gamma_{12}| \ll |M_{12}|$ one concludes from Eq. (69):

$$\left|\frac{q}{p}\right| \simeq 1 + \frac{1}{2}\left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi(\Delta B = 2) \quad . \tag{88}$$

Computing $\phi(\Delta B = 2)$ in the quark box ansatz, one finds[8] (ignoring QCD corrections):

$$M_{12} \simeq \frac{G_F^2 B_B f_B^2 m_B m_t^2}{12\pi^2} \, E'\left(\frac{m_t^2}{M_W^2}\right) \, \xi_t^2 \; , \tag{89}$$

$$\Gamma_{12} \simeq \frac{G_F^2 B_B f_B^2 m_B^3}{8\pi} \left[\xi_c^2 P(cc) + \xi_u^2 P(uu) + 2\xi_c \xi_u P(uc)\right] \; , \tag{90}$$

where

$$\xi_i = V(ib)V^*(id), \quad i = u, c, t \; ,$$

The functions $P(cc)$, $P(uu)$, $P(uc)$ denote the weight of the various intermediate configurations, see Fig. 4. Employing those diagrams one can express these functions in terms of quark masses:

$$\begin{aligned} P \;\simeq\; & \sqrt{\left(1 - \frac{(m_1+m_2)^2}{m_B^2}\right)\left(1 - \frac{(m_1-m_2)^2}{m_B^2}\right)} \\ & \left(1 - \frac{1}{3}\frac{m_1^2+m_2^2}{m_B^2} - \frac{2}{3}\frac{(m_1^2-m_2^2)^2}{m_B^4}\right) \; . \end{aligned} \tag{91}$$

With $m_u^2 \ll m_c^2 \ll m_b^2$ one gets

$$P(uu) \simeq 1 \tag{92}$$

$$P(uc) \simeq 1 - \frac{4}{3}\frac{m_c^2}{m_b^2} \tag{93}$$

$$P(cc) \simeq 1 - \frac{8}{3}\frac{m_c^2}{m_b^2} \; . \tag{94}$$

Hence

$$\Gamma_{12} \simeq \frac{G_F^2 B_B f_B^2 m_B^3}{8\pi} \left[((\xi_u + \xi_c)^2 - \frac{8}{3}\frac{m_c^2}{m_b^2} \, \xi_c(\xi_u + \xi_c)\right] \; . \tag{95}$$

Since

$$\xi_u + \xi_c = -\xi_t$$

holds for three families one arrives at

$$\frac{\Gamma_{12}}{M_{12}} \simeq \frac{3\pi}{2} \frac{m_B^2}{m_t^2 E'\left(\frac{m_t^2}{M_W^2}\right)} \left(1 + \frac{8}{3}\frac{m_c^2}{m_b^2}\frac{\xi_c}{\xi_t}\right) \quad , \tag{96}$$

$$\sin\phi(\Delta B = 2) \simeq \frac{8}{3}\frac{m_c^2}{m_b^2}\, \mathrm{Im}\frac{(V(cb)V^*(cd))}{(V(tb)V^*(td))} \quad . \tag{97}$$

The factor m_c^2/m_b^2 is produced by the GIM mechanism and actually reads $(m_c^2 - m_u^2)/m_b^2$. For there can be no CP violation in the KM scheme when $m_c = m_u$. Using the estimate given above for $|\Gamma_{12}/M_{12}|$, one finds in the Wolfenstein parametrization

$$\left|\frac{q}{p}\right| \simeq 1 + 1.3 \times 10^{-3} \frac{\eta}{(1-\rho)^2 + \eta^2} \text{ UNC} \quad . \tag{98}$$

The factor UNC appearing here represents the uncertainty inherent in this calculation. In the spirit of duality we use the quark box ansatz to estimate the inclusive rate for the reaction $B^0 \longleftrightarrow \overline{B}^0$. As discussed in detail in Ref. 11, this should provide a correct order of magnitude estimate. From a comparison of different hadronization models one infers the uncertainty in our estimate of $P(cc)$ vs. $P(uu)$ vs. $P(uc)$ not to exceed 30%. This uncertainty is greatly enhanced due to the cancellations taking place in (95) and we estimate $UNC \leq 2-3$.

Therefore [see Example 2 in Act I, Eqs. (35) and (36)],

$$\begin{aligned} a_{SL} &= \frac{\Gamma\left[B_d(t) \to \ell^-\nu X\right] - \Gamma\left[\overline{B}_d(t) \to \ell^+\nu X\right]}{\Gamma\left[B_d(t) \to \ell^-\nu X\right] + \Gamma\left[\overline{B}_d(t) \to \ell^+\nu X\right]} \\ &= \frac{\left|\frac{q}{p}\right|^2 - \left|\frac{p}{q}\right|^2}{\left|\frac{q}{p}\right|^2 + \left|\frac{p}{q}\right|^2} \lesssim 6 \times 10^{-3} \frac{\eta}{(1-\rho)^2 + \eta^2} \lesssim 10^{-3} \quad , \end{aligned} \tag{99}$$

i.e., this "pure superweak" CP asymmetry is expected to be tiny due to two concurrent reasons both of which are intimately connected to the KM ansatz:

- $\left|(\Gamma_{12}/M_{12})\right| \simeq (\Delta\Gamma/\Delta m) \ll 1$, and
- $\phi(\Delta B = 2) \propto (m_c^2/m_b^2)$.

The authors of Ref. 24 allow for much larger asymmetries, i.e., $a_{SL} \leq 10^{-2}$. We find their estimate overly cautious; nevertheless one should keep in mind that our estimate, Eq. (98), is not the result of a mathematical theorem. Future data on B decays will sharpen such a prediction.

New Physics will typically introduce new heavy quanta; since their mass would exceed m_b, they can affect only M_{12}, but not Γ_{12}. Thus they might well enhance $\phi(\Delta B = 2) = \arg(M_{12}/\Gamma_{12})$ and values like

$$a_{SL}(\text{New Physics}) \sim \mathcal{O}(1\%) \tag{100}$$

are at least conceivable. Yet, even in that case, it is quite unclear whether one can suppress systematic uncertainties sufficiently, in particular since the signal is not singled out by a special evolution in proper time.

(b) Im $[q/p\ \bar{\rho}(f)]$:

A quark level treatment of beauty decays should—according to the concept of duality—yield quite a good description of inclusive rates, when a (not too small) number of hadronic channels contribute. It was in this spirit that we estimated $\Delta m, \Delta\Gamma$ and q/p.

This procedure is however inadequate for dealing with $\overline{\rho}(f)$. The equality of inclusive B and $\overline{B}$ decay rates is frequently ensured by CPT invariance the impact of which goes beyond $\Gamma(B) = \Gamma(\overline{B})$: it enforces the equality already for whole subsets of decay channels, namely those that are "closed" under the effects of strong interactions, like semileptonic widths, etc. Therefore we are forced to analyse more or less exclusive modes.

Unfortunately this opens a Pandora's box since for exclusive modes one has to be concerned about the complexities caused by final state interactions.

Having stated this general caveat we hasten to add that predictions can be made with a confidence level that ranges from excellent to moderate:

(*i*) The transition $b \to c\bar{c}s$ is driven by an *isosinglet* weak operator; therefore it is described by a single amplitude as far as isospin is concerned. Yet even so there are channel mixing processes like

$$B_d \to D_s^+ D^- \Longrightarrow \psi K_S \tag{101}$$

that could intervene and introduce strong phase shifts, absorption, etc. Yet it is still a *single* quark level process that drives these processes. Therefore $g_1 = g_2$ in the notation of Eqs. (55)–(57) and

$$
\begin{aligned}
&\left|A\left(\psi K_S\right)\right| = \left|A\left(\psi K_S\right)\right| \, , \\
&\left|\bar{\rho}\left(\psi K_S\right)\right| = 1 \quad .
\end{aligned}
\tag{102}
$$

$(q/p)\ \bar{\rho}\,(\psi K_S)$ therefore represents a unit vector in the complex plane that is given by KM parameters alone:

$$
\frac{q}{p}\,\bar{\rho}\,(\psi K_S) \equiv e^{i\Phi_{CPV}(\psi K_S)} \ . \tag{103}
$$

For similar reasons one expects

$$
|\bar{\rho}\,(\psi K_S \pi^\circ)\,| \simeq 1 \simeq |\bar{\rho}\left(D\overline{D}K_S\right)| \, , \tag{104}
$$

$$
-\frac{q}{p}\,\bar{\rho}\,(\psi K_S) \simeq \frac{q}{p}\,\bar{\rho}\,(\psi K_S \pi^\circ) \simeq \frac{q}{p}\,\bar{\rho}\left(D\overline{D}K_S\right) \ , \tag{105}
$$

where the minus sign enters in Eq. (105) because $CP|\psi K_S\rangle = -|\psi K_S\rangle$ $CP|\psi K_S\pi^\circ\rangle = |\psi K_S\pi^\circ\rangle$, etc.

(*ii*) The transition $b \to c\bar{c}d$ is Cabibbo-suppressed and it changes the isospin by half a unit. Even so, we estimate

$$
\begin{aligned}
&|\ \bar{\rho}\,(\psi\pi\pi)\,| \simeq 1 \simeq |\ \bar{\rho}\left(D\overline{D}\right)| \, , \\
&\Phi_{CPV}\,(\psi\pi\pi) \simeq \Phi_{CPV}\left(D\overline{D}\right) \simeq \Phi_{CPV}\left(D\overline{D}K_S\right) \quad .
\end{aligned}
\tag{106}
$$

The final states $D\overline{D}$, $\psi\pi\pi$ could in principle be reached by the transition $b \to u\bar{u}d$ when coupled with $c\bar{c}$ excitation from the vacuum. Yet this represents a highly suppressed process.

(*iii*) There are other quark level transitions that can generate CP eigenstates as final states:

- $b \to u\bar{u}d$ can produce

$$B_d \to \pi\pi \; , \tag{107}$$

$$B_d \to p\bar{p} \; . \tag{108}$$

The $\pi\pi$ pair evidently forms an even CP eigenstate; since protons on the other hand carry spin they can be either in a P- or S-wave configuration in Eq. (108):

$$B_d \begin{matrix} \nearrow [p\bar{p}]_S \\ \searrow [p\bar{p}]_P \end{matrix} \quad . \tag{109}$$

Since

$$\begin{aligned} CP \big| \, [p\bar{p}]_S \rangle &= - \big| \, [p\bar{p}]_S \rangle \; , \\ CP \big| \, [p\bar{p}]_P \rangle &= + \big| \, [p\bar{p}]_P \rangle \; . \end{aligned} \tag{110}$$

we realize that the final state in Eq. (108) will in general not possess a definite CP parity. As discussed in Act I this has important consequences for the *sign* of the CP asymmetry—a point we will return to later. Here we want to add that related problems are encountered in dealings with multibody B decays. This is briefly illustrated by one example:

$$B_d \to \pi^+\pi^-\pi^+\pi^- \; .$$

If it is produced via

$$B_d \to \rho^\circ \rho^\circ \to \left(\pi^+\pi^-\right)\left(\pi^+\pi^-\right) \quad ,$$

then it represents a CP eigenstate which is even [odd] if $\rho^0\rho^0$ forms an s[p] wave. If it is however due to

$$B_d \to A_1^+\pi^- \to \left(\pi^+\pi^-\pi^+\right)\pi^- \quad ,$$

then it does *not* represent a CP eigenstate. Furthermore a soft π^0 that escapes detection

$$B_d \to \pi^+\pi^-\pi^+\pi^- \left(\pi^\circ\right) \; .$$

would affect the CP parity of the final state.

• At first sight, it seems obvious that $b \to c\bar{u}d$ and $\bar{b} \to \bar{c}u\bar{d}$ are always distinguishable. Yet a closer look reveals that this is not necessarily the case—it is again due to K°–$\overline{K}^\circ$ mixing that a no-go theorem is circumvented, as shown by the following example:

$$\begin{array}{l} B_d \to \overline{D}^\circ \pi^+\pi^- \searrow \\ \overline{B}_d \to D^\circ \pi^+\pi^- \nearrow \end{array} (K_S\pi^\circ)_D\ \pi^+\pi^- \quad , \tag{111}$$

for when a neutral D meson decays into a K_S plus pions its identity as a D° or $\overline{D}^\circ$ remains undetermined as a matter of principle (unless the neutral D meson reveals its flavor by coming from the decay of a *charged* D^*).

