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# A STATISTICAL STUDY OF TAU DECAY DATA*

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## ABSTRACT

In the measured decay properties of the tau there is a discrepancy between the total branching fraction for the one charged particle decay modes and the sum of the branching fractions for the known individual modes. This discrepancy is derived from about 60 different measurements of branching fractions and some use of weak interaction theory. Our statistical study of these 60 measurements shows there are problems in some of the measurements in the estimation of experimental bias or systematic error. But there is no evidence that the discrepancy derives from experimental bias or from incorrect estimation of systematic error.

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## I. INTRODUCTION

At present, the decay modes of the tau lepton containing 1-charged particle are not completely understood.[1–4] By definition, the total branching fraction for those modes, $B_1$, is the sum of the separate branching fractions, $B_\alpha$, for the individual modes containing 1-charged particle such as:

$$B_e \text{ for } \tau^- \to \nu_\tau + e^- + \bar{\nu}_e \quad ; \qquad (1a)$$

$$B_\mu \text{ for } \tau^- \to \nu_\tau + \mu^- + \bar{\nu}_\mu \quad ; \qquad (1b)$$

$$B_\pi \text{ for } \tau^- \to \nu_\tau + \pi^- \quad ; \qquad (1c)$$

$$B_\rho \text{ for } \tau^- \to \nu_\tau + \rho^- \quad ; \qquad (1d)$$

and

$$B_{\pi 2\pi^0} \text{ for } \tau^- \to \nu_\tau + \pi^- + 2\pi^0 \quad . \qquad (1e)$$

As emphasized by Gilman[1,2] the sum, $\sum_\alpha B_\alpha$, of present measurements of the individual branching fraction combined with theoretical constraints on unmeasured branching fractions does not fully explain the present measured value of $B_1$. A question raised by this problem is whether the errors, $\sigma_\alpha$, given for the measured branching fractions by the experimenters are correct, whether the appearance of a discrepancy between $\sum_\alpha B_\alpha$ and $B_1$ is caused by an underestimate of the size of one or more $\sigma_\alpha$'s or of $\sigma_1$.

We have examined this question by comparing the given errors, $\sigma_\alpha$, with the scatter of the measurements about the mean for each measurement set. We do this for $B_1$ and for the $B_\alpha$'s of the modes in Eqs. (1a)–(1d). Normal error distributions are used. We find that on the whole the errors estimated from the scatter are equal to or smaller than the given errors, $\sigma_\alpha$, according to this test.

In other words, some sets of measurements are overconsistent. By using just the statistical contribution to the measured errors, we can test in some cases whether the overconsistency is caused by overestimation of systematic errors or bias in the measurements.

As an aid to researchers in this area we present tables of the data we used. This is all the data published in journals, cataloged preprints, or Ph. D. thesis; the authors being the experimenters themselves. We also present a comparison of the measured $\tau$ lifetime, $\tau_\tau$, with the leptonic branching fractions, $B_e$ and $B_\mu$, for the decays in Eqs. (1a) and (1b).

The nature of the present apparent discrepancy[3–4] between $B_1$ and $\sum_\alpha B_\alpha$ is diagrammed in Table 1. There is no discrepancy if considerations of the various $B_\alpha$'s is limited to direct measurements. This is because there are no reliable measurements[4] for some modes contributing to the signature

$$\tau^- \to \nu_\tau + x^- + n\gamma \quad , \quad n > 4 \quad , \tag{2}$$

where $x$ is a charged particle. Also there are no comprehensive and sufficiently small experimental limits on unconventional 1-charged particle decay modes such as

$$\tau^- \to N^\circ + x^- \quad , \tag{3}$$

where $N^\circ$ is an unknown, massive, stable neutral particle.

The discrepancy appears when unconventional modes are excluded, and when conventional theory and other data is used[1,2] to set limits on the modes which could contribute to the event type in Eq. (2). Then $B_1$ is larger than $\sum_\alpha B_\alpha$ by about 6%.[4] Table 1 demonstrates the importance of the measurements and quoted errors on the branching fractions for $B_e$, $B_\mu$, $B_\pi$, $B_\rho$, and $B_1$. This motivated our study.

## II. DATA USED

We used the branching fraction data listed in Tables 2 through 5 and the lifetime data in Table 6. We have included in the tables all data presented by the experimenters themselves in journal articles, cataloged preprints, or Ph.D. thesis unless the experimenters have stated their measurement is replaced by their own later measurement. We have not included measurements which are reported only through private communication or in reviews. These criteria permit us to work with fixed measurements and permit the reader to examine the details of experiments.

Table 2 presents $B_3$ as well as $B_1$ although most measurements of $B_1$ and $B_3$ are strongly correlated. Often only one is measured and the other calculated by $B_1 + B_3 + B_5 = 1$, with $B_5 = 0.1\%$.

To insure the measurements for a specific branching ratio are statistically independent, we have excluded several measurements from the statistical analysis although they are included in the tables. The MARK II collaboration has published two measurements[5,6] of $B_1$, and two measurements[5,7] of $B_\rho$ that use the same data set but different analysis techniques. We use only the measurement with the smallest total error. The 1982 MARK II collaboration and 1984 TPC collaboration measurements[8,9] of $B_1$ are not independent of their more precise recent measurements and thus are also excluded.

## III. $B_e$, $B_\mu$, AND $e - \mu$ UNIVERSALITY

The use of the constraint on $B_e$ and $B_\mu$ from $e-\mu$ universality, $B_\mu = .973B_e$, must be carefully considered when comparing or averaging experimental measurements. Four cases occur: i) the experimenters measure the product branching ratio $B_e \cdot B_\mu$ and use the constraint to determine $B_e$ and $B_\mu$; ii) the experimenters measure $B_e$ and $B_\mu$ independently; iii) the experimenters measure $B_e$ and $B_\mu$ separately but the measurements are strongly correlated, perhaps because they simultaneously measure the product $B_e \cdot B_\mu$; iv) only $B_e$ or $B_\mu$ is measured. In the measurements listed in Table 3, there are four type i, six type ii, three type iii, and eight type iv experiments.

In our analysis of the $B_e$ and $B_\mu$ measurements, we first analyze only the subsets of experiments which do not make use of the universality constraint. The experiments in these sets can be equally treated in the analysis, and allow a test of the universality constraint to be made. We then apply, if necessary, the universality constraint to each experiment in Table 3 and determine a constrained branching ratio, $B_e'$. The third column of branching ratios listed in Table 3 are the results of this constraint procedure. The statistical analysis is then applied to the full set of constrained measurements.

In the constraint procedure, type i experiments and experiments which measure only $B_e$ are used directly. Experiments which measure only $B_\mu$ are scaled: $B_e' = B_\mu/.973$. Type ii experiments are constrained using the equations below:

$$B_e' = \left[B_e/\sigma^2_{B_e} + .973B_\mu/\sigma^2_{B_\mu}\right] \Big/ \left[1/\sigma^2_{B_e} + .973^2/\sigma^2_{B_\mu}\right] \quad , \tag{4}$$

and

$$\sigma_{B'_e} = \left[1/\sigma^2_{B_e} + .973^2/\sigma^2_{B_\mu}\right]^{-\frac{1}{2}} \quad . \tag{5}$$

The universality constraint can be applied to type iii experiments if the correlation between the $B_e$ and $B_\mu$ measurements is known. For the special case where $B_e$, $B_\mu$, and the product branching ratio $B_{e\mu} = B_e \cdot B_\mu$ are measured, then $B'_e$ is determined by minimizing the $\chi^2$

$$\chi^2 = \left(B'_e - B_e\right)^2/\sigma^2_{B_e} + \left(.973B'_e - B_\mu\right)^2/\sigma^2_{B_\mu} + \left(.973B'^2_e - B_{e\mu}\right)^2/\sigma^2_{B_{e\mu}} \quad . \tag{6}$$

This results in a cubic equation for $B'_e$ and an error given by

$$\sigma_{B'_e} = \left[1/\sigma^2_{B_e} + .973^2/\sigma^2_{B_\mu} + 2(.973)\cdot\left(3(.973)B'^2_e - B_{e\mu}\right)/\sigma^2_{B_{e\mu}}\right]^{-\frac{1}{2}} \quad . \tag{7}$$

Note that these constraint techniques average the systematic errors for $B_e$, $B_\mu$, and $B_{e\mu}$ within a single experiment. Thus, systematic errors which are common to the $B_e$, $B_\mu$, and $B_{e\mu}$ measurements will be averaged resulting, perhaps, in an *underestimate* of the systematic error on $B'_e$.

## IV. ANALYSIS METHOD

Consider a particular branching fraction, $B_\pi$ for example. As listed in Table 4 there are seven different measurements: $B_{\pi 1}$, $B_{\pi 2} \ldots B_{\pi i} \ldots$ . We want the weighted average $< B_\pi >$ and the error on that average, $\sigma_{B_\pi}$. Simplifying the notation: $y_i$ replaces $B_{\pi i}$, $y$ replaces $< B_\pi >$ and $\sigma$ replaces $\sigma_{B_\pi}$. The same notation is used for $B_e$, $B_\mu$, $B_\rho$ and $B_1$.

