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REGULATING $1/(k^+)^n$ SINGULARITIES IN LIGHT CONE THEORIES*

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ABSTRACT

Deficiencies with the principal value method of calculating $(1/i\partial^+)^n e^{-\frac{i}{2}k^+x^-}$ are described and a method of solving these difficulties using regulated Green functions and boundary conditions is proposed.

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Recent applications of discrete light cone quantization (DLCQ) by Pauli and Brodsky [1] to scalar QED in 1+1 dimensions, by Eller, Pauli and Brodsky (EPB) [2] to the Schwinger model (1+1 vector QED) and by Harindranath and Vary [3] to ϕ^4 theory in 1+1 dimensions have demonstrated the usefulness of DLCQ for determining the mass spectrum and wave functions of 1+1 theories. The success of DLCQ for these theories provides the hope of solving 3+1 theories in the near future. However, one feature common to these works and which must be understood before 3+1 theories can be tackled are the $1/k^+$ and $1/(k^+)^2$ singularities that arise from $1/i\partial^+$ and $1/(i\partial^+)^2$. A nice introduction to this problem and light cone gauge in general is provided by Leibbrandt.[4]

This paper is concerned with the problem of these singularities in the context of DLCQ. First, the principal value method of handling $1/(k^+)^n$ used by EPB is described. Then, a number of deficiencies with this method are considered, and finally, a few possible solutions are described. I will try to follow the notation of EPB throughout.

1. The Principal Value Method

QED in 1+1 dimensions is considered by EPB. Using light cone coordinates, one finds that the only independent field in this theory is ψ^+. The fields A^- and ψ^- can by determined in terms of ψ^+ if one can find an appropriate definition of the operators $1/i\partial^+$ and $1/(i\partial^+)^2$. EPB define these as

$$\phi_1(x^-) = \frac{1}{i\partial^+} g(x^-) = \frac{i}{4} \int\limits_{-\infty}^{\infty} dy^- \epsilon(x^- - y^-) g(y^-) + A \tag{1.1}$$

$$\phi_2(x^-) = \frac{1}{(i\partial^+)^2} g(x^-) = -\frac{1}{8} \int\limits_{-\infty}^{\infty} dy^- |x^- - y^-| g(y^-) + Bx^- + C. \tag{1.2}$$

EPB set the x^- independent functions A, B, C to be zero. The $\epsilon(x)$ function is defined to be -1 for $x > 0$ and $+1$ for $x < 0$. For the case $g(x^-) = e^{-\frac{i}{2}k^+x^-}$, they insert the regulator $e^{-\kappa|x^- - y^-|}, \kappa \to 0$, and derive

$$\begin{aligned}
\phi_1(x^-) &= \frac{i}{4}\int\limits_{-\infty}^{\infty} dy^- \epsilon(x^- - y^-) e^{-\kappa|x^- - y^-|} e^{-\frac{i}{2}k^+y^-} \\
&= \frac{i}{4} e^{-ik^+x^-} \int\limits_{-\infty}^{\infty} du\ \epsilon(-u) e^{-\kappa|u|} e^{-\frac{i}{2}k^+u} \\
&= \frac{1}{2} e^{-\frac{i}{2}k^+x^-} \frac{k^+/2}{(k^+/2)^2 + \kappa^2} \\
&= \frac{1}{2} e^{-\frac{i}{2}k^+x^-} \left[\frac{1}{k^+ + 2i\kappa} + \frac{1}{k^+ - 2i\kappa}\right]
\end{aligned} \tag{1.3}$$

and

$$\begin{aligned}
\phi_2(x^-) &= -\frac{1}{8}\int\limits_{-\infty}^{\infty} dy^- |x^- - y^-| e^{-\kappa|x^- - y^-|} e^{-\frac{i}{2}k^+y^-} \\
&= -\frac{1}{8} e^{-\frac{i}{2}k^+x^-} \int\limits_{-\infty}^{\infty} du\ |u| e^{-\kappa|u|} e^{-\frac{i}{2}k^+u} \\
&= \frac{1}{4} e^{-\frac{i}{2}k^+x^-} \frac{(k^+/2)^2 - \kappa^2}{((k^+/2)^2 + \kappa^2)^2} \\
&= \frac{1}{2} e^{-\frac{i}{2}k^+x^-} \left[\frac{1}{(k^+ + 2i\kappa)^2} + \frac{1}{(k^+ - 2i\kappa)^2}\right] .
\end{aligned} \tag{1.4}$$

One recognizes the last lines of (1.3) and (1.4) as the principal value prescriptions for $1/k^+$ and $1/(k^+)^2$.