There is no good reason to expect FSI to be negligible in the reactions (107), (108) and (111). In particular the process

$$B \to D\overline{D} \Longrightarrow \pi\pi$$

could well be quite important here since it proceeds via $b \to c$ rather than $b \to u$. Nevertheless,

$$|\, \overline{\rho}(\pi\pi)\, | \sim |\, \overline{\rho}(p\overline{p})\, | \sim |\, \overline{\rho}((K_S\pi's)_D\pi's\, | \sim\ 1 \tag{112}$$

are reasonable guestimates in the spirit of Buridan's conjecture.[21]

(*iv*) If there are two different quark level transitions to reach the same final state then those do not have to be CP eigenstates to be common to B_d and $\overline{B}_d$ decays. Two examples will suffice to illustrate such scenarios:

• The two quark transitions $b \to c\bar{u}d$ and $\bar{b} \to \bar{u}c\bar{d}$ produce for example $\overline{B}_d \to D^+\pi^-$ and $B_d \to \pi^-D^+$. Yet the second process is suppressed relative to the first one by small KM parameters

$$|\, \overline{\rho}(D^+\pi^-)\, | \sim \mathcal{O}\left(\frac{|\, V(cb)\, |}{|\, V(ub)V(cd)\, |}\right) \gg 1\ . \tag{113}$$

Similarly

$$|\, \overline{\rho}(D^-\pi^+)\, | \sim \mathcal{O}\left(\frac{|\, V(ub)V(cd)\, |}{|\, V(cb)\, |}\right) \ll 1\ . \tag{114}$$

Both of these cases are, as discussed before, very unfavorable for a CP asymmetry: the latter is suppressed by $1/\overline{\rho}$ or by $\overline{\rho}$ for $B_d(t) \rightarrow D^+\pi^-$ vs. $\overline{B}_d(t) \rightarrow D^-\pi^+$ or for $B_d(t) \rightarrow D^-\pi^+$ vs. $\overline{B}_d(t) \rightarrow D^+\pi^-$, respectively.

• Considerably larger CP asymmetries are expected in decay modes like $B_d, \overline{B}_d \rightarrow D^{*0}K_S, \overline{D}^{*0}K_S$ which proceed via the KM-suppressed transitions $b \rightarrow c\overline{u}s$ and $\overline{b} \rightarrow \overline{u}c\overline{s}$. More specifically one has in that case:

$$\begin{aligned}
\overline{A}(D^{o*}K_S) &= V(cb)\ V^*(us)\ M_1\ , \\
A(D^{o*}K_S) &= V^*(ub)\ V(cs)\ M_2\ , \\
\overline{A}(\overline{D}^{o*}K_S) &= V(ub)\ V^*(cs)\ M_2\ , \\
A(\overline{D}^{o*}K_S) &= V^*(cb)\ V(us)\ M_1\ ,
\end{aligned} \tag{115}$$

where we have factored out the KM parameters from the amplitudes; the "reduced" matrix elements M_1, M_2 contain the hadronization effects. In the factorization ansatz[9] one has $M_1 = M_2$ and therefore

$$\begin{aligned}
\mid \overline{\rho}(D^{*0}K_S) \mid &\simeq \mid \frac{V(cb)V(us)}{V(ub)V(cs)} \\
\mid &\simeq \mid \overline{\rho}(\overline{D}^{*0}K_S) \mid^{-1} \simeq \frac{1}{\sqrt{\rho^2+\eta^2}} \sim O(1)\ .
\end{aligned} \tag{116}$$

Two further observations should be made here: firstly, all decay modes of the neutral D mesons are acceptable here, not just $D^0 \rightarrow K_S + \pi's$ as it was the case in (111); secondly it is $K^0 - \overline{K}^0$ mixing one more time that allows interference between the two quark level transitions $b\overline{d} \rightarrow c\overline{u}s\overline{d}$ and $\overline{b}d \rightarrow \overline{u}c\overline{s}d$.

Now we are in a position to state our results which are given in Table I. A number of comments are in order here to elucidate its contents beyond what was already said.

Table I: Standard KM Predictions for B_d Decays

Quark level transition	Example of hadronic channel f	$\lvert\rho(f)\rvert$	$\eta(f)$
i) $b \to c\bar{c}s$	ψK_S	1	$-$
	$\psi K_S \pi^\circ$	$\simeq 1$	$\sim +$
	$D\overline{D}K_S$	$\simeq 1$	$\sim +$
ii) $b \to c\bar{c}d$	$\psi\pi\pi$	$\simeq 1$	$\sim +$
	$D\overline{D}$	$\simeq 1$	$+$
iii) $b \to u\bar{u}d$	$\pi\pi$	~ 1	$+$
	$[p\bar{p}]_S$	~ 1	$-$
	$[p\bar{p}]_P$	~ 1	$+$
iv) $b \to c\bar{u}d$	$D^\circ\pi^+\pi^- \to (K_S\pi^\circ)\pi^+\pi^-$	~ 1	$+$
v) $b \to c\bar{u}s$ and $\bar{b} \to \bar{u}c\bar{s}$	$\overline{D}^{\circ *} K_S$	$\simeq \mathcal{O}(1)$	$-$

$\sin\phi_{CPV}$

ad (i)

$$\mathrm{Im}\,\frac{(V^*(tb)V(td))^2}{|V(tb)V(td)|^2}\,\frac{(V(cb)V^*(cs))^2}{|V(cb)V(cs)|^2}\,\mathrm{F} \simeq -\frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2} \simeq \sin 2\Phi_1$$

ad (ii)

$$\mathrm{Im}\,\frac{(V^*(tb)V(td))^2}{|V(tb)V(td)|^2}\,\frac{(V(cb)V^*(cd))^2}{|V(cb)V(cd)|^2} \simeq -\frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2} \simeq \sin 2\Phi_1$$

ad (iii)

$$\mathrm{Im}\,\frac{(V^*(tb)V(td))^2}{|V(tb)V(td)|^2}\,\frac{(V(ub)V^*(ud))^2}{|V(ub)V(ud)|^2} \simeq -\frac{2\eta(\rho-\rho^2-\eta^2)}{((1-\rho)^2+\eta^2)\,(\rho^2+\eta^2)} \simeq -\sin 2\Phi_2$$

ad (iv)

$$\mathrm{Im}\,\frac{(V^*(tb)V(td))^2}{|V(tb)V(td)|^2}\,\frac{(V(cb)V^*(ud))^2}{|V(cb)V(ud)|^2}\,\frac{(V^*(cs)V(ud))^2}{|V(cs)V(ud)|^2}\,\mathrm{F} \simeq -\frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2} \simeq \sin 2\Phi_1$$

ad (v)

$$\mathrm{Im}\,\frac{(V^*(tb)V(td))^2}{|V(tb)V(td)|^2}\,\frac{V(ub)V^*(cs)V(cb)V^*(us)}{|V(ub)V(cs)V(cb)V(us)|}\,\mathrm{F} \simeq \frac{\eta(1-\rho^2-\eta^2)}{((1-\rho)^2+\eta^2)\,\sqrt{\rho^2+\eta^2}} \simeq -\sin(\Phi_1-\Phi_2)$$

(α) : A quantity like

$$K = [V^*(tb)V(td)]^2 \, [V(cb)V^*(cs)]^2 \tag{117}$$

is not invariant under changes in the *phase convention adopted for quark fields*: for

$$d \to d \;\exp\{i\delta_d\} \;,\; s \to s \;\exp\{i\delta_s\} \tag{118}$$

leads to

$$K \to K \;\exp\{(2i(\delta_d - \delta_s))\} \quad . \tag{119}$$

This change is compensated by

$$F \to \exp\{(2i(\delta_s - \delta_d))\} \, F \;, \tag{120}$$

since

$$F = \frac{\langle K_S|(\bar{d}s)\rangle\langle K_S|(\bar{s}d)\rangle^*}{|\langle K_S|(\bar{d}s)\rangle\langle K_S|(\bar{s}d)\rangle|} \quad . \tag{121}$$

The factor F explicitly denotes the projection onto K_S states that is essential for the potential existence of CP asymmetries in the transition classes (i), (iv) and (v) as emphasized before.

(β) : The factor $\eta(f)$ denotes the CP parity of the hadronic final state f, or more generally its properties under CP transformations. The symbol "$\sim +$" means that the available *phase space* strongly favors the CP even configuration.

(γ) : *Pure* phase space considerations give

$$\mathrm{BR}(B \to [p\bar{p}]_P) \simeq \beta^2 \; \mathrm{BR}(B \to [p\bar{p}]_S) \quad . \tag{122}$$

Since $\beta^2 = (v/c)^2 \simeq 0.87$ one expects the asymmetry to be *reduced* by almost an order of magnitude when one sums *indiscriminantly* over P- and S-wave configurations—unless FSI lead to a large suppression of one configuration relative to the other (which is quite conceivable, but certainly not guaranteed).

(δ) : Table I also shows quite clearly that B_d decays are expected to exhibit *direct* CP violation as well since Φ_{CPV} is *not* identical for all transitions.

(ϵ) : As far as the KM triangle is concerned, Fig. 1, one reads off from Table I that the three angles Φ_i are measured independently via $\sin 2\Phi_1$, $\sin 2\Phi_2$ and $\sin(\Phi_1 - \Phi_2) = \sin(2\Phi_1 + \Phi_3)$. A comprehensive study of B_d decays with sufficiently high sensitivity would thus allow to measure observables that *overdetermine* the Standard KM ansatz—a point we will return to in some detail in Act IV.

(ς) : There are still large uncertainties concerning the numerical size expected for the CP asymmetries:

- The KM parameters ρ and η are still allowed to vary within rather wide bounds.

- Hadronization effects (FSI like rescattering etc.) could well affect $|\,\bar{\rho}(f)\,|$ and Φ_{CPV} in a quite significant manner for the transitions $b \to u\bar{u}d$ and $b \to c\bar{u}s$, $\bar{b} \to \bar{u}c\bar{s}$.

3) On the Observable CP Asymmetry

Since one predicts quite confidently for B_d mesons $|q/p| \simeq 1, \Delta\Gamma \ll \Gamma$, see Eqs. (84) and (98), one deals with considerably simplified expressions:

$$\text{rate } (B_d(t) \to f) \propto \frac{1}{2} e^{-\Gamma t} \left\{1 + |\bar{\rho}(f)|^2 + 2\text{Im}\left[\frac{q}{p}\,\bar{\rho}(f)\right] \sin \Delta m t\right\} \tag{123}$$

$$\text{rate } (\overline{B}_d(t) \to \bar{f}) \propto \frac{1}{2}\, e^{-\Gamma\, t} \left\{1 + |\rho(\bar{f})|^2 + 2\text{Im}\left[\frac{p}{q}\rho(\bar{f})\right] \sin \Delta m t\right\}\ . \tag{124}$$

For $f = \psi K_S$ one has $|\bar{\rho}(\psi K_S)| \simeq 1$ as discussed before, and therefore (see example 3),

$$\text{rate } (B_d(t) \to \psi K_S) \propto e^{-\Gamma\, t} \{1 - \sin \Phi_{CPV}(b \to c\bar{c}s) \sin \Delta m t\}\ , \tag{125}$$

$$\text{rate } (\overline{B}_d(t) \to \psi K_S) \propto e^{-\Gamma\, t} \{1 + \sin \Phi_{CPV}(b \to c\bar{c}s) \sin \Delta m t\}\quad . \tag{126}$$

For $\Delta m/\Gamma = 0.75$, as suggested by the ARGUS data on like-sign di-leptons, and for the quite "reasonable" value $\sin \Phi_{CPV}(b \to c\bar{c}s) = -0.2$ one obtains the curves shown in Fig. 6. (Let us recall that a nonpure exponential evolution in

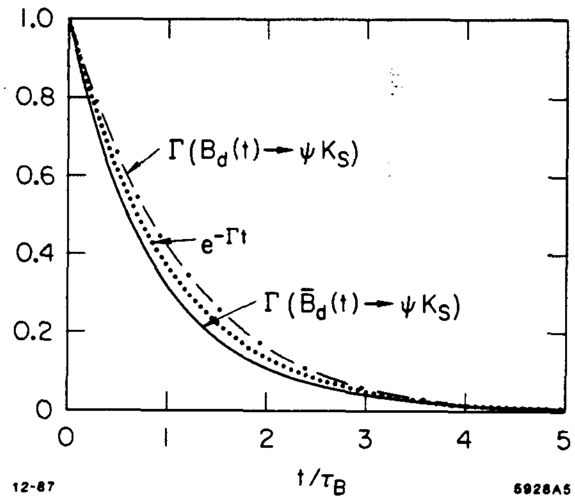

Fig. 6. Evolution in proper time for the decays $B_d, \overline{B}_d \to \psi K_S$ with $\Delta m/\Gamma = 0.75$, $\sin \Phi_{CPV} = 0.2$.

proper time of either $B_d \to \psi K_S$ or $\overline{B}_d \to \psi K_S$ alone already proves CP violation since ψK_S is a CP eigenstate.) In Fig. 7 we have plotted the asymmetry

$$\frac{G(t)}{\overline{G}(t)} - 1$$

as a function of proper time t. For $t < \tau_B$ relatively small asymmetries emerge, not surprisingly, since $B^\circ - \overline{B}^\circ$ mixing has to build up before such a CP asymmetry can become observable. The maximal asymmetry actually occurs for $t \cong 2\tau_B$ [or in general $(t/\tau_B) \cong (\pi/2)(\Gamma/\Delta m)$].