Most recent measurements, $y_i$, included a statistical error $\sigma_{stat,i}$ and a systematic error $\sigma_{sys,i}$. We follow the Particle Data Group's method[10] of combining these errors in quadrature

$$\sigma_i = (\sigma^2_{stat,i} + \sigma^2_{sys,i})^{\frac{1}{2}} \quad ; \tag{8}$$

and we use this combined error unless the experimenters provide a total error. The formal average is

$$y = \sum_i \left(y_i/\sigma_i^2\right) \Big/ \sum \left(1/\sigma_i^2\right) \quad , \tag{9}$$

and the formal combined error in $y$ is

$$\sigma = \left(\sum_i \left(1/\sigma_i^2\right)\right)^{-\frac{1}{2}} \quad . \tag{10}$$

The relative weight of a measurement $i$ is

$$w_i = \sigma^2/\sigma_i^2 \quad . \tag{11}$$

The scatter of the individual measurements, $y_i$, from $y$ are described by the pulls:

$$p_i = (y - y_i)/\left[\sigma_i^2 - \sigma^2\right]^{\frac{1}{2}} \quad . \tag{12}$$

For Gaussian distributed errors, $\sigma_i$, the distribution of pulls is a normal distribution of unit width and zero mean.

The standard deviation, $\sigma_{scat}$, of the weighted mean is calculated from the average variance, $s^2_{scat}$, for $N$ measurements:

$$s^2_{scat} = \sum_i \left(N w_i [y_i - y]^2\right) \Big/ (N-1) \quad ,$$

which reduces to

$$s^2_{scat} = N\Big(\sum_i w_i y_i^2 - y^2\Big) \Big/ (N-1) \quad .$$

Using $\sigma^2_{scat} = s^2_{scat}/N$, we have

$$\sigma_{scat} = \left[\Big(\sum_i w_i y_i^2 - y^2\Big) \Big/ (N-1)\right]^{\frac{1}{2}} \quad . \tag{13}$$

Observe that the errors $\sigma_i$ are used in this equation in the weighting of $[y_i - y]^2$, but are not directly used to calculate $\sigma_{scat}$.

Our interest centers on the relative sizes of $\sigma$ and $\sigma_{scat}$. As discussed in Sec. V, if $\sigma$ is significantly smaller than $\sigma_{scat}$, some of the experimenters have given $\sigma_i^2$ which are too small. Then the $\sigma$ used in Table 1 is too small, and the discrepancy problem is less certain. If $\sigma$ is significantly larger than $\sigma_{scat}$, there are three, not exclusive, explanations. Some experimenters may have overestimated their $\sigma_i^2$. Or, some experimenters may have corrected their raw measurements while biased toward a preconceived value for $y$, the preconception being based on the existing accepted value of $y$ or on theoretical considerations. Finally, systematic errors common to many experiments may exist which the experimenters have accounted for in their determination of $\sigma_i$. In this case the correlated contribution to $\sigma_i$ should be removed before comparing $\sigma$ and $\sigma_{scat}$. There are no sources of accounted correlated systematic errors common to many experiments in these data which are described in the referenced experimental papers. Although with intimate knowledge of all experiments such sources might be found, we have made no attempt here to hunt for them.

We use the ratio

$$r = \sigma_{scat}/\sigma \tag{14}$$

to measure the relative sizes of $\sigma_{scat}$ and $\sigma$ for a set of measurements. We are particularly interested if $r$ is significantly less than one or significantly greater than one. To determine the significance we calculate the probability, $P(< r)$, of finding a smaller value of $r$, and the converse, $P(> r)$, of finding a larger value of $r$. [Since $P(< r) + P(> r) = 1$, only $P(< r)$ need be calculated.] For example, suppose $r = .5$ because $\sigma$ is twice $\sigma_{scat}$. If $P(< .5)$ is 10%, then $r = .5$ has this statistical significance.

The formal average [Eq. (9)] is obtained by minimizing

$$\chi^2 = \sum_i \left(y_i - y\right)^2 / \sigma_i^2 \tag{15}$$

which has the minimum value

$$\chi^2_{min} = \sum_i \left(w_i y_i^2 - y^2\right) / \sigma^2 \quad . \tag{16}$$

From Eqs. (13) and (16) we see that

$$r = \left[\chi^2_{min} \middle/ (N-1)\right]^{\frac{1}{2}} \quad . \tag{17}$$

Thus, the probability $P(> r)$ is identical to the probability of having a larger $\chi^2$ for $N - 1$ degrees of freedom. Figure 1 plots the distribution of $r$ for several values of $N$.

We apply this analysis method first to each full set of data. However, $r$ can be very sensitive to a particular measurement which has a relatively large $\sigma_i$ even though that measurement has a small weight $w_i$ and little effect on $y$. Therefore, in each data set we select the minimum number of measurements $a$, $b$, $\ldots e$ such that

$$w_a + w_b + \ldots w_e > 0.81 \quad .$$

This smaller set of measurements will have a formal error no larger that $1/.9$ of $\sigma$ and will contain fewer measurements with relative large $\sigma_i$'s. We apply the same method of analysis to these smaller sets of data.

For both the full and small data sets we apply the same method of analysis using just the statistical errors. This tests the effects of systematic errors on the determination of $y$ and $\sigma$. We examine the relative importance of statistical and systematic errors in determining the formal error as follows: if $\sigma_{stat}$ is the formal error obtaining using only statistical errors, then we define the contribution to the formal error from systematic errors to be

$$\sigma_{sys} = \left[\sigma^2 - \sigma_{stat}^2\right]^{\frac{1}{2}} \quad . \tag{18}$$

Here $\sigma_{stat} = [\sum_i \sigma_{stat,i}^{-2}]^{-\frac{1}{2}}$. But note that $\sigma_{sys}$ is not $[\sum_i \sigma_{sys,i}^{-2}]^{-\frac{1}{2}}$.

In a few measurement sets the formal error is asymmetric: $\sigma_+ \neq \sigma_-$. In that event we use the arithmetic average. There is no change in our conclusions if we used the maximum or minimum of $\sigma_+$, $\sigma_-$ because in all cases their difference is relatively small.

## V. SYSTEMATIC ERRORS AND $r$

The combined error $\sigma_i$ of a measurement $y_i$ is obtained from $\sigma_i = (\sigma^2_{stat,i} + \sigma^2_{sys,i})^{\frac{1}{2}}$. The statistical error, $\sigma_{stat,i}$, depends on numbers of events and represents a normal error distribution. Hence our use of $\sigma_{stat,i}$ is straightforward. This is not true for the systematic error, $\sigma_{sys,i}$. There are a multitude of uncertainties in the estimate and use of $\sigma_{sys,i}$.

An obvious problem is that $\sigma_{sys,i}$ may not represent a normal error distribution. Suppose it represents an error distribution with tails relatively larger than those of a normal distribution. The use of $\sigma_i$ to calculate the weighted formal average is still acceptable. But it would be wrong to interpret the formal error, $\sigma$, as representing a normal distribution when one is considering discrepancies which are several $\sigma$ in magnitude. In a later paper[11] we will consider a method of treating errors which does not depend on the normal distribution assumption; in this paper we maintain the normal error distribution assumption for $\sigma_{sys,i}$ as well as $\sigma_{stat,i}$.

The determination of a branching fraction requires the counting in a data set of the number of $\tau$ decays with that decay mode. This number is then multiplied by factors $f$, $g$, $h$.... These factors include: normalization quantities such as total number of $\tau$ decays or total luminosity or total cross section, efficiency factors such as detector acceptance, and perhaps other quantities. A few of these factors are obtained by counting events in the data set — the total number of decays, for example — and are assigned a statistical error. But most of the factors are obtained by computation or from other data and are assigned a systematic error. (A few factors may have both types of errors.) Let $\sigma_i(f)$ be the systematic error assigned to the factor $f_i$ by the experimenters who reported branching fraction measurement $y_i$ with errors $\sigma_{stat,i}$ and $\sigma_{sys,i}$.

We are about to tabulate some of the problems that can occur in a measurement set $y_1$, $y_2$ ... $y_i$ ... from incorrect evaluation of $f_i$'s or $\sigma_i(f)$'s. We emphasize two aspects of these incorrect evaluations: a) we examine whether the formal error $\sigma$ will be smaller or larger than the *actual error* on $y$; b) we look at the ratio $r = \sigma_{scat}/\sigma$. If $\sigma_{scat}$ is significantly less than $\sigma$, that is $r < 1$, then the measurement set is *overconsistent.* If $\sigma_{scat}$ is significantly greater than $\sigma$, then the measure set is *inconsistent.*

(*i*) Overestimate of Some $\sigma_i(f)$'s: For the sake of caution and because of the difficulty of evaluating some $f$'s, experimenters may assign large $\sigma_i(f)$'s. Then:

a) $\sigma$ is larger than the actual error on $y$;

b) $\sigma_{scat}$ is smaller than $\sigma$ and the measurement set is overconsistent, therefore $r < 1$.

(*ii*) Underestimation of Some $\sigma_i(f)$'s: In spite of caution, the history of physics has many examples of underestimation of systematic errors. Then:

a) $\sigma$ is smaller than the actual error on $y$;

b) $\sigma_{scat}$ is larger than $\sigma$ and the measurement set is inconsistent, therefore $r > 1$.

