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SUPERSYMMETRIC CHIRAL BOSONS *

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ABSTRACT

We present the (1,0) supersymmetric chiral boson theories. Actions, including the coupling to abelian gauge superfields, and chiral super-currents are written down for self-dual bosons. The conditions for proper BRST quantization are discussed. The "reparametrization anomalies" are removed by the stringy prescription of considering multiplets of superfields and by the introduction of a Liouville term. The Siegel super-ghost system is super-bosonized. An anomaly free action which couples chiral bosons to supergravity, is proposed.

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I. Introduction

Bosonization of two-dimensional Dirac fermions which was discovered some time ago [1a], was found to be a very useful tool in various domains of theoretical physics [1b]. Within the last few years, the process has become an integral part of string theories [2]; leading to the bosonization of ghosts and anti-ghosts [3], for example. This process applied to matter fermions has also become important in superstrings. The heterotic string [4] has two formulations in which the internal coordinates are either chiral fermions or their bosonized form. For the bosonization of chiral bosons, however, only the Hamiltonian formulation was known. The Lagrangian picture was lacking. An action proposed by Siegel [5] for a self-dual boson (the so-called chiral boson) which propagates in one direction only, was a natural candidate for the chiral bosonization [6a,7a]. The existence of a "reparametrization anomaly" in this theory was pointed out in Refs. [7a,8]. Two mechanisms for canceling this anomaly were suggested. One involved the introduction of a Liouville term [7a,8] and the second proposed a "critical dimension" approach [7b]. It was shown, in Ref. [8], that two bosons are needed for a consistent quantization. The coupling of this model to gauge fields [6b,7b], to gravity [7a,7b,8] and the non-abelian version of it [6b,7b] were also introduced. Some supersymmetric extensions have been studied. These include the N = 4 and N = 2 theories [9].

Chiral boson actions are constructed from a "truncated two-dimensional gravity" theory coupled to scalars. The component of the graviton which remains after the truncation, acts as a lagrange multiplier whose equation of motion imposes the uni-directional condition. As a truncated gravity theory, some general coordinate invariance remains. This is the symmetry that is potentially anomalous. As a two-dimensional theory, the classical action is conformally invariant. Hence the existence of a Weyl or dilatation symmetry. We will see that in terms of calculations, the theory of chiral bosons can be treated in an analogous manner with ordinary conformal theories. This means that the potential Siegel anomaly can be shifted to a Weyl anomaly through the addition, to the one-loop effective action, of local counterterms which are functions of the lagrange multiplier and the analog of the conformal compensator. The Weyl anomaly is removed in the critical dimension. We will perform these calculations for the (1,0) supersymmetric chiral boson multiplets [10]. In so doing we will adopt the stringy prescription for curing anomalies. We will also discuss the supersymmetric extension of the Liouville term [7,8], especially with regards to the physical states of the theory.

In the next section, we will give the actions and axial and vector super-currents for both leftons and rightons. The nomenclatures *leftons* and *rightons* are used to label bosonic left movers (functions of $(\tau+\sigma)$) and right movers (functions of $(\tau-\sigma)$), respectively. That is, self-dual bosons are leftons when $\epsilon_{01} \equiv 1$ and rightons when $\epsilon_{01} \equiv -1$. Although the calculations will be performed in (1,0) superspace [11,12], we will give the component expressions wherever they are appropriate. The super-currents will be coupled to gauge superfields, in section III. Section IV will contain the Siegel transformation laws and discussions of the anomalies of the leftons and rightons. It will be noted that the anomaly in the latter symmetries can be shifted to that of super-dilatation invariance. BRST quantization [13,3] of the chiral bosons will be stated in section V. Our results will reproduce, in part, the results of [8]. A prescription for the super-bosonization of the Siegel symmetry (super) ghosts and anti-ghosts will be given in section VI. Section VII will provide a discussion of the more interesting case of leftons and rightons coupled to a curved background. This should be of use in superstring theories, as we will include D scalar superfields for the superstring coordinates. We will then reproduce the critical dimension formulas for $D \leq 10$ dimensional superstrings [14]. Our conclusions may be found in section VIII and our notation is explained in Ref. [11].

II. Actions And Currents

We will be considering superfields which propagate both to the right and to the left, separately. This means that we should treat each of the two theories independently. Axial and vector super-currents will be used in later sections where they will provide clues to super-bosonization and checks on results. The leftons are considered first.

II.1. Leftons:

The action for the leftons [5] is well known. In (1,0) superspace [11,12] it is

$$S_L \;=\; -i\tfrac{1}{2}\int d^2\sigma d\varsigma^- [D_+\Phi^{\hat\alpha}\partial_{--}\Phi^{\hat\beta} \;+\; \Lambda_+{}^{--}\partial_{--}\Phi^{\hat\alpha}\partial_{--}\Phi^{\hat\beta}]\eta_{\hat\alpha\hat\beta} \;, \qquad (2.1)$$

where $\hat\alpha = 1,\ldots,N_L$. This is a dressed down version of the lefton action given in Ref. [10], in that we have taken $\Lambda_+{}^{--}{}_{\hat\alpha\hat\beta} \equiv \Lambda_+{}^{--}\eta_{\hat\alpha\hat\beta}$. We adopt this simplification throughout the course of this work. This is a natural consequence of the scenario of "truncated world-sheet supergravity", as will be explained in section VII. However, it is remarked that much of the ensuing analysis holds true for the more general $\Lambda_{\hat\alpha\hat\beta}$. The superfield $\Lambda_+{}^{--}$ is the lagrange multiplier (Siegel gauge superfield) which imposes the left moving condition on $\Phi^{\hat\alpha}$. Our flat space-time metric $\eta_{\hat\alpha\hat\beta}$ may be either Minkowski or Euclidean or it may be the metric for some product of Euclidean and Minkowski manifolds; anyway we take $\eta^{\hat\alpha\hat\beta}\eta_{\hat\beta\hat\alpha} \equiv N_L$.

When we define the component fields by projection to be:

$$\begin{aligned} \phi^{\hat\alpha} \;&\equiv\; \Phi^{\hat\alpha}\big| \;, \qquad \beta_+{}^{\hat\alpha} \;\equiv\; D_+\Phi^{\hat\alpha}\big| \;, \\ \lambda_+{}^{--} \;&\equiv\; \Lambda_+{}^{--}\big| \;, \qquad i\lambda_{++}{}^{--} \;\equiv\; D_+\Lambda_+{}^{--}\big| \;, \end{aligned} \qquad (2.2)$$

and perform the $\int d\varsigma^-$ integral, the action reduces to

$$\begin{aligned} S_L \;=\; \tfrac{1}{2}\int d^2\sigma[&\partial_{++}\phi\cdot\partial_{--}\phi \;+\; i\beta_+\cdot\partial_{--}\beta_+ \\ &+\; \lambda_{++}{}^{--}\partial_{--}\phi\cdot\partial_{--}\phi \;+\; i2\lambda_+{}^{--}\partial_{--}\beta_+\cdot\partial_{--}\phi] \;. \end{aligned} \qquad (2.3)$$

The first and third terms constitute the usual bosonic action for left-chiral bosons and the other two terms are their (1,0) supersymmetric completion. The fields (ϕ, β_+) and $(\lambda_{++}{}^{--}, \lambda_+{}^{--})$ form supersymmetric multiplets, respectively.

It is trivial to see that the non-supersymmetric or (0,0) action is invariant under the global axial transformation $\phi^{\hat{\alpha}} \to \phi^{\hat{\alpha}} + \pi^{\hat{\alpha}}$. Such a symmetry is also manifested by Eqn. (2.1) or Eqn. (2.3). Using the superfield notation we find that $\Phi^{\hat{\alpha}} \to \Phi^{\hat{\alpha}} + \Pi^{\hat{\alpha}}$ leads to the axial super-current:

$$J_{(a)--}{}^{\hat{\alpha}} = \partial_{--}\Phi^{\hat{\alpha}} \ , \qquad J_{(a)+}{}^{\hat{\alpha}} = D_+\Phi^{\hat{\alpha}} + 2\Lambda_+{}^{--}\partial_{--}\Phi^{\hat{\alpha}} \ . \tag{2.4}$$

There is also a vector super-current dual to the axial super-current whose elements are

$$J_{(v)--}{}^{\hat{\alpha}} = -\partial_{--}\Phi^{\hat{\alpha}} \ , \qquad J_{(v)+}{}^{\hat{\alpha}} = D_+\Phi^{\hat{\alpha}} \ . \tag{2.5}$$

These currents possess the following components

$$\begin{aligned}
j_{(a)--}{}^{\hat{\alpha}} &\equiv J_{(a)--}{}^{\hat{\alpha}}\big| = \partial_{--}\phi^{\hat{\alpha}} = -j_{(v)--}{}^{\hat{\alpha}} \equiv -J_{(v)--}{}^{\hat{\alpha}}\big| \ , \\
j_{(a)-}{}^{\hat{\alpha}} &\equiv D_+J_{(a)--}{}^{\hat{\alpha}}\big| = \partial_{--}\beta_+{}^{\hat{\alpha}} = -j_{(v)-}{}^{\hat{\alpha}} \equiv -D_+J_{(v)--}{}^{\hat{\alpha}}\big| \ , \\
j_{(a)++}{}^{\hat{\alpha}} &\equiv -iD_+J_{(a)+}{}^{\hat{\alpha}}\big| = \partial_{++}\phi^{\hat{\alpha}} + 2\lambda_{++}{}^{--}\partial_{--}\phi^{\hat{\alpha}} + i2\lambda_+{}^{--}\partial_{--}\beta_+{}^{\hat{\alpha}}, \\
j_{(a)+}{}^{\hat{\alpha}} &\equiv J_{(a)+}{}^{\hat{\alpha}}\big| = \beta_+{}^{\hat{\alpha}} + 2\lambda_+{}^{--}\partial_{--}\phi^{\hat{\alpha}} \ , \\
j_{(v)++}{}^{\hat{\alpha}} &\equiv -iD_+J_{(v)+}{}^{\hat{\alpha}}\big| = \partial_{++}\phi^{\hat{\alpha}} \ , \\
j_{(v)+}{}^{\hat{\alpha}} &\equiv J_{(v)+}{}^{\hat{\alpha}}\big| = \beta_+{}^{\hat{\alpha}} \ .
\end{aligned} \tag{2.6}$$

One can explicitly check that the currents defined above form (1,0) supersymmetric multiplets under the global supersymmetry transformations [11,12]

$$\begin{aligned}
\delta_\epsilon\phi^{\hat{\alpha}} &= -\epsilon^+\beta_+{}^{\hat{\alpha}} \ , \\
\delta_\epsilon\beta_+{}^{\hat{\alpha}} &= -i\epsilon^+\partial_{++}\phi^{\hat{\alpha}} \ .
\end{aligned} \tag{2.7}$$

In general, the global (1,0) supersymmetry transformations will have this structure, up to factors of "i".

The superspace equation of motion which follow from the variation of the action in Eqn. (2.1) with respect to $\Phi^{\hat{\alpha}}$ is

$$\partial_{--}\hat{L}_{+}\Phi^{\hat{\alpha}} \quad = \quad 0 \quad . \tag{2.8a}$$

The second equation of motion associated with the variation of $\Lambda_{+}{}^{--}$ is

$$(\partial_{--}\Phi)^2 \quad = \quad 0 \quad . \tag{2.8b}$$

This last equation leads to the projection of only left-moving scalars and, as will be clarified below, is potentially anomalous. In order to solve these two equations, we take

$$\partial_{--}\Phi^{\hat{\alpha}} \quad = \quad 0 \quad . \tag{2.8c}$$

We have found it useful to define

$$\hat{L}_{+} \quad \equiv \quad D_{+} \quad + \quad \Lambda_{+}{}^{--}\partial_{--} \quad , \tag{2.9}$$

for later comparison with the supergravity theory. Axial current conservation demands that

$$\partial_{--}J_{(a)+}{}^{\hat{\alpha}} \quad + \quad D_{+}J_{(a)--}{}^{\hat{\alpha}} \quad = \quad 0 \quad . \tag{2.10}$$

Using Eqns. (2.4) and only the first equation of motion, namely Eqn. (2.8a), Eqn. (2.10) is quickly verified. As usual, the vector super-current conservation law is purely topological: $D_{+}J_{(v)--}{}^{\hat{\alpha}} + \partial_{--}J_{(v)+}{}^{\hat{\alpha}} = 0$. This is guaranteed since $[D_{+}, \partial_{--}\} = 0$.

