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Two Loop Partition Function in (Compactified) Heterotic String Vacua*

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ABSTRACT

Two loop partition function for the heterotic string theory, compactified on any background preserving space-time supersymmetry at the string tree level, is explicitly calculated. This includes $E_8 \times E_8$ or $SO(32)$ heterotic string theory in ten dimensional flat space-time.

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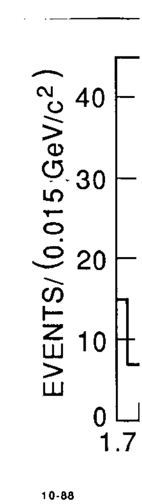
The issue of non-renormalization theorems and vacuum stability in compactified fermionic string theories has been the focus of considerable attention for the last few years. However, genuine progress in these directions has been hampered by the complexity of fermionic string perturbation theory. Until recently most of the progress has been achieved through general symmetry and effective lagrangian considerations [1]. Though illuminating, these arguments unfortunately do not expose the interplay between the relevant world-sheet dynamics and their space time manifestation, nor do they provide a check of the internal consistency of string theory. It is therefore very important at this stage to try to establish the validity of the non-renormalization theorems and understand vacuum stability directly through explicit string perturbative calculations.

The complexity of fermionic string perturbation theory beyond one loop can be essentially traced to the presence of the supermoduli [2]. These are modes of the world-sheet gravitino that cannot be gauged away by any world-sheet symmetry. On a world-sheet of genus $g \geq 2$ there are $2g-2$ of them. These are anologous to the moduli or the modes of the graviton that cannot be gauged away. Just as in the latter case, we have to integrate over them in the path integral, and the added complexity lies roughly speaking in finding the correct integration measure. In some natural setting these modes of the gravitino and graviton can be thought of as the odd and even coordinates of the supermoduli space parametrizing superconformally inequivalent super Riemann surfaces [3]. At this moment, however, not much is known about this space or the structures that can exist over it. Consequently a measure [4] on it is not yet known in any explicitness that would enable us to carry immediate perturbative calculations of string amplitudes.

An alternate approach to the problem has been to perform the integration over the odd moduli in advance in the path integral at the expense of introducing new operator insertions on the world-sheet. Although it may deprive us of important insights which could only be gained by working on supermoduli space, this procedure has the potential of being explicit: The path integral is now carried out on ordinary Riemann surfaces with spin structures, with which one is certainly more familiar. An associated problem in the path integral is posed by the zero modes of the commuting superconformal ghosts. On a world-sheet of genus $g \geq 2$ there are $2g-2$ zero modes for the superconformal ghost $\beta_{z\theta}$, the counting being the same as that for the gravitino zero modes. These have to be removed in a natural way before one can render the path integral well defined.

Recently Verlinde and Verlinde [5] have written down a path integral expression for the fermionic string measure after the integration over the supermoduli and soaking up of the ghost zero modes have been performed. Unfortunately, this does not yet imply that we possess a practical formalism to all orders in the perturbative expansion. As was pointed out in ref.[6], fermionic string perturbation seems to exhibit an inherent ambiguity steming essentially from ambiguities [7] in defining integration over the variables of a Grassmann algebra on a non-compact space—in this case the Grassmann valued coordinates of the supermoduli space. These ambiguities show up in the formalism of ref. [5] through the fact that the answer seems to depend on the choice of basis of the super-Beltrami differentials in terms of which we expand the gravitino field. The ambiguity is a total derivative in the moduli space which does not integrate to zero in general. At genus two nevertheless there exits a natural resolution of these ambiguities. Through various considerations of world-sheet supersymmetry, modular invariance, and decoupling of unphysical states it was shown in ref.[6] that the basis of super-Beltrami differentials at the boundary Δ_1 (where the surface degenerates into two tori) has to go to delta functions concentrated at the two nodes. The analogous statement at arbitrary genus is currently not known.

In this talk we shall use string perturbation to address the question of vacuum stability. In particular we shall see how to explicitly calculate the two loop cosmological constant in arbitrary supersymmetry preserving backgrounds. We shall see that it is zero except for compactifications which possess a Fayet-Iliopoulos D-term at one loop [8,9]. In that case we shall find that there is an induced cosmological constant given by the square of the one loop coefficient of



the Fayet-Iliopoulos terms. This cosmological constant was first calculated in ref. [10].