(*iii*) Biasing of $y_i$'s: The values of some $f_i$'s may be set unconsciously so that the resulting $y_i$ tends towards an already published or preconceived value of $y$. The $\sigma_i(f)$'s may not be set large enough to encompass this bias. Then:

a) $\sigma$ is smaller than the actual error on $y$;

b) depending on whether different experiments are biased in a similar direction or towards a similar value, $\sigma_{scat}$ might be smaller than $\sigma$ and the measurement set may be overconsistent, perhaps $r < 1$.

(*iv*) Uncorrelated, Unaccounted $\sigma_i(f)$'s: One experiment $i$ may have a mistake in $f_i$ not encompassed in its $\sigma_i(f)$, another experiment $j$ may have a different mistake in $f_j$ or may have a mistake in another factor $g_j$, neither may be encompassed in $\sigma_j(f)$ or $\sigma_j(g)$. Then:

a) $\sigma$ is smaller than the actual error on $y$;

b) $\sigma_{scat}$ is larger than $\sigma$ and the measurement set is inconsistent, therefore $r > 1$.

(*v*) Correlated, Unaccounted $\sigma_i(f)$'s: Suppose most measurements in a set use the same factor $f$, that it is slightly wrong, but the mistake is not encompassed in any of the $\sigma_i(f)$'s. This would shift the value of $y$. The error representing this shift would not be in $\sigma$, it might show up in $\sigma_{scat}$. Then:

a) $\sigma$ is smaller than the actual error on $y$;

b) $\sigma_{scat}$ might be larger than $\sigma$ and the measurement set may be inconsistent, perhaps $r > 1$.

All these factors may simultaneously exist in a specific data set, and competing effects may work together to make the data set appear consistent. For example, experimenters may be tempted to increase poorly understood systematic errors if their result *appears* to be inconsistent with other published results.

However, there is one instance when the existence of problem iii (bias) can be demonstrated: i.e., an overconsistent measurement set remains overconsistent when only the statistical errors, $\sigma_{stat,i}$, are used. This assumes that experimenters do not overestimate statistical errors. If systematic errors dominate the measurements, the overconsistency may cease when using only $\sigma_{stat,i}$ even if bias is present because the systematic errors exist and contribute to the scatter.

## VI. EXAMPLE

We clarify the method and our interpretation by the example of $B_1$, summarized in Table 7. We use only the higher energy measurements as described in Sec. VII A. The average values and errors for the 11 measurements are:

$$\begin{aligned} y &= 86.58\% \\ \sigma &= \pm.28\% \\ \sigma_{scat} &= 0.27\% \\ r &= .96 \\ &P(<.96) = 49\%,\ P(>.96) = 51\% \end{aligned} \tag{19}$$

We interpret these values of $r$ and $P(<r)$ to mean that $\sigma_i$'s given by the experimenters are the right size as measured by $\sigma_{scat}$.

We then analyze this data set using only $\sigma_{stat,i}$ to calculate $y$ and $\sigma$. We obtain:

$$\begin{aligned} y &= 86.79\% \\ \sigma_{stat} &= \pm0.14\% \\ \sigma_{scat} &= 0.21\% \\ r &= 1.48 \\ &P(<1.48) = 98.0\%,\ P(>1.48) = 2.0\% \end{aligned} \tag{20}$$

The formal error is now significantly smaller than the error determined from the scatter. This indicates that, as expected, systematic errors are indeed present in the experiments. Using Eq. (18), we obtain $\sigma_{sys} = \pm 0.24\%$, and the ratio of systematic to statistical errors is $\sigma_{sys}/\sigma_{stat} = 1.7$. Thus, the measurement of $B_1$ is dominated by systematic errors.

We now repeat the analysis using the five measurements with largest weights whose combined weight is greater than 0.81, Table 7. For this small set we find:

$$\begin{aligned} y &= 86.64\% \\ \sigma &= \pm 0.29\% \\ \sigma_{scat} &= 0.33\% \\ r &= 1.10 \\ P(&< 1.10) = 70\%,\ P(> 1.10) = 30\% \end{aligned} \tag{21}$$

The reduction from 11 measurements to 5 does not change $y$, a desirable feature in a set of measurements. The removal of measurements with small $w_i$'s and hence relatively large $\sigma_i$'s increases $r$ to 1.10. But $P(> 1.10) = 30\%$, therefore the difference of $r$ from 1 is not significant.

Finally, we analyze the small set using only $\sigma_{scat,i}$. The results are:

$$\begin{aligned} y &= 86.73\% \\ \sigma_{stat} &= \pm 0.15\% \\ \sigma_{scat} &= 0.23\% \\ r &= 1.55 \\ P(&< 1.55) = 95.3\%,\ P(> 1.48) = 4.7\% \end{aligned} \tag{22}$$

The small set, which contains only those experiments with the largest weight, has properties similar to the large set.

The results in Eqs. (19)–(22) show that the measurements used to find the formal average and error for $B_1$ in this example have reasonable errors attached to them by the experimenters.

## VII. RESULTS

This section consists of these parts: the results of the analysis for the individual measurement sets $B_1$, $B_3$, $B_e$, $B_\mu$, $B_\pi$, and $B_\rho$; a combined analysis for $B_e$, $B_\mu$, $B_\pi$, and $B_\rho$; and a comparison of $B_e$ and $B_\mu$ with $\tau_\tau$.

### A. Analysis of $B_1$, $B_3$, $B_e$, $B_\mu$, $B_\pi$, and $B_\rho$

Table 8 lists quantities found for each measurement set from which a reader can draw conclusions as to the quality of the set. We offer some comments as a guide.

Comments on $B_1$, $B_3$: The set is dominated by the measurement from the HRS collaboration[12] which contributes half the total weight. Looking at Table 2, the three lowest energy measurements are quite different from the formal average, but only the one from the DELCO collaboration[13] is by itself statistically inconsistent. The deviation of the low energy measurement is usually attributed to insufficient correction for background from the process $e^+e^- \to$ hadrons. However we cannot rule out the existence of an energy dependent, unknown process being confused with the events used to determine $B_1$ and $B_3$ at either low or high energy. The *average* of the other low energy experiments is also inconsistent with the formal average. In order to test the statistical properties of the precise high energy experiments, we exclude all low energy experiments from the $B_1$ and $B_3$ analyses in Table 8.

As discussed in Sec. VI, $\sigma_{scat}$ is consistent with $\sigma$, hence a large number of experiments agree on these relatively simple measurements and the formal average seems to be reliable.

Comment on $B_e$, $B_\mu$: As discussed in Sec. III, we first analyze the 10 unconstrained measurements of $B_e$ and the 16 unconstrained measurements of $B_\mu$ listed in Table 3. The results are given in Table 8. The measured ratio of $B_\mu/B_e$ is

$$B_\mu/B_e = 1.005 \pm .034 \tag{23}$$

which is consistent with the expected value of .973. Systematic and statistical errors are about equal: $\sigma_{sys}/\sigma_{stat} = .9$. When the full sets of measurements are used, the sets are consistent as defined in Sec. V. However, the small set of $B_\mu$ measurements tends to be overconsistent:

$$B_\mu,\ \text{small set}: \quad r = .47,\ P(< r) = 4.5\% \quad .$$

If only statistical errors are used, a hint of overconsistency remains:

$$B_\mu,\ \text{small set},\ \sigma_{stat,i}: \quad r = .67,\ P(< r) = 18.7\% \quad .$$

The $B_e'$ data set is the largest set, and due to the universality constraint, the formal errors are much smaller than for the $B_e$ or $B_\mu$ measurements. Both the full set and small set tend to be overconsistent:

$$B_e',\ \text{full set}: \quad r = .73,\ P(< .73) = 4.6\% \quad ,$$

$$B_e',\ \text{small set}: \quad r = .52,\ P(< .52) = 6.9\% \quad .$$

Either the experiments may have overestimated their errors, in which case the formal error is too large, or else there may be bias in the measurements in which case the formal error is too small. When using just the statistical errors, a hint of bias remains.

Comment on $B_\pi$: Systematic errors dominate these measurements: $\sigma_{sys}/\sigma_{stat} = 2.1$. Here again the full set of measurements tend toward overconsistency as defined in Sec. V.

$$B_\pi, \text{ full set}: \quad r = .59, \; P(<r) = 8.3\% \quad .$$

This overconsistency remains when the smaller sets are used, although the statistical significance is weaker since there are only four measurements in the small set.

$$B_\pi, \text{ small set}: \quad r = .56, \; P(<r) = 19.\% \quad .$$

The systematic errors are so much larger than the statistical errors, that when only the statistical errors are used, no hint of overconsistency remains.

Comment on $B_\rho$: Like the $B_\pi$ measurements, systematic errors dominate the measurements: $\sigma_{sys}/\sigma_{stat} = 2.1$. Both the full set and small set are very overconsistent:

$$B_\rho, \text{ full set}: \quad r = .21, \; P(<r) = 0.1\% \quad ,$$

$$B_\rho, \text{ small set}: \quad r = .19, \; P(<r) = 3.5\% \quad .$$

The overconsistency is so strong that even though the systematic errors are more than twice as large as the statistical ones, the data sets remain overconsistent when only statistical errors are used:

$$B_\rho, \text{ full set, } \sigma_{stat,i}: \quad r = .39, \; P(< r) = 2.0\% \quad ,$$

$$B_\rho, \text{ small set, } \sigma_{stat,i}: \quad r = .38, \; P(< r) = 13.8\% \quad .$$

Bias clearly exists in these measurements. The formal error on the average is *too small* since this bias is not included in the systematic errors.