This method has a few problems. First of all, one notes that the $k^+ \to 0$ limit of the PV regulated $1/(k^+)^n$,

$$1/(k^+)^n = \frac{1}{2}\left[\frac{1}{(k^+ + i\kappa)^n} + \frac{1}{(k^+ - i\kappa)^n}\right] , \tag{1.5}$$

is zero for odd n and infinite for even n. This is somewhat suspicious. Let us make this argument rigorous.

One knows that the sum of the three light cone perturbation theory (LCPTh) graphs shown in Figure 1 add to give the usual lowest order Feynman rule amplitude for e^-e^- scattering,

$$T_{fi}^{FR} = -ie^2\bar{u}(l_f)\gamma_\mu u(l_i)\bar{u}(k_f)\gamma_\nu u(k_i)\frac{g^{\mu\nu}}{q_{FR}^2 + i\epsilon} \tag{1.6}$$

where $q_{FR} = l_i - l_f = k_f - k_i$. One notes that as $q^+ \to 0$, this answer remains finite for $q_\perp \neq 0$.

On the other hand, the first light cone graph is

$$T_{fi}^{(1)} = e^2\bar{u}(l_f)\gamma_\mu u(l_i)\bar{u}(k_f)\gamma_\nu u(k_i)\frac{\eta^\mu\eta^\nu}{(q^+)^2_\kappa} \tag{1.7}$$

and the sum of the second and third is

$$\begin{aligned} T_{fi}^{(2+3)} &= e^2\bar{u}(l_f)\gamma_\mu u(l_i)\bar{u}(k_f)\gamma_\nu u(k_i) \\ &\times \frac{1}{q^+(l_i^- - l_f^-) - q_\perp^2 + i\epsilon}\left[-g^{\mu\nu} + \frac{\eta^\mu q^\nu + \eta^\nu q^\mu}{q_\kappa^+}\right] \end{aligned} \tag{1.8}$$

where $\eta^\mu = (\eta^+, \eta^-, \eta_\perp) = (0, 2, 0_\perp)$, and q_κ^+ and $(q^+)^2_\kappa$ are defined to be the above ϕ_1 and ϕ_2 with the factors of $e^{-\frac{i}{2}q^+x^-}$ removed.

Since the sum of the three graphs (1.6) is finite as $q^+ \to 0$ $(q_\perp \neq 0)$, it must be that either $T_{fi}^{(1)}$ and $T_{fi}^{(2+3)}$ are both finite or are both infinite in this limit. From this it then follows that q_κ^+ and $(q^+)^2_\kappa$, and therefore ϕ_1 and ϕ_2, also must satisfy this condition. This is inconsistent with the above definition of ϕ_1 and ϕ_2 for which $\phi_1 = 0$ and $\phi_2 = -1/\kappa^2$ at $q^+ = 0$.

Finally, one can in general set A, B, C in the definition of $1/i\partial^+$ and $1/(i\partial^+)^2$ to be any functions that are independent of x^-. The above method does not give a clear prescription on how to handle this. It will turn out, as one might expect, that the values of A, B, C are determined by the boundary conditions one places on ϕ_1 and ϕ_2.

2. Green Functions and Boundary Conditions

Let us now attempt a more thorough formulation of the problem. First, consider the case $n = 1$. The solution to

$$i\partial^+\phi_1(x^-) = g(x^-), \quad -c < x^- < c \tag{2.1}$$

is given by

$$\phi_1(x^-) = \int_{-c}^{c} dy^- G(x^-, y^-) g(y^-) \tag{2.2}$$

where $G(x^-, y^-)$ satisfies

$$i\partial^+ G(x^-, y^-) = \delta(x^- - y^-). \tag{2.3}$$

In general, $G(x^-, y^-)$ contains a number of terms that are solutions to the homogeneous differential equation. In this case, the one homogeneous term is $A(y^-)$, an arbitrary x^- independent function. This extra term is fixed by imposing boundary conditions on the problem.