To start our discussion we shall write down an expression for the two loop heterotic string partition function following ref.[5],

$$W = \int D[XBC]\Big(\prod_{i=1}^{6} dm_i\Big)\Big(\prod_{a=1}^{2} d\varsigma^a\Big)\Big(\prod_{i=1}^{6}(\eta_i \mid B)\Big) \\ \Big(\prod_{a=1}^{2}\delta\big((\chi_a \mid B)\big)\Big)e^{-S(X,B,C,m_i,\varsigma^a)}, \tag{1}$$

where $\{m_i\}$ are the six moduli for the genus two surface and $\{\varsigma^a\}$ are the two supermoduli [2]; B denotes the reparametrization ghosts b_{zz}, $\bar{b}_{\bar{z}\bar{z}}$ and the super-reparametrization ghost $\beta_{z\theta}$. Similarly C stands for the reparametrization ghost fields c^z, $\bar{c}^{\bar{z}}$ as well as the super-reparametrization ghost field γ^θ. X denotes the set of all the matter fields. The inner products $(\eta_i \mid B)$, $\delta((\chi_a \mid B))$ in (1), which are there to soak up the various ghost zero modes, are defined as follows:

$$(\eta_i \mid B) = \int d^2z\{\eta_{i\bar{z}}{}^z b_{zz} + \bar{\eta}_{iz}{}^{\bar{z}}\bar{b}_{\bar{z}\bar{z}} + \eta_{i\bar{z}}{}^\theta \beta_{z\theta}\} \\ (\chi_a \mid B) = \int d^2z \chi_{a\bar{z}}^{\theta}\beta_{z\theta} \tag{2}$$

where η_i, χ_a form a basis for the super-Beltrami differentials and are defined through the following equations:

$$\eta_{i\bar{z}}{}^z = g_{z\bar{z}}\frac{\delta g^{zz}}{\delta m_i}, \quad \bar{\eta}_{iz}{}^{\bar{z}} = g_{z\bar{z}}\frac{\delta g^{\bar{z}\bar{z}}}{\delta m_i}, \\ \eta_{i\bar{z}}{}^\theta = \frac{\delta \chi_{\bar{z}}^\theta}{\delta m_i}, \quad \chi_{a\bar{z}}{}^\theta = \frac{\delta \chi_{\bar{z}}^\theta}{\delta \varsigma^a}. \tag{3}$$

and we have assumed that the world-sheet metric $g^{\alpha\beta}(m_i)$ is independent of the odd coordinates. At this stage we can carry out the integration over the odd moduli. It is easy to see that the expression we arrive at is given by:

$$W = \int D[XBC] \prod_{i=1}^{6} dm_i e^{-S_0} \prod_{a=1}^{2} \delta((\chi_a \mid B))$$
$$\left[\prod_{a=1}^{2} \{(\chi_a \mid T) + \frac{\partial}{\partial \varsigma^a}\} \prod_{i=1}^{6} (\eta_i \mid B) \right]_{\varsigma^a=0}, \tag{4}$$

The above expression can be made more explicit through a particular choice of basis for the super-Beltrami differentials $\chi_{a\bar{z}}^{\ \theta}$ given by:

$$\chi_{a\bar{z}}^{\ \theta} = \delta^{(2)}(z - z_a) \quad (a = 1, 2) \tag{5}$$

where $\{z_a\}$ are apriori arbitrary points on the Riemann surface. Also we shall group the six real moduli into three complex moduli $(m_i, m_{\bar{i}})$ and choose the metric such that $\eta_{\bar{i}\bar{z}}^{\ \ z} = \bar{\eta}_{iz}^{\ \ \bar{z}} = 0$. In this basis the above expression takes the following form[6]:

