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ON WEAK DECAYS OF HEAVY FLAVORS, MIXING AND CP VIOLATION*

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ABSTRACT

Detailed studies of weak decays serve not only to confirm the Standard Model, but possess also a high sensitivity to New Physics: tau and top decays are discussed in this vein, with some short remarks on beauty and charm. The sensitivity to New Physics is even higher in delicate phenomena like mixing and CP violation: a fairly detailed discussion on $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$, and $B^0 - \bar{B}^0$ mixing and on CP violation in K^0 and B decays is presented.

P. als "langsames Bohren von harten Brettern mit Leidenschaft und Augenmass" Max Weber

PROLOGUE

There is a strongly held belief in the High Energy Physics community that the "Standard Model" based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)$ with three families [1] represents only an incomplete reflection of Nature's forces (gravity is ignored throughout these lectures). At the same time there exists little agreement on the kind of New Physics expected to fill out the picture. It is quite natural to rely on further experimental efforts to elicit some hints from Nature on the New Physics. There are two approaches to this problem:

- The "High Road to New Physics" where one employs collisions at the highest energies to produce new quanta, new gauge bosons, new fermions, and so on.
- The "Low Road to New Physics" where one searches for new forces; since these are due to the exchange of *virtual* new quanta one is not necessarily driven towards higher and higher collision energies. Yet the searched for

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signal is typically of a rather indirect type, for one looks for a significant difference between a measured rate and a rate that is calculated within the Standard Model. Precision both on the experimental and the theoretical side is thus essential for such an analysis. On the former I have nothing to say; the latter will occupy us again and again: how much and under which conditions can we really trust expectations based on theoretical calculations? In Chapter I I discuss τ decays; Chapter II deals with heavy flavor decays, mainly of top hadrons with some short remarks on beauty and charm decays; in Chapter III I analyze flavor mixing — $B^0 - \bar{B}^0$, $D^0 - \bar{D}^0$ and $K^0 - \bar{K}^0$ mixing — before addressing the issue of CP violation in Chapter IV.

I. τ DECAYS

If one attributes the highest priority to our ability to perform truly reliable calculations then at present one cannot find a better field of study than heavy lepton decays.

1. A First Profile

τ leptons give off a strong feeling of "deju vu":

(i) As with previous neutrinos ν_τ has a vanishing mass.

(ii) Their charged current coupling is — up to a small radiative corrections — determined by the Fermi constant G_F. The width for the purely leptonic transitions $\tau \to \nu_\tau \ell \bar{\nu}_\ell$, $\ell = e, \mu$ is thus fixed by the τ mass

$$\begin{aligned} \Gamma\left(\tau \to \nu_\tau \ell \bar{\nu}_\ell\right) &= \frac{G_F^2 m_\tau^5}{192\pi^3}\, K\left(\frac{m_\ell^2}{m_\tau^2}\right) \\ K(y) &= 1 - 8y + 8y^3 - y^4 - 12y^2 \log y \quad . \end{aligned} \tag{1}$$

Inserting the experimental values for m_τ, m_ℓ one obtains

$$\begin{aligned} \Gamma\left(\tau \to \nu_\tau e \bar{\nu}_e\right) &= \Gamma(\mu)\left(\frac{m_\tau}{m_\mu}\right)^5 = \left[1.595 \times 10^{-12}\ \mathrm{sec}\right]^{-1} \\ \Gamma\left(\tau \to \nu_\tau \mu \bar{\nu}_\mu\right) &\simeq 0.97\, \Gamma\left(\tau \to \nu_\tau e \nu_e\right) \quad . \end{aligned} \tag{2}$$

(iii) The energy spectrum of the leptons ℓ is fixed as well. Its shape is most conveniently characterized by the Michel parameter ρ_M

$$\frac{1}{\Gamma}\frac{d}{dx_\ell}\Gamma\left(\tau \to \ell\bar{\nu}_\ell\nu_\tau\right) \simeq x_\ell^2\left\{3\left(1-x_\ell\right)+2\rho_M\left(\frac{4}{3}x_\ell-1\right)\right\}$$
$$0 \leq x_\ell = \frac{2E_\ell}{m_\tau} \leq 1 \quad . \tag{3}$$

ρ_M is easily expressed in terms of left-handed and right-handed current-current couplings

$$\rho_M = \frac{3}{4}\frac{g_L^2}{g_L^2+g_R^2} = \begin{cases} \frac{3}{4} & \text{V}-\text{A} \\ \frac{3}{8} \quad \text{for} & \text{V;A} \\ 0 & \text{V}+\text{A} \end{cases} \quad . \tag{4}$$