For the problem at hand, the Green function is

$$G(x^-, y^-) = \frac{i}{4}\epsilon(x^- - y^-) + A(y^-). \tag{2.4}$$

Let us impose the boundary condition

$$\phi_1(c) = -\phi_1(-c) \tag{2.5}$$

on $\phi_1(x^-)$. This can be met if one imposes an analagous condition on $G(x^-, y^-)$,

$$G(c, y^-) = -G(-c, y^-). \tag{2.6}$$

To satisfy this problem, one easily derives

$$
\begin{aligned}
A(y^-) = 0 \qquad G(x^-, y^-) &= \frac{i}{4}\epsilon(x^- - y^-) \\
\phi_1(x^-) &= \frac{i}{4}\int_{-c}^{c} dy^- \epsilon(x^- - y^-) g(y^-).
\end{aligned}
\tag{2.7}
$$

One recognizes this as the EPB prescription (1.1) . If one had instead asked for the boundary condition

$$
\begin{aligned}
\phi_1(c) &= 0 \\
G(c, y^-) &= 0,
\end{aligned}
\tag{2.8}
$$

one finds

$$
\begin{aligned}
A(y^-) = \frac{i}{4} \qquad G(x^-, y^-) &= -\frac{1}{2i}\theta(y^- - x^-) \\
\phi_1(x^-) &= -\frac{1}{2i}\int_{x^-}^{\infty} dy^- g(y^-).
\end{aligned}
\tag{2.9}
$$

Now consider a boundary value problem for $n = 2$. The problem

$$
\begin{aligned}
\phi_2(x^-) = \int_{-c}^{c} dy^- G(x^-, y^-) g(y^-), \quad -c < x^- < c \\
(i\partial^+)^2 G(x^-, y^-) = \delta(x^- - y^-)
\end{aligned}
\tag{2.10}
$$

has the general solution

$$
G(x^-, y^-) = -\frac{1}{8}|x^- - y^-| + B(y^-) + C(y^-)x^-. \tag{2.11}
$$

Imposing the boundary conditions

$$
\begin{aligned}
\phi_2(c) = -\phi_2(-c), \quad i\partial^+\phi_2(c) = -i\partial^+\phi_2(-c) \\
G(c, y^-) = -G(-c, y^-), \quad i\partial^+G(c, y^-) = -i\partial^+G(-c, y^-)
\end{aligned}
\tag{2.12}
$$

fixes $B(y^-)$ and $C(y^-)$ and gives a final answer of

$$
\begin{aligned}
G(x^-,y^-) &= -\frac{1}{8}|x^- - y^-| + \frac{c}{8} \\
&= -\frac{1}{16}\int_{-c}^{c} dz^- \epsilon(x^- - z^-)\epsilon(z^- - y^-) \\
\phi_2(x^-) &= -\frac{1}{16}\int_{-c}^{c} dy^- \int_{-c}^{c} dz^- \epsilon(x^- - y^-)\epsilon(y^- - z^-)g(z^-).
\end{aligned}
\tag{2.13}
$$

One notes that this Green function differs from the EPB prescription (1.2) by the term $\frac{c}{8}$. It turns out that the boundary conditions satisfied by the EPB Green function are the somewhat peculiar set

$$
\phi_2(c) - \frac{c}{2}\partial^+\phi_2(c) = -\phi_2(-c) - \frac{c}{2}\partial^+\phi_2(-c), \quad \partial^+\phi_2(c) = -\partial^+\phi_2(-c). \tag{2.14}
$$

A collection of Green functions and their corresponding boundary conditions for $n = 1, 2$ is given in the Appendix.