Integrating over *all* decay times produces only a rather small reduction in the asymmetry [see Eq. (54)].

$$\begin{aligned} \langle \text{Asym} \rangle &= \frac{\Gamma(B_d \to \psi K_S) - \Gamma(\overline{B}_d \to \psi K_S)}{\Gamma(B_d \to \psi K_S) + \Gamma(\overline{B}_d \to \psi K_S)} , \\ &= \sin \Phi_{CPV}(\psi K_S) \times \frac{x}{1+x^2} \quad . \end{aligned} \tag{127}$$

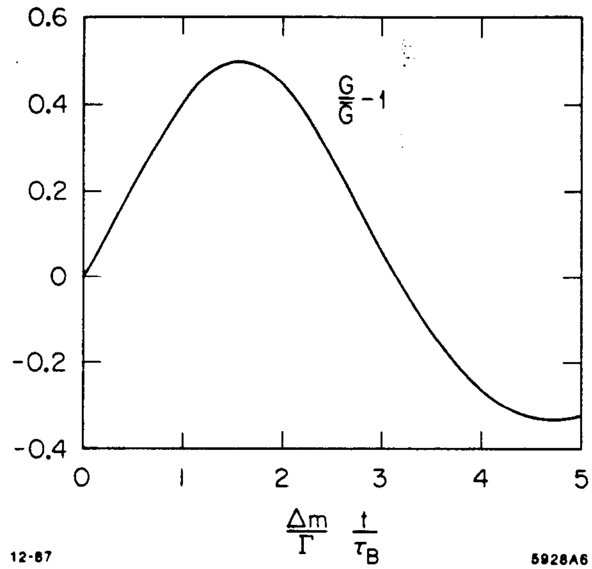

Fig. 7. CP asymmetry for $\Gamma(B_d(t) \to \psi K_S)/\Gamma(\overline{B}_d(t) \to \psi K_S)$ as a function of $\frac{\Delta m}{\Gamma} \cdot \frac{t}{\tau B}$ with $\sin \Phi_{CPV} = 0.2$.

The ratio $x = \Delta m/\Gamma$ is clearly not a well-measured quantity at present. There is every reason to believe that this situation will improve considerably in the future. Yet even so one should keep the following observation in mind:

- B°–$\overline{B}^\circ$ mixing is conventionally studied by searching for semileptonic B° decays producing leptons with the "wrong" charge:

$$r = \frac{\Gamma(B^\circ \to l^- \nu X)}{\Gamma(B^\circ \to l^+ \nu X)} \simeq \frac{x^2}{2 + x^2} , \tag{128}$$

or

$$e^{\Gamma t} \text{ rate } \left[B^\circ(t) \to l^- \nu X\right] \propto (1 - \cos \Delta m t) \quad . \tag{129}$$

- The CP asymmetries we are discussing here have quite a different dependance on x:

$$\langle \text{Asym} \rangle \propto \frac{x}{1 + x^2} , \tag{130}$$

$$e^{\Gamma t} \text{ rate } \left[B_d(t) \to \psi K_S\right] \propto \left[1 + \sin \Phi_{CPV}(\psi K_S) \sin \Delta m\, t\right] \ . \tag{131}$$

This difference is to be expected on rather general grounds since the term in Eq. (131) that contains $\sin \Phi_{CPV}$ reflects CP and thus T violation; therefore it has to be odd under $t \to -t$. Equation (129), on the other hand, is insensitive to T violation and therefore has to be even under $t \to -t$.

This difference has a practical consequence as well: changing r by a factor of two leads to a considerably smaller change in x.

4)On The Relevant Branching Ratios

So far we have stressed the good parts of our message: according to the KM ansatz B° decays should exhibit large CP asymmetries and this should happen in a fair number of different decay channels. Of course there is a downside as well to our message as the reader will have realized by now: the relevant branching ratios are nothing to brag about! Predictions on them actually tend to contain larger uncertainties than those on the asymmetries, yet we give them anyway

$$\mathrm{BR}(B_d \to \psi K_S) 5 \times 10^{-4}\ ,\ \sim \tag{132}$$

$$\mathrm{BR}(B_d \to \psi \rho^\circ)\ \sim\ 10^{-4}\ , \tag{133}$$

$$\mathrm{BR}(B_d \to D\overline{D} K_S)\ \sim\ 10^{-3} - 10^{-2}\ , \tag{134}$$

$$\mathrm{BR}(B_d \to D\overline{D})\ \sim\ 10^{-3}\ , \tag{135}$$

$$\mathrm{BR}(B_d \to \pi^+\pi^-)\ \sim\ 2 \times 10^{-3} \left|\frac{V(ub)}{V(cb)}\right|^2\ , \tag{136}$$

$$\mathrm{BR}(\mathrm{B}_d \to \mathrm{p}\bar{\mathrm{p}})\ \lesssim\ \mathrm{few} \times 10^{-4} \left|\frac{V(ub)}{V(cb)}\right|^2\ , \tag{137}$$

$$\mathrm{BR}(B_d \to \overline{D}^\circ \pi^+\pi^- \to (K_S\pi^\circ)_D\ \pi^+\pi^-)\ \sim\ \mathcal{O}(10^{-4})\ , \tag{138}$$

$$\mathrm{BR}(B_d \to D^{-*}\pi^+)\ \sim\ 5 \times 10^{-3}\ , \tag{139}$$

$$\mathrm{BR}(B_d \to \overline{D}^{\circ *} K_S)\ \sim\ \mathrm{few} \times 10^{-4}\ . \tag{140}$$

These numbers have been obtained by employing the factorization ansatz as described in Ref. 9; they should therefore be considered as guestimates only.

Nevertheless, one should keep in mind that the factorization ansatz works quite well for D decays; furthermore there is some experimental evidence emerging that supports at least two of these numbers, namely Eqs. (132) and (139).

The branching ratios listed above are very small which should come as no surprise: all nonleptonic B branching ratios (with the exception of $\overline{B} \to D^{(*)}\overline{D}_s^{(*)}$ and $\overline{B}_s \to D_s^{(*)}\overline{D}_s^{(*)}$) are small: it is 10^{-3} rather than 1% or even 10% that provides the natural unit, since there is such a multitude of nonleptonic decay modes. Then there are further suppression factors like phase space, as in $b \to c\bar{c}s$ vs. $b \to c\bar{u}d$ — or small KM parameters, as in $b \to u$ vs. $b \to c$.

These small branching ratios already indicate the severity of the challenge one has to face. In addition, the final states are rather complex, i.e., typically involve multi-prong topologies, in particular for the dominant $b \to c$ transitions.

Determining these branching ratios experimentally will of course represent a major step forward: firstly, it would allow us to develop a clearer idea on the statistics required for a meaningful search for CP violation and on which are the most promising decay modes; secondly, a body of well-measured branching ratios would yield information on the weight of FSI in general and on $\overline{\rho}(f)$ in particular. This is—as alluded to above—of high importance when one is attempting a *quantitative* analysis of CP violation in B decays.

(B) CP Asymmetries and FSI

As discussed in general terms in Act I, Part B, decays that proceed via the quark level transitions $b\bar{d} \to s\bar{u}u\bar{d}$ have all the ingredients to exhibit direct CP violation. As Eq. (64) shows

- KM parameters with a large complex phase, V(ub), appear and
- Penguin operators with internal charm quarks can generate the required complex phases—in addition to what is achieved by the usual (and expected) FSI.

Interesting examples for such transitions are

$$\overline{B}_d \to K^-\pi^+ \text{ vs. } B_d \to K^+\pi^- \ , \tag{141}$$

or

$$\overline{B}_d \to K^-\rho^+ \text{ vs. } B_d \to K^+\rho^- \quad . \tag{142}$$

Within the Standard Model these decay modes are flavor specific, i.e., they can occur only in either B_d or $\overline{B}_d$ decays. A rough guestimate yields

$$\mathrm{BR}(\overline{B}_d \to K^- \pi^+) \sim \mathcal{O}(10^{-5}) \ .$$

In the following we will undertake to estimate the difference that is expected between $\overline{B}_d \to K^- \pi^+$ and $B_d \to K^+ \pi^-$. The purpose of this exercise is *not* to obtain reliable numbers — instead it will serve to illustrate the uncertainties one typically encounters in any such calculation.

The main problems arise in a computation of the relevant hadronic matrix elements. There are two relevant operators, namely the usual current-current operator

$$\overline{O}_{LL} = [\overline{u}\gamma_\mu(1-\gamma_5)b][\overline{s}\gamma_\mu(1-\gamma_5)u] \ ,$$

and the Penguin operator

$$\overline{O}_P = \left[\overline{s}\gamma_\mu(1-\gamma_5)\frac{t^a}{2}b\right]\left(\overline{q}\gamma_\mu\frac{t^a}{2}q\right) \ .$$

Since we are unable at present to evaluate $\langle K\pi \mid \overline{O}_{LL}, \overline{O}_P \mid B\rangle$ in a rigorous fashion we have to employ various *approximations*:

(*i*) *Factorization:*

$$\begin{aligned}\langle K^+\pi^- \mid \overline{O}_{LL} \mid B\rangle &\simeq \langle K^+ \mid \overline{s}\gamma_\mu(1-\gamma_5)u \mid 0\rangle \\ &\quad \times \langle \pi^- \mid \overline{u}\gamma_\mu(1-\gamma_5)b \mid B_d\rangle \\ &\simeq i f_K \left[(m_B^2 - m_\pi^2) f_+^{B\pi}(m_K^2) + m_K^2 f_-^{B\pi}(m_K^2)\right] \ , \end{aligned} \tag{143}$$

with the usual form factors f_+, f_-. A considerably more complicated expression is found for the Penguin transition due to the different chiral and flavor structure of $\overline{O}_P$:

$$
\begin{aligned}
\langle K^+\pi^- \mid \overline{O}_P \mid B_d\rangle &= \left\langle K^+\pi^- \left| \left[\bar{s}\gamma_\mu(1-\gamma_5)\frac{t^a}{2}b\right]\left(\bar{u}\gamma_\mu\frac{t^a}{2}u + \bar{d}\gamma_\mu\frac{t^a}{2}d\right)\right| B_d\right\rangle \\
&= \frac{i}{4}\left(1-\frac{1}{N_C^2}\right)\left\{\left[f_K + 2f_K\frac{m_K^2}{(m_s+m_u)(m_b-m_u)}\right]\right. \\
&\times\left[(m_B^2-m_\pi^2)f_+^{B\pi} + m_K^2 f_-^{B\pi}(m_K^2)\right] \\
&- \left[f_B + 2f_B\frac{m_B^2}{(m_b+m_d)(m_s-m_u)}\right] \\
&\times\left.\left[(m_K^2-m_\pi^2)f_+^{K\pi}(m_B^2) + m_B^2 f_-^{K\pi}(m_B^2)\right]\right\} \\
&\simeq \frac{i}{4}m_B^2\left(1-\frac{1}{N_C^2}\right)\left\{f_K f_+^{B\pi}(m_K^2)\left[1+\frac{2m_K^2}{m_b(m_s+m_u)}\right]\right. \\
&\left.- 2f_B\frac{1}{m_b(m_s-m_u)}\left[m_K^2 f_+^{K\pi}(m_B^2) + m_B^2 f_-^{K\pi}(m_B^2)\right]\right\}
\end{aligned}
\tag{144}
$$

where we have used the obvious approximations $m_\pi^2 \ll m_K^2 \ll m_B^2$, $m_s \ll m_B$ in the last line.