Left and right super-currents, $J_{\binom{l}{r}} \equiv \frac{1}{2}[J_{(v)} \pm J_{(a)}]$, may also be defined. Using Eqns. (2.4) and (2.5) we find

$$\begin{aligned} J_{(l)--}{}^{\hat{\alpha}} \quad &= \quad 0 \quad , \qquad J_{(r)+}{}^{\hat{\alpha}} \quad = \quad -\Lambda_{+}{}^{--}\partial_{--}\Phi^{\hat{\alpha}} \quad , \\ J_{(l)+}{}^{\hat{\alpha}} \quad &= \quad \hat{L}_{+}\Phi^{\hat{\alpha}} \quad , \qquad J_{(r)--}{}^{\hat{\alpha}} \quad = \quad -\partial_{--}\Phi^{\hat{\alpha}} \quad . \end{aligned} \tag{2.11}$$

Their conservation law

$$\partial_{--} J_{\binom{l}{r}+}{}^{\hat{\alpha}} \; + \; D_+ J_{\binom{l}{r}--}{}^{\hat{\alpha}} \; = \; 0 \; , \tag{2.12}$$

follows from Eqn. (2.8a).

In ordinary superymmetric extensions of free scalar fields, there are separate super Kac-Moody invariances in the right and left sectors. These symmetries are reflected by the fact that there is only one super-current in each sector $J_{(l)+}$ (which depends on σ^{++} and ς^{+}) and $J_{(r)--}$ (which depends only on σ^{--}). In our case, as expected, we see from Eqns. (2.11) and (2.12) that the affine symmetry survives only in the left sector. The right current has a $J_{(r)+}$ component and hence it depends on both σ^{--}, σ^{++} and ς^{+}.

II.2. Rightons:

Turning our attention to the right-chiral boson (righton) theory, we find the action [10]

$$S_R \;=\; -i\tfrac{1}{2}\int d^2\sigma d\varsigma^{-}[D_+\Phi^{\hat{a}}\partial_{--}\Phi^{\hat{b}} \;+\; \Lambda_{--}{}^{++}D_+\Phi^{\hat{a}}\partial_{++}\Phi^{\hat{b}}]\eta_{\hat{a}\hat{b}} \; , \tag{2.13}$$

where $\hat{a} = 1, \ldots, N_R$. Reducing this to components we find the following

$$\begin{aligned}
&\phi^{\hat{a}} \;\equiv\; \Phi^{\hat{a}}\big| \; , \qquad\qquad \beta_+{}^{\hat{a}} \;\equiv\; D_+\Phi^{\hat{a}}\big| \; , \\
&\lambda_{--}{}^{++} \;\equiv\; \Lambda_{--}{}^{++}\big| \; , \qquad i\lambda_{--}{}^{+} \;\equiv\; D_+\Lambda_{--}{}^{++}\big| \; , \\
S_R \;=\; &\tfrac{1}{2}\int d^2\sigma[\partial_{++}\phi\cdot\partial_{--}\phi \;+\; i\beta_+\cdot\partial_{--}\beta_+ \\
&\quad +\; \lambda_{--}{}^{++}\partial_{++}\phi\cdot\partial_{++}\phi \;+\; i\lambda_{--}{}^{++}\beta_+\cdot\partial_{++}\beta_+ \\
&\quad +\lambda_{--}{}^{+}\beta_+\cdot\partial_{++}\phi] \; .
\end{aligned} \tag{2.14}$$

The first and third terms give the usual bosonic action for rightons.

The axial super-current which again is related to $\phi^{\hat{a}} \to \phi^{\hat{a}} + \pi^{\hat{a}}$, is given by

$$\begin{aligned}
J_{(a)--}{}^{\hat{a}} \;&=\; \partial_{--}\Phi^{\hat{a}} \;+\; \Lambda_{--}{}^{++}\partial_{++}\Phi^{\hat{a}} - iD_+(\Lambda_{--}{}^{++}D_+\Phi^{\hat{a}}) \; , \\
J_{(a)+}{}^{\hat{a}} \;&=\; D_+\Phi^{\hat{a}} \; .
\end{aligned} \tag{2.15}$$

Following the definitions of the components given in Eqn. (2.6), we have

$$
\begin{aligned}
j_{(a)--}{}^{\hat{a}} &= \partial_{--}\phi^{\hat{a}} + 2\lambda_{--}{}^{++}\partial_{++}\phi^{\hat{a}} + \lambda_{--}{}^{+}\beta_{+}{}^{\hat{a}} \ , \\
j_{(a)-}{}^{\hat{a}} &= \partial_{--}\beta_{+}{}^{\hat{a}} + i\lambda_{--}{}^{+}\partial_{++}\phi^{\hat{a}} + 2\lambda_{--}{}^{++}\partial_{++}\beta_{+}{}^{\hat{a}} \\
&\quad + \beta_{+}{}^{\hat{a}}\partial_{++}\lambda_{--}{}^{++} \ , \\
j_{(a)++}{}^{\hat{a}} &= \partial_{++}\phi^{\hat{a}} \ , \qquad j_{(a)+}{}^{\hat{a}} = \beta_{+}{}^{\hat{a}} \ .
\end{aligned}
\tag{2.16}
$$

The vector super-current is identical to the one given for the leftons, namely Eqn. (2.5) with its components given by Eqn. (2.6). The vector super-current, as before, is topologically conserved while the axial super-current conservation demands that

$$
D_{+}J_{(a)--}{}^{\hat{a}} + \partial_{--}J_{(a)+}{}^{\hat{a}} = 0 \ . \tag{2.17}
$$

This is true on-shell with the use of the following equation of motion

$$
\begin{aligned}
D_{+}\hat{L}_{--}\Phi^{\hat{a}} &= 0 \ , \\
\hat{L}_{--} &\equiv \partial_{--} + \Lambda_{--}{}^{++}\partial_{++} - i\tfrac{1}{2}D_{+}\Lambda_{--}{}^{++}D_{+} \ ,
\end{aligned}
\tag{2.18a}
$$

The equation of motion associated with the variation of $\Lambda_{--}{}^{++}$, which may also be endangered by an anomaly, is

$$
D_{+}\Phi \cdot D_{+}D_{+}\Phi = 0 \ . \tag{2.18b}
$$

To solve these equations, we take

$$
D_{+}\Phi^{\hat{a}} = 0 \ . \tag{2.18c}
$$

Note that Eqn. (2.18c) implies the weaker condition $\partial_{++}\Phi^{\hat{a}} = 0$. Furthermore, we have $D_{+}\Phi^{\hat{a}}\big| = \beta^{\hat{a}} = 0$. So the spinor super-partner of $\phi^{\hat{a}}$ does not propagate. This is simply an artifact of the absence of supersymmetry in this direction, as the theory is embedded in a (1,0) superspace.

Using the same definitions for the left and right currents we now get:

$$
\begin{aligned}
J_{(l)--}{}^{\hat{a}} &= \Lambda_{--}{}^{++}\partial_{++}\Phi^{\hat{a}} \quad - \quad i\tfrac{1}{2}D_{+}\Lambda_{--}{}^{++}D_{+}\Phi^{\hat{a}} \ , \\
J_{(r)+}{}^{\hat{a}} &= 0 \ , \\
J_{(l)+}{}^{\hat{a}} &= D_{+}\Phi^{\hat{a}} \ , \\
J_{(r)--}{}^{\hat{a}} &= -\hat{L}_{--}\Phi^{\hat{a}} \ .
\end{aligned}
\tag{2.19}
$$

Obviously, both the right and the left currents are conserved here in a fashion similar to Eqn.(2.12). Notice that now only the right super-current (prior to using the equations of motion) is in the form of an affine Kac-Moody current.

III. Couplings To Abelian Gauge Superfields

In complete analogy with the coupling of a single chiral boson to abelian gauge fields [6.b], we now present the coupling of the vector and left currents in the lefton case and the vector and right currents for the rightons. See Ref. [11] for a general discussion of (1,0) vector super-multiplets.

III.1. Leftons:

We start by introducing the gauge supermultiplets Γ_{--} and Γ_{+}. These are the fundamental gauge superfields. The remaining one, Γ_{++} is given by $\Gamma_{++} = -iD_{+}\Gamma_{+}$, for the abelian theory. We absorb the "electric charge" into the definitions of the gauge superfields. This means that their dimensions are $[\Gamma_{--}] = 1$ and $[\Gamma_{+}] = \frac{1}{2}$.

The action for the system with the vector current coupled to a $U(1)$ gauge field is given simply by adding a current gauge field interaction term to the free action given by Eqn.(2.1) (for one lefton), namely

$$
\begin{aligned}
S_{LU(1)(v)} &= S_{L} \quad - \quad i\tfrac{1}{2}\int d^{2}\sigma d\varsigma^{-}\{\Gamma_{+}J_{(v)--} \quad + \quad \Gamma_{--}J_{(v)+}\} \\
&= S_{L} \quad + \quad i\tfrac{1}{2}\int d^{2}\sigma d\varsigma^{-}\{\Gamma_{+}\partial_{--}\Phi \quad - \quad \Gamma_{--}D_{+}\Phi\} \ .
\end{aligned}
\tag{3.1}
$$

Where we have substituted the expression for the vector current given in Eqn. (2.5). The invariance of this action under super-Siegel transformations (see section IV for a complete discussion) is guaranteed if one adopts the following transformation laws for the superfields

$$
\begin{aligned}
\delta_\Upsilon \Phi &= \Upsilon^{--}\partial_{--}\Phi \ , \\
\delta_\Upsilon \Gamma_{+} &= (D_{+}\Upsilon^{--})\Gamma_{--} \ + \ \Upsilon^{--}\partial_{--}\Gamma_{+} \ , \\
\delta_\Upsilon \Gamma_{--} &= (\partial_{--}\Upsilon^{--})\Gamma_{--} \ + \ \Upsilon^{--}\partial_{--}\Gamma_{--} \ .
\end{aligned}
\tag{3.2}
$$

Defining the components of the gauge superfields via

$$
\begin{aligned}
V_{--} &\equiv \Gamma_{--}| \ , \qquad \lambda_{-} \equiv D_{+}\Gamma_{--}| \ - \ \partial_{--}\Gamma_{+}| \ , \\
\rho_{+} &\equiv \Gamma_{+}| \ , \qquad iV_{++} \equiv D_{+}\Gamma_{+}| \ ,
\end{aligned}
\tag{3.3}
$$

we obtain the following component expression for the action (3.1):

$$
S_{LU(1)(v)} \ = \ S_L \ - \ \tfrac{1}{2}\int d^2\sigma[V_{++}\partial_{--}\phi \ - \ V_{--}\partial_{++}\phi \ + \ i\lambda_{-}\beta_{+}] \ , \tag{3.4}
$$

where S_L is now given by Eqn.(2.3). We see that λ_- is auxiliary. The Siegel transformations of the component gauge fields reads ($v^{--} \equiv \Upsilon^{--}|$, $\nu_{+}{}^{--} \equiv D_{+}\Upsilon^{--}|$)

$$
\begin{aligned}
\delta_v V_{++} &= v^{--}\partial_{--}V_{++} \ + (\partial_{++}v^{--})V_{--} \ , \\
\delta_\nu V_{++} &= i\nu_{+}{}^{--}\lambda_{-} \ , \\
\delta_v \rho_{+} &= v^{--}\partial_{--}\rho_{+} \ , \\
\delta_\nu \rho_{+} &= \nu_{+}{}^{--}V_{--} \ , \\
\delta_v V_{--} &= v^{--}\partial_{--}V_{--} \ + \ (\partial_{--}v^{--})V_{--} \ , \\
\delta_\nu V_{--} &= 0 \ , \\
\delta_v \lambda_{-} &= v^{--}\partial_{--}\lambda_{-} \ + \ (\partial_{--}v^{--})\lambda_{-} \ , \\
\delta_\nu \lambda_{-} &= 0 \ .
\end{aligned}
\tag{3.5}
$$

Note that the variations $\delta_v V_{++}$ and $\delta_v V_{--}$ are just the transformation of a vector field under the coordinate transformation $\sigma^{--} \rightarrow \sigma^{--} - v^{--}$. The variation of the auxiliary spinor, λ_-, under infinitesimal Lorentz transformations is given by $\delta_{Lorentz}\lambda_- = -\frac{1}{2}\eta\lambda_-$. Thus the second term on the right-hand-side of the $\delta_v\lambda_-$ variation is understood as $\delta_{Lorentz}\lambda_-$ with the identification $\eta = -2(\partial_{--}v^{--})$. The term $(v^{--}\partial_{--}\lambda_-)$ is simply the effect of the coordinate transformations for a spin-$\frac{1}{2}$ field.

Since the vector current remains unchanged, it continues to be topologically conserved. Using the equation of motion deduced from the action (3.1) we get (in analogy with Eqn. (2.10)) the following expressions for the divergence of the axial current

$$\partial_{--}J_{(a)+} + D_+J_{(a)--} = \partial_{--}\Gamma_+ - D_+\Gamma_{--} = -W_- , \tag{3.6}$$

where W_- is the (1,0) super Yang-Mills field strength [11]. In terms of components we obtain the following expression for the axial current and its superpartner:

$$\begin{aligned} \partial_{--}j_{(a)++} + \partial_{++}j_{(a)--} &= \partial_{--}V_{++} - \partial_{++}V_{--} , \\ \partial_{--}j_{(a)+} + j_{(a)-} &= - \lambda_- . \end{aligned} \tag{3.7}$$

The first is the same as the anomalous divergence of the axial current constructed for one left Weyl fermion. As in Ref. [6.b], this suggests the equivalence between the bosonic component of our supersymmetric model and a free left Weyl fermion. Furthermore, notice the occurrence of gauge covariant expressions. Supersymmetry relates the usual $U(1)$ field-strength, $\partial_{[--}V_{++]}$, to the spinorial field-strength, $D_{[+}\Gamma_{--]}| = \lambda_-$. This motivated the choice of components in Eqn. (3.3).