3. Regulated Green Functions for $e^{-\frac{i}{2}k^+x^-}$

Let us now focus on the specific choice $g(x^-) = e^{-\frac{i}{2}k^+x^-}$. $\phi_1 = (1/i\partial^+)\, e^{-\frac{i}{2}k^+x^-}$ and $\phi_2 = (1/i\partial^+)^2 e^{-\frac{i}{2}k^+x^-}$ both contain singularities at $k^+ = 0$ that need to be regulated. We do this by following a method similar to that used by EPB. Define regulated Green functions for $1/i\partial^+$ and $1/(i\partial^+)^2$ by including damping factors $e^{-\kappa|x^- - y^-|}, \kappa \to 0$. For example, the regulated Green function for the problem

$$
i\partial^+\phi_1(x^-) = e^{-\frac{i}{2}k^+x^-}, \quad -c < x^- < c, \quad \phi_1(c) = -\phi_1(-c) \tag{3.1}
$$

is

$$G(x^-,y^-) = \frac{i}{4}\epsilon(x^- - y^-)e^{-\kappa|x^- - y^-|} \,. \qquad (3.2)$$

In the situation $c \to \infty$, it is understood that the limit $\kappa \to 0$ is taken after the limit $c \to \infty$. The answer for ϕ_1 for arbitrary c using this regulated Green function is

$$\begin{aligned}\phi_1(x^-) = -\frac{i}{4}\Bigg\{ &\frac{1}{\kappa - \frac{i}{2}k^+}\left[e^{-\frac{i}{2}k^+x^-} - e^{-\kappa(c+x^-)}e^{\frac{i}{2}k^+c}\right] \\ &+ \frac{1}{\kappa + \frac{i}{2}k^+}\left[-e^{-\frac{i}{2}k^+x^-} + e^{-\kappa(c-x^-)}e^{-\frac{i}{2}k^+c}\right]\Bigg\} \,.\end{aligned} \qquad (3.3)$$

This result satisfies $\phi_1(c) = -\phi_1(-c)$ after setting $\kappa = 0$. Now consider $c \to \infty$. In the region $|x^-| < c$, the factors $e^{-\kappa(c \pm x^-)}$ go to zero, reproducing the EPB answer

$$\phi_1(x^-) = \frac{1}{2}e^{-\frac{i}{2}k^+x^-}\frac{k^+/2}{(k^+/2)^2 + \kappa^2} \,, \qquad |x^-| < c. \qquad (3.4)$$

For $|x^-| = c$, the answer is

$$\begin{aligned}\phi_1(c) &= \frac{-i/4}{\kappa - \frac{i}{2}k^+}e^{-\frac{i}{2}k^+c}, \qquad x^- = c \\ \phi_1(-c) &= \frac{i/4}{\kappa + \frac{i}{2}k^+}e^{\frac{i}{2}k^+c}, \qquad x^- = -c.\end{aligned} \qquad (3.5)$$

Let us look at one example for $n = 2$,

$$\begin{aligned}(i\partial^+)^2\phi_2(x^-) &= e^{-\frac{i}{2}k^+x^-}, \qquad -c < x^- < c \\ \phi_2(c) = -\phi_2(-c), &\quad i\partial^+\phi_2(c) = -i\partial^+\phi_2(-c).\end{aligned} \qquad (3.6)$$

The regulated Green function for this problem is given by

$$G(x^-,z^-) = -\frac{1}{16}\int_{-c}^{c} dy^-\epsilon(x^- - y^-)\epsilon(y^- - z^-)e^{-\kappa|x^- - y^-|}e^{-\kappa|y^- - z^-|} \,. \qquad (3.7)$$

The answer for $\phi_2(x^-)$ using this Green function after taking $c \to \infty$ is

$$\phi_2(x^-) = \frac{1}{4} e^{-\frac{i}{2}k^+x^-} \left[\frac{k^+/2}{(k^+/2)^2 + \kappa^2}\right]^2, \quad |x^-| < c. \tag{3.8}$$

The results (3.4) and (3.8) for ϕ_1 and ϕ_2 are zero for $k^+ = 0$. This is consistent with the requirement described at the end of Chapter 1 that ϕ_1 and ϕ_2 be both finite or both infinite as $k^+ \to 0$, and differs from the EPB answer. The results for $g(x^-) = e^{-\frac{i}{2}k^+x^-}$ for various regulated Green functions in the $c \to \infty$ limit are given in the Appendix.

4. Concluding Remarks

The principal value method of calculating $\phi_1 = (1/i\partial^+)e^{-\frac{i}{2}k^+x^-}$ and $\phi_2 = (1/i\partial^+)^2 e^{-\frac{i}{2}k^+x^-}$ used by EPB is found to be deficient. As described in Chapter 1, ϕ_1 and ϕ_2 must be both finite or both infinite as $k^+ \to 0$ so that the Feynman amplitude for e^-e^- scattering remain finite.