(iv) The neutral current couplings of τ leptons are fixed as well, both in their vector and axial-vector parts

$$\begin{aligned} a_\tau &= -\frac{1}{2} \\ v_\tau &= -\frac{1}{2} + 2\sin^2\theta_W \simeq -0.06 \end{aligned} \tag{5}$$

which can be measured by studying forward-backward asymmetries in e^+e^- annihilation

$$\frac{d\sigma}{d\Omega}\left(e^+e^- \to \tau^+\tau^-\right) = \frac{\pi\alpha^2}{2s}\left[A(1+\cos^2\theta)+B\cos\theta\right]$$

$$A = 1 + 2v_e v_\tau \mathrm{Re}\,\chi + \left(v_e^2+a_e^2\right)\left(v_\tau^2+a_\tau^2\right)|\chi|^2$$

$$B = 4a_e a_\tau \mathrm{Re}\,\chi + 8v_e v_\tau a_e a_\tau |\chi|^2 \tag{6}$$

$$\chi = \frac{G_F}{2\sqrt{2}\,\pi\alpha}\,\frac{m_Z^2 s}{s-m_Z^2+im_Z\Gamma_Z} \quad .$$

There exists rather good experimental verifications of these gross features [2]:

(a) A decent upper bound on the τ neutrino mass has been obtained:

$$m(\nu_\tau) \leq 70\,\mathrm{MeV} \qquad 95\%\ \mathrm{C.L.} \tag{7}$$

(b) Using the World Average on the τ lifetime

$$\langle\tau_\tau\rangle = (3.07 \pm 0.2) \times 10^{-13}\ \mathrm{sec} \tag{8}$$

together with the prediction for $\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)$ leads to

$$BR\,(\tau \to \nu_\tau e \bar{\nu}_e) = 19.2 \pm 1.2\% \tag{9}$$

which is not in clear conflict with a world average on the directly measured leptonic branching ratio

$$BR_{\mathrm{direct}}\,(\tau \to \nu_\tau e \bar{\nu}_e) = 17.9 \pm 0.4\% \quad . \tag{10}$$

(c) The Michel parameter has been measured as well:

$$\rho_M = 0.73 \pm 0.07 \tag{11}$$

again in quite good agreement with the Standard Model expectation $\rho_M(\mathrm{V}-\mathrm{A}) = 0.75$.

(d) From the measured forward-backward asymmetries one concludes[3]

$$a_e a_\tau = 0.24 \pm 0.031 \tag{12}$$

The gross features as they are expected for a sequential lepton are thus confirmed within present experimental accuracy. So far, however, we have basically ignored more than 60% of τ decays, namely those that contain hadrons in the final state.

2. Semihadronic Decays

As soon as hadrons enter the stage we cannot rely any longer on purely theoretical calculations to obtain reliable results. Instead we invoke general concepts like CVC to relate τ decay widths to e^+e^- cross sections that can be measured independently: one starts from the usual expression for the decay width for $\tau \to \nu_\tau f$ where f denotes a hadronic state[4,5]

$$\Gamma\left(\tau^- \to \nu_\tau f^-\right) = \frac{G_F^2}{4m_\tau}\int \frac{d^3p_\nu}{2E_\nu}\frac{1}{(2\pi)^3}\frac{1}{2}tr\left(\not{p}_\tau + m_\tau\right)\left(1+\gamma_5\right)\gamma_\rho\ \not{p}_\nu\gamma_\sigma\left(1-\gamma_5\right)$$
$$\times \langle 0|J_\rho(0)|f\rangle\langle f|J_\sigma^\dagger(0)|0\rangle(2\pi)^4\delta^{(4)}\left(p_\tau - p_\nu - p_f\right) \tag{13}$$

where

$$J_\mu = \left[\left(F_\mu^1 + iF_\mu^2\right) - \left(F_{\mu,5}^1 + iF_{\mu,5}^2\right)\right]\cos\theta_c + \left[\left(F_\mu^4 + iF_\mu^5\right) - \left(F_{\mu,5}^4 + iF_{\mu,5}^5\right)\right]\sin\theta_c$$

the F_μ^i [$F_{\mu,5}^i$] denote SU(3)$_{FL}$ octet vector [axialvector] currents. Lorentz covariance leads to

$$\begin{gathered}\langle 0|F_\mu^1(0) + iF_\mu^2(0)|f\rangle\langle f|F_\nu^1(0) - iF_\nu^2(0)|0\rangle(2\pi)^4\delta^{(4)}(q-p_f) = \\ (q_\mu q_\nu - g_{\mu\nu}q^2)v_1(q^2)\end{gathered} \tag{14}$$

$$\begin{gathered}\langle 0|F_{\mu,5}^1(0) + iF_{\mu,5}^2(0)|f\rangle\langle f|F_{\nu,5}^1(0) - iF_{\nu,5}^2(0)|0\rangle(2\pi)^4\delta^{(4)}(q-p_f) = \\ (q_\mu q_\nu - g_{\mu\nu}q^2)a_1(q^2) + q_\mu q_\nu a_0(q^2)\end{gathered} \tag{15}$$