B. Combined Analysis of $B_e$, $B_\mu$, $B_\pi$, and $B_\rho$

The three data sets $B'_e$, $B_\pi$, and $B_\rho$, show evidence of overconsistency as measured by $r$. However, $r$ is most sensitive to points which are furthest from the mean and can change considerably if one measurement is far from the mean. Another indicator of the consistency of a data set is the distribution of pulls [Eq. (12)], which should be a normal distribution of unit width and zero mean for a data set with Gaussian errors. $r$ is very nearly equal to the rms deviation of the pull distribution. Figure 2 shows the sum of the pull distributions for the three data sets $B'_e$, $B_\pi$, and $B_\rho$ along with the expected distribution. Here also there is clear evidence of the overconsistency of the data sets. Figure 3 shows the same distribution for the small sets. Of the 13 measurements in the three small sets, none is more than one sigma away from the small set mean.

We quantify the overconsistency of the summed pull distribution by evaluating the rms deviation, $R_\Sigma$. For the full sets $R_\Sigma = .636$. The probability that $R_\Sigma$ is less than or equal to .636 for an equivalent set of experiments having Gaussian errors is $.14 \pm .01\%$. For the small sets, $R_\Sigma = .484$. The probability of finding a smaller $R_\Sigma$ is $.64 \pm .04\%$.

Another method to measure the combined statistical significance of the observed overconsistency is to study the sum of the $r$ values, $\Sigma r$, for the three data sets. There is no reason to expect the overconsistency to be of the same magnitude in the three different types of measurements. For example, the ratio of systematic to statistical errors is twice as large for $B_\pi$ and $B_\rho$ as it is for $B'_e$. The summed pull distribution will not be sensitive to a very overconsistent data set if that data set has relatively few measurements. The value of $\Sigma r$ for the full sets is 1.53. The probability that $\Sigma r$ is less than or equal to 1.53 for an equivalent set of experiments having Gaussian errors is $.017 \pm .005\%$. For the small sets, $\Sigma r = 1.27$. The probability of a smaller $\Sigma r$ is $.54 \pm .03\%$.

C. Comparison of $B_e$, $B_\mu$, and $\tau_\tau$

The analysis of the $\tau_\tau$ set of measurements, Table 9, shows again some evidence for overestimation of some $\sigma_i$ or biasing of some $y_i$:

$$\tau_\tau,\ \text{full set}: \quad r = .65,\ P(< r) = 6.2\% \quad .$$

This overconsistency remains when the smaller sets are used, although the statistical significance is weaker since there are only five measurements in the small set:

$$\tau_\tau,\ \text{small set}: \quad r = .65,\ P(< r) = 21.\% \quad .$$

The assumption of $e - \mu - \tau$ universality leads to the prediction

$$\tau_\tau = \left(\frac{m_\mu}{m_\tau}\right)^5 \tau_\mu B_e \quad , \tag{24}$$

$$B_\mu = 0.973\ B_e \tag{25}$$

when the $e$ mass and all neutrino masses are set to 0. From Table 8, the full set formal average for $B'_e$ is

$$B'_e = (17.96 \pm 0.26)\% \quad . \tag{26}$$

Then from Eqs. (24) and (26)

$$\tau_\tau(\text{predicted}) = (2.874 \pm 0.042) \times 10^{-13} \text{ s} \tag{27}$$

compared with the full set measured value from Table 9

$$\tau_\tau(\text{measured}) = (3.026 \pm 0.085) \times 10^{-13} \text{ s} \ . \tag{28}$$

The difference is

$$\tau_\tau(\text{measured}) - \tau_\tau(\text{predicted}) = (0.152 \pm 0.095) \times 10^{-13} \text{ s} \tag{29}$$

and is not statistically significant. We have noted the errors in $B_e$, $B_\mu$, and $\tau_\tau$ may be overestimated or there may be biases in $B_e$, $B_\mu$, and $\tau_\tau$. Given these uncertainties, the difference in Eq. (29) cannot be interpreted as requiring larger values of $B_e$ and $B_\mu$.

# VIII. CONCLUSIONS

We studied the measurements of various decay branching fractions and the lifetime of the $\tau$ lepton for statistical consistency, assuming normal error distributions. There is clear evidence for overestimation of errors or bias in the individual measurements for $B_e'$, $B_\pi$, and particularly for $B_\rho$. By considering only the statistical errors, there is clear evidence for bias in the $B_\rho$ measurements, and hints of bias in other measurements. Therefore, the formal error on the average of the $B_\rho$ measurements is too small. Since the error on the $\rho$ branching ratio is the largest contribution to the error on the sum of the well measured one prong decay modes, the significance of the one prong discrepancy is reduced.

While we find evidence for bias, there is no evidence that the bias causes the discrepancy in summing the branching fractions. For example, although the $B_\rho$ measurements cluster too much, they may still cluster about the true value of $B_\rho$. Or the true value of $B_\rho$ may be larger, decreasing the discrepancy; or the true value of $B_\rho$ may be smaller, increasing the discrepancy. We do not know the size or sign of the bias.

There is no evidence for widespread underestimation of systematic errors in the sets of measurements examined here. Hence the discrepancy should not be ignored simply by claiming that the errors should be set larger.

In summary, our examination of the branching fraction measurements has not resolved the existing problem in understanding the 1-charged particle decay modes of the tau. Resolution of this discrepancy requires new information such as measurements with greatly improved statistical and systematic precision, or explicit measurement of as yet unmeasured or poorly measured modes.

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Table 1a. Summary of measured branching fractions of modes with 1-charged particle.

| Type of Measurement | Row | Decay Mode | Branching Fraction (%) | Reference |
|---|---|---|---|---|
| Exclusive Measurements of Modes with 0 or $\pi^0$ | A | $e^-\bar{\nu}_e\nu_\tau$ | $17.6 \pm 0.4$ | Table 8 |
| | B | $\mu^-\bar{\nu}_\mu\nu_\tau$ | $17.7 \pm 0.4$ | Table 8 |
| | C | $\pi^-\nu_\tau$ | $10.8 \pm 0.6$ | Table 8 |
| | D | $\rho^-\nu_\tau$ | $22.5 \pm 0.9$ | Table 8 |
| | E | $K^-\nu_\tau$ | $0.7 \pm 0.2$ | Ref. 3 |
| | F | $K^{*-}\nu_\tau$ | $1.4 \pm 0.1$ | Ref. 3 |
| Sum of rows A–F Called $B_{1e\mu\pi\rho K}$ | G | | $70.7 \pm 1.2$ | |
| Sum of Modes with $> 1\pi^0$ or with $\eta$'s Called $B_{1\,\text{mult neut}}$ | H | $\pi^- n\pi^0\nu_\tau,\ n > 1$<br>$\pi^- n\eta\nu_\tau,\ n > 0$<br>$\pi^- m\pi^0 n\eta\nu_\tau,\ m+n > 1$<br>$K^- n\pi^0\nu_\tau,\ n > 1$<br>⋮ ⋮ | 8. to 16. | Ref. 4 |

Table 1b. Summary of 1-charged particle branching fractions in percent.

| Decay Mode Category | Branching Fraction (%) and Origin | |
|---|---|---|
| $B_{1e\mu\pi\rho K}$ | $70.8 \pm 1.2$ from measurement (Table 1a) | |
| $B_{1\,\text{mult neut}}$ | 8 to 16 from measurement (Ref. 4) | $\leq 9.8$ from theory and other data (Refs. 1, 2, 4) |
| $B_{1e\mu\pi\rho K} + B_{1\,\text{mult neut}}$ | 79. to 87. | $\leq 80.5 \pm 1.2$ |
| $B_1$ | $86.6 \pm 0.3$ from measurement (Table 8) | |

Table 2. $\tau$ topological branching fractions in percent. The statistical error is given first, the systematic error second. We list all measurements provided the measurement is described in a preprint, journal article, or Ph.D. thesis authored by the experimenters, and the authors have not stated the measurement is superseded by a more recent measurement.