$$W = \int \prod_{i=1}^{3} [dm_i d\bar{m}_i] D[XBC] e^{-S_0} \prod_{i=1}^{3} (\bar{\eta}_{\bar{i}} \mid \bar{b}) \xi(z_0) \Big[Y(z_1) Y(z_2) \prod_{i=1}^{3} (\eta_i \mid b)$$
$$+ Y(z_1) \partial\xi(z_2) \sum_{j=1}^{3} (-1)^{j+1} \frac{\partial z_2}{\partial m_j} \prod_{i \neq j} (\eta_i \mid b)$$
$$+ Y(z_2) \partial\xi(z_1) \sum_{j=1}^{3} (-1)^{j+1} \frac{\partial z_1}{\partial m_j} \prod_{i \neq j} (\eta_i \mid b)$$
$$- \partial\xi(z_1) \partial\xi(z_2) \sum_{j=1}^{3} \sum_{k>j} (-1)^{j+k} \prod_{i \neq j,k} (\eta_i \mid b) \left(\frac{\partial z_1}{\partial m_j} \frac{\partial z_2}{\partial m_k} - \frac{\partial z_1}{\partial m_k} \frac{\partial z_2}{\partial m_j} \right) \Big] \tag{6}$$

where $Y(z_i)$ is the picture changing operator given by[11]:

$$Y =: e^{\phi} T_F := c\partial\xi + e^{\phi} T_F^{matter} - \frac{1}{4}\{\partial\eta e^{2\phi} b + \partial(\eta e^{2\phi} b)\} = \{Q_B, \xi\} \tag{7}$$

with ξ, ϕ, η related to the superconformal ghosts through the bosonization prescription: $\beta = \partial\xi e^{-\phi}$, $\gamma = \eta e^{\phi}$. Q_B is the BRST charge. The factor of $\xi(z_0)$

is needed to soak up the ξ zero mode. The final answer is independent of z_0. In writing down eq.(6) we have dropped terms which vanish by $(b,\bar{b})$ ghost charge conservation.

It is perhaps worth emphasizing at this point that $(z,\bar{z})$ denotes a coordinate system such that $g^{zz}=g^{\bar{z}\bar{z}}=0$ everywhere on the Riemann surface at the particular point $\{m_i\}$ in the moduli space where we are evaluating the string integrand. Thus the choice of coordinates $(z,\bar{z})$ varies with the moduli. In defining $\frac{\partial z_a}{\partial m_i}$ or the various derivatives appearing in (3) we must keep the coordinate system fixed. In other words, after we choose the specific coordinate system $(z,\bar{z})$ we evaluate $z_a(m_i+\delta m_i)$, $g^{zz}(m_i+\delta m_i)$, $g^{\bar{z}\bar{z}}(m_i+\delta m_i)$, $\chi_{\bar{z}}{}^{\theta}(m_i+\delta m_i)$ in this coordinate system and take $\delta m_i \to 0$ limit of appropriate ratios to calculate $\frac{\partial z_a}{\partial m_i}$ and the various super Beltrami differentials. As has become clear from the analysis of ref.[6,12,13], the choice $\frac{\partial z_a}{\partial m_i}=0$ is not in general consistent with modular invariance. So we shall not drop these terms from our analysis. Furthermore $z_a(m,\bar{m})$ is in general not a holomorphic function of the moduli. Terms with $\frac{\partial z_a}{\partial \bar{m}_i}$ though drop out from expression (6) by ghost charge conservation of the $\bar{b}$ system. We should also point out that in view of the results of ref. [6] the only constraint we shall impose on $z_a(m_i,\bar{m}_i)$ is that at the boundary Δ_1 of the moduli space the points z_1 and z_2 should coincide with the nodes p_1,p_2 on the tori T_1,T_2 respectively.

Before we evaluate the cosmological constant consider the following amplitude:

$$\Lambda = \int\limits_{M-\{z_1,z_2\}} d^2y \langle \partial X(y) \bar{\partial} X(y) \rangle \tag{8}$$