where CVC (*i.e.*, the conserved vector current hypothesis) has already been used to obtain $v_0(q^2) \equiv 0$. Thus

$$\begin{aligned}\Gamma(\tau \to \nu_\tau f) = {} & \frac{G_F^2}{(2\pi)^2(2m_\tau)^3}\int_0^{m_\tau^2} dq^2\left(m_\tau^2 - q^2\right)^2 \\ & \times\left[\left\{\left(m_\tau^2 + 2q^2\right)\left(v_1(q^2) + a_1(q^2)\right) + m_\tau^2 a_0(q^2)\right\}\cos^2\theta_c\right. \\ & \left. + \left\{\left(m_\tau^2 + 2q^2\right)\left(v_1^S(q^2) + a_1^S(q^2)\right) + m_\tau^2\left(v_0^S(q^2) + a_0^S(q^2)\right)\right\}\sin^2\theta_c\right]\end{aligned} \tag{16}$$

Here v^S, a^S denote the strangeness changing analogues of v, a. CVC tells us also how to relate the vector currents F_μ^i listed above with F_μ^3 which describes the isovector part of the photon [4,5]:

$$\begin{aligned}&2\langle 0|F_\mu^3(0)|f^0\rangle\langle f^0|F_\nu^3(0)|0\rangle(2\pi)^4\delta^{(4)}(q-p_f) \\ =&\langle 0|F_\mu^1(0) + iF_\mu^2(0)|f^-\rangle\langle f^-|F_\nu^1(0) - iF_\nu^2(0)|0\rangle(2\pi)^4\delta^{(4)}(q-p_f) \quad .\end{aligned} \tag{17}$$

For a hadron state f with vector and isovector quantum numbers one then finds after a rescaling of variables

$$\begin{gathered}\frac{\Gamma(\tau^- \to \nu_\tau f^-)}{\Gamma(\tau^- \to \nu_\tau e^-\bar{\nu}_e)} = 2\cos^2\theta_c\int_0^1 dx(1-x)^2(1+2x)\frac{\sigma(e^+e^- \to f^0)}{\sigma_{pt}} \\ \sigma_{pt} = \frac{4\pi\alpha^2}{3q^2} \quad , \quad x = \frac{q^2}{m_\tau^2} \quad .\end{gathered} \tag{18}$$

This shows that a measured cross section for $e^+e^- \to f^0$ allows us to determine quite reliably $\Gamma(\tau \to \nu_\tau f^-)$ by doing the appropriate integration. When f is produced by the axialvector current then the theoretical machinery available (Weinberg's sum rules, and so on) is somewhat less reliable, yet still adequate.

Since a detailed comparison between theoretical predictions and experimental findings is given in M. Perl's lectures at this school, I will not repeat it here; instead I make a few more general observations:

(i) Basically all predictions on branching ratios were indeed made before measurements existed.

(ii) It is a pleasing experience to note, channel by channel, the agreement between theory and experiment.

(iii) Yet when one adds up all the numbers one encounters a surprise [5]:

	Exclusive [%]	Inclusive [%]
BR ($\tau^- \to$ 1 prong)	≤ 80	86.6±0.3
BR ($\tau^- \to$ 3 prongs)	12.4±0.6	13.3±0.3

Taking these numbers at face value one would conclude that roughly six percent of τ decays have not been identified yet and that those "missing" decays are one-prong modes, *i.e.*, $\tau^- \to \nu_\tau \pi^-$ + neutrals. This by itself would not be so surprising; the dilemma becomes apparent when one tries to come up with channels that could account for this deficit [5]:

– The decay $\tau^- \to \nu_\tau \pi^- \eta$ has to be produced by a second class current: $\pi\eta$ form a vector state, 1^-, yet are odd under G parity. The Standard Model allows such transitions only via radiative electromagnetic corrections which makes them very weak indeed. Despite some earlier excitement it is now fair to say that data from different groups strongly support the theoretical expectation that this mode does not help significantly in resolving our problem.

– It is very hard to see how modes like $\tau^- \to \nu_\tau \pi^- + \eta's + \pi^0's$ could make up a few percent of all τ decay while their isospin related modes would not make their presence felt in 5-prong, etc. decays.

Considering this dilemma one feels inclined to ask how confident one should be about the experimental numbers; more specifically, are all systematic uncertainties properly taken into account when averaging over the results of different groups? The branching ratios for semihadronic τ decays are typically calibrated by relating them to $BR(\tau \to \nu_\tau e \bar{\nu}_e)$; for the latter one uses a branching ratio of 17–18%.