| $B_1$ | | | $B_3$ | | Energy (GeV) | Experimental Group | Reference |
|---|---|---|---|---|---|---|---|
| Measurement | Combined Error | Weight | Measurement | Combined Error | | | |
| $70.^{*}$ | $\pm 10.$ | — | $30.^{\dagger *}$ | $\pm 10.$ | 3.6 to 5.0 | PLUTO | J. Burmester *et al.*, Phys. Lett. **68B**, 297 (1977) |
| $68.^{\dagger *}$ | $\pm 5.$ | — | $32.^{*}$ | $\pm 5.$ | 3.1 to 7.4 | DELCO | W. Bacino *et al.*, Phys. Rev. Lett. **41**, 13 (1978) |
| $65.^{\dagger *}$ | $\pm 11.$ | — | $35.^{*}$ | $\pm 11.$ | 3.9 to 5.2 | DASP | R. Brandelik *et al.*, Phys. Lett. **73B**, 109 (1978) |
| $82.^{\dagger *}$ | $\pm 6.5$ | — | $18.^{*}$ | $\pm 6.5$ | 6 to 7.4 | MARK I | J. Jaros *et al.*, Phys. Rev. Lett. **40**, 1120 (1978) |
| $76.^{*}$ | $\pm 6.$ | — | $24.^{*}$ | $\pm 6.$ | 12 to 31.6 | TASSO | R. Brandelik *et al.*, Phys. Lett. **92B**, 199 (1980) |
| 84.0 | $\pm 2.0$ | .019 | 15.0 | $\pm 2.0$ | 32.0 to 36.8 | CELLO | H. J. Behrend *et al.*, Phys. Lett. **114B**, 282 (1982) |
| $86.0 \pm 2.0 \pm 1.0^{*}$ | $\pm 2.2$ | — | $14.0 \pm 2.0 \pm 1.0^{*}$ | $\pm 2.2$ | 29.0 | MARK II | C. A. Blocker *et al.*, Phys. Rev. Lett. **49**, 1369 (1982) |
| $85.2 \pm 0.9 \pm 1.5^{*}$ | $\pm 1.7$ | — | $14.8 \pm 0.9 \pm 1.5^{*}$ | $\pm 1.7$ | 29.0 | TPC | H. Aihara *et al.*, Phys. Rev. **D30**, 2436 (1984) |
| $85.2 \pm 2.6 \pm 1.3$ | $\pm 2.9$ | .009 | $14.8 \pm 2.0 \pm 1.3$ | $\pm 2.4$ | 14.0 | CELLO | H. J. Behrend *et al.*, Z. Phys. **C23**, 103 (1984) |
| $85.1 \pm 2.8 \pm 1.3$ | $\pm 3.1$ | .008 | $14.5 \pm 2.2 \pm 1.3$ | $\pm 2.6$ | 22.0 | CELLO | H. J. Behrend *et al.*, Z. Phys. **C23**, 103 (1984) |
| $87.8 \pm 1.3 \pm 3.9$ | $\pm 4.1$ | .005 | $12.2 \pm 1.3 \pm 3.9$ | $\pm 4.1$ | 34.6 average | PLUTO | Ch. Berger *et al.*, Z. Phys. **C28**, 1 (1985) |
| $84.7 \pm 1.1^{+1.6}_{-1.3}$ | $^{+1.9}_{-1.7}$ | .024 | $15.3 \pm 1.1^{+1.3}_{-1.6}$ | $^{+1.7}_{-1.9}$ | 13.9 to 43.1 | TASSO | M. Althoff *et al.*, Z. Phys. **C26**, 521 (1985) |
| $86.7 \pm 0.3 \pm 0.6$ | $\pm 0.7$ | .157 | $13,3 \pm 0.3 \pm 0.6$ | $\pm 0.7$ | 29.0 | MAC | E. Fernandez *et al.*, Phys. Rev. Lett. **54**, 1624 (1985) |
| $86.9 \pm 0.2 \pm 0.3$ | $\pm 0.4$ | .482 | $13.0 \pm 0.2 \pm 0.3$ | $\pm 0.4$ | 29.0 | HRS | C. Akerlof *et al.*, Phys. Rev. Lett. **55**, 570 (1985) |
| $86.1 \pm 0.5 \pm 0.9$ | $\pm 1.0$ | .077 | $13.6 \pm 0.5 \pm 0.8$ | $\pm 0.9$ | 30.0 to 46.8 | JADE | W. Bartel *et al.*, Phys. Lett. **161B**, 188 (1985) |
| $87.9 \pm 0.5 \pm 1.2$ | $\pm 1.3$ | .046 | $12.1 \pm 0.5 \pm 1.2$ | $\pm 1.3$ | 29.0 | DELCO | W. Ruckstuhl *et al.*, Phys. Rev. Lett. **56**, 2132 (1986) |
| $87.2 \pm 0.5 \pm 0.8$ | $\pm 0.9$ | .095 | $12.8 \pm 0.5 \pm 0.8$ | $\pm 0.9$ | 29.0 | MARK II | W. B. Schmidke *et al.*, Phys. Rev. Lett. **57**, 527 (1986) |
| $87.1 \pm 1.0 \pm 0.7^{\dagger *}$ | $\pm 1.2$ | — | $12.8 \pm 1.0 \pm 0.7^{*}$ | $\pm 1.2$ | 29.0 | MARK II | P. R. Burchat *et al.*, Phys. Rev. **D35**, 27 (1987) |
| $84.7 \pm 0.8 \pm 0.6$ | $\pm 1.0$ | .077 | $15.1 \pm 0.8 \pm 0.6$ | $\pm 1.0$ | 29.0 | TPC | H. Aihara *et al.*, Phys. Rev. **D35**, 1553 (1987) |

$^{\dagger}$Calculated from $B_1$ or $B_3$ measurement using $B_1 + B_3 + B_5 = 1.$ with $B_5 = 0.1\%$.

$^{*}$Not included in average.

Table 3. $\tau$ leptonic branching fractions in percent. The statistical error is given first, the systematic error second. We list all measurements provided the measurement is described in a preprint, journal article, or Ph.D. thesis authored by the experimenters, and the authors have not stated the measurement is superseded by a more recent measurement. The first two columns of branching ratios are published measurements. The third column contains values we have determined using the $\mu - e$ universality constraint $B_\mu = .973\ B_e$ as described in the text.

| Use $e-\mu$ Universality | $B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$ | | | $B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)$ | | | $B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$ Assuming $B_\mu$=.973 $B_e$ | | | Energy (GeV) | Experimental Group | Reference |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Measurement | Combined Error | Weight | Measurement | Combined Error | Weight | Measurement | Combined Error | Weight | | | |
| Yes† | 18.9±1.0±2.8 | ±3.0 | — | 18.3±1.0±2.8 | ±3.0 | — | 18.9±1.0±2.8 | ±3.0 | .008 | 3.8 to 7.8 | MARK I | M. L. Perl *et al.*, Phys. Lett. **70B**, 487 (1977) |
| No | | | | 17.5±2.7±3.0 | ±4.0 | .011 | 18.0±2.8±3.1 | ±4.2 | .004 | 3.8 to 7.8 | MARK I | Same Data as above |
| No | | | | 22. | $^{+10.}_{-7.}$ | .002 | 22.6 | $^{+10.3}_{-7.2}$ | .0009 | 4.8 | | M. Cavalli-Sforza *et al.*, Lett. Nuovo Cimento **20**, **337** (1977) |
| Yes† | 22.7 | ±5.5 | — | 22.1 | ±5.5 | — | 22.7 | ±5.5 | .002 | 4.1 to 7.4 | Lead Glass Wall | A. Barbaro-Galtieri *et al.*, Phys. Rev. Lett. **39**, 1058 (1977) |
| No | | | | 15. | ±3.0 | .019 | 15.4 | ±3.1 | .007 | 3.6 to 5.0 | PLUTO | J. Burmester *et al.*, Phys. Lett. **68B**, 297 (1977) |
| No | 16.0 | ±1.3 | .116 | | | | 16.0 | ±1.3 | .041 | 3.1 to 7.4 | DELCO | W. Bacino *et al.*, Phys. Rev. Lett. **41**, 13 (1978) |
| No | | | | 22. | $^{+7.}_{-8.}$ | .004 | 22.6 | $^{+7.2}_{-8.2}$ | .0012 | 6.4 to 7.4 | Iron Ball | J. G. Smith *et al.*, Phys. Rev. **D18**, 1 (1978) |
| Yes† | 18.5±2.8±1.4 | ±3.1 | — | 18.0±2.8±1.4 | ±3.1 | — | 18.5±2.8±1.4 | ±3.1 | .007 | 3.9 to 5.2 | DASP | R. Brandelik *et al.*, Phys. Lett. **73B**, 109 (1978) |
| No | | | | 21±5±3 | ±6. | .005 | 21.6±5.1±3.1 | ±6.0 | .002 | 3.6 to 7.4 | DELCO | W. Bacino *et al.*, Phys. Rev. Lett. **42**, 6 (1979) |
| No | 19. | ±9.0 | .002 | 35. | ±14. | .0009 | 23.8 | ±7.6 | .0012 | 12 to 31.6 | TASSO | R. Brandelik *et al.*, Phys. Lett. **92B**, 199 (1980) |
| No | | | | 17.8±2.0±1.8 | ±2.7 | .023 | 18.3±2.1±1.8 | ±2.8 | .009 | 9.4 to 31.6 | PLUTO | Ch. Berger *et al.*, Phys. Lett. **99B**, 489 (1981) |