Among the terms in the square bracket the quantity $f_K f_+^{B\pi}$ is in principle the leading one in the limit $m_b \to \infty$. However for the actual mass values the coefficient of the "nonleading" term is $2m_K^2/(m_s m_b) \sim 0.7$, i.e., it does not provide a large suppression. The weight of these extra terms therefore depends crucially on the form factors $f_+^{B\pi}, f_+^{K\pi}$. Again their exact values are beyond our grasp. Instead we have to employ another approximation of untested reliability:

(*ii*) *The nonrelativistic quark model* suggests

$$
f_K f_+^{B\pi}(m_K^2) \sim O(f_B f_+^{K\pi}(m_B^2)) \; . \tag{145}
$$

In that case the extra terms largely cancel and we obtain

$$
< K^+\pi^- \mid \overline{O}_P \mid B_d > \sim \frac{1}{4}\left(1-\frac{1}{N_C^2}\right)\langle K^+\pi^- \mid \overline{O}_{LL} \mid B_d\rangle \; . \tag{146}
$$

Then one can write

$$< K^+\pi^- \mid \mathcal{L}(\Delta B = 1) \mid B_d > \propto \langle K^+\pi^- \mid \overline{O}_{LL} \mid B_d\rangle \times \left[V^*(ub)V(us) + \frac{i}{4}\tilde{c}_P V^*(cb)V(cs)\right] , \tag{147}$$

$$\langle K^-\pi^+ \mid \mathcal{L}(\Delta B = 1) \mid \overline{B}_d\langle \propto \langle K^-\pi^+ \mid \overline{O}^\dagger_{LL} \mid \overline{B}_d\rangle \times \left[V(ub)V^*(us) + \frac{i}{4}\tilde{c}_P V(cb)V^*(cs)\right] . \tag{148}$$

Accordingly

$$\frac{\Gamma(B_d \to K^+\pi^-) - \Gamma(\overline{B}_d \to K^-\pi^+)}{\Gamma(B_d \to K^+\pi^-) + \Gamma(\overline{B}_d \to K^-\pi^+)} \simeq \frac{\tilde{c}_q \eta}{2\lambda^2(\rho^2 + \eta^2) + \frac{\tilde{c}_q}{8\lambda^2}} . \tag{149}$$

A positive value for η is inferred from ϵ_K, see Fig. 2. If one takes the Penguin computation, as stated in Eq. (62), at face value, one finds

$$\Gamma(B_d \to K^+\pi^-) > \Gamma(\overline{B}_d \to K^-\pi^+) .$$

Furthermore for the "reasonable" values $\rho = -1$, $\eta = 0.2$ one obtains

$$\frac{\Gamma(B_d \to K^+\pi^-) - \Gamma(\overline{B}_d \to K^-\pi^+)}{\Gamma(B_d \to K^+\pi^-) + \Gamma(\overline{B}_d \to K^-\pi^+)} \sim 0.12 . \tag{150}$$

This is certainly a large CP asymmetry! Yet at the same time one has to keep in mind that quite a few approximations entered our computation: in addition to the ones already mentioned explicitly under (i), (ii) we have ignored soft FSI that are not reproduced by Penguins. As discussed in Act I there is no *firm* basis for such assumptions.

Synopsis of Act II

(*i*) It is unlikely that one will ever be able to measure CP violation in *semileptonic* B_d decays: one predicts CP asymmetries not to exceed the 10^{-3} level there; even New Physics could push it up only to $\mathcal{O}(1\%)$.

(*ii*) Sizeable CP asymmetries, i.e., $\sim$ 1–10%, could occur in truly rare ***nonleptonic B*** decays which are ***flavor specific*** like $\overline{B}_d \to K^-\pi^+$ vs. $B_d \to K^+\pi^-$. They by themselves would establish direct CP violation. Yet the numbers involved are quite uncertain.

(*iii*) ***Nonleptonic*** channels that are shared by B_d and $\overline{B}_d$ decays are expected to exhibit large CP asymmetries, i.e., of $\sim$ 5–30% if B_d–$\overline{B}_d$ mixing is indeed as strong as it appears now. The ***evolution*** of the decay rate in ***proper time*** provides a striking signature for these asymmetries. The predictions can be refined quite considerably once

- the B_d–$\overline{B}_d$ mixing strength as expressed by $\Delta m/\Gamma$ is well-measured
- the top mass is known to within 10 GeV and
- branching ratios have been determined.

In that case, B_d decays would provide not just a mere hunting ground for CP violation, they would be transformed into a quantitative laboratory for studying CP noninvariance.

ACT III. THE DARK HORSE: B_s AND ITS DECAYS

There are three reasons to believe that finding CP violation in B_s decays represents an even more formidable task than in B_d decays.

- There are fewer B_s than B_d produced, typically by a factor two to three;
- B_s–$\overline{B}_s$ mixing is expected to be close to "maximal" in the Standard Model, as explained below. This creates the need for excellent time resolution.
- In the KM ansatz with three families, one predicts on very general grounds that CP asymmetries in B_s decays are either Cabibbo-suppressed or can be large only in KM-suppressed decay modes.

In the following, we will make these statements more quantitative where our discussion will closely parallel that of Act II.

1) Estimates on $\mathbf{B_s}$–$\overline{\mathbf{B}}_\mathbf{s}$ Mixing

(a) Δm

There is a simple relation between B_d–$\overline{B}_d$ and B_s–$\overline{B}_s$ mixing as read off from Fig. 5:

$$\frac{\Delta m(B_s)}{\Delta m(B_d)} = \frac{|V(ts)|^2}{|V(td)|^2}\,\frac{Bf_B^2(B_s)}{Bf_B^2(B_d)} \geq \frac{|V(ts)|^2}{|V(td)|^2} \simeq \frac{1}{\lambda^2[(1-\rho)^2+\eta^2]} \quad , \tag{151}$$

since all model calculations yield $Bf_B^2(B_s) \geq Bf_B^2(B_d)$.[18] Scanning the allowed parameter space, one predicts[13]

$$\Delta m(B_s) \gtrsim 6\Delta m(B_d) \quad . \tag{152}$$

Using the ARGUS bound[10]

$$\left.\frac{\Delta m}{\Gamma}\right|_{B_d} \gtrsim 0.44 \quad ,$$

one is then led to expect (for $\tau(B_s) = \tau(B_d)$)

$$\left.\frac{\Delta m}{\Gamma}\right|_{B_s} \gtrsim 2.8 \quad ; \tag{153}$$

i.e., the mixing rate should be considerably larger than the decay rate for B_s mesons! A violation of Eq. (153) would signal the presence of New Physics contributing destructively to $\Delta m(B_s)$.

Yet measuring $x = \Delta m/\Gamma$ is not quite straightforward for $x > 1$, since the time-integrated quantity

$$r_s = \frac{\Gamma(B_s \to \ell^- X)}{\Gamma(B_s \to \ell^+ X)} \simeq \frac{x^2}{2+x^2}$$

is rather insensitive to the exact value of x. Equation (153) translates into

$$r_s \geq 0.80 \quad . \tag{154}$$

On the other hand, $x = 2$ produces $r_s \simeq 0.67$, which differs from $r_s = 0.8$ by only less than 20%. To measure the intrinsic strength of B_s–$\overline{B}_s$ mixing with satisfactory accuracy, one therefore has to study the proper time evolution of $B_s(t) \to \ell^- X$!

(b) $\Delta\Gamma$

As for B_d mesons, one expects $\Delta\Gamma(B_s) \ll \Delta m(B_s)$. Yet unlike the B_d case, one has $\Delta m(B_s) \gg \Gamma(B_s)$, Eq. (153); therefore one cannot necessarily infer $\Delta\Gamma(B_s) \ll \Gamma(B_s)$ and a more careful estimate is needed. Comparing the box diagrams of Figs. 3 and 4, one finds as for B_d mesons, see Eqs. (81)–(83)

$$\left|\frac{\Delta\Gamma}{\Delta m}\right| \sim 0.05 \quad , \tag{155}$$

if $m_t = 50$ GeV. Yet Eq. (153) shows that

$$\frac{\Delta\Gamma}{\Gamma} \sim 0.15 \tag{156}$$

does not represent an unreasonable expectation.

Again one arrives at essentially the same estimate in a complementary way: there are some nonleptonic two-body decay modes with substantial branching ratios of a few percent; for example, $B_s \to D_s^* \overline{D}_s^*$ with, say

$$\mathrm{BR}(B_s \to D_s^* \overline{D}_s^*) \sim 0.05 \quad .$$

This channel by itself would produce a lifetime difference,

$$\frac{\Delta\Gamma}{\Gamma} \sim 2\mathrm{BR}(B_s \to D_s^* \overline{D}_s^*) \sim 0.1 \quad . \tag{157}$$

These estimates are meant to show that $\Delta\Gamma/\Gamma$ could be as large as 10% or so for B_s mesons. It could be significantly smaller, too, due to cancellations taking place between

$$B_s \to D_s^* \overline{D}_s^* \to \overline{B}_s$$

and

$$B_s \to D_s \overline{D}_s^* + \mathrm{h.c.} \to \overline{B}_s \ , \quad \text{etc.,}$$

transitions.

2) Estimates on the Intrinsic Strength of CP Violation

(*a*) $|q/p|$

Because of Eq. (157), one has

$$\left|\frac{q}{p}\right| \simeq 1 + \frac{1}{2}\left|\frac{\Delta\Gamma}{\Delta m}\right| \sin\phi(\Delta B = 2) \quad . \tag{158}$$

Following the same procedure as for B_d mesons one infers from the quark box diagrams

$$\sin\phi(\Delta B = 2) \simeq \frac{8}{3}\frac{m_c^2}{m_b^2}\,\mathrm{Im}\,\frac{V(cb)\,V^*(cs)}{V(bt)\,V^*(ts)} \simeq -\frac{8}{3}\frac{m_c^2}{m_b^2}\,\lambda^2\eta \quad ,$$

and therefore

$$\left|1 - \left|\frac{q}{p}\right|\right| \lesssim 4 \times 10^{-4}\,\eta \quad ; \tag{159}$$

i.e., typically smaller than for B_d mesons. This happens despite of the larger $|\Delta\Gamma/\Delta m|$, since $\sin\phi(\Delta B = 2, B_s)$ is additionally suppressed by λ^2. This is easily understood: on the leading level, only quarks of the third and second family contribute (b, t, c, s); accordingly, there can be no CP violation to the leading order in KM parameters.

$$\begin{aligned} a_{SL}(B_s) &= \frac{\Gamma[\overline{B}_s(t) \to \ell^+\nu X] - \Gamma[B_s(t) \to \ell^-\nu X]}{\Gamma[\overline{B}_s(t) \to \ell^+\nu X] + \Gamma[B_s(t) \to \ell^-\nu X]} \\ &\lesssim 8 \times 10^{-4}\,\eta \sim 2 \times 10^{-4} \quad . \end{aligned} \tag{160}$$

Yet the presence of New Physics could quite likely enhance $\phi(\Delta B = 2, B_s)$, in particular, by more than an order of magnitude, allowing for[22]

$$a_{SL}(\text{New Physics}) \sim \mathcal{O}(1\%) \tag{161}$$

(*b*) $\mathrm{Im}\,[(q/p)\,\bar{\rho}(f)]$

Since all the appropriate caveats have already been made in Act II, we can go *medias in res* and list the results in Table 2.