with the correlator defined using the path integral in expression (6), and the y integration runs over the Riemann surface with the points z_1,z_2 removed. The correlator in (8) is given by: $(Im\Omega)^{-1}_{ij}\omega_i(y)\bar{\omega}_j(\bar{y})\langle\ I\ \rangle$ where ω_i are the normalized abelian differentials and Ω is the period matrix, plus terms which come from the contraction of $\partial X(y)$ with $\partial X(z_1)$ in $Y(z_1)$ and $\bar{\partial} X(\bar{y})$ with $\partial X(z_2)$ in $Y(z_2)$ and vice versa. It is easy to see that the y dependence in these latter correlators is given by a total derivative of the form: $\partial_y(\langle \partial X(z_1)X(y)\rangle\langle\partial X(z_2)\bar{\partial}X(y)\rangle + (z_1 \leftrightarrow z_2))$. This total derivative could contribute at the boundary of the y integral, namely at (z_1, z_2), iff the correlator in question develops a pole of the form $(\bar{y} - \bar{z}_a)^{-1}$. By examining the correlator in question we can see that such poles do not arise. Consequently these total derivatives integrate to zero. Using the fact that $\int d^2y\omega_i(y)\bar{\omega}_j(\bar{y}) = (Im\Omega)_{ij}$, we immediately conclude that on a given genus surface, Λ as defined in (8) is the partition function $\langle\ I\ \rangle$ up to an overall numerical factor.

At this stage we can express $\partial X\bar{\partial}X$ as the contour integral of the suprsymmetry current around the dilatino vertex:

$$\delta_\alpha^\beta\langle\partial X(y)\bar{\partial}X(y)\rangle = \oint \frac{dx}{2\pi i}\langle J_\alpha(x)V^\beta(y)\rangle \tag{9}$$

where $J_\alpha(= e^{-\frac{\phi}{2}}\hat{S}^+S_\alpha)$ is the +ve chirality 4-dimensional supersymmetry current, and $V^\alpha(y)$ is given by:

$$\begin{aligned} V^\alpha(y) =&\bar{\partial}X^\mu(\gamma_\mu)^{\alpha\dot{\beta}}[e^{\frac{1}{2}\phi}\partial X^\nu(\gamma_\nu)_{\dot{\beta}\gamma}\hat{S}^-S^\gamma \\ &+\frac{1}{2}e^{\frac{3}{2}\phi}\eta b\hat{S}^-S_{\dot{\beta}} + e^{\frac{1}{2}\phi}S_{\dot{\beta}}\lim_{w\to z}(w-z)^{1/2}T_F^{\text{int}}(w)\hat{S}^-(z)] \end{aligned} \tag{10}$$

in the notation of ref. [10]. We can now deform the supersymmetry contour and attempt to shrink it to a point. If the supersymmetry current $J_\alpha(x)$ had only the physical pole at $x = y$ then contour deformation would lead to zero answer for (9). We would then conclude that the cosmological constant is zero in all theories which possess a space-time super current $J_\alpha(x)$[14]. However as was first pointed out in ref. [5] the supersymmetry current possesses spurious poles—poles not dictated by the operator product expansion. The origin of those poles was explained in ref.[10]. Their implication to contour deformation is that (9) may be expressed as the sum of residues at the spurious poles. Let $\{r_l\}$ denote the location of these poles on the surface. For $g=2$ it turns out that $l=1,\cdots 8$ (in general there are $2^{2g-2}g$ of these on a genus g surface [10]) and they are given in this case by the zeros of the function $f(x)=\prod_\delta \vartheta[\delta](-\frac{1}{2}\vec{x}+\frac{1}{2}\vec{y}+\sum_{a=1}^{2}\vec{z}_a-2\vec{\Delta})$ in the x-plane. After contour deformation the expression for the cosmological constant takes the form:

$$\Lambda=-\int[\prod_i dm_i d\bar{m}_i]\sum_l\int d^2y\oint_{r_l}\frac{dx}{2\pi i}D[XBC]e^{-S_0}J_\alpha(x)V^\alpha(y)\prod_{i=1}^{3}(\bar{\eta}_{\bar{i}}\mid\bar{b})\xi(z_0)$$
$$\Big[Y(z_1)Y(z_2)\prod_i(\eta_i\mid b)+Y(z_1)\partial\xi(z_2)\sum_{j=1}^{3}(-)^{j+1}\frac{\partial z_2}{\partial m_j}\prod_{i\neq j}(\eta_i\mid b)$$
$$+Y(z_2)\partial\xi(z_1)\sum_{j=1}^{3}(-)^{j+1}\frac{\partial z_1}{\partial m_j}\prod_{i\neq j}(\eta_i\mid b)\Big] \tag{11}$$