A method of calculating ϕ_1 and ϕ_2 using regulated Green functions has been demonstrated. In this method, the arbitrary functions A, B, C described by EPB are fixed by boundary conditions and are, in general, not zero. Using the set of conditions

$$\begin{aligned} \phi_1(c) &= -\phi_1(-c) \\ \phi_2(c) = -\phi_2(-c), &\quad i\partial^+\phi_2(c) = -i\partial^+\phi_2(-c) \end{aligned} \tag{4.1}$$

provides answers for $(1/i\partial^+)e^{-\frac{i}{2}k^+x^-}$ and $(1/i\partial^+)^2 e^{-\frac{i}{2}k^+x^-}$ that are consistent with the condition that they both be finite or infinite as $k^+ \to 0$. In fact, with this definition, they are both zero. This method can easily be extended to arbirary

n,

$$(i\partial^+)^n \phi_n(x^-) = g(x^-)$$

$$(i\partial^+)^m \phi_n(c) = -(i\partial^+)^m \phi_n(-c), \quad m = 0, 1, \ldots, n-1$$

$$\phi_n(x^-) = \left(\frac{i}{4}\right)^n \int_{-c}^{c} dy_1^- \int_{-c}^{c} dy_2^- \ldots \int_{-c}^{c} dy_n^- \epsilon(x^- - y_1^-) \ldots \epsilon(y_{n-1}^- - y_n^-) g(y_n^-). \tag{4.2}$$

Using this definition with $g(x^-) = e^{-\frac{i}{2}k^+x^-}$ and $c \to \infty$,

$$\phi_n(x^-) = \frac{1}{(i\partial^+)^n} e^{-\frac{i}{2}k^+x^-} = \left[\frac{k^+}{(k^+)^2 + 4\kappa^2}\right]^n e^{-\frac{i}{2}k^+x^-}. \tag{4.3}$$

Although a physical motivation for the boundary conditions (4.1) has not been provided (ideally, the correct boundary conditions should be determined by some physical argument), it has been shown that it is possible to construct Green functions that obey the finiteness condition described above.

Using the EPB prescription (1.3) and (1.4) to define $1/i\partial^+$ and $(1/i\partial^+)^2$ in 1+1 dimensional QED forces one to only consider charge-zero Fock states because the light-cone energy has a singularity $\propto \phi_2(k^+ = 0)Q(Q + 2\Lambda) \propto -\frac{1}{\kappa^2}Q(Q + 2\Lambda)$ (see eq. (2.18) and the discussion following eq. (3.3) in Ref. 2). With the new definition (4.3) for which ϕ_1 and ϕ_2 are both zero for $k^+ = 0$, this singularity is removed and the light-cone energy for 1+1 QED is finite for all values of the charge. This opens the possibility of considering charged states using the formalism of discrete light cone quantization.

Furthermore, if one applies DLCQ to 3+1 QED, one finds that the corresponding term proportional to $\phi_2(k^+ = 0)$ does *not* vanish for $Q = 0$. However, this term is identically zero if one uses the definition (4.3) for ϕ_2, and thus removes this singularity from the 3+1 dimensional QED light-cone energy.

One final remark concerning the difference between the EPB definition of ϕ_2 and the definition (4.2): they differ by a constant term $\frac{c}{8}$ (see eq. (2.13)). This added term only adds a constant term to the field A^- and as a result, has no effect on on the physical field $F_{\mu\nu}$ (see eq. (2.2) of Ref. 2).

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APPENDIX

The following are a collection of i) Green functions, ii) their corresponding boundary conditions, and iii) the result for $g(x^-) = e^{-\frac{i}{2}k^+x^-}$ using a regulated version of i) with $c = \infty$.