On the other hand, using the measured τ lifetime together with the calculated width for $\tau \rightarrow \nu_\tau e \bar{\nu}_e$ leads to a leptonic branching ratio of 18–20% [Eqs. (2),(8) and (9)]—certainly not inconsistent with the directly determined value, but somewhat higher. If, for example, $BR(\tau \rightarrow \nu_\tau e \bar{\nu}_e) = 19.5\%$ were employed as input instead of 17.9% there would not be any missing 1-prong decays left! An unbiased interpretation of the data is further complicated by that fact that some experimental analysis employed the universality relation $\Gamma(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau) = 0.97\,\Gamma(\tau \rightarrow e \bar{\nu}_e \nu_\tau)$ as *constraint* while others did not.

Uncertainties like these and others allow only the following conclusions:

- The data on τ decays that have been compiled over the last few years contain intriguing hints that heavy lepton decays are *not* fully understood.
- It is, however, premature to claim any established incompleteness in the standard description.
- Unfortunately (or fortunately?) no reasonable, let alone attractive, theoretical scenario has so far been uncovered that would allow to close a gap once that were established to exist.
- Clearly we need more data!

3. The Future of τ Physics

There is a triple motivation for continuing a vigorous research program on τ physics:

(i) An experiment dedicated to τ studies is best suited to eliminate the uncertainties in the present data samples.

(ii) It is quite desirable to improve the experimental sensitivity for $m(\nu_\tau)$. According to the "see-saw mechanism"[6]

$$\frac{m(\nu_\tau)}{m(\nu_e)} \sim \frac{m_\tau^2}{m_e^2} \quad . \tag{19}$$

If, for example, $m(\nu_e) \sim 3$ eV which is well below the present upper bound then (1.18) leads to $m(\nu_\tau) \sim 40$ MeV which is again below the present upper bound of 70 MeV, but not by a large margin. It hardly needs emphasizing what profound impact the observation of a nonvanishing τ neutrino mass would have on our understanding of Nature's forces.

(iii) It is highly desirable to subject τ decays to a general dynamical analysis without Standard Model constraints built in *ab initio*. The most general current-current coupling is given as follows

$$\mathcal{L} = \sum_{i=1}^{5} \bar{\psi}\left(\alpha_i \Gamma_i + \beta_i \Gamma_i \gamma_5\right) \psi \bar{\psi} \Gamma_i \psi \tag{20}$$

where the Γ_i represent the Lorentz structure of the various "currents" $\bar{\psi} \ldots \psi$. Thus there are ten complex coupling constants $\{\alpha_i, \beta_i\}$. Since one overall phase is unphysical we end up with 19 real physical parameters of which the Michel parameter ρ introduced earlier is just one! The latter is not even a particularly sensitive one since its present value — $\rho = 0.73 \pm 0.07$ — still allows for very substantial right-handed couplings, namely

$$\frac{g_R}{g_L} \leq 0.47 \quad . \tag{21}$$

Other parameters are much more sensitive for the presence of New Physics.

A very recent and very comprehensive study [7] of muon decays can serve as a case study to illustrate the scope of such an analysis. The results are summarized in Fig. 1. From it one concludes that the data on μ decays are not just consistent with the Standard Model, but that they even *imply* it (within experimental errors): the V-A coupling has, with excellent accuracy, weight +1, whereas all the other couplings are quite consistent with zero.

One clearly wants to repeat such an analysis for τ decays since

- one is dealing with an, in principle, completely different dynamical system;
- $m(\nu_\tau)$ could quite possibly be much larger than $m(\nu_\mu)$ and,
- finally, if the New Physics one is searching for, contains charged Higgs fields then τ decays are favored over μ decays by many orders of magnitude up to $(m_\tau/m_\mu)^2 (m_s/m_e)^2 \sim 2 \times 10^7$!

Such an analysis seems feasible to a reasonable degree if two requirements are met:

(α) Large statistics.

(β) One is able to extract spin information by measuring the μ polarization in $\tau \to \nu_\tau \mu \bar{\nu}_\mu$ or studying angular correlations in $e^+e^- \to \tau^+\tau^- \to (\rho^+\bar{\nu}_\tau)(\rho^-\nu_\tau)$, *etc.*[8]

May I just add that beauty and charm factories which have been suggested with increased vigor over the last two years will serve concurrently as proficient τ factories.