In expression (11) we have dropped terms that vanish by independent ghost charge conservation in the holomorphic and the anti-holomorphic sectors. Also, no sum over α is implied in this equation. We shall now show that (11) is a total derivative on the moduli space. Let us first observe that replacing z_1 by $\tilde{z}_1$, the spurious poles will shift to $\{r'_l\}$. By choosing $\tilde{z}_1$ appropriately we can ensure that $\{r_l\}\bigcap\{r'_l\}=0$. Consequently in equation (11) for the cosmological constant we can replace $Y(z_1)$ and $\partial\xi(z_1)\frac{\partial z_1}{\partial m_j}$ by $Y(z_1)-Y(\tilde{z}_1)$ and $\{\partial\xi(z_1)\frac{\partial z_1}{\partial m_j}-\partial\xi(\tilde{z}_1)\frac{\partial\tilde{z}_1}{\partial m_j}\}$ respectively, without changing the answer for the Λ.

Now we write

$$Y(z_1)-Y(\tilde{z}_1)=\oint\frac{dz}{2\pi i}J_{BRST}(z)(\xi(z_1)-\xi(\tilde{z}_1)) \tag{12}$$

and deform the BRST contour and try to shrink it to a point in the first two terms in eq. (11). The obstructions to this deformation are the poles in the argument of the BRST current at the locations of $V^{\alpha}(y)$, $\partial\xi(z_2)$, $(\eta_i \mid b)$ and $\xi(z_0)$. The latter residue vanishes identically due to the absence of a ξ mode in the correlator that can be used to soak up the zero mode. Had we not carried out the above subtraction we would have had to worry about this residue. $V^{\alpha}(y)$ in (10) is BRST invariant up to total derivative in y. Consequently the residue of the BRST current at the location y will be a total derivative in y. This could contribute at the boundary (z_1, z_2) of the y integration only if the integrand develops a $(\bar{y} - \bar{z}_a)^{-1}$ pole. Remembering that the only $\bar{y}$ dependence in the problem comes through the $\bar{\partial}X(y)$ factor in (10) we can see that no such singularity exists. Finally the residue of J_{BRST} at $\partial\xi(z_2)$ is $\partial Y(z_2)$ and at the location of $(\eta_i \mid b)$ is given by $(\eta_i \mid T)$, where T is the stress tensor on the world sheet. The latter insertion in the correlator may in turn be expressed as $-\frac{\partial S_0}{\partial m_i}$. Combining these results together, we may express Λ as,

$$\Lambda = \int [\prod_{i=1}^{3} dm_i d\bar{m}_i] \sum_j \frac{\partial}{\partial m_j} M_j \tag{13}$$

where the density M_j is given by:

$$\begin{aligned} M_j = \int d^2y \oint_{r_l} \frac{dx}{2\pi i} \int D[XBC] e^{-S_0} \xi(\tilde{z}_1)\xi(z_1)(-1)^j \\ Y(z_2) \prod_{i \neq j} (\eta_i \mid b) \prod_{i=1}^{3} (\bar{\eta}_{\bar{i}} \mid \bar{b}) J_{\alpha}(x) V^{\alpha}(y) \end{aligned} \tag{14}$$

In writing down eq. (14) we have dropped terms that vanish by $(b, \bar{b})$ ghost charge conservation, and have set $z_0 = \tilde{z}_1$. Notice that so far we have been working entirely in a model independent setting. We have carried out world-sheet manipulations in a manner that is independent of the background fields except that they must allow the existence of a spacetime supersymmetry current at the string tree level.

Being a total derivative in moduli, expression (14) can be written as a boundary term. The boundary of genus two moduli space has two distinct components denoted by Δ_0 and Δ_1. The first is where the Riemann surface degenerates by pinching a nontrivial homology cycle. This is conformally equivalent to one of the handles becoming infinitely long. It is not difficult to see that no contribution can result from this boundary* : If s stands for the length of the handle in some appropriate coordinates, then Δ_0 corresponds to $s \to \infty$. The string integrand in these coordinates behaves like $\frac{1}{s^5}e^{-m^2 s}$. The polynomial factor comes from the degeneration of the $(det Im\Omega)^{-5}$ factor in the measure and $e^{-m^2 s}$ is the effect of space-time propagation of the string modes in the long tube of length s. In the absence of tachyons the limit $s \to \infty$ yields a vanishing contribution.