*continued ...*

Table 3. *continued ...*

| † Use $e-\mu$ Universality | $B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$ Measurement | Combined Error | Weight | $B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)$ Measurement | Combined Error | Weight | $B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$ Assuming $B_\mu$=.973 $B_e$ Measurement | Combined Error | Weight | Energy (GeV) | Experimental Group | Reference |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Yes | 17.6±0.6±1.0 | ±1.3 | — | 17.1±0.6±1.0 | ±1.3 | — | 17.6±0.6±1.0 | ±1.3 | .048 | 3.5 to 6.7 | MARK II | C. A. Blocker *et al.*, Phys. Lett. **109B**, 119 (1982) |
| No | 18.3±2.4±1.9 | ±3.1 | .021 | 17.6±2.6±2.1 | ±3.3 | .016 | 18.2±1.8±1.4 | ±2.3 | .013 | 34.0 | CELLO | H. J. Behrend *et al.*, Phys. Lett. **127B**, 270 (1983) |
| No | $20.4\pm3.0^{+1.4}_{-0.9}$ | $^{+3.3}_{-3.1}$ | .019 | $12.9\pm1.7^{+0.7}_{-0.5}$ | ±1.8 | .052 | $15.0\pm1.5^{+.7}_{-.5}$ | ±1.6 | .027 | 13.9 to 43.1 | TASSO | M. Althoff *et al.*, Z. Phys. **C26**, 521 (1985) |
| No | 13.0±1.9±2.9 | ±3.5 | .016 | 19.4±1.6±1.7 | ±2.3 | .032 | 17.8±1.2±1.5 | ±1.9 | .019 | 34.6 average | PLUTO | Ch. Berger *et al.*, Z. Phys. **C28**, 1 (1985) |
| No | 18.2±0.7±0.5 | ±0.9 | .243 | 18.0±1.0±0.6 | ±1.2 | .117 | 18.3±0.5±0.4 | ±0.6 | .191 | 3.8 | MARK III | R. M. Baltrusaitis *et al.*, Phys. Rev. Lett. **55**,1842 (1985) |
| No | 17.4±0.8±0.5 | ±0.9 | .243 | 17.7±0.8±0.5 | ±0.9 | .208 | — | — | — | 29.0 | MAC | W. W. Ash *et al.*, Phys. Rev. Lett. **55**, 2118 (1985) |
| Yes | 17.8±0.4±0.3‡ | ±0.5 | — | 17.3±0.4±0.3‡ | ±0.5 | — | 17.8±0.4±0.3‡ | ±0.5 | .275 | 29.0 | MAC | Same Data as above |
| No | | | | 17.4±0.6±0.8 | ±1.0 | .169 | 17.9±0.6±0.8 | ±1.0 | .069 | 4.0 to 46.8 | MARK J | B. Adeva *et al.*, Phys. Lett. **179B**, 177 (1986) |
| No | 17.0±0.7±0.9 | ±1.1 | .163 | 18.8±0.8±0.7 | ±1.1 | .139 | 18.2±0.5±0.6 | ±0.8 | .107 | 34.6 average | JADE | W. Bartel *et al.*, Phys. Lett. **182B**, 216 (1986) |
| No | 18.4±1.2±1.0 | ±1.6 | .077 | 17.7±1.2±0.7 | ±1.4 | .086 | — | — | — | 29.0 | TPC | H. Aihara *et al.*, Phys. Rev. **D35**, 1553 (1987) |
| Yes | 18.3±0.7±0.5 | ±0.9 | — | 17.8±0.7±0.5 | ±0.9 | — | 18.3±0.7±0.5 | ±0.9 | .085 | 29.0 | TPC | Same Data as above |
| No | 19.1±0.8±1.1 | ±1.4 | .100 | 18.3±0.9±0.8 | ±1.2 | .117 | 18.9±0.6±0.7 | ±0.9 | .085 | 29.0 | MARK II | P. R. Burchat *et al.*, Phys. Rev. **D35**, 27 (1987) |

†Adjusted for $B(\tau^- \to \mu \bar{\nu}_\mu \nu_\tau)/B(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = .973$

‡We have determined the breakdown of statistical and systematic errors.

Table 4. $\tau^- \to \pi^- \nu_\tau$ branching ratio in percent. The statistical error is given first, the systematic error second. We list all measurements provided the measurement is described in a preprint, journal article, or Ph.D. thesis authored by the experimenters, and the authors have not stated the measurement is superseded by a more recent measurement.

| Measurement | Combined Error | Weight | Energy (GeV) | Experimental Group | Reference |
|---|---|---|---|---|---|
| $9.0 \pm 2.9 \pm 2.5$ | $\pm 3.8$ | .025 | 4.1 to 5.0 | PLUTO | G. Alexander *et al.*, Phys Lett. **78B**, 162 (1978) |
| $8.0 \pm 3.2 \pm 1.3$ | $\pm 3.5$ | .029 | 3.6 to 7.4 | DELCO | W. Bacino *et al.*, Phys Lett. **42**, 6 (1978) |
| $11.7 \pm 0.4 \pm 1.8$ | $\pm 1.8$ | .109 | 3.5 to 6.7 | MARK II | C. A. Blocker *et al.*, Phys. Lett. **109B**, 119 (1982) |
| $9.9 \pm 1.7 \pm 1.3$ | $\pm 2.1$ | .080 | 34.0 | CELLO | H. J. Behrend *et al.*, Phys. Lett. **127B**, 270 (1983) |
| $11.8 \pm 0.6 \pm 1.1$ | $\pm 1.3$ | .210 | 34.6 average | JADE | W. Bartel *et al.*, Phys. Lett. **182B**, 216 (1986) |
| $10.7 \pm 0.5 \pm 0.8$ | $\pm 0.9$ | .438 | 29.0 | MAC | W. T. Ford *et al.*, Phys. Rev. **D35**, 408 (1987) |
| $10.0 \pm 1.1 \pm 1.4$ | $\pm 1.8$ | .109 | 29.0 | MARK II | P. R. Burchat *et al.*, Phys. Rev. **D35**, 27 (1987) |

Table 5. $\tau^- \rightarrow \rho^- \nu_\tau$ branching ratio in percent. The statistical error is given first, the systematic error second. We list all measurements provided the measurement is described in a preprint, journal article, or Ph.D. thesis authored by the experimenters, and the authors have not stated the measurement is superseded by a more recent measurement.

| Measurement | Combined Error | Weight | Energy (GeV) | Experimental Group | Reference |
|---|---|---|---|---|---|
| $24. \pm 6. \pm 7.^\dagger$ | ±9. | .009 | 3.6 to 5.2 | DASP | R. Brandelik *et al.*, Z. Phys. **C1**, 233 (1979) |
| $21.5 \pm 1.7 \pm 3.0^\dagger$ | ±3.4 | .063 | 3.7 to 6.0 | MARK II | C. A. Blocker, Thesis, LBL–10801 (1980) |
| $22.1 \pm 1.9 \pm 1.6$ | ±2.5 | .116 | 14.0 to 34.0 | CELLO | H. J. Behrend *et al.*, Z. Phys. **C23**, 103 (1984) |
| $22.3 \pm 0.6 \pm 1.4$ | ±1.5 | .323 | 29.0 | MARK II | J. M. Yelton *et al.*, Phys. Rev. Lett. **56**, 812 (1986) |
| $23.0 \pm 1.3 \pm 1.7$ | ±2.1 | .165 | 3.8 | MARK III | J. Adler *et al.*, Phys. Rev. Lett. **59**, 1527 (1987) |
| $25.8 \pm 1.7 \pm 2.5^*$ | ±3.0 | — | 29.0 | MARK II | P. R. Burchat *et al.*, Phys. Rev. **D35**, 27 (1987) |
| $22.6 \pm 0.5 \pm 1.4$ | ±1.5 | .323 | 9.4 to 10.6 | CRYSTAL BALL | S. T. Lowe *et al.*, SLAC–PUB–4449 (1987) |

*All $\tau \rightarrow \pi^- \pi^0 \nu_\tau$ included in $\tau^- \rightarrow \rho^- \nu_\tau$.
Not included in formal average.

†We have determined the breakdown of statistical and systematic errors.

Table 6. $\tau$ lifetime in units of $10^{-13}$ s. The statistical error is given first, the systematic error second. We list all measurements provided the measurement is described in a preprint, journal article, or Ph.D. thesis authored by the experimenters and the authors have not stated the measurement is superseded by a more recent measurement.

| Lifetime | Errors combined in quadrature | Weight | Energy (GeV) | Experimental Group | Reference |
|---|---|---|---|---|---|
| 4.6 | $\pm1.9$ | .002 | 29.0 | MARK II | G. Feldman *et al.*, Phys. Rev. Lett. **48**, 66 (1982) |
| 4.9 | $\pm2.0$ | .002 | 29.0 | MAC | W. Ford *et al.*, Phys. Rev. Lett. **49**, 106 (1982) |
| 4.7 | $^{+3.9}_{-2.9}$ | .0006 | 17.1 average | CELLO | H. J. Behrend *et al.*, Nucl. Phys. **B211**, 369 (1983) |
| $3.18^{+0.59}_{-0.75} \pm 0.56$ | $^{+0.81}_{-0.94}$ | .010 | 39.8–45.2 | TASSO | M. Althoff *et al.*, Phys. Lett. **141B**, 264 (1984) |
| $3.15 \pm 0.36 \pm 0.40$ | $\pm0.54$ | .025 | 29.0 | MAC | E. Fernandez *et al.*, Phys. Rev. Lett. **54**, 1624 (1985) |
| $2.63 \pm 0.46 \pm 0.20$ | $\pm0.50$ | .029 | 29.0 | DELCO | D. E. Klem *et al.*, SLAC–Report–300 (1986), p. 67 |
| $2.88 \pm 0.16 \pm 0.17$ | $\pm0.23$ | .134 | 29.0 | MARK II | D. Amidei *et al.*, SLAC–PUB–4362 (1987) |
| 3.09 | $\pm0.19$ | .202 | 29.0 | MAC | H. R. Band *et al.*, Phys. Rev. Lett. **59**, 415 (1987) |
| $2.99 \pm 0.15 \pm 0.10$ | $\pm0.18$ | .225 | 29.0 | HRS | S. Abachi *et al.*, Phys. Rev. Lett. **59**, 2519 (1987) |
| $3.25 \pm 0.14 \pm 0.18$ | $\pm0.23$ | .140 | 10.5 | CLEO | C. Bebek *et al.*, Phys. Rev. **D36**, 690 (1987) |
| $2.95 \pm 0.14 \pm 0.11$ | $\pm0.18$ | .230 | 9.3–10.6 | ARGUS | H. Albrecht *et al.*, Phys. Lett. **199B**, 580 (1987) |