Next we couple an abelian gauge superfield to the left supercurrent. In this case the generalization of the non-supersymmetric results [6.b] leads to:

$$\begin{aligned} S_{LU(1)(l)} &= S_L - i\tfrac{1}{2}\int d^2\sigma d\varsigma^- \{\Gamma_{--}J_{(l)+} + \tfrac{1}{4}[\Lambda_+{}^{--}\Gamma_{--}\Gamma_{--} + \Gamma_{--}\Gamma_+]\} \\ &= S_L - i\tfrac{1}{2}\int d^2\sigma d\varsigma^- \{\Gamma_{--}\hat{L}_+\Phi + \tfrac{1}{4}[\Lambda_+{}^{--}\Gamma_{--}\Gamma_{--} + \Gamma_{--}\Gamma_+]\} . \end{aligned} \tag{3.8a}$$

The corresponding component field action reads

$$
\begin{aligned}
S_{LU(1)(l)} \quad = \quad S_L \quad + \quad \tfrac{1}{2}\int d^2\sigma\{ & V_{--}[\partial_{++}\phi \quad + \quad \lambda_{++}{}^{--}\partial_{--}\phi \\
& + \quad \tfrac{1}{4}(\lambda_{++}{}^{--}V_{--} \quad + \quad V_{++}) \quad + \quad i\lambda_{+}{}^{--}\partial_{--}\beta_{+}] \\
& - \quad i(\lambda_{-} \quad + \quad \partial_{--}\rho_{+})[\beta_{+} \quad + \quad \lambda_{+}{}^{--}\partial_{--}\phi \\
& + \quad \tfrac{1}{4}(2\lambda_{+}{}^{--}V_{--} \quad + \quad \rho_{+})]\} \ .
\end{aligned} \tag{3.8b}
$$

By taking the variation of the action with respect to Γ_{--} and Γ_{+}, the super-currents are found to be:

$$
\tilde{J}_{(l)--} \quad = \tfrac{1}{4}\Gamma_{--} \ , \qquad \tilde{J}_{(l)+} \quad = J_{(l)+} \quad + \tfrac{1}{4}(\Gamma_{+} + 2\Lambda_{+}{}^{--}\Gamma_{--}) \ . \tag{3.9a}
$$

Which in terms of the components, are

$$
\begin{aligned}
\tilde{\jmath}_{(l)--} \quad &= \quad \tfrac{1}{4}V_{--} \ , \\
\tilde{\jmath}_{(l)-} \quad &= \quad \tfrac{1}{4}(\lambda_{-} \quad + \quad \partial_{--}\rho_{+}) \ , \\
\tilde{\jmath}_{(l)++} \quad &= \quad \partial_{++}\phi \quad + \quad \lambda_{++}{}^{--}\partial_{--}\phi \quad + \quad i\lambda_{+}{}^{--}\partial_{--}\beta_{+} \\
& \qquad + \quad \tfrac{1}{4}[V_{++} \quad + \quad 2\lambda_{++}{}^{--}V_{--} \quad + \quad i2\lambda_{+}{}^{--}(\lambda_{-} \quad + \quad \partial_{--}\rho_{+})] \ , \\
\tilde{\jmath}_{(l)+} \quad &= \quad \beta_{+} \quad + \quad \lambda_{+}{}^{--}\partial_{--}\phi \quad + \quad \tfrac{1}{4}(\rho_{+} \quad + \quad 2\lambda_{+}{}^{--}V_{--}) \ .
\end{aligned} \tag{3.9b}
$$

The divergence of the left current has the form:

$$
\partial_{--}\tilde{J}_{(l)+} \quad + \quad D_{+}\tilde{J}_{(l)--} \quad = \tfrac{1}{4}[\partial_{--}\Gamma_{+} \quad - \quad D_{+}\Gamma_{--}] \quad = \quad -\tfrac{1}{4}W_{-} \ . \tag{3.10}
$$

Once again the bosonic component of Eqn. (3.10) leads to the anomalous divergence of the (left) current. This can be deduced from the one loop calculation for one left Weyl fermion coupled to a $U(1)$ gauge field.

III.2. Rightons:

For the rightons we once again first couple the vector super-current to a vector $U(1)$ superfield and then we couple the right chiral current to an abelian gauge superfield. The action for the vector coupling is given by

$$\begin{aligned} S_{RU(1)(v)} &= S_R \;-\; i\tfrac{1}{2}\int d^2\sigma d\varsigma^- \{\Gamma_+ J_{(v)--} + \Gamma_{--} J_{(v)+}\} \\ &= S_R \;+\; i\tfrac{1}{2}\int d^2\sigma d\varsigma^- \{\Gamma_+ \partial_{--}\Phi - \Gamma_{--} D_+ \Phi\} \ . \end{aligned} \tag{3.11}$$

This action is invariant under Siegel transformation if the variations of Φ and the gauge superfields are of the following forms:

$$\begin{aligned} \delta_\Upsilon \Phi &= \Upsilon^{++}\partial_{++}\Phi \;-\; i\tfrac{1}{2} D_+ \Upsilon^{++} D_+ \Phi \ , \\ \delta_\Upsilon \Gamma_+ &= -i\tfrac{1}{2}(D_+\Upsilon^{++}) D_+ \Gamma_+ \;+\; \Upsilon^{++}\partial_{++}\Gamma_+ \;+\; \tfrac{1}{2}(\partial_{++}\Upsilon^{++})\Gamma_+ \ , \\ \delta_\Upsilon \Gamma_{--} &= \Upsilon^{++}\partial_{++}\Gamma_{--} \;-\; i(\partial_{--}\Upsilon^{++}) D_+ \Gamma_+ \\ &\quad - i\tfrac{1}{2}[(\partial_{--} D_+ \Upsilon^{++})\Gamma_+ \;+\; D_+\Upsilon^{++} D_+ \Gamma_{--}] \ . \end{aligned} \tag{3.12a}$$

In terms of the components of the gauge superfields the transformations under Siegel symmetry take the forms:

$$\begin{aligned} \delta_v V_{++} &= (\partial_{++} v^{++}) V_{++} \;+\; v^{++}\partial_{++} V_{++} \ , \\ \delta_\nu V_{++} &= \tfrac{1}{2}\partial_{++}(\nu_- \rho_+) \ , \\ \delta_v \rho_+ &= v^{++}\partial_{++}\rho_+ \;+\; \tfrac{1}{2}(\partial_{++} v^{++})\rho_+ \ , \\ \delta_\nu \rho_+ &= \tfrac{1}{2}\nu_- V_{++} \ , \\ \delta_v V_{--} &= v^{++}\partial_{++} V_{--} \;+\; (\partial_{--} v^{++}) V_{++} \ , \\ \delta_\nu V_{--} &= -i\tfrac{1}{2}[\nu_-\lambda_- \;+\; \partial_{--}(\nu_-\rho_+)] \ , \\ \delta_v \lambda_- &= v^{++}\partial_{++}\lambda_- \;+\; \tfrac{1}{2}(\partial_{++} v^{++})\lambda_- \ , \\ \delta_\nu \lambda_- &= \tfrac{1}{2}\nu_-(\partial_{++} V_{--} \;-\; \partial_{--} V_{++}) \ . \end{aligned} \tag{3.12b}$$

The vector current here is also topologically conserved in the presence of the vector gauge superfields. The divergence of the axial current superfield, on the other hand is again given by Eqn. (3.6). For the coupling of the righton to an abelian gauge superfield we add, in addition to the current-gauge superfield term, terms bilinear in the gauge superfields. The action is

$$\begin{aligned} S_{RU(1)(r)} &= S_R + i\tfrac{1}{2}\int d^2\sigma d\varsigma^- \{\Gamma_+ J_{(r)--} - \tfrac{1}{4}[-i\Lambda_{--}{}^{++}\Gamma_+ D_+\Gamma_+ + \Gamma_{--}\Gamma_+]\} \\ &= S_R - i\tfrac{1}{2}\int d^2\sigma d\varsigma^- \{\Gamma_+[\partial_{--}\Phi + \Lambda_{--}{}^{++}\partial_{++}\Phi - i\tfrac{1}{2}D_+\Lambda_{--}{}^{++}D_+\Phi] \\ &\qquad + \tfrac{1}{4}[-i\Lambda_{--}{}^{++}\Gamma_+ D_+\Gamma_+ + \Gamma_{--}\Gamma_+]\} \ . \end{aligned} \tag{3.13a}$$

The corresponding component field action reads

$$\begin{aligned} S_{RU(1)(r)} = S_R - \tfrac{1}{2}\int d^2\sigma [& i(\partial_{--}\beta_+ + i\tfrac{1}{2}\lambda_{--}{}^{+}\partial_{++}\phi \\ & + \lambda_{--}{}^{++}\partial_{++}\beta_+ + \tfrac{1}{2}\beta_+\partial_{++}\lambda_{--}{}^{++})\rho_+ \\ & - (\partial_{--}\phi + \lambda_{--}{}^{++}\partial_{++}\phi + \tfrac{1}{2}\lambda_{--}{}^{+}\beta_+)V_{++} \\ & - \tfrac{1}{4}V_{++}(V_{--} + \lambda_{--}{}^{++}V_{++}) \\ & - i\tfrac{1}{4}\rho_+(\lambda_- + \partial_{--}\rho_+ + i\lambda_{--}{}^{+}V_{++} \\ & + \lambda_{--}{}^{++}\partial_{++}\rho_+)] \ . \end{aligned} \tag{3.13b}$$

By taking the variation of the action with respect to Γ_{--} and Γ_+ we get the currents $\tilde{J}_{(r)--}$ and $\tilde{J}_{(r)+}$, as follows:

$$\begin{aligned} \tilde{J}_{(r)+} &= -\tfrac{1}{4}\Gamma_+ \ , \\ \tilde{J}_{(r)--} &= J_{(r)--} - \tfrac{1}{4}[\Gamma_{--} - i2\Lambda_{--}{}^{++}D_+\Gamma_+ - iD_+\Lambda_{--}{}^{++}\Gamma_+] \ . \end{aligned} \tag{3.14a}$$

Projecting this into components, we get

$$\begin{aligned} \tilde{\jmath}_{(r)++} &= -\tfrac{1}{4}V_{++} \ , \\ \tilde{\jmath}_{(r)+} &= -\tfrac{1}{4}\rho_+ \ , \\ \tilde{\jmath}_{(r)--} &= -\partial_{--}\phi - \lambda_{--}{}^{++}\partial_{++}\phi - \tfrac{1}{2}\lambda_{--}{}^{+}\beta_+ \\ &\quad - \tfrac{1}{4}(V_{--} + 2\lambda_{--}{}^{++}V_{++} + \lambda_{--}{}^{+}\rho_+) \ , \\ \tilde{\jmath}_{(r)-} &= \tfrac{1}{4}(\partial_{--}\rho_+ - \lambda_-) \ . \end{aligned} \tag{3.14b}$$

The divergence of the right current has the form:

$$\partial_{--}\tilde{J}_{(r)+} \;+\; D_{+}\tilde{J}_{(r)--} \;=\; \tfrac{1}{4}[\partial_{--}\Gamma_{+} \;-\; D_{+}\Gamma_{--}] \;=\; -\tfrac{1}{4}W_{-} \;. \tag{3.15}$$

The bilinear terms in Eqns. (3.8) and (3.13) are the superspace mass terms for the vector supermultiplet. Of course, this breaks the gauge invariance, but we need this "tree level anomaly" in order to correctly bosonize chiral fermions.

Actions for self-dual bosons coupled to $U(1)$ gauge superfields ((1,0)) were given in Ref. [10b]. The latter work corresponds to studies of chiral bosons *in absentia* of bosonization. Thus the $U(1)$ anomaly was not of any concern there. Finally, we note that in the superstring, the vector supermultiplet is replaced by the pullback: $\Gamma_A \rightarrow D_A X^{\underline{m}} A_{\underline{m}}(X)$, where $X^{\underline{m}}$ is the space-time, coordinate superfield and $A_{\underline{m}}$ is the space-time, gauge field.