The other boundary Δ_1 is where the Riemann surface degenerates by pinching a trivial homology cycle and can be pictured as a genus two surface breaking up into two tori T_1 and T_2 connected by a thin (long) tube. We shall next see that Λ receives a contribution from this boundary: In the neighborhood of Δ_1 we can parametrize the moduli space by $(\tau_1 = \Omega_{11}, \tau_2 = \Omega_{22}, t = \Omega_{12})$ where τ_1, τ_2 are the modular parameters of the tori T_1 and T_2 respectively. t can be thought of as the pluming fixture variable (see for e.g. [17]) and can be written as $t = re^{i\theta}$ where r is the radius of the cylinder connecting the two tori. The boundary in question corresponds to $t \to 0$. Since the boundary is a point of measure zero the integral

$$\Lambda = \int dt d\bar{t} \frac{\partial}{\partial t} M_t(t, \bar{t}) \tag{15}$$

would be non-vanishing on the boundary only if

$$\lim_{t \to 0} M_t(t, \bar{t}) \sim \frac{1}{\bar{t}}. \tag{16}$$

We shall now briefly sketch the calculation of M_t (for details see ref.[10]). The fastest way to determine the behaviour of M_t near the boundary is to use

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⋆ J. J. A would like to thank G. Moore for discussions on this point the factorization theorem [15][†]

$$\langle A_1(z_1)A_2(z_2)\rangle_{g=2} \sim \sum_{\Phi} \langle A_1(z_1)\Phi(p_1)\rangle_{T_1} \langle \Phi^\dagger(p_2)A_2(z_2)\rangle_{T_2} t^{h_\Phi}\bar{t}^{\bar{h}_\Phi} \tag{17}$$

where $(h_\Phi, \bar{h}_\Phi)$ are the conformal dimensions of the field Φ propagating in the narrow tube connecting the two tori; p_1, p_2 are the nodes on T_1 and T_2 respectively.

In view of the results of ref. [6] we shall take in the correlator in eq. (14) z_1 and z_2 to lie on T_1 and T_2 respectively. Ultimately we have to take the $z_1 \to p_1$ and $z_2 \to p_2$ limit. We shall also take $\tilde{z}_1$ to lie on T_2 for convenience, although the final result may be shown to be manifestly independent of the location of $\tilde{z}_1$. The y integration runs over the tori T_1 and T_2. Let us take for definiteness y on T_1. It is not hard to prove that the other case gives the same contribution [10]. In this case, the positions r_l of the spurious poles in the $t \to 0$ limit may be found by analyzing the behavior of $\vartheta[\delta](-\frac{1}{2}\vec{x} + \frac{1}{2}\vec{y} + \sum_{a=1}^{2} \vec{z}_a - 2\vec{\Delta})$ in this limit. It can be seen that four of these poles lie on T_1, and the other four lie on T_2[10].

Let us first take x on T_2. Application of the factorization theorem to the correlator in eq. (14) yields:

$$\begin{aligned} &t^{h_\Phi}\bar{t}^{\bar{h}_\Phi - 2}\overline{\eta(\tau_1)}^2\,\overline{\eta(\tau_2)}^2 \langle \xi(z_1)b(w_1)V^\alpha(y)\Phi(p_1)\rangle_{T_1} \\ &\qquad \langle \Phi^\dagger(p_2)b(w_2)e^{-\phi(x)/2}\hat{S}^+(x)S_\alpha\xi(\tilde{z}_1)Y(z_2)\rangle_{T_2} \end{aligned} \tag{18}$$

where we have explicitly exhibited the contribution from the factorization of the antiholomorphic ghost determinant. We also used the fact that $(\eta_1 \mid b)_0$, $(\eta_2 \mid b)_0$ in the limit $t \to 0$ reduce to $b(w_1)$, $b(w_2)$ inserted at arbitrary points w_1, w_2