Table 7. Example of the calculation of statistical quantities using the topological branching fraction $B_1$ in percent.

| Measurement | Combined Error | Weight | Pull | Used in Largest Weights Analysis |
|---|---|---|---|---|
| 84. | $\pm 2.0$ | .019 | 1.27 | |
| 85.2 | $\pm 2.9$ | .009 | .46 | |
| 85.1 | $\pm 3.1$ | .008 | .46 | |
| 87.8 | $\pm 4.1$ | .005 | −.31 | |
| 84.7 | $^{+1.9}_{-1.7}$ | .024 | 1.02 | |
| 86.7 | $\pm 0.7$ | .157 | −.28 | Yes |
| 86.9 | $\pm 0.4$ | .482 | -1.32 | Yes |
| 86.1 | $\pm 1.0$ | .077 | .44 | Yes |
| 87.9 | $\pm 1.3$ | .046 | -1.09 | |
| 87.2 | $\pm .9$ | .095 | −.79 | Yes |
| 84.7 | $\pm 1.0$ | .075 | 1.89 | Yes |

Table 8. Calculated values of $y$, $|\sigma|$, $|\sigma_{scat}|$, $r$, $P(<r)$, $\sigma_{sys}/\sigma_{stat}$, and number of measurements for $B_1$, $B_3$, unconstrained $B_e$ and $B_\mu$, $B'_e$, $B_\pi$, and $B_\rho$. Values of $y$, $\sigma$, and $\sigma_{scat}$ are in percent.

| Branching Fraction | Measurement Selection | Number of Measurements | $y$ Formal Average | $\|\sigma\|$ Formal Error | $\|\sigma_{scat}\|$ | $r$ | $P(<r)$ | $\frac{\sigma_{sys}}{\sigma_{stat}}$ |
|---|---|---|---|---|---|---|---|---|
| $B_1$ | full set | 11 | 86.58 | .28 | .27 | .96 | .491 | 1.7 |
| | $\sigma_{stat}$, only | 10 | 86.79 | .14 | .21 | 1.48 | .980 | — |
| | small set | 5 | 86.64 | .29 | .33 | 1.10 | .699 | 1.7 |
| | $\sigma_{stat}$, only | 5 | 86.73 | .15 | .23 | 1.55 | .953 | — |
| $B_3$ | full set | 11 | 13.32 | .28 | .24 | .87 | .313 | 1.7 |
| | $\sigma_{stat}$, only | 10 | 13.13 | .14 | .19 | 1.40 | .957 | — |
| | small set | 5 | 13.27 | .29 | .30 | 1.03 | .618 | 1.7 |
| | $\sigma_{stat}$, only | 5 | 13.18 | .15 | .21 | 1.42 | .908 | — |
| $B_e$ | full set | 10 | 17.62 | .44 | .37 | .83 | .297 | .9 |
| | $\sigma_{stat}$, only | 8 | 17.81 | .34 | .45 | 1.31 | .898 | — |
| | small set | 5 | 17.56 | .48 | .44 | .93 | .514 | .9 |
| | $\sigma_{stat}$, only | 4 | 17.88 | .37 | .45 | 1.22 | .784 | — |
| $B_\mu$ | full set | 16 | 17.71 | .41 | .37 | .91 | .345 | .9 |
| | $\sigma_{stat}$, only | 12 | 17.80 | .31 | .33 | 1.05 | .645 | — |
| | small set | 6 | 17.95 | .45 | .21 | .47 | .045 | .9 |
| | $\sigma_{stat}$ only | 6 | 17.92 | .34 | .23 | .67 | .187 | — |
| $B'_e$ | full set | 21 | 17.96 | .26 | .19 | .73 | .047 | 1.0 |
| | $\sigma_{stat}$, only | 15 | 18.07 | .19 | .15 | .79 | .158 | — |
| | small set | 6 | 18.13 | .29 | .15 | .52 | .069 | .9 |
| | $\sigma_{stat}$, only | 6 | 18.16 | .21 | .16 | .73 | .245 | — |
| $B_\pi$ | full set | 7 | 10.78 | .60 | .35 | .59 | .083 | 2.1 |
| | $\sigma_{stat}$, only | 7 | 11.25 | .26 | .28 | 1.07 | .671 | — |
| | small set | 4 | 11.00 | .64 | .36 | .56 | .190 | 2.2 |
| | $\sigma_{stat}$, only | 4 | 11.33 | .27 | .33 | 1.23 | .789 | — |
| $B_\rho$ | full set | 6 | 22.45 | .85 | .18 | .21 | .001 | 2.1 |
| | $\sigma_{stat}$, only | 6 | 22.47 | .35 | .14 | .39 | .020 | — |
| | small set | 3 | 22.56 | .95 | .18 | .19 | .035 | 2.4 |
| | $\sigma_{stat}$ only | 3 | 22.52 | .37 | .14 | .38 | .138 | — |

Table 9. Calculated values of $y$, $|\sigma|$, $|\sigma_{scat}|$, $r$, $P(< r)$, $\frac{\sigma_{sys}}{\sigma_{stat}}$, and number of measurements for the $\tau$ lifetime $\tau_T$. Values of $y$, $\sigma$, and $\sigma_{scat}$ are in $10^{-13}$ sec units.

| Measurement Selection | Number of Measurements | $y$ Formal Average | $\|\sigma\|$ Formal Error | $\|\sigma_{scat}\|$ | $r$ | $P(< r)$ | $\frac{\sigma_{sys}}{\sigma_{stat}}$ |
|---|---|---|---|---|---|---|---|
| full set | 11 | 3.026 | .085 | .055 | .65 | .062 | .8 |
| $\sigma_{stat}$, only | 7 | 3.024 | .070 | .062 | .88 | .411 | — |
| small set | 5 | 3.025 | .089 | .058 | .65 | .208 | .8 |
| $\sigma_{stat}$, only | 4 | 3.027 | .073 | .082 | 1.12 | .710 | — |

# FIGURE CAPTIONS

1. $r$ distribution for different values of $N$.