IV. Siegel Invariance

As was mentioned in the introduction and further discussed in the previous section, the lefton and righton actions possess a so called Siegel gauge invariance [5]. The presence of such an invariance is seen by the fact the Λ drops out of the equations of motion. Viewed as a chiral general coordinate transformation, we would expect these actions to be potentially anomalous. We will see that, like conformal two-dimensional field theories, this anomaly can be shifted by adding local counterterms to the effective action. This new effective action is anomalous under the analogous Weyl transformation. However, this anomaly can be removed by considering a multiplet of chiral bosons, a stringy prescription or by adding a Liouville term [7,8] to the classical action. In this section, we will calculate the effective actions $\Gamma(\Lambda_{+}{}^{--})$ and $\Gamma(\Lambda_{--}{}^{++})$ in the absence of any super-gravitational or $U(1)$ background superfields.

IV.1. Leftons:

Resuming with the leftons, the action given in Eqn. (2.1) is invariant [10] under the (1,0) Siegel symmetry

$$\begin{aligned} \delta_\Upsilon \Phi^{\hat{\alpha}} &= \Upsilon^{--}\partial_{--}\Phi^{\hat{\alpha}} \ , \\ \delta_\Upsilon \Lambda_+{}^{--} &= -D_+\Upsilon^{--} + \Upsilon^{--}\overleftrightarrow{\partial}_{--}\Lambda_+{}^{--} \ . \end{aligned} \tag{4.1}$$

In components defined by: $v^{--} \equiv \Upsilon^{--}|$ and $\nu_+{}^{--} \equiv D_+\Upsilon^{--}|$, this reads

$$\begin{aligned} \delta_v \phi^{\hat{\alpha}} &= v^{--}\partial_{--}\phi^{\hat{\alpha}} \ , \\ \delta_v \beta_+{}^{\hat{\alpha}} &= v^{--}\partial_{--}\beta_+{}^{\hat{\alpha}} \ , \\ \delta_v \lambda_{++}{}^{--} &= -\partial_{++}v^{--} + v^{--}\overleftrightarrow{\partial}_{--}\lambda_{++}{}^{--} \ , \\ \delta_v \lambda_+{}^{--} &= v^{--}\overleftrightarrow{\partial}_{--}\lambda_+{}^{--} \ , \\ \delta_\nu \beta_+{}^{\hat{\alpha}} &= \nu_+{}^{--}\partial_{--}\phi^{\hat{\alpha}} \ , \\ \delta_\nu \lambda_+{}^{--} &= -\nu_+{}^{--} \ , \\ \delta_\nu \lambda_{++}{}^{--} &= -i\nu_+^{--}\overleftrightarrow{\partial}_{--}\lambda_+{}^{--} \ . \end{aligned} \tag{4.2}$$

The δ_v in (4.2) arise from the coordinate transformation $\sigma^{--} \to \sigma^{--} - v^{--}$. So we view $\lambda_{++}{}^{--}$ as a component of the graviton and $\lambda_+{}^{--}$ as a pure gauge field which is removed in the Wess-Zumino gauge. The action (2.1) is the action for a truncated or "chiral supergravity" theory, written to *non-linear* order in the supergravity multiplet.

Now we fix the symmetry by imposing the quantum gauge condition $\Lambda_+{}^{--} \equiv 0$. We do this in analogy with the super-conformal field theory. The Faddeev-Popov procedure leads to the super-ghost action

$$S_{LGH} = \int d^2\sigma d\varsigma^- A_{--}{}^{++}[D_+G^{--} + \Lambda_+{}^{--}\partial_{--}G^{--} - (\partial_{--}\Lambda_+{}^{--})G^{--}] \ , \tag{4.3}$$

where G^{--} is the Siegel ghost superfield and $A_{--}{}^{++}$ is the anti-ghost superfield. The latter are quantum, while $\Lambda_+{}^{--}$ is a background superfield. Upon defining

$$\begin{aligned} \mathcal{G}^{--} &\equiv G^{--}| \ , \qquad i\gamma_+{}^{--} \equiv D_+G^{--}| \ , \\ \mathcal{A}_{--}{}^{++} &\equiv A_{--}{}^{++}| \ , \qquad \alpha_{--}{}^{+} \equiv D_+A_{--}{}^{++}| \ , \end{aligned} \tag{4.4}$$

we find

$$\begin{aligned} S_{LGH} = -\int d^2\sigma \{ &\mathcal{A}_{--}{}^{++}[i\partial_{++}\mathcal{G}^{--} + i\lambda_{++}{}^{--}\partial_{--}\mathcal{G}^{--} \\ &- i(\partial_{--}\lambda_{++}{}^{--})\mathcal{G}^{--} - i\lambda_+{}^{--}\partial_{--}\gamma_+{}^{--} \\ &+ i(\partial_{--}\lambda_+{}^{--})\gamma_+{}^{--}] \\ &\alpha_{--}{}^{+}[\lambda_+{}^{--}\partial_{--}\mathcal{G}^{--} - (\partial_{--}\lambda_+{}^{--})\mathcal{G}^{--} + i\gamma_+{}^{--}] \ , \end{aligned} \tag{4.5}$$

as the component action.

With $\Phi^{\hat{\alpha}}$ as a quantum superfield, we perform the $\int[\mathbf{D}\Phi^{\hat{\alpha}}][\mathbf{D}A_{--}{}^{++}][\mathbf{D}G^{--}]$ integrals, use the super-propagator ($z \equiv (\sigma^{\pm\pm}, \varsigma^+)$)

$$\begin{aligned} \langle 0|TA_{--}{}^{++}(z)G^{--}(z')|0\rangle &= \frac{\partial_{--}}{\Box}D_+\delta_-(z - z') \ , \\ \delta_-(z - z') &= \delta^2(\sigma - \sigma')\delta(\varsigma^+ - \varsigma^{+\prime}) \ , \end{aligned} \tag{4.6}$$

and follow the analysis of Ref. [16] (using the Adler-Rosenberg method insisting on Lorentz invariance) to find

$$\begin{aligned} \Gamma_{\text{`}ANOM\text{'},L}(\Lambda_+{}^{--}) &= \frac{(N_L - 26)}{96\pi}\int d^2\sigma d\varsigma^- [D_+\Lambda_+{}^{--}\frac{(\partial_{--})^4}{\Box}\Lambda_+{}^{--}] \ , \\ &= -\frac{(N_L - 26)}{96\pi}\int d^2\sigma[\lambda_{++}{}^{--}\frac{(\partial_{--})^4}{\Box}\lambda_{++}{}^{--} \\ &\qquad + i\lambda_+{}^{--}(\partial_{--})^3\lambda_+{}^{--}] \ , \end{aligned} \tag{4.7}$$

as the potential anomalous contribution to the one-loop effective action. The (anti)ghost component fields $\alpha_{--}{}^{+}$ and $\gamma_+{}^{--}$ are auxiliary and there is no "gravitino" here. Hence the -26 ghost contribution.

Since $\Gamma_{\text{`}ANOM\text{'},L}$ was computed from an action in the background superfields, we must check for its invariance under the linearized transformation: $\delta_\Upsilon \Lambda_+{}^{--} =$

$-D_+\Upsilon^{--}$. As mentioned above, it is not so invariant. In order to restore invariance to the theory, we must study the (1,0) supergravity theory [11] with $H_{--}{}^{++} \equiv 0$; this results in the truncated or chiral theory (see section VII). This last superfield is one of the fundamental superfields of the theory, along with $H_+{}^{--}$ and the conformal compensator, Ψ. We remove the superfield $H_{--}{}^{++}$ and restrict the theory to be invariant under the local gauge generator $K \equiv K^{--}\partial_{--} + \Omega M$ (M is the Lorentz generator). Next we make the identifications $\Lambda_+{}^{--} \equiv H_+{}^{--}$, $\Lambda_{--}{}^{++} \equiv H_{--}{}^{++}$ ($= 0$ for leftons) and $\rho \equiv \Psi$. The action for a scalar multiplet coupled to (1,0) supergravity then reduces — to non-linear order — exactly to Eqn. (2.1). The superfield ρ drops out of the action as it is the analog of the super-Weyl mode. Furthermore, under the identifications, $\Upsilon^{--} \equiv -K^{--}$ and $\Upsilon \equiv -\Omega$, the truncated (1,0) supergravity transformations become identical to the Siegel transformations of Eqn. (4.1) along with

$$
\begin{aligned}
\delta_\Upsilon \rho &= \Upsilon^{--}\partial_{--}\rho + \tfrac{1}{2}\Upsilon \ , \\
\delta_S \rho &= \tfrac{1}{2}S \ , \qquad \delta_S \Lambda_+{}^{--} = 0 \ .
\end{aligned}
\tag{4.8}
$$

The first transformation is the variation of ρ under the extended generator which includes "local Lorentz" transformations with parameter Υ. Super-dilatation transformations are given by δ_S variations. The Lagrange multiplier, $\Lambda_+{}^{--}$, transforms as in Eqn. (4.1) under the extended generator, *i.e.* its transformation is unchanged.

Now all of this means that we can follow Ref. [16] in adding local counterterms to the effective Lagrangian in order to restore the invariance under Siegel transformations. We can do this if $\Upsilon \equiv \partial_{--}\Upsilon^{--}$. So we have one parameter only (the other parameter, S, trivially gauges away ρ) and can gauge away only one superfield. The counterterms lead to the new effective action

$$\begin{aligned}\Gamma_{ANOM,L} &= -\frac{(N_L-26)}{24\pi}\int d^2\sigma d\varsigma^-[S^+\frac{D_+}{\Box}S^+] \ , \\ &= -\frac{(N_L-26)}{96\pi}\int d^2\sigma[r\frac{1}{\Box}r \ + \ i\tfrac{1}{4}\lambda_+{}^{--}(\partial_{--})^3\lambda_+{}^{--} \\ &\qquad - \ \lambda_{++}{}^+(\partial_{--})^2\lambda_+{}^{--} \ + \ i\lambda_{++}{}^+\partial_{--}\lambda_{++}{}^+] \ , \qquad (4.9) \\ S^+ &= i\tfrac{1}{2}\partial_{--}\partial_{--}\Lambda_+{}^{--} \ - \ i\partial_{--}D_+\rho \ , \\ r &= 2D_+S^+| \ = \ -\partial_{--}\partial_{--}\lambda_{++}{}^{--} \ + \ 2\Box\mu_0 \ .\end{aligned}$$

The superfield S^+ is the analog of the linearized (1,0) super-curvature [11], Σ^+, with $H_{--}{}^{++}=0$. The component field, r, is the linearized, "truncated bosonic curvature", $\mu_0 \equiv \rho|$ is the Weyl mode and $\lambda_{++}{}^+ \equiv -iD_+\rho|$ is the "gravitino" component. The fermionic piece vanishes in the Wess-Zumino gauge. $\Gamma_{ANOM,L}$ is anomalous under the δ_S variation. We must then have $N_L = 26$ as a condition which removes the latter anomaly.

The counterterms, $\partial_{--}\Lambda_+{}^{--}\partial_{--}\rho$ and $D_+\rho\partial_{--}\rho$, appear with coefficients which are proportional to (N_L-26), so they do not contribute to the Lagrangian of an anomaly free theory. The extended Siegel transformation for μ_0 is equivalent to that of μ given in the first work of Ref. [8], for the "anomaly free theory". Written to quadratic order in the bosonic background fields, our counterterm lagrangian agrees with theirs (see Eqn. (2.32) there). We are also in agreement with the fact that this additional lagrangian removes the "Siegel anomaly". Clearly, a Liouville term (see section VI for its form) can be added to the classical action. Such a term leads to the effective action in Eqn. (4.7), but with the coefficient shifted. It is then possible to have an anomaly free theory in less than 26 dimensions, *a la* Polyakov [17].