Table 2: Standard KM Predictions for B_S Decays

Quark level transition	Example of hadronic channel f	$\rho\,|(f)|$	$\eta(f)$
i) $b \to c\bar{c}s$	$\psi\phi$	1	$\sim +$
	$\psi\eta$	1	$+$
	$\psi\eta'$	1	$+$
	$D_s\overline{D}_s$	~ 1	$+$
	$D_s^*\overline{D}_s^*$	~ 1	$\sim +$
ii) $b \to u\bar{u}d$	$K_S\pi^\circ$	~ 1	$-$
	$K_S\rho^\circ$	~ 1	$-$
iii) $b \to u\bar{u}s$	$\phi\rho^\circ$	~ 1	$\sim +$
iv) $b \to c\bar{u}s \& \bar{b} \to \bar{u}c\bar{s}$	$D^\circ\phi$[14]	$\simeq \mathcal{O}(1)$	$-$
	$D_s^+K^-$	$\simeq \mathcal{O}(1)$	$+$

$\sin\Phi_{CPV}$

ad (i)

$$\mathrm{Im}\,\frac{(V^*(tb)\,V(ts))^2}{|V(tb)\,V(ts)|^2}\,\frac{(V(cb)\,V^*(cs))^2}{|V(cb)\,V(cs)|^2} \simeq 2\lambda^2\eta$$

ad (ii)

$$\mathrm{Im}\,\frac{(V^*(tb)\,V(ts))^2}{|V(tb)\,V(ts)|^2}\,\frac{(V(ub)\,V^*(ud))^2}{|V(ub)\,V(ud)|^2}\,F^* \simeq -\frac{2\rho\eta}{\rho^2+\eta^2} \simeq \sin 2\Phi_3$$

ad (iii)

$$\mathrm{Im}\,\frac{(V^*(tb)\,V(ts))^2}{|V(tb)\,V(ts)|^2}\,\frac{(V(ub)\,V^*(us))^2}{|V(ub)\,V(us)|^2} \simeq -\frac{2\rho\eta}{\rho^2+\eta^2} \simeq \sin 2\Phi_3$$

ad (iv)

$$\mathrm{Im}\,\frac{(V^*(tb)\,V(ts))^2}{|V(tb)\,V(ts)|^2}\,\frac{V(cb)\,V^*(us)\,V(ub)\,V^*(cs)}{|V(cb)\,V(us)\,V(ub)\,V(cs)|} \simeq -\frac{\eta}{\sqrt{\rho^2+\eta^2}} \simeq \sin\Phi_3$$

A few comments are in order to elucidate the content of Table 2 :

(α) The (Cabibbo) suppressed size of $\Phi_{CPV}(b \to c\bar{c}s)$ is easily understood: again it is only the second and third families that contribute on the leading level in the KM parameters to $B_s \longleftrightarrow \overline{B}_s$ (see Fig. 5 for $q = s$) and to $\overline{B}_s \to c\bar{c}s\bar{s}$. A relative phase between the quark couplings can therefore be rotated away — up to corrections of order λ^2. However, New Physics—like the existence of a fourth family (t', b') with $m(t') \sim \mathcal{O}$ (few hundred GeV)—could contribute significantly to B_s–$\overline{B}_s$ mixing and thus enhance $\Phi_{CPV}(b \to c\bar{c}s, B_s)$ considerably.

(β) Both spectator and Penguin diagrams contribute to $b\bar{s} \to u\bar{u}s\bar{s}$ transitions, yet they will do so with quite a different weight in different hadronic channels: $B_s \to \phi\rho^\circ$ should be quite insensitive to Penguins. Those might contribute somewhat to $B_s \to \phi\omega$ and will represent a sizeable, if not dominant, factor in $B_s \to K^+K^-$. The CP asymmetry in $B_s \to K^+K^-$ could, therefore, be quite different from that in $B_s \to \phi\rho^\circ$.

3) On the Observable CP Asymmetry

Our estimates gave $|q/p| \simeq 1$ and $\Delta\Gamma(B_d) < \Delta\Gamma(B_s) \ll \Gamma_B$, again allowing us to use very simple expressions:

$$\text{rate}\,[\,B_s(t) \to f\,] \propto \frac{1}{2}e^{-\Gamma t}\left\{1 + |\bar{\rho}(f)|^2 + 2|\bar{\rho}(f)|\sin\Phi_{CPV}(f)\,\sin\Delta m t\right\} \;, \quad (162)$$

$$\text{rate}\left[\,\overline{B}_s(t) \to \bar{f}\,\right] \propto \frac{1}{2}e^{-\Gamma t}\left\{1 + |\rho(\bar{f})|^2 - 2|\rho(f)|\sin\Phi_{CPV}(\bar{f})\,\sin\Delta m t\right\} \;. \quad (163)$$

The main qualitative difference to the B_d case lies in the much larger mixing strength expected for the $B_s\ \overline{B}_s$ system relative to the $B_d\ \overline{B}_d$ system. This is demonstrated in Fig. 8 where we have plotted Eqs. (162) and (163) for $\sin\Phi_{CPV} = 0.4$ and $\Delta m/\Gamma = 5$! The resulting curves are easy to grasp. Because of the rapid mixing time, one expects a very striking signal from a number of back and forth oscillations. (One should note in passing that such a pattern of more than one oscillation is not observed in $K^\circ\ \overline{K}^\circ$ decays since $\Gamma_S \gg \Gamma_L$, and would represent quite a novel feature.) This has two very important consequences.

- An extremely high premium obviously has to be placed on excellent time resolution. This point is made explicit by considering the *time-integrated* asymmetry

$$\langle\text{Asym}\rangle \;=\; \frac{\Gamma(B_s \to \psi\phi) - \Gamma(\overline{B}_s \to \psi\phi)}{\Gamma(B_s \to \psi\phi) + \Gamma(\overline{B}_s \to \psi\phi)} \;=\; \frac{x}{1+x^2}\,\sin\Phi_{CPV}(\psi\phi) \quad , \quad (164)$$

  which is suppressed like $1/x$ for $x \gg 1$.

- As discussed previously, *time-integrated* B_s decays into "wrong sign" leptons do not allow a good measurement of x since they exhibit little sensitivity to the precise value of x for $x^2 \gg 1$.

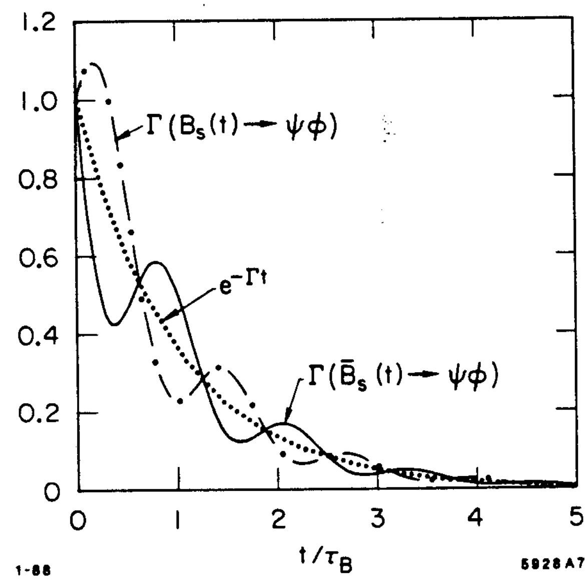

Fig. 8. Evolution in proper time for the decays $B_s, \overline{B}_s \to \psi\phi$ with $\Delta m/\Gamma = 5$, $sin\ \Phi_{CPV} = 0.4$.

4) On the Relevant Branching Ratios

Since so far not a single B_s decay has been reconstructed, very little is really known about the branching ratios and we have to depend almost completely on theoretical guestimates, where we typically find, using factorization,

$$\mathrm{BR}(B_s \to \psi\phi) \sim 5 \times 10^{-3} , \tag{165}$$

$$\mathrm{BR}(B_s \to D_s\overline{D}_s) \sim \mathcal{O}(10^{-2}) , \tag{166}$$

$$\mathrm{BR}(B_s \to D_s^*\overline{D}_s^*) \sim \text{few} \cdot 10^{-2} , \tag{167}$$

$$\mathrm{BR}(B_s \to K_S^+\rho^\circ) \sim 10^{-5}\text{–}10^{-4} , \tag{168}$$

$$\mathrm{BR}(B_s \to \phi\rho^\circ) \sim 10^{-5} , \tag{169}$$

$$\mathrm{BR}(B_s \to D^\circ\phi) \sim 10^{-5}\text{–}10^{-4} , \tag{170}$$

$$\mathrm{BR}(B_s \to D_s^+K^-) \sim \mathcal{O}(10^{-4}) . \tag{171}$$

Again, most exclusive branching ratios are very small due to the huge number of available channels, and due to some KM suppression. There is one noteworthy exception; namely, the two-body mode $B_s \to D_s^* \overline{D}_s^*$.

The numbers listed in Eqs. (165) and (171) should be seen in proper context, i.e., the absence of experimental information. We will be able to give more reliable estimates once a few branching ratios have been measured.

5) CP Asymmetries and FSI

Penguin-like contributions are presumed to be quite significant in decay modes such as

$$B_s \to K^+K^- \ , \ \phi\phi \quad . \tag{172}$$

The branching ratios could conceivably be as high as roughly 10^{-4} and CP asymmetries might reach the few percent regime.

Synopsis of Act III

(*i*) It is quite unlikely that CP violation will be measurable in *semileptonic* B_s decays, unless New Physics intervenes to generate asymmetries of $\mathcal{O}(1\%)$.

(*ii*) *Nonleptonic* channels that are common to B_s and $\overline{B}_s$ decays should exhibit large CP asymmetries of order 10–50% if their branching ratios are KM-suppressed by $|V(ub)/V(cb)|^2$.

(*iii*) Smallish asymmetries of at most a few percent are predicted for KM-allowed modes such as $B_s \to \psi\phi, D_s^* \overline{D}_s^*$, unless New Physics intervenes. In that case CP asymmetries can quite conceivably reach the 10% level or beyond even for these modes.

(*iv*) Since B_s–$\overline{B}_s$ mixing is expected to proceed quite rapidly, *excellent resolution in proper time is essential* for such studies.

ACT IV: THE DARK SIDE—SEARCH SCENARIOS

So far we have dealt with B decays as if we had intense single beams of B mesons at our disposal. This represents of course an unrealistic scenario since in e^+e^- annihilation and in hadronic collisions beauty hadrons are always produced in pairs. The additional complexities that arise accordingly will be addressed in this chapter.

One generic problem to which we will return time and again can be stated already at this point: a final state f that is common to both B° and $\overline{B}^\circ$ decays can by its very nature not reveal whether it came from a B° or a $\overline{B}^\circ$; in that case no CP asymmetry can be defined (at least as long as $\Delta\Gamma$ effects are beyond reach). Therefore, independent information on the flavor identity of the decaying neutral B meson has to be obtained before a CP asymmetry can be observed. This is referred to as "flavor tagging."

A) $e^+e^- \to B\overline{B},\ B\overline{B}^* +$ h.c.

The *exclusive* production of a pair of beauty mesons leads to a coherent quantum state; in

$$e^+e^- \to \text{“}1\gamma\text{”} \to B\overline{B} \tag{173}$$

the $B\overline{B}$ pair forms a C (=charge conjugation) *odd* state; in

$$e^+e^- \to \text{“}1\gamma\text{”} \to B^*\overline{B} + \text{h.c.} \to B\overline{B}\gamma \quad , \tag{174}$$

on the other hand, the $B\overline{B}$ are in a C *even* state. The proper quantum mechanical initial state then reads

$$|i\rangle_0 = \frac{1}{\sqrt{2}}\left(|B^\circ(k)\overline{B}^\circ(\bar{k})\rangle_0 + (-1)^C\ |B^\circ(\bar{k})\overline{B}^\circ(k)\rangle_0\right) \tag{175}$$

where $k, \bar{k}$ denote momenta and $(-1)^C$ reflects the C parity of the $B\overline{B}$ pair. A straightforward though lengthy calculation yields

$$\begin{aligned}
\text{rate } & \left(B^\circ(t)\overline{B}^\circ(\bar{t})|_{C=\mp} \to f_a f_b\right) \propto e^{-\Gamma_B(t+\bar{t})} \times |A(a)A(b)|^2 \\
& \left\{ \left[1 + \cos\left(\Delta \mathrm{m}(t \mp \bar{t})\right)\right] |\rho(b) \mp \overline{\rho}(a)|^2 \right. \\
& + \left[1 - \cos\left(\Delta \mathrm{m}(t \mp t)\right)\right] \left|\frac{p}{q} \mp \frac{q}{p}\overline{\rho}(a)\overline{\rho}(b)\right|^2 \\
& \left. \mp\ 2\sin\left(\Delta \mathrm{m}(t \mp \bar{t})\right) \mathrm{Im} \left[\left(\frac{p}{q} \mp \frac{q}{p}\overline{\rho}(a)\overline{\rho}(b)\right)\left(\overline{\rho}(b) \mp \overline{\rho}(a)\right)^*\right]\right\}
\end{aligned} \tag{176}$$

where we have set $\Delta\Gamma = 0$; $t[\bar{t}]$ denotes the time of decay of the meson that was born as $B^\circ[\overline{B}^\circ]$.