2. Sum of the pull distributions for the $B_e'$, $B_\pi$, and $B_\rho$ full data sets.

3. Sum of the pull distributions for the $B_e'$, $B_\pi$, and $B_\rho$ small data sets.

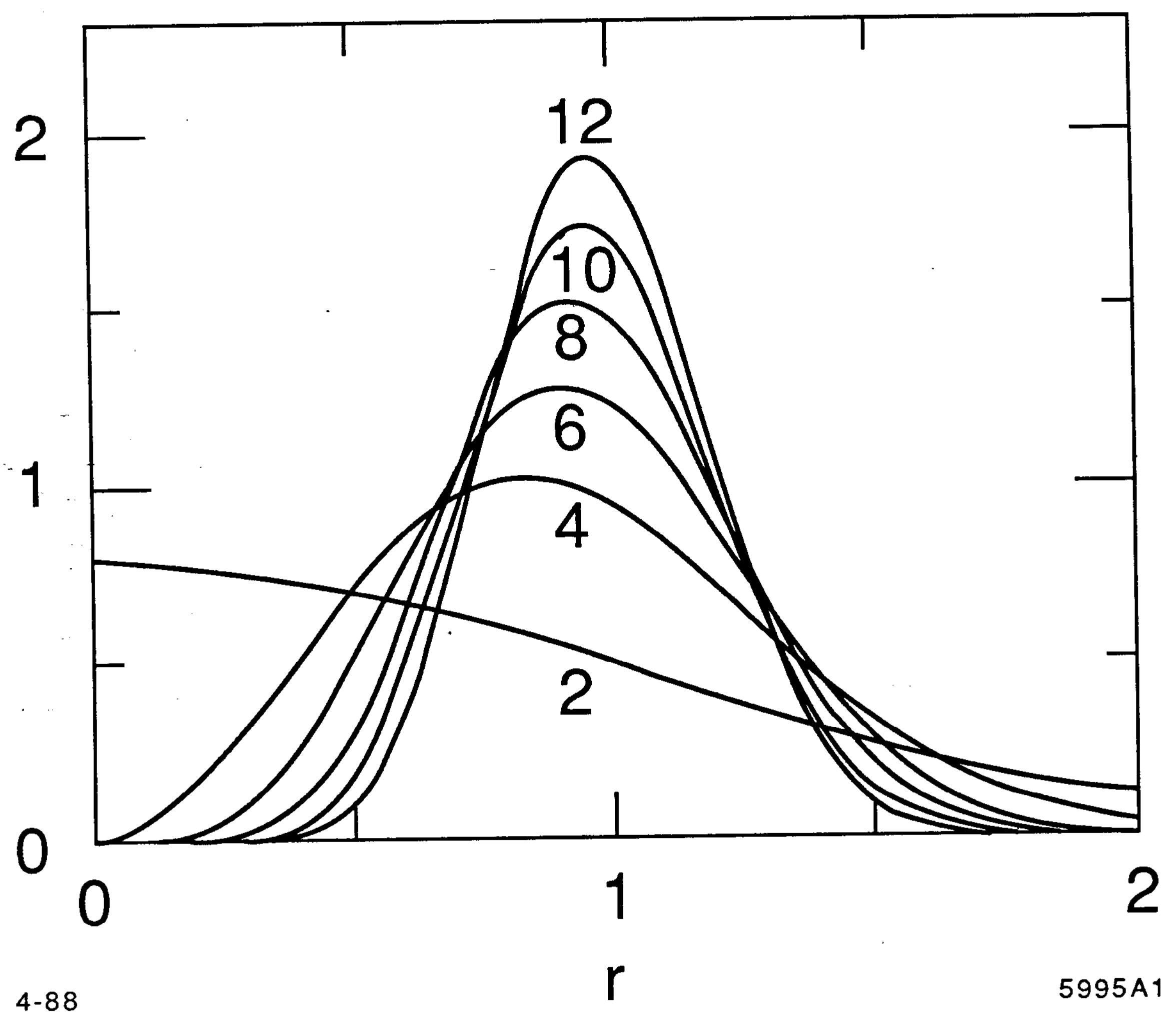


Fig. 1

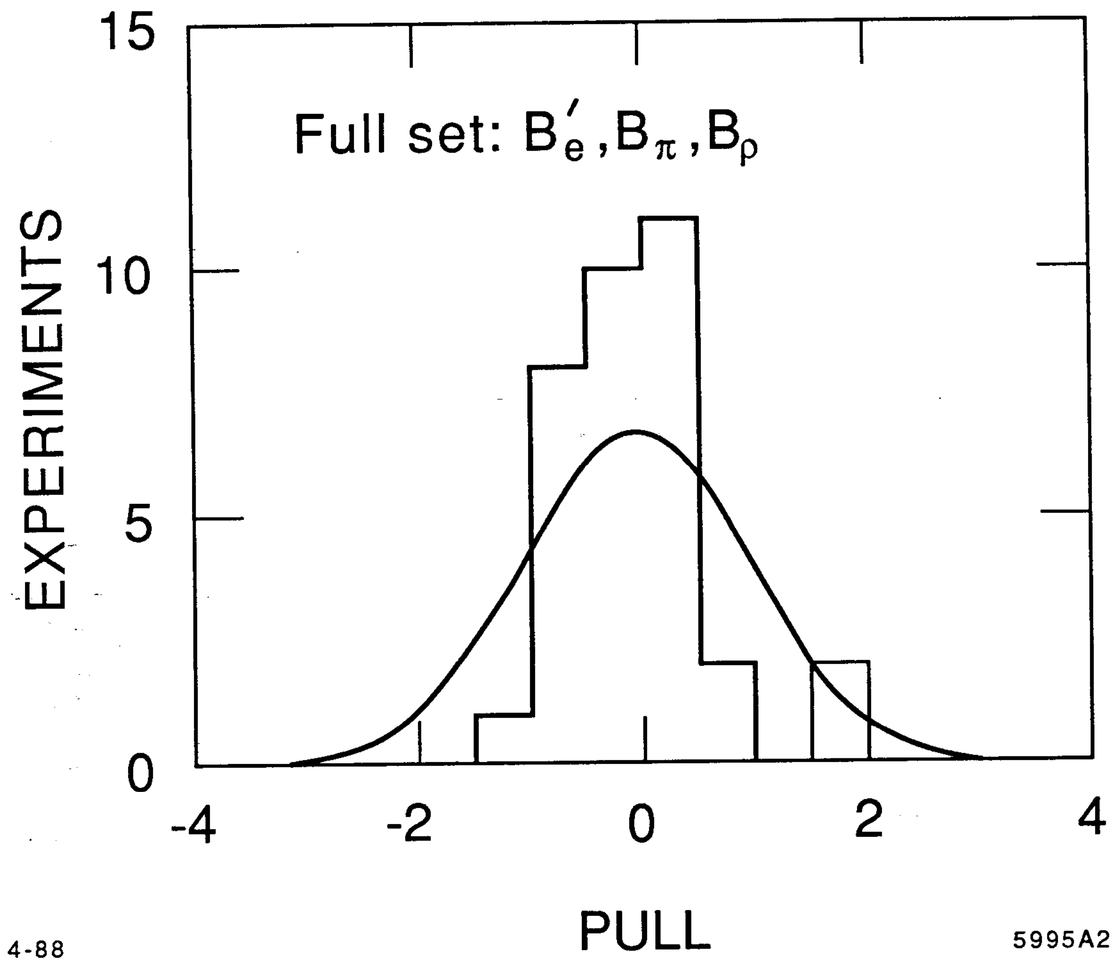
15
10
5
0
EXPERIMENTS
Full set: $B_e'$, $B_\pi$, $B_\rho$
-4
-2
0
2
4
PULL
4-88
5995A2

Fig. 2

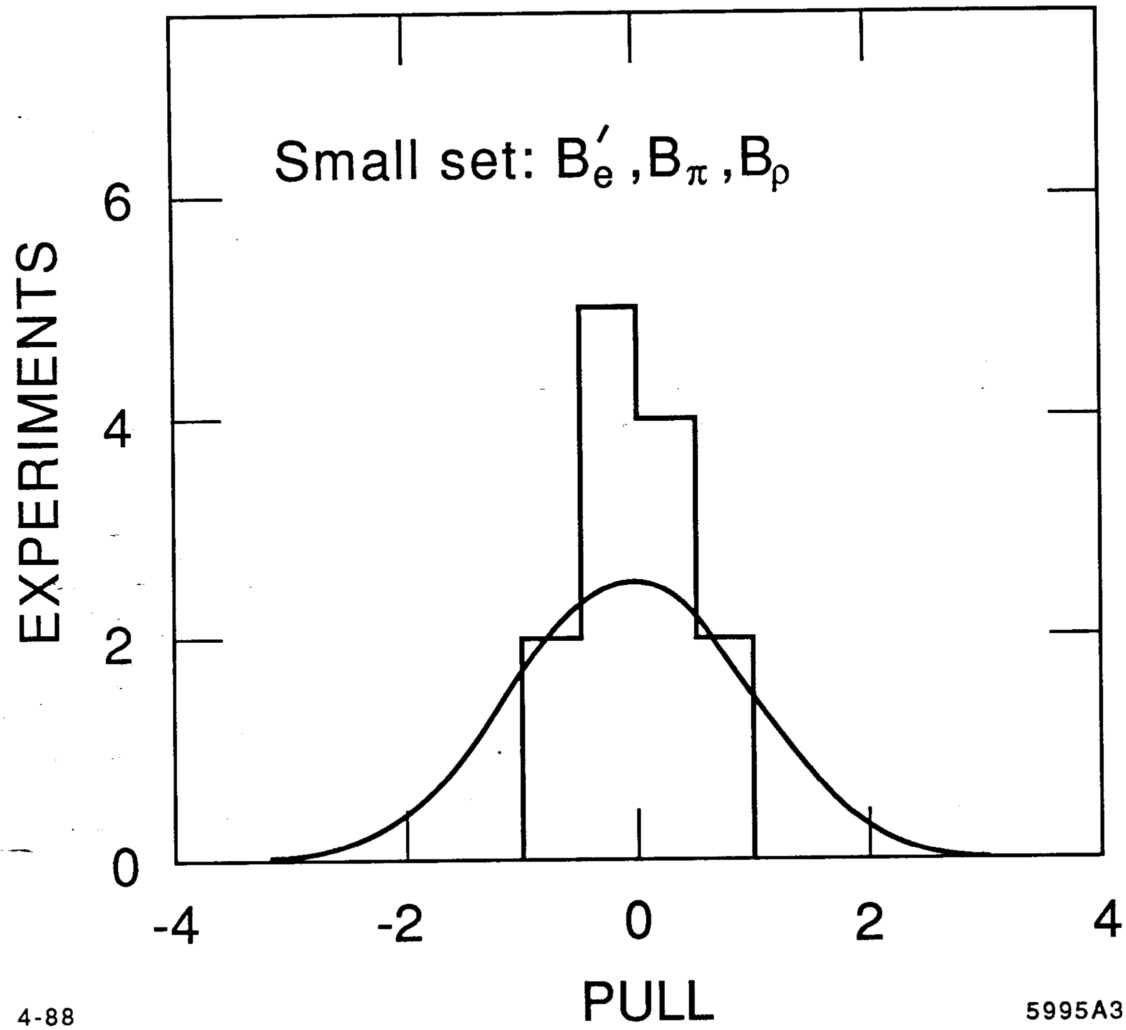


Fig. 3