IV.2. Rightons:

The situation for rightons is as follows. The Siegel transformations which leave Eqn. (2.13) invariant are

$$\begin{aligned}\delta_\Upsilon \Phi^{\hat{a}} &= \Upsilon^{++}\partial_{++}\Phi^{\hat{a}} - i\tfrac{1}{2}D_+\Upsilon^{++}D_+\Phi^{\hat{a}} ,\\ \delta_\Upsilon \Lambda_{--}{}^{++} &= -\partial_{--}\Upsilon^{++} + \Upsilon^{++}\overleftrightarrow{\partial}_{++}\Lambda_{--}{}^{++} \\ &\quad + i\tfrac{1}{2}D_+\Lambda_{--}{}^{++}D_+\Upsilon^{++} .\end{aligned} \tag{4.10}$$

In terms of components defined by: $v^{++} \equiv \Upsilon^{++}|$ and $\nu_- \equiv D_+\Upsilon^{++}|$, we have the pair of symmetries

$$\begin{aligned}\delta_v \phi^{\hat{a}} &= v^{++}\partial_{++}\phi^{\hat{a}} ,\\ \delta_v \beta_+{}^{\hat{a}} &= v^{++}\partial_{++}\beta_+{}^{\hat{a}} + \tfrac{1}{2}(\partial_{++}v^{++})\beta_+{}^{\hat{a}} ,\\ \delta_v \lambda_{--}{}^{++} &= -\partial_{--}v^{++} + v^{++}\overleftrightarrow{\partial}_{++}\lambda_{--}{}^{++} ,\\ \delta_v \lambda_{--}{}^{+} &= v^{++}\overleftrightarrow{\partial}_{++}\lambda_{--}{}^{+} + \tfrac{1}{2}\lambda_{--}{}^{+}\partial_{++}v^{++} ,\\ \delta_\nu \phi^{\hat{a}} &= -i\tfrac{1}{2}\nu_-\beta_+{}^{\hat{a}} ,\\ \delta_\nu \beta_+{}^{\hat{a}} &= \tfrac{1}{2}\nu_-\partial_{++}\phi^{\hat{a}} ,\\ \delta_\nu \lambda_{--}{}^{++} &= \tfrac{1}{2}\nu_-\lambda_{--}{}^{+} ,\\ \delta_\nu \lambda_{--}{}^{+} &= i\partial_{--}\nu_- - i\tfrac{1}{2}\nu_-\partial_{++}\lambda_{--}{}^{++} + i(\partial_{++}\nu_-)\lambda_{--}{}^{++} .\end{aligned} \tag{4.11}$$

Here we have the interpretation of $\lambda_{--}{}^{++}$ as the other graviton component (as opposed to $\lambda_{++}{}^{--}$ in the lefton theory) and $\lambda_{--}{}^{+}$ as its gravitino. As we saw before, the trace of the "graviton", μ_0, and the other "gravitino" component, $\lambda_{++}{}^{+}$, reside in the "conformal compensator", ρ.

Fixing the invariance of Eqn. (4.8) by imposing $\Lambda_{--}{}^{++} \equiv 0$, we find the super-ghost action

$$\begin{aligned}S_{RGH} &= -i\int d^2\sigma d\varsigma^- A_+{}^{--}[\partial_{--}G^{++} - i\tfrac{1}{2}D_+\Lambda_{--}{}^{++}D_+G^{++} \\ &\quad + \Lambda_{--}{}^{++}\partial_{++}G^{++} - (\partial_{++}\Lambda_{--}{}^{++})G^{++}] .\end{aligned} \tag{4.12}$$

Its component form is found by defining

$$
\begin{aligned}
\mathcal{G}^{++} \;&\equiv\; G^{++}| \;, \qquad \gamma^{+} \;\equiv\; D_+G^{++}| \;, \\
\alpha_+{}^{--} \;&\equiv\; A_+{}^{--}| \;, \qquad \mathcal{A}_{++}{}^{--} \;\equiv\; D_+A_+{}^{--}| \;,
\end{aligned}
\tag{4.13}
$$

so that

$$
\begin{aligned}
S_{RGH} \;=\; -i\int d^2\sigma\{&\mathcal{A}_{++}{}^{--}[\partial_{--}\mathcal{G}^{++} \;+\; \lambda_{--}{}^{++}\partial_{++}\mathcal{G}^{++} \\
&\qquad -\; (\partial_{++}\lambda_{--}{}^{++})\mathcal{G}^{++} \;+\; \tfrac{1}{2}\lambda_{--}{}^{+}\gamma^{+}] \\
&+\; \alpha_+{}^{--}[\partial_{--}\gamma^{+} \;-\; \tfrac{1}{2}(\partial_{++}\lambda_{--}{}^{++})\gamma^{+} \;+\; \lambda_{--}{}^{++}\partial_{++}\gamma^{+} \\
&\qquad +\; i\tfrac{1}{2}\lambda_{--}{}^{+}\partial_{++}\mathcal{G}^{++} \;-\; i(\partial_{++}\lambda_{--}{}^{+})\mathcal{G}^{++}]\} \;.
\end{aligned}
\tag{4.14}
$$

As before, we take $\Phi^{\hat{a}}$, $A_+{}^{--}$ and G^{--} to be quantum superfields to be integrated over. Using the super-propagator

$$
\langle 0|TA_+{}^{--}(z)G^{++}(z')|0\rangle \;=\; -\frac{\partial_{++}}{\Box}\delta_-(z \;-\; z') \;, \tag{4.15}
$$

we find

$$
\begin{aligned}
\Gamma_{ANOM',R}(\Lambda_{--}{}^{++}) \;&=\; -i\frac{\frac{3}{2}(N_R-10)}{96\pi}\int d^2\sigma d\varsigma^-[D_+\Lambda_{--}{}^{++}\frac{(\partial_{++})^3}{\Box}\Lambda_{--}{}^{++}] \;, \\
&=\; -\frac{\frac{3}{2}(N_R-10)}{96\pi}\int d^2\sigma[\lambda_{--}{}^{++}\frac{(\partial_{++})^4}{\Box}\lambda_{--}{}^{++} \\
&\qquad +\; i\lambda_{--}{}^{+}\frac{(\partial_{++})^3}{\Box}\lambda_{--}{}^{+}] \;,
\end{aligned}
\tag{4.16}
$$

where the $\lambda_{--}{}^{+}$ term is the supersymmetric completion. This is not invariant under the linearized transformations obtained from Eqn. (4.10). When we add the local counterterms: $D_+\Lambda_{--}{}^{++}\partial_{++}\rho$ and $D_+\rho\partial_{--}\rho$, we can restore the invariance obtaining the new effective action

$$
\begin{aligned}
\Gamma_{ANOM,R}(\Lambda_{--}{}^{++}) \;&=\; -\frac{\frac{3}{2}(N_R-10)}{24\pi}\int d^2\sigma d\varsigma^- S^+\frac{D_+}{\Box}S^+ \;, \\
S^+ \;&=\; i\tfrac{1}{2}\partial_{++}D_+\Lambda_{--}{}^{++} \;-\; i\partial_{--}D_+\rho \;.
\end{aligned}
\tag{4.17}
$$

This violates the $\delta_S\rho = \frac{1}{2}S$ and $\delta_S\Lambda_{--}{}^{++} = 0$ super-dilatation variations, unless $N_R = 10$.

IV.3. Siegel versus Conformal Anomaly:

We have thus produced the critical dimension results by following the standard analysis of the superspace effective action [16,18]. In the process, the foundations for the more important calculation — coupling to a curved background — have been laid. However, this has led to interesting results for the N's.

Although we call the (1,0) supersymmetry a right-handed supersymmetry, the fermion, β_+, which is a member of the supersymmetric multiplet, is actually a left-mover ($\partial_{--}\beta_+ = 0$). The right moving fields do not form supersymmetric multiplets. Now, consider that for a chiral boson, the component current $j_{--}{}^{\hat{a}}$ can only be constructed out of a set of "minus-Weyl-spinors", $\eta_-{}^{\hat{I}}$, as $(j_{--})_{\hat{I}\hat{J}} = -i\eta_-{}^{\hat{I}}\eta_-{}^{\hat{J}}$. There is a superfield for $\eta_-{}^{\hat{I}}$, it is [11] $\Psi_-{}^{\hat{I}} = \eta_-{}^{\hat{I}} + i\varsigma^+ F^{\hat{I}}$, where $F^{\hat{I}}$ is an auxiliary boson which, for a free theory, satisfies $F^{\hat{I}} = 0$. So we write $J_{--}{}^{\hat{a}} \equiv \partial_{--}\Phi^{\hat{a}} \equiv -i\Psi_-{}^{\hat{I}}(M^{\hat{a}})_{\hat{I}\hat{J}}\Psi_{-\hat{J}}$, where the matrices $M^{\hat{a}}$ form a vector representation of $O(2)$, for example. For this to be a non-trivial result, we must have $\partial_{--}\Phi \neq 0$, so Φ must at least be a function of σ^{--}. A righton is a candidate for such a superfield. This is as it should be since, $\Psi_-{}^{\hat{I}}$ is in fact a pure right mover. Thus the righton should have the conformal anomaly (26) of half the anomaly of the right moving fermions. This is not the result we have obtained, as we did not calculate the conformal anomaly (see section VII).

Analogously for the leftons, we have that $(j_{++})_{\underline{ab}} = -i\beta_+{}^{\underline{a}}\beta_+{}^{\underline{b}}$. However, here β_+ is the upper component of a scalar superfield. So we have the superfield current $J_+ = D_+\Phi$. The requirement that this does not vanish leads to Φ being a superfunction of σ^{++} and ς^+, at least. Taking the lefton superfield as a candidate, we should have an anomaly of 10. Again, as we did not compute the conformal anomaly, we did not obtain this number.

So the "Siegel" anomaly and conformal anomalies are antipodal. This behavior would be much harder to detect in non-supersymmetric or (p,q) supersymmetric theories where $p = q$.

V. BRST Quantization

Extensive work has been done on the BRST quantization of two-dimensional conformal field theories, especially with applications to the bosonic string [13]. The supersymmetric theories (both (1,0) [19] and (1,1) [3,19,20]) have also been studied. For non-supersymmetric chiral bosons, the literature is sparse [7,8]. Needless to say, the Siegel symmetries of (1,0) supersymmetric chiral bosons have not been previously BRST quantized. We start this process here. As in Refs. [7a,8,19,20], we follow the work of Kato and Ogawa [13] generalized to (1,0) superspace.

V.1. Leftons:

After making the replacement $\Upsilon^{--} \to i\xi G^{--}$ where ξ is the anti-commuting parameter, and defining $\delta_\Upsilon \equiv i\xi\delta'$, we find

$$\begin{aligned}
\delta'\Phi^{\hat{\alpha}} &= G^{--}\partial_{--}\Phi^{\hat{\alpha}} \ , \\
\delta'\Lambda_+{}^{--} &= D_+G^{--} \ + \ G^{--}\overleftrightarrow{\partial}_{--}\Lambda_+{}^{--} \ , \\
\delta' G^{--} &= G^{--}\partial_{--}G^{--} \ , \\
\delta' A_{--}{}^{++} &= \mathcal{B}_{--}{}^{++} \ ,
\end{aligned} \tag{5.1}$$

where $\mathcal{B}_{--}{}^{++}$ is an auxiliary field with $\delta'\mathcal{B}_{--}{}^{++} = \ 0$. The action is now

$$\begin{aligned}
S_L \ = \ -i\int d^2\sigma d\varsigma^- \{ & \tfrac{1}{2}[D_+\Phi\cdot\partial_{--}\Phi \ + \ \Lambda_+{}^{--}\partial_{--}\Phi\cdot\partial_{--}\Phi] \\
& - i \ \delta'(A_{--}{}^{++}\Lambda_+{}^{--})\} \ .
\end{aligned} \tag{5.2}$$

Then we shift $\mathcal{B}_{--}{}^{++}$ in such a way that all the terms with $\Lambda_+{}^{--}$ will be cancelled out. The variation of $A_{--}{}^{++}$ becomes

$$\begin{aligned} \delta' A_{--}{}^{++} &= -i\tfrac{1}{2}\partial_{--}\Phi\cdot\partial_{--}\Phi \ + \ G^{--}\partial_{--}A_{--}{}^{++} \\ &\quad - \ 2A_{--}{}^{++}\partial_{--}G^{--} \ . \end{aligned} \tag{5.3}$$

Reducing Eqns. (5.1) and (5.3) to components, we obtain

$$\begin{aligned} \delta'\phi^{\hat\alpha} &= \mathcal{G}^{--}\partial_{--}\phi^{\hat\alpha} \ , \\ \delta'\beta_+{}^{\hat\alpha} &= -i\gamma_+{}^{--}\partial_{--}\phi^{\hat\alpha} \ + \ \mathcal{G}^{--}\partial_{--}\beta_+{}^{\hat\alpha} \ , \\ \delta'\lambda_+{}^{--} &= i\gamma_+{}^{--} \ + \ \mathcal{G}^{--}\overleftrightarrow{\partial}_{--}\lambda_+{}^{--} \ , \\ \delta'\lambda_{++}{}^{--} &= -\partial_{++}\mathcal{G}^{--} \ - \ \gamma_+{}^{--}\overleftrightarrow{\partial}_{--}\lambda_+{}^{--} \ + \ \mathcal{G}^{--}\overleftrightarrow{\partial}_{--}\lambda_{++}{}^{--} \ , \\ \delta'\mathcal{G}^{--} &= \mathcal{G}^{--}\partial_{--}\mathcal{G}^{--} \ , \\ \delta'\gamma_+{}^{--} &= \mathcal{G}^{--}\overleftrightarrow{\partial}_{--}\gamma_+{}^{--} \ , \\ \delta'\mathcal{A}_{--}{}^{++} &= -i\tfrac{1}{2}\partial_{--}\phi\cdot\partial_{--}\phi + \mathcal{G}^{--}\partial_{--}\mathcal{A}_{--}{}^{++} \\ &\quad -2\mathcal{A}_{--}{}^{++}\partial_{--}\mathcal{G}^{--} \ , \\ \delta'\alpha_{--}{}^{+} &= i\partial_{--}\phi\cdot\partial_{--}\beta_+ \ + \ \mathcal{G}^{--}\partial_{--}\alpha_{--}{}^{+} \\ &\quad + \ 2\alpha_{--}{}^{+}\partial_{--}\mathcal{G}^{--} \ - \ i\gamma_+{}^{--}\partial_{--}\mathcal{A}_{--}{}^{++} \\ &\quad - \ i2\mathcal{A}_{--}{}^{++}\partial_{--}\gamma_+{}^{--} \ . \end{aligned} \tag{5.4}$$