The physics contained in Eq. (176) can best be illustrated by three complementary examples:

(1) *Both f_a and f_b are flavor specific*, like

$$f_a = l^- \ \nu\ D^*,\ f_b = K^+\pi^- \quad .$$

Since $\overline{A}(K^+\pi^-) = 0 = A(l^- \ \nu\ D^*)$, Eq. (176) simplifies greatly:

$$\begin{aligned}
\text{rate } & \left(B^\circ(t)\overline{B}^\circ(\bar{t})|_{C=\mp} \to (l^- \ \nu\ D^*)_B\ (K^+\pi^-)_B\right) \propto e^{-\Gamma(t+\bar{t})} \\
& \times\left[1 + \cos\left(\Delta \mathrm{m}\ (t \mp \bar{t})\right)\right] \left|\overline{A}(l^- \ \nu\ D^*)A(K^+\pi^-)\right|^2
\end{aligned} \tag{177}$$

and, analogously for the CP conjugate process,

$$\begin{aligned}
\text{rate } & \left(B^\circ(t)\overline{B}^\circ(\bar{t})|_{C=\mp} \to (l^+ \ \nu\ \overline{D}^*)_B(K^-\pi^+)_B\right) \propto e^{-\Gamma(t+\bar{t})} \times \\
& \left[1 + \cos\left(\Delta \mathrm{m}(t \mp \bar{t})\right)\right] \left|A(l^+ \ \nu\ \overline{D}^*)A(K^-\pi^+)\right|^2 .
\end{aligned} \tag{178}$$

An asymmetry is produced if $|A(K^+\pi^-)|^2 \neq |\overline{A}(K^-\pi^+)|^2$ and is independent of the proper decay time t or $\bar{t}$, as expected.

(2) *f_a is flavor specific, while f_b is not*, like

$$f_a = l^- \ \nu \ D^*, \ f_b = \psi \ K_S \quad .$$

With $A(l^- \ \nu \ D^*) = 0$ one obtains

$$\begin{aligned}
\text{rate} \ & \left(B^\circ(t)\overline{B}^\circ(\bar{t})|_{C=\mp} \to (l^- \ \nu \ D^*)_B \ (\psi \ K_S)_B\right) \propto e^{-\Gamma(t+\bar{t})} \\
& \times \left\{ \left[1 + \cos\left(\Delta \mathrm{m}(t \mp \bar{t})\right)\right] \left|A(\psi \ K_S)\overline{A}(l^- \ \nu \ D^*)\right|^2 \right. \\
& + \left[1 - \cos\left(\Delta \mathrm{m}(t \mp \bar{t})\right)\right] \left|\frac{q}{p}\right|^2 \left|\overline{A}(l^- \ \nu \ D^*)\overline{A}(\psi \ K_S)\right|^2 \\
& \mp \ 2 \sin\left(\Delta \mathrm{m}(t \mp \bar{t})\right) \\
& \left. \times \operatorname{Im} \left(\frac{q}{p}\overline{A}(l^- \ \nu \ D^*)\overline{A}(\psi \ K_S)A^*(\psi \ K_S)\overline{A}^*(l^- \ \nu \ D^*)\right) \right\} \\
& \simeq 2e^{-\Gamma(t+\bar{t})}\left|A(\psi \ K_S)\overline{A}(l^- \ \nu \ D^*)\right|^2 \\
& \times \left(1 \mp \sin\left(\Delta \mathrm{m}(t \mp \bar{t})\right) \ \operatorname{Im}\left(\frac{q}{p}\bar{\rho}(\psi \ K_S)\right)\right) \quad .
\end{aligned} \tag{179}$$

Analogously,

$$\begin{aligned}
\text{rate} \ & \left(B^\circ(t)\overline{B}^\circ(\bar{t})|_{C=\mp} \to (l^+ \ \nu \ D^*)_B \ (\psi \ K_S)_B\right) \propto 2e^{-\Gamma(t+\bar{t})} \\
& \times \left|A(\psi \ K_S)\overline{A}(l^- \ \nu \ D^*)\right|^2 \left(1 \pm \sin\left(\Delta \mathrm{m}(t \mp \bar{t})\right) \ \operatorname{Im}\left(\frac{q}{p}\bar{\rho}(\psi \ K_S)\right)\right) .
\end{aligned} \tag{180}$$

The important observation to make at this point concerns the proper time structure of the CP asymmetry, namely

$$\text{CP ASYM} \ \propto e^{-\Gamma(t+\bar{t})} \sin \Delta \ \mathrm{m}(t \mp \bar{t})$$

for $C[B^\circ\overline{B}^\circ] = \mp 1$. For a C odd configuration of $B^\circ\overline{B}^\circ$ the asymmetry contains a term that is *odd* under $t \leftrightarrow \bar{t}$. It will therefore vanish as long as the times of decay t and $\bar{t}$ are treated symmetrically in defining the sample. This is true in particular when one integrates over all decay times

$$\int_0^\infty dt \int_0^\infty d\bar{t}\; e^{-\Gamma(t+\bar{t})} \sin\Delta\,\mathrm{m}\;(t \mp t) = \begin{cases} 0 & \text{C odd} \\ & \text{for} \\ \frac{2x}{(1+x^2)^2} & \text{C even} \end{cases} \tag{181}$$

From Eq. (181) we learn two things

- In

$$e^+e^- \to \Upsilon(4S) \to B\overline{B} \tag{182}$$

one has the $B\overline{B}$ pair in a C odd configuration. *None of the CP asymmetries that involve mixing can then be observed as a matter of principle — unless one can at least partially resolve the proper time evolution,* say by employing asymmetric e^+e^- collisions.

- In

$$e^+e^- \to B^\circ\overline{B}^{\circ *} + \text{h.c.} \to B^\circ\overline{B}^\circ\gamma$$

one has realized the C even case that is quite favorable for detecting a CP asymmetry, unless the B°–$\overline{B}^\circ$ mixing rate is too rapid. For the term $2x/(1+x^2)^2$ vanishes like $(1/x)^3$ for $x \gg 1$, i.e., much more dramatically than when one is dealing with the decays of single B°'s where the corresponding factor reads $x/(1+x^2)$, see Eq. (130). The origin of this difference is intuitively quite clear: in $e^+e^- \to B^\circ\overline{B}^\circ$ *both* B mesons can mix (into each other); therefore one integrates over two concurrent oscillation processes. This means also that the process

$$e^+e^- \to B_s\overline{B}_s^* \to B_s\; B_s\gamma \tag{183}$$

is ill-suited for searching for CP asymmetries since $x(B_s) \geq 3$ is expected. The reaction

$$e^+e^- \to B_d\overline{B}_d^* \to B_d\overline{B}_d\gamma \quad , \tag{184}$$

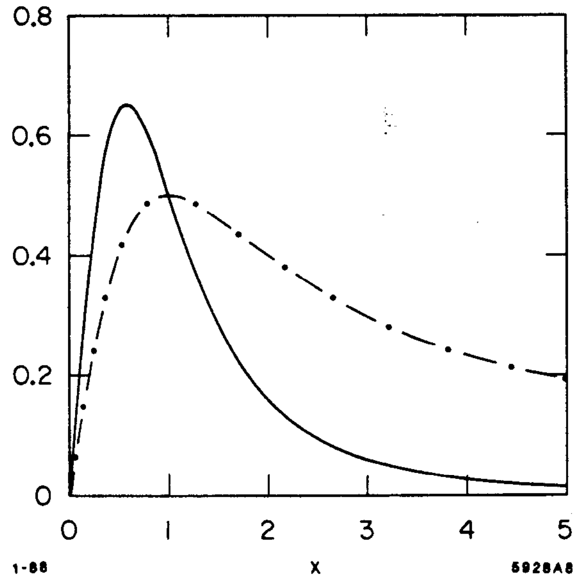

Fig. 9. x dependent weight functions: $2x/(1+x^2)^2$, solid line, $x/(1+x^2)$, dash-dot line.

on the other hand, is quite promising if indeed $x(B_d) \simeq 0.75$. This is shown explicitly in Fig. 9 where the two x dependant weight functions that enter in Eqs. (130) and (181) are plotted.

(3) There is another interesting case where one looks for a *rate and not an asymmetry:* the reaction[15,16,17]

$$e^+e^- \to B^\circ \overline{B}^\circ \to f_1 f_2 \tag{185}$$

can proceed only via a violation of CP invariance *if f_1, f_2 denote CP eigenstates with the same CP parity.* For the initial state is CP even (ignoring two photon contributions), the final state CP odd:

$$\begin{aligned}\mathrm{CP}[1\gamma] &= \mathrm{CP}\,[\Upsilon(4S)] = +1\\ \mathrm{CP}[f_1 f_2] &= \mathrm{CP}[f_1]\mathrm{CP}[f_2](-1)^l = -1\end{aligned}$$

since $B\overline{B}$ are produced in a P-wave. Integrating (176) over all t and $\bar{t}$ and using $|q|^2 = |p|^2$ one finds

$$\text{rate } \left(B^\circ(t)\overline{B}^\circ(\bar{t})|_{C=-} \to f_1 f_2\right) \propto \left|A(f_1)A(f_2)\right|^2$$
$$\left\{\left(1+\frac{1}{1+x^2}\right)|\overline{\rho}(f_1)-\overline{\rho}(f_2)|^2 + \frac{x^2}{1+x^2}|1-\left(\frac{q}{p}\right)^2\overline{\rho}(f_1)\overline{\rho}(f_2)|^2\right\} . \tag{186}$$

The last term in (176) has of course been integrated out.

The physics contained in (186) is quite transparent:

- If

$$\overline{\rho}(f_1) = \overline{\rho}(f_2) = \overline{\rho}$$

as in a superweak scenario for CP violation where there is only one universal phase in the $\overline{\rho}(i)$ then the reaction (185) can proceed only in the presence of mixing, i.e., $x \neq 0$.

$$\mathrm{BR}(B^\circ\overline{B}^\circ|_{C=-} \to f_1 f_2) = \mathrm{BR}(B^\circ \to f_1)\ \mathrm{BR}(B^\circ \to f_2)F$$
$$F \simeq \frac{x^2}{1+x^2}\left(2\ \mathrm{Im}\frac{q}{p}\overline{\rho}\right)^2 \quad . \tag{187}$$

- If there is no mixing, i.e., $x = 0$, yet there are nontrivial relative phases between the different amplitudes

$$\frac{\overline{\rho}(f_2)}{\overline{\rho}(f_1)} = e^{i\alpha_{12}} \quad , \alpha_{12} \neq 0, \pi$$

then

$$\mathrm{BR}(B^\circ\overline{B}^\circ|_{c=-} \to f_1 f_2) = \mathrm{BR}(B^\circ \to f_1)\ \mathrm{BR}(B^\circ \to f_2)\widetilde{F}$$
$$\widetilde{F} \simeq 2\left(2\sin\frac{\alpha_{12}}{2}\right)^2 \quad . \tag{188}$$

Tables I and II clearly exhibit the fact that the KM ansatz does not represent a superweak scenario for B decays, that there are relative phases between different classes of transitions. Nevertheless, Eq. (187) is quite adequate for our present discussion:

– For small mixing, i.e., $x^2 \ll 1$, the rate is suppressed by x^2.

– Sizeable and *a fortiori* rapid mixing, i.e., $x^2 \sim 1$ or $x^2 \gg 1$, hardly suppresses the rate; one finds typically

$$F \sim 0.02 - 0.7 \quad . \tag{189}$$

The feasibility of a search for the reaction (185) depends largely on the branching ratios $\mathrm{BR}(B^\circ \to f_1)$, $\mathrm{BR}(B^\circ \to f_2)$. From the numbers given in Act II one concludes that the "usable" branching ratios into CP eigenstates amount to $\leq 10^{-4}$ for both B_d and B_s decays; "usable" means that secondary branching ratios like $\psi \to l^+ l^-$ or $D^\circ \to K^- \rho^+$ are included. This would mean that the huge number of roughly $10^9 \Upsilon(4S) \to B_d B_d$ or $\Upsilon(5S) \to B_s \overline{B}_s$ had to be collected to perform a meaningful search. Yet this conclusion might well be overly pessimistic; for it was obtained by assuming that one considers just one specific channel $B^\circ \overline{B}|_{C=-} \to f_1 f_2$. On the other hand, there are many channels that qualify as CP eigenstates. They can all be included in a search. As explained in some more detail later on *summing* over all these modes if properly executed could lead to

$$\sum_i \mathrm{BR}(B \to f_i) \sim 10^{-3} - 10^{-2} \quad . \tag{190}$$

In that case, 10^6–10^7 $\Upsilon(4S) \to B^\circ \overline{B}^\circ$ might be sufficient for revealing a signal.