† One of course can calculate all the ghost correlators and the free matter part at two loops explicitly in terms of ϑ-functions and prime forms and then the effect of these correlators on the behaviour of M near the boundary can be inferred using well known formulae for degeneration of ϑ-functions [16]. Nevertheless one would still have to use the factorization theorem to exhibit the contribution of the interacting part of the matter correlators. In ref.[10] the analysis is carried along these lines.

on T_1, T_2 respectively. Using various ghost charge conservations and the fact that the operator Φ has to have dimension (0,1) in order to contribute at the boundary we can determine what Φ has to be:

$$\Phi(p_1) = c(p_1)e^{-\phi(p_1)/2}\hat{S}^{+}(p_1)S_{\alpha}(p_1)U(\bar{z}) \tag{19}$$

where $U(\bar{z})$ is any operator of conformal dimension $(0,1)$ and neutral under all ghost charges. The only such operators in the heterotic string are associated with gauge currents (recall that for every dim. $(0,1)$ operator we can construct the vertex operator for a massless gauge boson by adjoing it with $(\partial X + ik\cdot\psi\psi)e^{ik\cdot X}$). It is clear that the only nonvanishing contribution to that matrix element could come from the $(0,1)$ operators associated with the abelian factors of the unbroken gauge group. We now have to compute the one loop matrix elements appearing in eq.(18), with Φ as given in eq. (19). The superconformal ghost correlator can be calculated readily. In that one finds that there exists a simple pole in x on T_2. This pole accounts for four of the spurios poles on the genus two surface before degeneration since it exists on T_2 for each spin structure on T_1. Finally the one-loop matrix elements of the interacting matter fields can be computed by applying the results of ref. [9], and the answer can be exhibited entirely in terms of properties of the massless spectrum. From this one can easily calculate the residue of the correlator at the spurious poles.

The other four poles in the x plane lie on the torus T_1. The contribution to M_t from these poles may be analyzed by taking x on the torus T_1, and analyzing the resulting correlator using factorization theorem. The relevant intermediate operator $\Phi(p_1)$ is given by $c(p_1) : \xi(p_1)\eta(p_1) : U(p_1)$. We find a simple pole on T_1, whose position is independent of the spin structure on the torus T_2. This accounts for the other four poles. The residue at this pole however can be seen to vanish after summing over spin structures on T_2.

Combining all the results, we find that the cosmological constant at two loops is given by:

$$\Lambda \sim \sum_a c^{(a)} c^{(a)} \tag{20}$$

where $c^{(a)}$ is the coefficient of the Fayet-Iliopoulos D term associated with the a'th abelian factor in the gauge group, induced at one loop [8,9]. More explicitly it is given by

$$c^{(a)} = \frac{g}{192\pi^2} \sum_i n_i q_i^{(a)} h_i \tag{21}$$

where n_i is the number of massless fermions (bosons) with chirality h_i and $U^{(a)}(1)$ charge $q_i^{(a)}$. Needless to say this means that the cosmological constant at two loops vanishes for all string vacua which have tree-level supersymmetry and which do not develop a one loop Fayet-Iliopoulos D-term. These include among other things flat space-time and the standard compactifications of the $E_8 \times E_8$ heterotic string on Calabi-Yau backgrounds.

It is important to see whether one can push string perturbation theory at higher loops to the same level of explicitness that we have witnessed at two loops. One obstacle in that direction seems to be the ambiguities alluded to earlier. Another interesting question is the connection between the boundary terms in the moduli space and the vev's of the auxiliary fields. In our analysis above we found that the two loop boundary term was related to the square of the vev of the auxialiary D term. One is tempted at this stage to conjecture a correlation between the two. It is therefore important to try to calculate within string perturbation directly the vev's of the auxiliary fields, *e.g.* $\langle F \rangle$ and $\langle D \rangle$ at higher loops. At one loop it can be shown through some variant of contour deformation arguments that $\langle F \rangle = 0$. However at higher genus no such statement exists so far. It is therefore safe to say that the F-term non renormalization theorems at higher loops have not yet explicitly been established in string perturbation theory.

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