The left over action is

$$S_{LGH} \ = \ \int d^2\sigma d\varsigma^-[-i\tfrac{1}{2}D_+\Phi\cdot\partial_{--}\Phi \ + \ A_{--}{}^{++}D_+G^{--}] \ . \tag{5.5}$$

The component field action may be deduced from Eqns. (2.3) and (4.5) after imposing the gauge conditions $\lambda_{++}{}^{--} = \lambda_+{}^{--} = 0$. The BRST super-current $J_{(BRST)--}$ is now deduced via a Noether procedure

$$\delta_{BRST}S_{LGH} \ = \ \int d^2\sigma d\varsigma^-[D_+\xi \ \ J_{(BRST)--}] \ . \tag{5.6}$$

It is given by

$$J_{(BRST)--} \;=\; G^{--}[\tfrac{1}{2}\partial_{--}\Phi\cdot\partial_{--}\Phi \;-\; iA_{--}{}^{++}\partial_{--}G^{--}] \ . \tag{5.7}$$

Upon applying the equations of motion obtained from Eqn. (5.5):

$$\partial_{--}D_{+}\Phi \;=\; 0 \ , \qquad D_{+}G^{--} \;=\; D_{+}A_{--}{}^{++} \;=\; 0 \ , \tag{5.8}$$

it is straightforward to verify that the BRST super-current is conserved and

$$\begin{aligned} \jmath_{(BRST)-} &\equiv -iD_{+}J_{(BRST)--}| = 0 \ , \\ \jmath_{(BRST)--} &\equiv J_{(BRST)--}| \;=\; \mathcal{G}^{--}[\tfrac{1}{2}\partial_{--}\phi\cdot\partial_{--}\phi \;-\; i\mathcal{A}_{--}{}^{++}\partial_{--}\mathcal{G}^{--}] \ , \\ \mathcal{S} &= \int_{0}^{\pi} d\sigma : \jmath_{(BRST)--} : \ . \end{aligned} \tag{5.9}$$

The calculation of the BRST operator, $\mathcal{S}$, is now the usual one for the bosonic string [13]. In particular the BRST charge $\mathcal{S}$ can be written in the form above, where : : is the normal ordered operator. This leads to the introduction of the intercept, a. The necessary conditions of the nilpotency of the BRST charge are the cancellation of the anomaly and that the intercept must obey the following expression:

$$a = \tfrac{1}{24}(N_L - 2) \ . \tag{5.10}$$

These two conditions can be fulfilled in two different ways: (1) by adopting the "critical dimension" $N_L = 26$ and $a = 1$ or (2) by using the Liouville term in the form presented in Ref. [8]. The latter allows us to have $N_L = 2$ and $a = 0$. It was shown in ref. [8] that in the non-supersymmetric case for $a > 0$ the physical spectrum includes states with right-handed momenta in addition to the left handed modes. Therefore for a quantum system describing only left modes one has to use the second way of fulfilling the conditions for the nilpotency. The derivation of above Eqn. (5.9) shows that for the lefton case the result of Ref. [8] also holds here.

Later, in the next section, we will encounter a Liouville term in the bosonization of the Siegel ghosts systems.

It is interesting that the stringy prescription has allowed us to cancel the Siegel anomaly but in so doing it has led to "undesired states". When separately right and left moving bosons are included in a single theory, we should presumably find that there is no longer a problem. As we know [21], modular invariance imposes a relationship on the energy spectra of the states of the left and right movers. It would be interesting to look for a correspondence between these results and string propagation on asymmetric orbifolds [22]. We may then find that $N_L > 2$ is consistent with the spectrum of states.

V.2. Rightons:

By repeating the procedure for obtaining the gauge-fixed action, as was done for the leftons, we find the following action for the rightons

$$S_{RGH} = -i\int d^2\sigma d\varsigma^- [\tfrac{1}{2}D_+\Phi\cdot\partial_{--}\Phi + A_+{}^{--}\partial_{--}G^{++}] \ . \tag{5.11}$$

This is associated with the following BRST transformations

$$\begin{aligned}
\delta'\Phi^{\hat{a}} &= G^{++}\partial_{++}\Phi^{\hat{a}} + i\tfrac{1}{2}(D_+G^{++})D_+\Phi^{\hat{a}} \ , \\
\delta'\Lambda_{--}{}^{++} &= -\partial_{--}G^{++} + G^{++}\overleftrightarrow{\partial}_{++}\Lambda_{--}{}^{++} \\
&\quad + i\tfrac{1}{2}(D_+\Lambda_{--}{}^{++})D_+G^{++} \ , \\
\delta'G^{++} &= G^{++}\partial_{++}G^{++} + i\tfrac{1}{4}(D_+G^{++})D_+G^{++} \ , \\
\delta'A_+{}^{--} &= \tfrac{3}{2}A_+{}^{--}\partial_{++}G^{++} + (\partial_{++}A_+{}^{--})G^{++} \\
&\quad + i\tfrac{1}{2}(D_+A_+{}^{--})D_+G^{++} + \tfrac{1}{2}D_+\Phi\cdot\partial_{++}\Phi \ .
\end{aligned} \tag{5.12a}$$

Projecting onto components we get Eqn. (2.14) plus Eqn. (4.14) with the gauge conditions, $\lambda_{--}{}^{++} = \lambda_{--}{}^{+} = 0$, imposed. The component form of Eqn. (5.12a) is

$$
\begin{aligned}
\delta'\phi &= \mathcal{G}^{++}\partial_{++}\phi + i\tfrac{1}{2}\gamma^{+}\beta_{+} \ , \\
\delta'\beta_{+} &= -\gamma^{+}\partial_{++}\phi + \mathcal{G}^{++}\partial_{++}\beta_{+} + \tfrac{1}{2}(\partial_{++}\mathcal{G}^{++})\beta_{+} + \tfrac{1}{2}\gamma^{+}\partial_{++}\phi \ , \\
\delta'\lambda_{--}{}^{++} &= -\partial_{--}\mathcal{G}^{++} + \mathcal{G}^{++}\overleftrightarrow{\partial}_{++}\lambda_{--}{}^{++} - \tfrac{1}{2}\lambda_{--}{}^{+}\gamma^{+} \ , \\
\delta'\lambda_{--}{}^{+} &= -i\partial_{--}\gamma^{+} + i\gamma^{+}\overleftrightarrow{\partial}_{++}\lambda_{--}{}^{++} + \mathcal{G}^{++}\overleftrightarrow{\partial}_{++}\lambda_{--}{}^{+} \\
&\quad - i\tfrac{1}{2}(\partial_{++}\lambda_{--}{}^{++})\gamma^{+} - \tfrac{1}{2}\lambda_{--}{}^{+}\partial_{++}\mathcal{G}^{++} \ , \\
\delta'\mathcal{G}^{++} &= \mathcal{G}^{++}\partial_{++}\mathcal{G}^{++} + i\tfrac{1}{4}\gamma^{+}\gamma^{+} \ , \\
\delta'\gamma^{+} &= \mathcal{G}^{++}\overleftrightarrow{\partial}_{++}\gamma^{+} + \tfrac{1}{2}\gamma^{+}\partial_{++}\mathcal{G}^{++} \ , \\
\delta'\mathcal{A}_{++}{}^{--} &= \mathcal{G}^{++}\partial_{++}\mathcal{A}_{++}{}^{--} - 2\mathcal{A}_{++}{}^{--}\partial_{++}\mathcal{G}^{++} - \tfrac{3}{2}\alpha_{+}{}^{--}\partial_{++}\gamma^{+} \\
&\quad - \tfrac{1}{2}(\partial_{++}\alpha_{+}{}^{--})\gamma^{+} - \tfrac{1}{2}(i\partial_{++}\phi\cdot\partial_{++}\phi - \beta_{+}\cdot\partial_{++}\beta_{+}) \ , \\
\delta'\alpha_{+}{}^{--} &= \mathcal{G}^{++}\partial_{++}\alpha_{+}{}^{--} + \tfrac{3}{2}\alpha_{+}{}^{--}\partial_{++}\mathcal{G}^{++} \\
&\quad + i\tfrac{1}{2}\mathcal{A}_{++}{}^{--}\gamma^{+} + \tfrac{1}{2}\beta_{+}\cdot\partial_{++}\phi \ .
\end{aligned}
\tag{5.12b}
$$

By taking the variation of the action (5.11) and in accord with a definition similar to Eqn. (5.6), we get the following BRST super-current:

$$
\begin{aligned}
J_{(BRST)+} &= -iG^{++}[\tfrac{1}{2}D_{+}\Phi\cdot\partial_{++}\Phi + A_{+}{}^{--}\partial_{++}G^{++}] \\
&\quad + \tfrac{1}{4}A_{+}{}^{--}(D_{+}G^{++})(D_{+}G^{++}) \ .
\end{aligned}
\tag{5.13}
$$

Using the equations of motion from Eqn. (5.11), it is again easy to verify that $\partial_{--}J_{(BRST)+} = 0$ and that

$$
J_{(BRST)--} = -i\tfrac{1}{2}G^{++}D_{+}\Phi\cdot\partial_{--}D_{+}\Phi + i\tfrac{1}{2}A_{+}{}^{--}(D_{+}G^{++})\partial_{--}G^{--} \ , \tag{5.14}
$$

vanishes. The components of the super-currents are given by:

$$
j_{(BRST)++} \equiv D_{+}J_{(BRST)+}\big| \ ,
$$

$$
\begin{aligned}
&= -\mathcal{G}^{++}[\tfrac{1}{2}\partial_{++}\phi\cdot\partial_{++}\phi \;+\; i\tfrac{1}{2}\beta_+\cdot\partial_{++}\beta_+ \\
&\quad -\; i\mathcal{A}_{++}{}^{--}\partial_{++}\mathcal{G}^{++} \;-\; i\alpha_+{}^{--}\partial_{++}\gamma^+] \\
&\quad -\; i\gamma^+[\tfrac{1}{2}\beta_+\cdot\partial_{++}\phi \;+\; \tfrac{1}{2}\alpha_+{}^{--}\partial_{++}\mathcal{G}^{++} \;+\; i\tfrac{1}{4}\mathcal{A}_{++}{}^{--}\gamma^+]\;,
\end{aligned}
$$

$$
\begin{aligned}
\jmath_{(BRST)+} &\equiv J_{(BRST)+}\big| \\
&= -i\tfrac{1}{2}\mathcal{G}^{++}\beta_+\cdot\partial_{++}\phi \;-\; i\alpha_+{}^{--}(\mathcal{G}^{++}\partial_{++}\mathcal{G}^{++} \;+\; i\tfrac{1}{4}\gamma^+\gamma^+) \quad (5.15)
\end{aligned}
$$

Now the analysis is almost the usual one for the NSR superstring [2]. We have two sectors: Ramond (R) sector and Neveu-Schwarz (NS) sector. In the R sector the intercept is $a = 0$ with $N_R = 10$. While the NS sector has $a = \frac{1}{2}$. So there is no problem with having left moving states in the R sector of the righton theory. However, the NS sector has the same problem as the purely bosonic theory. Recall from our study of the leftons and Ref. [8], we must have $a = 0$ to remove "undesired states". In order to preserve this criterion in the righton theory, we must introduce a Liouville term so that $N_R = 2$ yields an anomaly free theory. (See sub-section VI.2 for the form of this term.) We choose $N_R = 2$ since $a = \frac{1}{16}(N_R - 2) \to 0$.

As an aside, we note that the surface term involving the lagrange multiplier leads to $\lambda_{--}{}^+(\tau,0) = \lambda_{--}{}^+(\tau,\pi)$ when $\beta_+(\tau,0) = \beta_+(\tau,\pi)$ in the R sector. Along with $\lambda_{--}{}^+(\tau,0) = -\lambda_{--}{}^+(\tau,\pi)$ when β_+ is anti-periodic in the NS sector.

VI. Bosonization Of Siegel Super-ghosts

We now turn our attention to the bosonization of the Siegel super-ghosts. However, before we look for such a prescription, we first recall that the ghost system (b,c) of the bosonic string has a ghost conjugation symmetry: $b \to c$ and $c \to b$, symbolically. The ghost number current $J = ibc$ is odd under this symmetry. Looking for such a symmetry in the (1,0) ghost actions above, we instead find: $A \to -G$, $G \to A$, or *vice versa*. This is a global $O(2)$ symmetry. So as not to confuse the latter with the $U(1)$ ghost-number symmetry, we will refer to the "ghost-number symmetry" by the expression in quotes. As is well known for the bosonic string, the ghost number symmetry ($b \rightarrow be^{-i\theta}$, $c \rightarrow e^{i\theta}c$) is anomalous: $\partial J \propto \sqrt{g}r$, where r is the curvature.