B) e^+e^-, $h_1 h_2 \to B^\circ + X$

The pair of beauty hadrons produced well above beauty threshold is not (necessarily) particle and its *own* antiparticle anymore, e.g., in

$$e^+e^-, h_1 h_2 \to B^\circ \; X \tag{191}$$

where h_1, h_2 denote hadrons one has

$$X = \overline{B}_u \text{ or } \overline{B}_d \text{ or } \overline{B}_s \text{ or } \Lambda_b + X' \quad .$$

At the same time even a $B_d \overline{B}_d$ pair will not form a quantum state of definite C parity anymore: C even and odd configurations will enter with equal weight.

(1) e^+e^-, $h_1h_2 \to B^\circ + (\overline{B}_u \text{ or } \Lambda_b) + X'$

Since only the neutral B meson can mix, one can carry over the time evolutions stated in Acts I, II and III for single B° decays in a straightforward way.

Flavor tagging when necessary can be achieved in several different ways: it could be performed by nature via

- the forward-backward asymmetry in $e^+e^- \to$ beauty jets that is generated by the γ–Z^0 interference, or
- a production asymmetry in hadronic collisions caused by leading particle effects or associated $B\Lambda_b$ production;

or it can be enforced by human intervention via

- polarized beams in e^+e^-annihilation, or
- the (partial) reconstruction of the $\overline{B}_u$ or Λ_b decay.

A difference in the observed rate for $\overline{B}_d \to K^-\pi^+$ vs. $B_d \to K^+\pi^-$ cannot automatically be equated with *direct CP violation,* i.e., $\mathrm{BR}(\overline{B}_d \to K^-\pi^+) \neq \mathrm{BR}(B_d \to K^+\pi^-)$: *a priori* one cannot rule out a production asymmetry, i.e., $N(B_d) \neq N(\overline{B}_d)$. Instead one has to compare these rates to decay channels where no or only a small CP asymmetry is expected, like $\overline{B}_d \to l^- \; \nu \; D^*$, $B_d \to l^+ \; \nu \; \overline{D}^*$.

Production asymmetries are actually a double blessing, when $B^0 - \overline{B}^0$ mixing can be resolved in proper time: not only do they provide flavor tagging, but they can unambiguously be measured as well. One example can illustrate the general method:

(α): For a flavor *specific* mode one finds (with $\Delta\Gamma = 0$)

$$\text{rate}\Big(B_d(t) \to \psi K^+\pi^-\Big) \propto e^{-\Gamma t}\Big(1 + P\cos(\Delta m t)\Big) \tag{192}$$

$$P = \frac{N - \overline{N}}{N + \overline{N}} \tag{193}$$

with $\overline{N}[N]$ denoting the number of $\overline{B}^0[B^0]$ mesons present at t=0. Thus the size of the production asymmetry P as well as the strength of mixing can be determined independently.

(β): For a flavor ***nonspecific*** mode one then obtains

$$\text{rate}\,(B_d(t) \to \psi K_S) \propto e^{-\Gamma t}\left(1 + P\ \text{Im}\ \left(\frac{q}{p}\overline{\rho}(\psi K_S)\right)\sin(\Delta m t)\right) \quad . \tag{194}$$

2) $e^+e^-,\ h_1h_2 \to B^\circ\overline{B}^\circ + X'$

The two neutral B mesons can mix into each other and one applies Eq. (176) by summing over $C = \pm$ configurations with equal weight, as appropriate well above the threshold region. For example,

$$\begin{aligned}\text{rate}\ &\left(B_d(t)\overline{B}_d(\bar{t}) \to (l^-\ \nu\ D^*)_B(\psi\ K_S)_B\right) \propto\ e^{-\Gamma(t+\bar{t})}|A(\psi\ K_S)\overline{A}(l^-\ \nu\ D^*)|^2 \\ &\times\left\{2 - \left[\sin\left(\Delta\text{m}(t-\bar{t})\right) - \sin\left(\Delta\text{m}(t+\bar{t})\right)\right]\text{Im}\left(\frac{q}{p}\overline{\rho}(\psi\ K_S)\right)\right\} \quad .\end{aligned} \tag{195}$$

Integration over all times of decay yields

$$\begin{aligned}\text{rate}\ &\left(B_d\overline{B}_d \to (l^-\ \nu\ D^*)_B(\psi\ K_S)_B\right) \propto \frac{2}{\Gamma^2}\left|A(\psi\ K_S)\overline{A}(l^-\ \nu\ D^*)\right|^2 \\ &\times\left(1 + \frac{x}{(1+x^2)^2}\ \text{Im}\left(\frac{q}{p}overline\rho(\psi\ K_S)\right)\right) \quad .\end{aligned} \tag{196}$$

Correspondingly

$$\begin{aligned}\text{rate}\ &\left(B_d\overline{B}_d \to (l^+\ \nu\ \overline{D}^*)_B(\psi\ K_S)_B\right) \propto \frac{2}{\Gamma^2}|A(\psi\ K_S)\overline{A}(l^-\ \nu\ D^*)|^2 \\ &\times\left(1 - \frac{x}{(1+x^2)^2}\ \text{Im}\left(\frac{q}{p}\overline{\rho}(\psi\ K_S)\right)\right) \quad .\end{aligned} \tag{197}$$

In Eqs. (195)–(197) flavor tagging by explicit reconstruction of both neutral B decays was assumed. It should be kept in mind, however, that other methods are also conceivable.

3) e^+e^-, $h_1h_2 \to B_d\overline{B}_s + X', B_s\overline{B}_d + X'$

Both neutral B mesons can oscillate, yet this time not into each other. Assuming as usual $\Delta\Gamma = 0, |\frac{q}{p}|^2 = 1$ one finds

$$\begin{aligned}
\text{rate}\ \ &(B_s(t_s)\overline{B}_d(t_d) \to f_s f_d) \propto e^{-\Gamma_d t_d}\, e^{-\Gamma_s t_s} \times |A(f_s)|^2 |A(f_d)|^2 \\
&\times \Bigg\{ K_-(t_s)K_+(t_d) + K_+(t_s)K_+(t_d)|\overline{\rho}(f_s)|^2 \\
&+ K_-(t_s)K_-(t_d)|\overline{\rho}(f_d)|^2 + K_+(t_s)K_-(t_d)|\overline{\rho}(f_d)|^2|\overline{\rho}(f_s)|^2 \\
&- 4\sin(\Delta m_d t_d)\sin(\Delta m_s t_s)\mathrm{Im}\left(\left(\frac{q}{p}\right)_s \overline{\rho}(f_s)\right)\mathrm{Im}\left(\left(\frac{q}{p}\right)_d \overline{\rho}(f_d)\right) \\
&- 2\left(K_+(t_d) + |\overline{\rho}(f_d)|^2\ K_-(t_d)\right)\sin(\Delta m_s t_s)\mathrm{Im}\left(\left(\frac{q}{p}\right)_s \overline{\rho}(f_s)\right) \\
&+ 2\left(K_-(t_s) + |\overline{\rho}(f_s)|^2\ K_+(t_s)\right)\sin(\Delta m_d t_d)\ \mathrm{Im}\left(\left(\frac{q}{p}\right)_d \overline{\rho}(f_d)\right)\Bigg\}
\end{aligned} \tag{198}$$

$$K_\pm(t_i) = 1 \pm \cos\Delta\ m_i t_i \quad . \tag{199}$$

The expression for $B_d(t_d)\overline{B}_s(t_s) \to f_s f_d$ is obtained by switching the subscripts $s \leftrightarrow d$. The two rates are then added in a probalistic prescription: for the two states f_s and f_d are, at least in principle, distinguishable due to $M(B_d) \neq M(B_s)$ even if they are made up by the same hadrons.

C) Inclusive B Decays

So far we have discussed CP asymmetries in exclusive channels of B decays. The careful reader will be painfully aware of the enormity of the experimental task ahead: the asymmetries are expected to be large, yet the effective branching ratios in the promising modes do not exceed the 10^{-4} level. Further cuts that will be required to purify the sample will decrease this number significantly. And finally there is the requirement of flavor tagging.

One is quite naturally lead to the question whether one could not gain in statistics by summing over many decay channels. We will see that in principle this is indeed possible — but only under carefully controlled circumstances.

This caveat is illustrated by one example: Since

$$\mathrm{BR}(B \to \psi + X) \simeq 1\% \text{ vs. } \mathrm{BR}(B_d \to \psi\, K_S) \sim 5 \times 10^{-4}$$

one would be quite tempted to search for CP asymmetries in the inclusive decays $B_d \to \psi\, X$ vs. $\overline{B}_d \to \psi\, X$. Yet one has to keep in mind that the sign of the asymmetry depends, among other things, on the CP parity of the final state. Therefore,

$$\text{Asym}\ (B \to \psi\, K_S) = -\text{Asym}\ (B \to \psi\, K_L) \quad . \tag{200}$$

Accordingly,

$$\text{Asym}\ (B_d \to \psi\, X^0) \simeq 0 \quad . \tag{201}$$

Similarly,

$$\text{Asym}\ (B_d \to [p\bar{p}]_S) = -\text{Asym}\ (B_d \to [p\bar{p}]_P) \tag{202}$$

in the absence of nontrivial FSI.

This does not mean however that an analysis of inclusive decays is always bound to be fruitless. Three examples to support this claim:[18]

(1) $B_s \to \psi\, X$:

The CP parity of the final states $\psi\,\eta$, $\psi\,\eta'$, $\psi\,\phi$, $\psi'\,\eta$, $\psi'\,\eta'$, $\psi'\,\phi$ is even. At the same time these six processes might well almost saturate the inclusive width estimated as

$$\mathrm{BR}(B_s \to \psi\, X) \sim 1\% \quad . \tag{203}$$

Comparing the inclusive transitions $B_s \to \psi\, X$ vs. $\overline{B}_s \to \psi\, X$ would then not dilute an asymmetry that exists in the exclusive modes. (Of course, in the KM ansatz with three families only one predicts fairly small CP asymmetries here.)

(2) $B_d, \overline{B}_d \to D^\circ, \overline{D}^\circ + \pi' s \to (K_S + \pi' s) + \pi' s$

Consider the two-step decay

$$B_d \to D^\circ\ M \to (K_S\ N)_{D^\circ} M \tag{204}$$

where N, M denote an arbitrary member of the pseudoscaler, vector or axialvector nonets. One convinces oneself easily that

$$\begin{aligned} \mathrm{CP}|(K_S N)_{D^\circ}\rangle &= -1 \\ \mathrm{CP}|(K_S N)_{D^\circ} M\rangle &= +1 \quad . \end{aligned} \tag{205}$$

Therefore, all decays of the type (204) should contribute with the same sign to the asymmetry (unless some special FSI intervene). Unfortunately some of these decays would be very hard to identify, such as

$$B_d \to D^0\ \omega \to K_S \eta'\ \omega\ .$$

One can try to go one step further and compare $\overline{B}_d \to D + \pi' s \to K_S + \pi' s$ with $B_d \to \overline{D} + \pi' s \to K_S + \pi' s$. A CP asymmetry will be diluted in these inclusive modes, yet the resulting reduction might amount to a factor of two or three only.