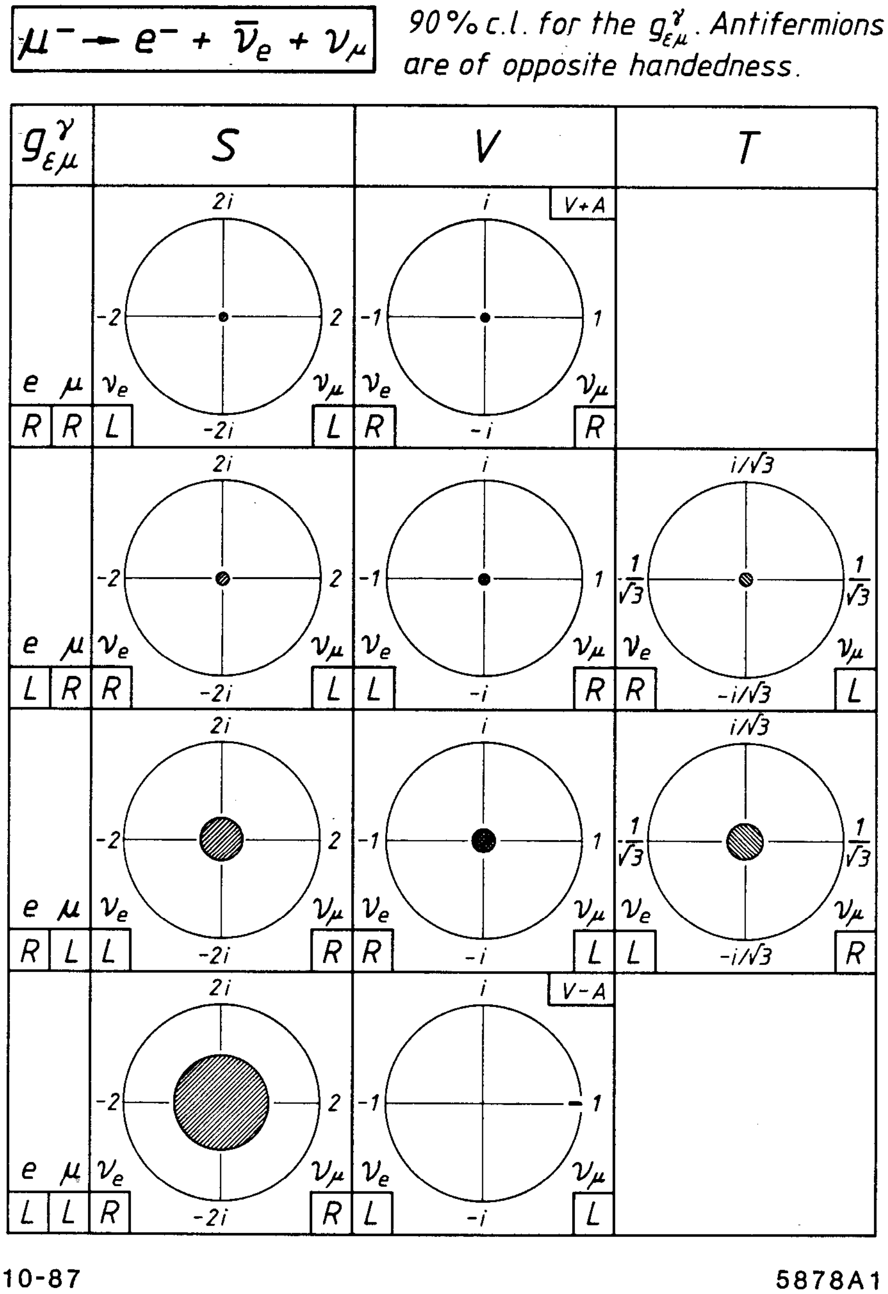

Fig. 1. Experimental information on the ten independant complex couplings possible in μ decay. The shaded areas denote the allowed regions (courtesy of W. Fetscher).

II. HEAVY FLAVOR DECAYS

1. Top

Of the six quark flavors in the Standard Model, five — u, d, s, c, b — have been observed so far. We can feel quite assured that the sixth flavor — top — exists as well. This confidence is based mainly on two pieces of indirect evidence combining observations with theoretical reasoning (a combination which I personally do not find objectionable):

- Banishing the top quark from the Standard Model represents a highly non-trivial undertaking: its removal creates some awkward problems with the Adler–Bell–Jackiw anomaly since b and τ are already known to exist — unless one changes the b quark couplings in a judicious fashion. This would affect beauty decays in a rather dramatic fashion [9], e.g., the flavor changing neutral current transition $B \to \ell^+\ell^- X$ would be a major decay mode. Such effects have not been observed [10].
- The forward-backward asymmetry of beauty jets in e^+e^- annihilation allows to determine g_A^b, the axial coupling of beauty quarks. In the Standard Model g_A is given by the third component of weak isospin of the quark in question. Present data yield $g_A^b \simeq -0.5 \pm 0.1$ and thus $I_3(b) = -\frac{1}{2}$; therefore there has to be an isodoublet partner of b, otherwise weak isospin invariance would be broken in an unacceptable fashion; this partner can be neither c nor u. Top is then a good name for it.