VI.1. Leftons:

For the lefton, Siegel transformation, ghost system we have the ghost number current

$$\mathbf{U}_{--} \;=\; iA_{--}{}^{++}G^{--} \quad , \tag{6.1a}$$

$$\begin{aligned} U_{--} \;&=\; i\mathcal{A}_{--}{}^{++}\mathcal{G}^{--} \;, \\ \mu_{-} \;&=\; i\alpha_{--}{}^{+}\mathcal{G}^{--} \;+\; \mathcal{A}_{--}{}^{++}\gamma_{+}{}^{--} \;, \end{aligned} \tag{6.1b}$$

where the (b) equations are the component expressions from the superfield current in (a). The spinor component current μ_{-} vanishes on-shell where $\alpha = \gamma = 0$. This current has an anomaly of the form

$$D_{+}\mathbf{U}_{--} \;=\; -\vartheta_{L}S^{+} \;, \tag{6.2}$$

written to linear order in the gauge superfields (see Eqn. (4.9)). We would like to bosonize the A-G system in such a manner that this anomaly is reproduced and in the process find a value for ϑ_L. Additionally, we will maintain the global $O(2)$ symmetry of the super-ghost system. (As there is no supersymmetry in this sector, we need not maintain this symmetry here.) In Ref. [3] it was shown, that for the bosonic string, the former can be done if the boson is non-minimally coupled to gravity. This leads to the Liouville term introduced in Ref. [7a]. So we write

$$\mathbf{U}_{--} \;\equiv\; (\partial_{--} \;+\; \Lambda_{--}{}^{++}\partial_{++} \;-\; i\tfrac{1}{2}D_{+}\Lambda_{--}{}^{++}D_{+})\chi \;, \tag{6.3}$$

and ask for an action which leads to the equation of motion given by Eqn. (6.2), written in terms of the bosonic superfield χ and the correct Siegel anomaly. The bosonized form (in terms of a single scalar field ω) of the ghost conjugation symmetry of the bosonic string reads $\omega \to -\omega$, since the current is odd. To have an $O(2)$ symmetry we take two bosonic superfields $\chi^{\check{\alpha}}$ ($\check{\alpha} = 1, 2$) which transform as $O(2)$ vectors. Let χ in Eqn. (6.3) be χ^1.

Putting all of this together and using our experience from section II and Ref. [11], we write

$$S_{LGH} = \int d^2\sigma d\varsigma^- [-i\tfrac{1}{2}(D_+\chi\cdot\partial_{--}\chi + \Lambda_{--}{}^{++}D_+\chi\cdot\partial_{++}\chi) + i\vartheta_L S^+\chi^1] \ , \quad (6.4)$$

which yields Eqn. (6.2) as its χ^1 equation of motion. So this is a candidate action for the super-bosonized form of the ghost term in Eqn. (5.5). The S^+ factor (see Eqn. (4.17)) is the Liouville term written in superspace, it breaks the $O(2)$ symmetry. In order to get an anomaly contribution of -26 from Eqn. (6.4), we need $\vartheta_L = \sqrt{\frac{28}{3}}$.

For a "superfield" which satisfies $D_+\chi^1 = 0$, we may use the results of Ref. [3] for the bosonic string to write $G^{--}(\sigma^{--}) = \exp[\chi^1(\sigma^{--})]$ and $A_{--}{}^{++}(\sigma^{--}) = \exp[-\chi^1(\sigma^{--})]$. We do this since the component ghosts and anti-ghosts, $\alpha_{--}{}^{+}$ and $\gamma_+{}^{--}$, are auxiliary. Then the various commutation/operator product expansions are as given in the latter work.

VI.2. Rightons:

The ghost-number super-current for the rightons is treated similarly. The superfield current and its components are

$$\begin{aligned} \mathbf{U}_+{}^1 &= A_+{}^{--}G^{++} \ , \\ U_{++}{}^1 &= \mathcal{A}_{++}{}^{--}\mathcal{G}^{++} + \alpha_+{}^{--}\gamma^+ \ , \\ \mu_+{}^1 &= \alpha_+{}^{--}\mathcal{G}^{++} \ . \end{aligned} \quad (6.5)$$

In analogy with the analysis of the lefton theory, the $U(1)$ anomaly is of the form

$$\partial_{--}\mathbf{U}_+{}^1 = -\vartheta_R S^+ \ , \quad (6.6)$$

where ϑ_R is to be determined. Super-bosonization of this system is given in terms of the pair of bosonic superfields, $\tilde{\chi}^{\hat{a}}$, by

$$
\begin{aligned}
\mathbf{U}_+{}^{\hat{a}} &= (D_+ + \Lambda_+{}^{--}\partial_{--})\tilde{\chi}^{\hat{a}} \;, \\
S_{RGH} &= \int d^2\sigma d\varsigma^- [\, -i\tfrac{1}{2}(D_+\tilde{\chi}\cdot\partial_{--}\tilde{\chi} + \Lambda_+{}^{--}\partial_{--}\tilde{\chi}\cdot\partial_{--}\tilde{\chi}) \\
&\qquad + i\vartheta_R S^+\tilde{\chi}^1] \;.
\end{aligned}
\tag{6.7}
$$

The righton, Siegel, super-ghost system has an anomalous contribution of -10. This means we must have $\vartheta_R = \sqrt{\frac{28}{3}}$.

A remark about the $O(2)$ symmetry is in order. This symmetry is purely a by-product of supersymmetry. For example, look at the component ghost action in Eqn. (4.14). This action has the usual ghost conjugation symmetry: $\mathcal{A}_{++}{}^{--} \to \mathcal{G}^{++}$, $\mathcal{G}^{++} \to \mathcal{A}_{++}{}^{--}$. However it has: $\alpha_+{}^{--} \to -\gamma^+$, $\gamma^+ \to \alpha_+{}^{--}$ as a symmetry. This is the source of the $O(2)$ symmetry and leads to the analogous transformations on the superfields. It was then seen that this meant that we needed two bosonic superfields to super-bosonize the super-ghost system. These superfields contain a total of two component bosons and two component fermions. It has been known [3] for some time that these four fields are needed for the bosonization of the super-conformal ghost system. What is interesting is the fact that we have found this result in a manner which is explicitly supersymmetric.

Just as we have the operator expressions for the bosonization of the lefton ghosts, we expect that we should be able to find similar expressions in the righton theory. There are two subtleties here. The first is that in the righton theory, all of the component ghost and anti-ghost fields propagate. Thus the need for the four fields as described in the previous paragraph. The second point is that the dimension of the anti-ghost superfield, $A_+{}^{--}$, is $d = \frac{3}{2}$. The component operator expressions are known [3]. They are $\mathcal{G} = \exp[\omega_1(\sigma^{++})]$ and $\gamma = \exp[\omega_2(\sigma^{++})]\eta$ for the ghost super-multiplet along with $\mathcal{A} = \exp[-\omega_1(\sigma^{++})]$ and $\alpha = \exp[-\omega_2(\sigma^{++})]\partial_{++}\xi$

for the anti-ghost super-multiplet. We delay giving the corresponding superfield expressions until a later work [23]. At this point it is sufficient to note that the $\omega_{\check{\alpha}}$ are bosons which are the lower components of $\chi^{\check{\alpha}}$ and the fermions, η and ξ, compose the upper components. The operator product expansions are as given in Ref. [3].

VII. Leftons And Rightons Coupled To Supergravity

A part of the definition of superstring theories is the stringent requirement that they must be superconformally invariant. So we now couple the actions of section II to (1,0) supergravity. In particular, we now investigate the superconformal anomalies for leftons and rightons plus a set of D (1,0) superstring coordinates. The resulting theory will lead to the compactification of the heterotic superstring to $D \leq 10$ dimensions with gauge group $G_L \times G_R$ [14]. An example of such a group is $SO(2N_L) \times SO(2N_R)$. A sketch of the anomaly results was given in Ref. [10].

To covariantly couple, in a minimal manner, the actions for the various "matter" superfields to supergravity, we take the action

$$\begin{aligned} S_{CLR} \;=\; -i\tfrac{1}{2}\int d^2\sigma d\varsigma^- E^{-1}[&\nabla_+ X^{\underline{a}}\nabla_{--}X^{\underline{b}}\eta_{\underline{ab}} \;+\; \nabla_+{\Phi_L}^{\hat{\alpha}}\nabla_{--}{\Phi_L}^{\hat{\beta}}\eta_{\hat{\alpha}\hat{\beta}} \\ &+\; \nabla_+{\Phi_R}^{\hat{a}}\nabla_{--}{\Phi_R}^{\hat{b}}\eta_{\hat{a}\hat{b}} \;+\; \Lambda_+{}^{--}\nabla_{--}{\Phi_L}^{\hat{\alpha}}\nabla_{--}{\Phi_L}^{\hat{\beta}}\eta_{\hat{\alpha}\hat{\beta}} \\ &+\; \Lambda_{--}{}^{++}\nabla_+{\Phi_R}^{\hat{a}}\nabla_{++}{\Phi_R}^{\hat{b}}\eta_{\hat{a}\hat{b}}] \;. \end{aligned} \tag{7.1}$$

This action was constructed [10] based on the requirements that it should be super-dilatation, general super-coordinate, locally Lorentz and Siegel invariant. It is the sum of the locally covariant forms of Eqns. (2.1) and (2.13) along with the action for the space-time coordinates, $X^{\underline{a}}$.

Computation of the anomaly requires that we write Eqn. (7.1) to linear order in the background superfields. We consider fluctuations about the background superfields, H^B, where

$$
\begin{aligned}
H_+{}^{--} &= H_+^{B--} + H_+^{Q--} , \qquad & H_{--}{}^{++} &= H^B_{--}{}^{++} + H^Q_{--}{}^{++} , \\
\Lambda_+{}^{--} &= -H_+^{B--} + \Lambda_+^{Q--} , \qquad & \Lambda_{--}{}^{++} &= -H^B_{--}{}^{++} + \Lambda^Q_{--}{}^{++} ,
\end{aligned}
\tag{7.2}
$$

is the split. This is done keeping in mind the fact that the background H's and Λ's are not required to satisfy any equations of motion. We treat this as a toy model, useful only as a guide to the construction of another action (see below) which couples leftons and rightons to supergravity. When linearized, the action S_{CLR} becomes

$$
\begin{aligned}
S_{CLR} = -i\tfrac{1}{2}\int d^2\sigma d\varsigma^- [& D_+X\cdot\partial_{--}X + D_+\Phi_L\cdot\partial_{--}\Phi_L + D_+\Phi_R\cdot\partial_{--}\Phi_R \\
& + H_+^{B--}(\partial_{--}X\cdot\partial_{--}X + \partial_{--}\Phi_R\cdot\partial_{--}\Phi_R) \\
& + H^B_{--}{}^{++}(D_+X\cdot\partial_{++}X + D_+\Phi_L\cdot\partial_{++}\Phi_L)]
\end{aligned}
\tag{7.3}
$$

where we have chosen the quantum gauges $\Lambda^Q = H^Q = 0$, symbolically. As a result of the choice of the background to expand about, the coupling of the Φ's to the H's has been reduced. Furthermore, under the quantum gauge choice chosen, we find the ghost actions

$$
\begin{aligned}
S_{SGGH} &= \int d^2\sigma d\varsigma^- E^{-1}[-iB_+{}^{--}\nabla_{--}C^{++} + B_{--}{}^{++}\nabla_+C^{--}] , \\
S_{LGH} &= \int d^2\sigma d\varsigma^- E^{-1}A_{--}{}^{++}[\nabla_+G^{--} - H_+^{B--}\nabla_{--}G^{--} \\
&\qquad + (\nabla_{--}H_+^{B--})G^{--}] , \\
S_{RGH} &= -i\int d^2\sigma d\varsigma^- E^{-1}A_+{}^{--}[\nabla_{--}G^{++} + i\tfrac{1}{2}\nabla_+H^B_{--}{}^{++}\nabla_+G^{++} \\
&\qquad - H^B_{--}{}^{++}\nabla_{++}G^{++} + (\nabla_{++}H^B_{--}{}^{++})G^{++}] ,
\end{aligned}
\tag{7.4}
$$

where the ∇_A and E^{-1} are background quantities. The first is the superconformal gauge, supergravity ghost action as given in Ref. [24]. The other two are the covariantized forms of Eqns. (4.3) and (4.12), as the new Siegel transformations are the covariantized versions of those of section IV. When linearized and with the use of Eqn. (7.2), these become

$$\begin{aligned}
S_{SGGH} &= \int d^2\sigma d\varsigma^- [-iB_+{}^{--}\partial_{--}C^{++} + B_{--}{}^{++}D_+C^{--} \\
&\quad + \tfrac{1}{2}D_+H^B_{--}{}^{++}B_+{}^{--}D_+C^{++} \\
&\quad - iH^B_{--}{}^{++}B_+{}^{--}\partial_{++}C^{++} \\
&\quad + i\partial_{++}H^B_{--}{}^{++}B_+{}^{--}C^{++} \\
&\quad - H^{B}_{+}{}^{--}B_{--}{}^{++}\partial_{++}C^{--} \\
&\quad + \partial_{--}H^{B}_{+}{}^{--}B_{--}{}^{++}C^{--}] \ , \\
S_{LGH} &= \int d^2\sigma d\varsigma^- [A_{--}{}^{++}D_+G^{--}] \ , \\
S_{RGH} &= \int d^2\sigma d\varsigma^- [A_+{}^{--}\partial_{--}G^{++}] \ .
\end{aligned} \tag{7.5}$$