(3) $\Upsilon(4S, 5S) \to B^\circ \overline{B}^\circ \to f_1 f_2$

When searching for a $B^\circ \overline{B}^\circ$ pair that had been produced in a 1^{--} (or a 1^{++}) state to decay into two CP eigenstates f_1 and f_2 one does not have to restrict oneself to identical states, like

$$\Upsilon(4S) \to B_d \overline{B}_d \to (\psi\ K_S)(\psi\ K_S) \quad .$$

Any CP eigenstate will do as long as $CP[f_1] = CP[f_2]$. Yet some subtleties have to be kept in mind, for example,

$$B_s \to D_s \overline{D}_s \gamma$$

does *not* produce a CP eigenstate, if it proceeded via one of the two distinguishable routes

$$B_s \to D_s^* \overline{D}_s \text{ or } B_s \to D_s \overline{D}_s^* \quad .$$

This, however, does not mean that the reaction

$$\Upsilon(5S) \to B_s \overline{B}_s \to (D_s \overline{D}_s \gamma)_B \; (D_s \overline{D}_s \gamma)_B \tag{206}$$

is insensitive to CP violation: the initial state is still CP even which has to reflect itself in the kinematical distributions in the final state. This points actually to a wider problem: so far we have largely concentrated on two-body decay modes of various types where the kinematics are trivial. Starting with three-body decay modes there is dynamical information contained in kinematical distributions, as expressed in the Dalitz plot. More phenomenological and theoretical work is needed in this direction.

D) Direct CP Violation in $\mathbf{\overline{B}_d \to (S = -1, C = 0)}$

It is at least tempting to search for direct CP violation in *inclusive* decays of neutral B mesons, like in $\overline{B}_d \to K^- + \pi's$ vs. $B_d \to K^+ + \pi's$. CPT invariance per se does not rule out a difference there. Simple duality concepts would suggest that the conventional soft FSI are averaged out in the sum over hadronic states contained in this process and that the asymmetry is driven purely by the quark level transitions as discussed in Act II, Part B:

$$\Gamma(\overline{B}_d \to K^- + \pi's) < \Gamma(B_d \to K^+ + \pi's) \tag{207}$$

with the inclusive asymmetry ranging from $\sim 1\%$ to $\sim 10\%$ represents a conceivable scenario.

E) CP Asymmetries in the Decays of Neutral D Mesons

It was already stated in the beginning that the KM ansatz causes only tiny CP asymmetries in charm decays. Nevertheless we want to mention it here since

- New Physics might increase CP asymmetries significantly and
- the phenomenology developed so far can be applied here in the same way.

(*i*) There are CP eigenstates that are produced with decent branching ratios in D° decays; i.e.,

$$\mathrm{BR}(D^\circ \to K^+\ K^-) \sim \mathrm{BR}(D^\circ \to K_S\ \phi) \sim 0.5\% \quad .$$

(*ii*) Their evolution in proper time is approximately given by

$$\begin{aligned} \mathrm{rate}\left(D^\circ(t) \to K^+K^-\right) &\propto\ e^{-\Gamma t}\left(1 - \sin(\Delta \mathrm{m} t)\mathrm{Im}\left(\frac{q}{p}\overline{\rho}(K^+K^-)\right)\right) \\ \mathrm{rate}\left(\overline{D}^\circ(t) \to K^+K^-\right) &\propto\ e^{-\Gamma t}\left(1 + \sin(\Delta \mathrm{m} t)\mathrm{Im}\left(\frac{q}{p}\overline{\rho}(K^+K^-)\right)\right) \quad . \end{aligned} \tag{208}$$

An asymmetry can emerge for Δ m $\neq 0$, i.e., if mixing occurs.

The Standard Model predicts very little $D^\circ - \overline{D}^\circ$ mixing and the E691 collaboration has placed a very stringent upper bound on it

$$r_D = \frac{\Gamma(D^\circ \to \overline{D}^\circ \to \bar{f})}{\Gamma(D^\circ \to f)} < 0.5\% \quad . \tag{209}$$

Yet one has to keep in mind that

$$r_D \simeq \frac{x^2}{2 + x^2} \tag{210}$$

where we have put $\Delta\Gamma = 0$ for simplicity. Therefore, the bound (209) implies

$$x \leq 0.1 \quad .$$

Accordingly, one has for $x = 0.1$,

$$\mathrm{rate}\left(D^\circ(t) \to K^+K^-\right) \propto\ e^{-\Gamma t}\left(1 - 0.1 \times \frac{t}{\tau_D}\ \mathrm{Im}\left(\frac{q}{p}\overline{\rho}(K^+K^-)\right)\right)$$

i.e., CP asymmetries of order 5–10% are still allowed in principle and should be searched for.

Synopsis of Act IV

(*i*) A search for CP asymmetries in

$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B_d\overline{B}_d$$

is very limited in scope, unless one obtains some (partial) information on the proper time evolution of the neutral B decays by employing asymmetric collisions:

- direct CP violation can be looked for.
- One can search for events of the type

$$\Upsilon(4s) \rightarrow B_d\overline{B}_d \rightarrow f_1 f_2$$

where f_1, f_2 denote CP eigenstates of the same CP parity.

- All other asymmetries are washed out by the integration over times of decay.

(*ii*) The reaction

$$e^+e^- \rightarrow B^\circ\overline{B}^{\circ *} \rightarrow B^\circ\overline{B}^\circ\gamma$$

is very favorable for finding CP asymmetries. A high premium is placed on good time resolution if B°–$\overline{B}^\circ$ mixing is rapid since a (double) time integration introduces a factor $2x/(1+x^2)^2$.

(*iii*) Using high energy e^+e^-annihilation or hadronic collisions for producing beauty hadrons should allow to resolve the proper time evolution of these decays. Of course one has to deal then with the problems caused by complex final states.

(*iv*) Flavor tagging

- can be done by partially reconstructing the decays of both beauty hadrons or
- it can be provided by a *production* asymmetry due to $\gamma - Z^\circ$ interference, polarized beams in

$$e_L^- e^+ \to Z^\circ \to b\bar{b}$$

or by leading particle effects, etc. in hadronic collisions.

(*v*) Searches for CP asymmetries in *inclusive* B decays are meaningful only if proper care is applied.

(*vi*) Analogous CP asymmetries should be looked for in the decays of neutral D mesons.

ACT V: CONCLUSIONS AND OUTLOOK

The industrious reader who has made it this far will by now have grasped one message: a meaningful search for CP violation in heavy flavor decays can be conducted only after truly awesome experimental challenges have been overcome and it is not clear at present how they can be met. Yet the reader should not forget about the second message we were conveying: that there are bound to be large CP asymmetries in beauty decays that could reach as high as 60 % — if the KM ansatz indeed describes at least a major part of the CP violation in K_L decays.

We can actually go beyond just stating that in some B decay channels there have to be CP asymmetries that are at least a few percent, and could reach even sixty percent. In principle, one should be in a position to make reliable and reasonably precise predictions. There are two categories of reasons why we are presently unable to do so: firstly, some of the underlying electroweak model parameters are insufficiently known; secondly, intrinsic uncertainties due to the impact of strong interactions arise.

A) Electroweak Model Parameters

(*i*) If it is the intervention of $B^\circ - \overline{B}^\circ$ mixing that is required, then one better knows the real size of the relevant parameters, namely Δm and $\Delta\Gamma$. This is not the case—yet we can expect that future measurements will narrow, or even close, this gap. A measurement of the top mass would be of great theoretical help for a crosscheck.

(*ii*) Great uncertainties concerning $|V(ub)/V(cb)|$ exist. Once a certain body of various well-measured B branching ratios has been established, such uncertainties will decrease significantly.

(*iii*) To make a quantitative prediction on CP asymmetries, one obviously has to know the value of the KM phase, i.e., η in our notation. In principle, η can be extracted from the measured value of ϵ_K; at present, however, this procedure resembles more a form of art than a craft, let alone a science. The situation would improve significantly if the top mass were known.

B) Impact of Strong Interactions

The term

$$\mathrm{Im}\,\frac{q}{p}\,\bar{\rho}\,(f) \simeq \left|\frac{q}{p}\bar{\rho}(f)\right| \sin\Phi_{CPV}$$

drives CP asymmetries and can be observed. The connection between the observable and the underlying electroweak parameters is, in general, obscured by the presence of strong interactions.

As pointed out before, there are a few cases where these complications can be ignored and one has

$$\left|\frac{q}{p}\,\bar{\rho}\,(f)\right| \simeq 1 \tag{211}$$

while Φ_{CPV} is given by KM parameters alone; this is true in decays that are driven by $b \to c\bar{c}s$ transitions, e.g.,

$$B_d \to \psi K_S \tag{212}$$

$$B_d \to D\overline{D}K_S \tag{213}$$

$$B_s \to \psi\phi \tag{214}$$

$$B_s \to D_s\overline{D}_s \quad . \tag{215}$$

Equation (211) is not the result of a strict mathematical theorem; yet, by the standards of our field it is quite conclusive and, in any case, sufficiently convincing to us. This already allows the first unequivocal conclusion:

- The existence of New Physics is established if decays like

$$B_s \to \psi\phi, D_s\overline{D}_s$$

exhibited CP asymmetries in excess of a few percent.

In decay modes that are driven by $b \to u\bar{u}d, u\bar{s}d$ transitions, we can guestimate

$$\left|\frac{q}{p}\bar{\rho}(f)\right| \sim 1 \quad . \tag{216}$$

Yet FSI (like rescattering and channel mixing) could well violate (216) by significant amounts in, e.g.,

$$B_d \to \pi^+\pi^- \tag{217}$$

$$B_s \to K^+K^- \quad . \tag{218}$$

On the other hand, we can learn a lot about the significance of FSI from the branching ratios once they are measured. *Once a body of well-measured branching ratios—including $BR(B^\circ \to \pi^+\pi^-$, $K^\mp\pi^\pm$, $K^+K^-)$—has been established,* we will be able to cross-check relations like Eq. (216); this allows further conclusions:

- When *experimental* data of sufficient sensitivity become available, we can extract $\mathrm{Im}\, q/p\bar{\rho}(f)$ from the data with rather good theoretical confidence.

- In that case we can probe the KM triangle, Fig. 1, in a very sensitive way. For example:

$$-\mathrm{Im}\,\frac{q}{p}\bar{\rho}\,(B_d \to \psi K_S) \simeq \sin 2\Phi_1 \tag{219}$$

$$-\mathrm{Im}\,\frac{q}{p}\bar{\rho}\left(B_d \to \pi^+\pi^-\right) \simeq \left|\frac{q}{p}\bar{\rho}\left(B_d \to \pi^+\pi^-\right)\right| \sin 2\Phi_2 \tag{220}$$

$$\mathrm{Im}\,\frac{q}{p}\bar{\rho}\left(B_s \to \Phi\rho^0\right) \simeq \left|\frac{q}{p}\bar{\rho}\left(B_d \to \Phi\rho^0\right)\right| \sin 2\Phi_3 \quad . \tag{221}$$

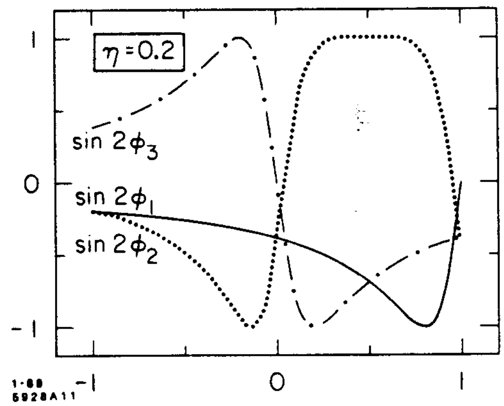

Fig. 10. The three asymmetry parameters sin $2\Phi_i$ as a function of ρ with $\eta = 0.2$.

Any violation of planar trigonometry, like

$$\phi_1 + \phi_2 + \phi_3 \neq 180^\circ \quad ,$$

would establish the presence of New Physics. It should be emphasized that *these trigonometric relations are independent of the problem of relating the KM phase to ϵ_K.*

The absolute size of the sin $2\Phi_i$ are shown in Fig. 10 as a function of ρ for $\eta = 0.2$.

- We would learn to interpret a difference between $\overline{B}_d \to K^-\pi^+, K^-\rho^+$ and $B_d \to K^+\pi^-, K^+\rho^-$ (if observed) in an at least semiquantitative fashion.

EPILOGUE

Experiments of sufficiently high sensitivity have to turn up rather large CP asymmetries in beauty decays. Once detailed data on $B^\circ - \overline{B}^\circ$ mixing and on branching ratios have become available, we will be able to state a large number of rather precise predictions. If those failed, there would be no "plausible deniability"—the KM scheme could no longer be maintained as the sole, or even dominant, source of CP violation, *there had to be New Physics.*

If on the other hand they were confirmed we would finally have arrived at a *tested* description of CP violation. It would represent a crucial step forward towards the goal of developing a deeper understanding of one of the most mysterious elements in nature's design.

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