1. $n = 1$

   (a) $G(x^-, y^-) = \frac{i}{4}\epsilon(x^- - y^-)$

   $\phi_1(c) = -\phi_1(-c)$

   $\phi_1(x^-) = \frac{k^+}{(k^+)^2 + 4\kappa^2} e^{-\frac{i}{2}k^+x^-}$

   (b) $G(x^-, y^-) = -\frac{1}{2i}\theta(y^- - x^-)$

   $\phi_1(c) = 0$

   $\phi_1(x^-) = \frac{1}{k^+ - 2i\kappa} e^{-\frac{i}{2}k^+x^-}$

   (c) $G(x^-, y^-) = F(x^-, y^-)$

   $k^- > 0 : \phi(-c) = 0, \quad k^- < 0 : \phi(c) = 0$

   $\phi_1(x^-) = \frac{1}{k^+ + 2i\kappa sgn(k^-)} e^{-\frac{i}{2}k^+x^-}$

2. $n = 2$

(a) $G(x^-,y^-) = -\frac{1}{8}|x^- - y^-| + \frac{c}{8}$

$= -\frac{1}{16}\int_{-c}^{c} dz^- \epsilon(x^- - z^-)\epsilon(z^- - y^-)$

$\phi_2(c) = -\phi(-c), \quad i\partial^+\phi_2(c) = -i\partial^+\phi_2(-c)$

$\phi_2(x^-) = \left[\frac{k^+}{(k^+)^2+4\kappa^2}\right]^2 e^{-\frac{i}{2}k^+x^-}$

(b) $G(x^-,y^-) = -\frac{1}{8}|x^- - y^-| + \frac{c}{8} - \frac{1}{8c}x^-y^-$

$\phi_2(c) = 0, \quad \phi_2(-c) = 0$

$\phi_2(x^-) = \left[\frac{k^+}{(k^+)^2+4\kappa^2}\right]^2 e^{-\frac{i}{2}k^+x^-}$

(c) $G(x^-,y^-) = -\frac{1}{8}|x^- - y^-| + \frac{1}{8}(x^- - y^-)$

$= -\frac{1}{4}\int_{-c}^{c} dz^- \theta(y^- - z^-)\theta(z^- - x^-)$

$\phi_2(c) = 0, \quad i\partial^+\phi_2(c) = 0$

$\phi_2(x^-) = \left[\frac{1}{k^+-2i\kappa}\right]^2 e^{-\frac{i}{2}k^+x^-}$

(d) $G(x^-,y^-) = -\frac{1}{8}|x^- - y^-|$

$\phi_2(c) - \frac{c}{2}\partial^+\phi_2(c) = -\phi_2(-c) - \frac{c}{2}\partial^+\phi_2(-c), \quad \partial^+\phi_2(c) = -\partial^+\phi_2(-c).$

$\phi_2(x^-) = \frac{(k^+)^2-4\kappa^2}{((k^+)^2+4\kappa^2)^2} e^{-\frac{i}{2}k^+x^-}$

(e) $G(x^-,y^-) = \theta(k^-)\left[-\frac{1}{8}|x^- - y^-| - \frac{1}{8}(x^- - y^-)\right]$

$+ \theta(-k^-)\left[-\frac{1}{8}|x^- - y^-| + \frac{1}{8}(x^- - y^-)\right]$

$= \int_{-c}^{c} dz^- F(x^- - z^-)F(z^- - y^-)$

$k^- > 0 : \phi_2(-c) = 0, \quad i\partial^+\phi_2(-c) = 0$

$k^- < 0 : \phi_2(c) = 0, \quad i\partial^+\phi_2(c) = 0$

$\phi_2(x^-) = \left[\frac{1}{k^+ + 2i\kappa sgn(k^-)}\right]^2 e^{-\frac{i}{2}k^+x^-}$

The last Green function in the two sections above are from a paper by S. Mandelstam.[5] For these two cases, the function $F(x^-,y^-)$ is given by

$$F(x^-,y^-) = \frac{1}{2i}\left[\theta(k^-)\theta(x^- - y^-) - \theta(-k^-)\theta(y^- - x^-)\right].$$

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FIGURE CAPTIONS

1) Graphs for e^-e^- scattering. The graph on the left is the usual Feynman rule graph and the three graphs on the right arise from LCPTh.

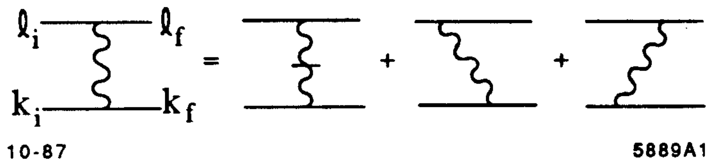

Fig. 1