While the existence of top is thus hardly in doubt, the value of its mass is. PETRA data tell us that $m_t \gtrsim 23$ GeV while very recent TRISTAN data extend this to

$$m_t \gtrsim 26 \text{ GeV} \quad . \tag{22}$$

Another very recent analysis, this time by UA1, extends it even further, namely to $m_t \gtrsim 44$ GeV [11]. A quite comprehensive analysis of neutral current data (see W. Marciano's lectures) puts limits on the allowed amount of SU(2) breaking leading to

$$m_t \sim m_t - m_b \gtrsim 180 - 200 \text{ GeV} \quad . \tag{23}$$

In the range 26 GeV $\leq m_t \lesssim 200$ GeV there are actually two regions (plus a transition region) that should be treated separately; there is the case of "ultraheavy" top quarks when $m_t > M_W + m_q$ thus allowing $t \to q + W$ to proceed as an *on-shell* process; and then there is the case of merely "heavy" top with $m_t < M_W$ involving *virtual* W bosons only, *i.e.*, $t \to b``W" \to b q_1 \bar{q}_2$.

(i) Ultraheavy top:

A straightforward calculation yields for generic quarks Q, q [12]:

$$\Gamma(Q \to qW) = \frac{G_F m_Q^3}{8\pi\sqrt{2}} |V(Qq)|^2 \frac{2k}{m_Q}$$
$$\times \left\{ \left[1 - \left(\frac{m_q}{m_Q}\right)^2\right]^2 + \left[1 + \left(\frac{m_q}{m_Q}\right)^2\right] \times \left(\frac{m_W}{m_Q}\right)^2 - 2\left(\frac{m_W}{m_Q}\right)^4 \right\} \tag{24}$$

where

$$k = \sqrt{m_Q^2 - (m_W + m_q)^2}\sqrt{m_Q^2 - (m_W - m_q)^2} \Big/ 2m_Q$$

denotes the W momentum in the Q rest frame. This width is plotted in Fig. 2 as a function of m_Q with $V(Qq) = 1$, $m_q = m_b = 5$ GeV as appropriate for top. [For comparison, it shows also the curves for $\Gamma(Q \to qH^{ch})$ where H^{ch} denotes a charged (physical) Higgs.] One reads off that the width quickly reaches its asymptotic form

$$\Gamma(Q \to qW) \simeq 180 \text{ MeV } \times |V(Qq)|^2 \left(\frac{m_Q}{m_W}\right)^3 \quad . \tag{25}$$

The step dependence on m_Q is easily understood: one power of m_Q is due to phase space; the other two powers of m_Q are produced by the emission of *longitudinal* W bosons (the reincarnation of the unphysical Higgs fields).

Just as a note in passing: since

$$\frac{\Gamma(Q \to qW)}{m_Q} \simeq 2.2 \times 10^{-3} \left(\frac{m_Q}{m_W}\right)^2 \tag{26}$$

one has encountered another reason why the usual concepts of quarks cease to be meaningful for quark masses exceeding the 1 TeV scale.

Of more direct relevance for our discussion is the observation that Eq. (24) translates into

$$\tau(Q \to qW) \sim \mathcal{O}(10^{-23} \text{ sec}) \frac{1}{|V(Q_q)|^2} \left(\frac{m_W}{m_Q}\right)^3$$

For ultraheavy top quarks where $|V(tb)| \simeq 1$ is expected to hold one then finds

$$\tau_t < \tau \text{ (hadronization) } \sim \mathcal{O}(10^{-22} \text{ sec}) \tag{27}$$

i.e., such quarks decay *before* they can hadronize by picking up (light) antiquarks. Ultraheavy top quarks therefore behave very much like heavy leptons:

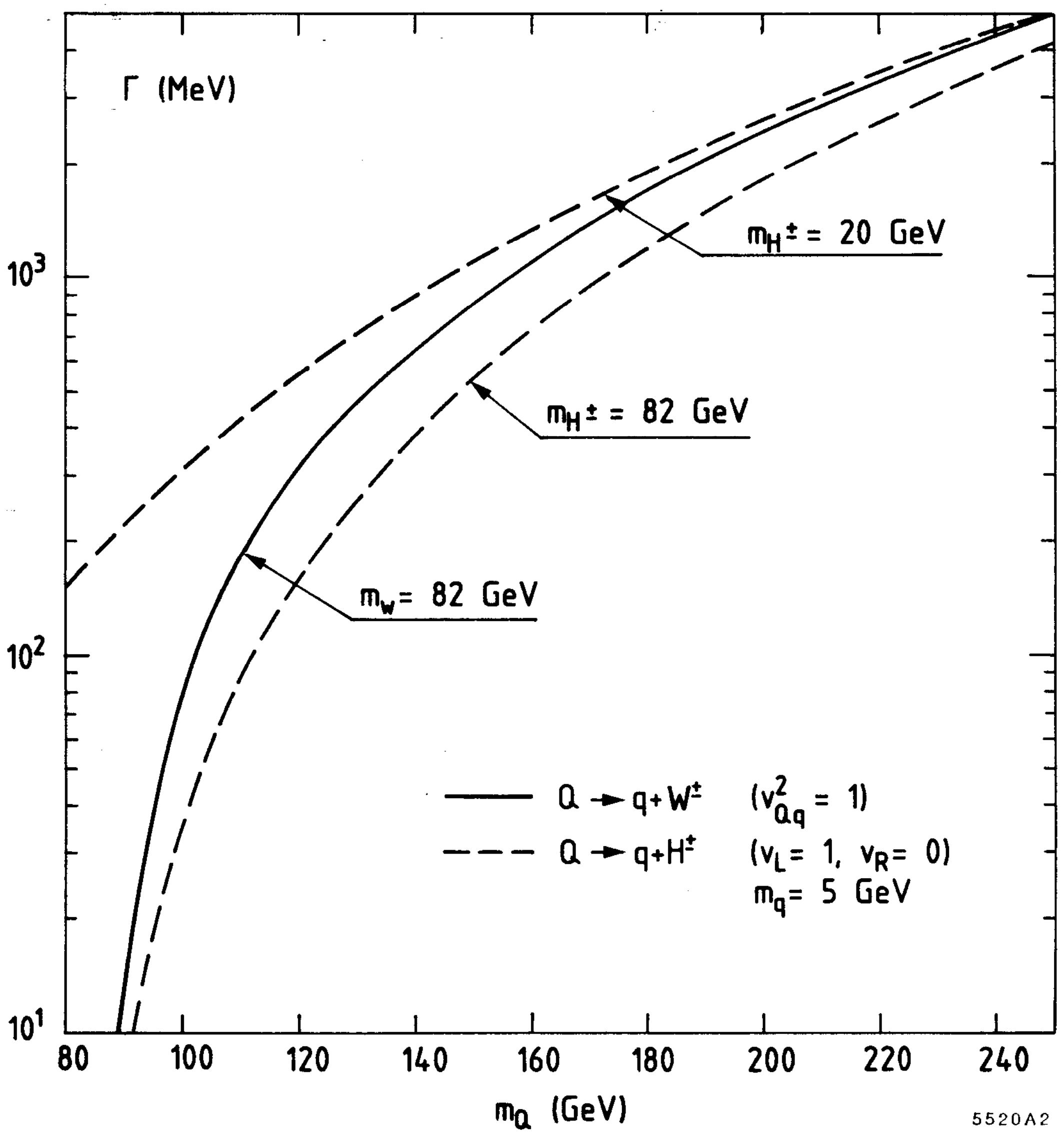

Fig. 2. The total width for $Q \to q^{W^\pm}, q^{H^\pm}$ as a function of m_Q.

(α) They have a very hard fragmentation function with a radiative tail due to gluon bremsstrahlung[12] . The only difference to the lepton case is of a quantitative nature: $\alpha_s > \alpha$.

(β) The polarization of the ultraheavy top quark can be measured by determining the polarization of W bosons in $t \to bW$. This is in marked contrast to the first five flavors: the hadronization that occurs there washes out any effect since the weakly decaying hadron is most of the time a pseudoscalar meson without any memory of the original quark polarization (only in those cases where hadronization leads to a baryon has one some handle on the quark polarization).

(γ) Quarkonia states cannot form as can be seen from two complementary arguments: common to both is the observation that for sufficiently heavy quarks and sufficiently large $|V(Qq)|$ — conditions that are clearly satisfied for ultra-heavy top quarks — quarkonia decays are dominated by single quark decays (= SQD) of the "quasifree" heavy quarks. Thus

$$\Gamma_{t\bar{t}} \simeq 2\Gamma_t \geq 800 \text{ MeV} \tag{28}$$

for $m_t \geq 120$ GeV.

- The inter-quark potential is highly Coulombic for the mass range under study here. The revolution time of the $Q\bar{Q}$ bound state is then given by

$$\tau_{\text{rev}} \simeq \frac{9}{4\alpha_s^2 m_Q} \quad . \tag{29}$$

  Quarkonia cannot form if their lifetime is shorter than their revolution time. Since

$$\tau_{\text{rev}} \simeq \frac{9}{4\alpha_s^2 m_t} > \tau_{t\bar{t}} \simeq \frac{1}{2}\tau_t \simeq \frac{1}{360 \text{ MeV}} \left(\frac{m_W}{m_t}\right)^3 \tag{30}$$

  for $m_t \gtrsim 120$ GeV, toponium formation ceases to be meaningful for such heavy masses.

- Alternatively, one can point out that when the uncertainty in the quark mass becomes larger than the $(Q\bar{Q})$ level spacing, estimated to be around 800 MeV in potential models, the nonperturbative binding forces become ineffective. Any resonance structure in $Q\bar{Q}$ production near threshold is washed out, and the properties of the process $e^+e^- \to Q\bar{Q}$ follow literally the predictions of the free quark model, modified only by perturbative QCD corrections. Nature thus performs the duality integration on her own.