The utility of the choice of the Λ-background is now clear. The (A, G) ghosts are non-interacting (to linear order)! This means that ghost contributions to the anomaly will come only from S_{SGGH}. Also, the absence of Λ means that the effective action will be a functional only of the H's [25]. Thus the absence of a Siegel anomaly. In fact, the non-local part of the one-loop effective action is

$$\begin{aligned}
\Gamma_{\text{`}ANOM\text{'}} &= \frac{1}{96\pi}\int d^2\sigma d\varsigma^- [(D + N_R - 26)D_+H^{B}_{+}{}^{--}\frac{(\partial_{--})^4}{\Box}H^{B}_{+}{}^{--} \\
&\quad - i\tfrac{3}{2}(D + N_L - 10)D_+H^B_{--}{}^{++}\frac{(\partial_{++})^3}{\Box}H^B_{--}{}^{++}] \ .
\end{aligned} \tag{7.6}$$

As in Ref. [16], the addition of the local counterterms restores the gauge invariance under the action of the supergravity gauge generator but breaks super-dilatation invariance. The removal of the latter anomaly requires that the pair of equations

$$\begin{aligned}
D + N_R &= 26 \ , \\
D + N_L &= 10 \ ,
\end{aligned} \tag{7.7}$$

be satisfied.

We have a reversal of the N's in the above equation as compared to the "Siegel anomaly" critical dimensions in section IV. This last equation is the expected result; compare this with the heterotic superstring ($N_L = 0$), for example. It was arrived at from a more qualitative argument in Ref. [10]. As discussed in section IV, these are the superconformal anomalies, not the "Siegel" anomalies. Notice the flipping of the coupling to the background superfields between Eqn. (7.3) and the analogous actions in section II. We have a Φ_R where a lefton is expected, and *vice versa.* If we had not chosen the backgrounds in Eqn. (7.2), we would not have computed the anomaly which must be removed for consistent superstring propagation. In a general background, we would have pure Siegel, pure superconformal and mixed anomalies. The anomaly removal equations analogous to Eqn. (7.7) turn out to be a set of inconsistent equations. With Eqns. (7.2) and (7.7), the theory is free of both the "Siegel" and superconformal anomalies.

Looking at Eqn. (7.3) we see that it should be possible to obtain the same action by defining the supergravity generator (general super-coordinate, $K^M D_M$, plus local Lorentz , ΩM) to act schizophrenically on the matter superfields. Keeping in mind the left $\leftrightarrow$ right reversal, we take the truncated supergravity transformations:

$$\begin{aligned} \delta_K \Phi_L{}^{\hat\alpha} &= -[K^{++}\partial_{++} - i\tfrac{1}{2}(D_+K^{++})D_+]\Phi_L{}^{\hat\alpha} \quad , \\ \delta_K \Phi_R{}^{\hat a} &= -K^{--}\partial_{--}\Phi_R{}^{\hat a} \quad , \end{aligned} \tag{7.8}$$

where we have gone to the Wess-Zumino gauge. With these transformations, we can obtain the latter action by using

$$\begin{aligned} \hat{E}_+ X^{\underline{a}} &= (D_+ \ + \ H_+{}^{--}\partial_{--})X^{\underline{a}} \quad , \\ \hat{E}_{--} X^{\underline{a}} &= (\partial_{--} \ + \ H_{--}{}^{++}\partial_{++})X^{\underline{a}} \quad , \\ \hat{E}_+ \Phi_L{}^{\hat\alpha} &= D_+\Phi_L{}^{\hat\alpha} \quad , \\ \hat{E}_{--} \Phi_L{}^{\hat\alpha} &= (\partial_{--} \ + \ H_{--}{}^{++}\partial_{++})\Phi_L{}^{\hat\alpha} \quad , \\ \hat{E}_+ \Phi_R{}^{\hat a} &= (D_+ \ + \ H_+{}^{--}\partial_{--})\Phi_R{}^{\hat a} \quad , \\ \hat{E}_{--} \Phi_R{}^{\hat a} &= \partial_{--}\Phi_R{}^{\hat a} \quad , \end{aligned} \tag{7.9}$$

where $\hat{E}_+$ and $\hat{E}_{--}$ are the (1,0) semi-zweibeins [11] fully defined by their action on the space-time coordinates, $X^{\underline{a}}$. Our task now is to construct the zweibiens based on the representation in Eqn. (7.8) and the (1,0) supergravity constraints [11]. Before we state the results, we define the supergravity covariant derivatives $\nabla_A \equiv E_A{}^M D_M \; + \; \omega_A M$, where $E_A = E_A{}^M D_M$ and the ω_A are the Lorentz spin super-connections. It is also necessary to introduce the conformal compensator, Ψ. The utility of the latter superfield was discussed in section IV. We will now use these to construct a theory invariant under Eqn. (7.8).

As usual, we commence with the leftons. Carrying out the calculations as outlined in Ref. [11], we find

$$\begin{aligned}
E_+ &\equiv e^{\Psi} D_+ \ , \\
E_{++} &= e^{2\Psi}[\partial_{++} \; - \; i2(D_+\Psi)D_+] \ , \\
E_{--} &= e^{2\Psi}[\hat{E}_{--} \; - \; i\tfrac{1}{2}(D_+ H_{--}{}^{++})D_+] \ , \\
\omega_+ &= 2e^{\Psi} D_+ \Psi \ , \\
\omega_{++} &= 2e^{2\Psi}\partial_{++}\Psi \ , \\
\omega_{--} &= e^{2\Psi}[\partial_{++}H_{--}{}^{++} \; - \; 2\hat{E}_{--}\Psi] \ .
\end{aligned} \tag{7.10}$$

For the rightons we have

$$\begin{aligned}
E_+ &\equiv e^{\Psi}\hat{E}_+ \ , \\
E_{++} &= e^{2\Psi}[\hat{E}_{++} \; + \; i\tfrac{1}{2}(\partial_{--}H_+{}^{--})\hat{E}_+ \; - \; i2(\hat{E}_+\Psi)\hat{E}_+] \ , \\
E_{--} &= e^{2\Psi}\partial_{--} \ , \\
\omega_+ &= e^{\Psi}[2\hat{E}_+\Psi \; - \; \partial_{--}H_+{}^{--}] \ , \\
\omega_{++} &= ie^{2\Psi}[\partial_{--}D_+H_+{}^{--} \; + \; H_+{}^{--}\partial_{--}\partial_{--}H_+{}^{--} \\
&\qquad - \; \partial_{--}H_+{}^{--}\hat{E}_+\Psi \; - \; i2\hat{E}_{++}\Psi] \ , \\
\omega_{--} &= -\,2e^{2\Psi}\partial_{--}\Psi \ .
\end{aligned} \tag{7.11}$$

These expressions, along with

$$\begin{aligned}
\Sigma^{+} &= i\tfrac{1}{2}[E_{+}\omega_{--} - E_{--}\omega_{+} - \tfrac{3}{2}\omega_{+}\omega_{--}] \ , \\
\mathcal{R} &= -[E_{++}\omega_{--} + iE_{+}E_{--}\omega_{+} - \omega_{++}\omega_{--} + i\tfrac{1}{2}\omega_{+}\omega_{--}] \ , \qquad (7.12) \\
E &\equiv \mathrm{sdet}(E_{A}{}^{M}) = e^{-3\Psi} \ ,
\end{aligned}$$

and the supergravity, graded commutation relations

$$\begin{aligned}
[\nabla_{+}, \nabla_{+}\} &= i2\nabla_{++} \ , \qquad [\nabla_{+}, \nabla_{++}\} = 0 \ , \\
[\nabla_{+}, \nabla_{--}\} &= -i2\Sigma^{+}M \ , \qquad (7.13) \\
[\nabla_{++}, \nabla_{--}\} &= -(\Sigma^{+}\nabla_{+} + \mathcal{R}M) \ ,
\end{aligned}$$

define the (anti)holomorphic realization of the (1,0) supergravity algebra on the leftons and rightons.

The compactified heterotic string action then reads (in compact notation)

$$S_{CLR} = -i\tfrac{1}{2}\int d^2\sigma d\varsigma^{-} E^{-1}[\nabla_{+}\Phi^{\hat{A}}\nabla_{--}\Phi^{\hat{B}}]\eta_{\hat{A}\hat{B}} \ , \qquad (7.14)$$

where $\Phi^{\hat{A}} \in \{X^{\underline{a}}, \Phi_{L}{}^{\hat{\alpha}}, \Phi_{R}{}^{\hat{a}}\}$ and $\eta_{\hat{A}\hat{B}} = \eta_{\underline{ab}} \oplus \eta_{\hat{\alpha}\hat{\beta}} \oplus \eta_{\hat{a}\hat{b}}$. Reduced to compoents, this becomes

$$\begin{aligned}
S_{CLR} = \tfrac{1}{2}\int d^2\sigma e^{-1}[\mathcal{D}_{++}x^{\hat{A}}\mathcal{D}_{--}x^{\hat{B}} &+ i\beta_{+}{}^{\hat{A}}\mathcal{D}_{--}\beta_{+}{}^{\hat{B}} \\
&- 2\beta_{+}{}^{\hat{A}}\psi_{--}{}^{+}\mathcal{D}_{++}\beta_{+}{}^{\hat{B}}]\eta_{\hat{A}\hat{B}} \ , \qquad (7.15)
\end{aligned}$$

where $\mathcal{D}_{\pm\pm}$ is the gravity covariant derivative and $\psi_{--}{}^{+}$ is a gravitino component. In this last expression, one must use the component analogs of Eqn. (7.9) or (7.10) or (7.11).

As a result of super-conformal invariance and the realizations in Eqns. (7.10) and (7.11), Eqn. (7.14) (with $X \equiv 0$) is identical to Eqn. (7.3) (with $X \equiv 0$). This is not a linearized result, it is the non-linear theory. Of course, including X in the action does not change the $\Phi_{(\substack{l \\ r})}$ contribution to the action, but it gives the action for superstring, space-time coordinates and leftons and rightons coupled to world-sheet supergravity. The H's then act as lagrange multipliers for the $\Phi_{\binom{l}{r}}$ imposing their uni-directional motion. The Siegel constraint algebra has been "transferred" to the Virasoro algebra. Furthermore, the Φ_R anomaly is one-half that of the minus-spinor fermion it bosonized. As mentioned before, there is no plus-spinor, matter superfield in (1,0) superspace. So such a statement cannot be made for the leftons. The righton propagator, when exponentiated, is equivalent to that of a minus spinor. Thus Eqn. (7.14) is proposed as the bosonized action for the compactified heterotic string. Before this statement can be fully asserted, the spectrum of states in the theory must be checked. We will do so in a later work [23].

VIII. Conclusions

We have studied two-dimensional, (1,0) supersymmetric, self-dual bosons. This was done with an eye on two facets of the theory: (1) bosonization in a two-dimensional superfield theory and (2) applications to superstrings. As shown in earlier works [7,8] on the purely bosonic theory, a Liouville term is needed in the first picture in order to obtain the correct spectrum of states. It may not be needed in the second picture. It is needed in the Neveu-Schwarz sector but not the Ramond sector of the superstring. The vector and chiral super-currents were coupled to abelian gauge superfields. This led to the (1,0) supersymmetric completion of the purely bosonic results of Ref. [6b]. Following this, the conditions for Siegel anomaly removal were obtained through a calculation of the effective action by the supergraph method of Ref. [16]. The Siegel super-ghosts, in the analog of the super-conformal gauge, were bosonized. A Liouville term was needed there. Finally, an action which couples chiral bosons to world-sheet supergravity, was proposed. This action was obtained by demanding that the supergravity generator acted schizophrenically on the bosonic superfields. The Siegel constraint algebra was co-opted into the Virasoro algebra. Although our primary calculations were done in superspace, we have given the component expressions for most of our results.

In future works, we plan to study (1) the non-abelian chiral super-bosonization, (2) the spectrum of states in the theory outlined in this work and (3) the bosonization of the supersymmetric charge. We hope to then obtain a more complete comprehension of the bosonization of supersymmetric theories, both global and local. This should be of some importance to superstring theories.

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