Considering all these features of "quasifree" quarks I find it hard to imagine that Nature would deny us such a nice laboratory.

(ii) (Merely) Heavy Top:

If $m_t < M_W$ then top decays conventionally, *i.e.*, via virtual W emission: $t \to bq_1\bar{q}_2$. The partial width $t \to be\nu$, for instance, via virtual (or real) W emission can be written as[12]

$$\Gamma\left(t \to b + W(\to e\nu)\right) = \frac{G_F^2 m_t^5}{192\pi^3} |V(tb)|^2 \times f\left(\frac{m_t^2}{m_W^2}, \frac{m_b^2}{m_t^2}, \frac{\Gamma_W^2}{m_W^2}\right)$$

$$f(\rho,\mu,\gamma) = 2 \int_0^{(1-\sqrt{\mu})^2} \frac{dx}{(1-x\rho)^2+\gamma} \tag{31}$$

$$\times \left[(1-\mu)^2 + (1+\mu)\,x - 2x^2\right]\sqrt{1+\mu^2+x^2-2(\mu+\mu x+x)} \quad .$$

Here we have included finite width effects. This function is shown in Fig. 3 which exhibits two interesting, though obviously not surprising features:

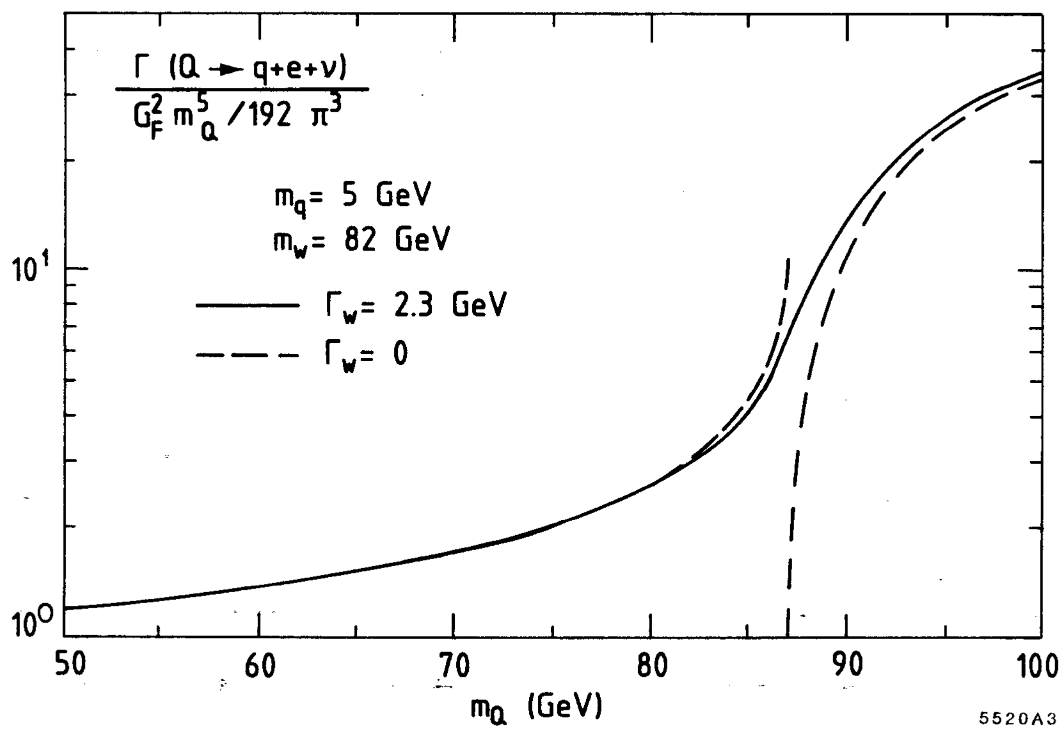

Fig. 3. The semileptonic width in the transition region around M_W

- The width for $m_t < M_W$ is much smaller than for $m_t > M_W$ since the latter involves on-shell W emission and involves two-body instead of three-body phase space.

- The finite width of the W boson, $\Gamma_W > 0$, leads to a smooth connection between the two regimes $m_t \lesssim M_W$ and $m_t \gtrsim M_W$.

(α) The lifetime of top quarks depends very steeply on m_t, yet it is still longer than the hadronization time $\sim \mathcal{O}(10^{-23}$ sec). Thus top hadronizes after production and then decays as a top hadron, *i.e.*, meson or baryon. In principle, this raises the question whether there is a universal top lifetime or not, *i.e.*, whether $\tau(T_u) = \tau(T_d)$, *etc.*, or not where $T_q = (t\bar{q})$. Without going into details — the interested reader can satisfy his/her curiosity by looking up Ref. 13 — let me just state that on rather general grounds one expects the lifetimes of the weakly decaying top hadrons to be equal to a very high accuracy.

(β) The polarization of the original top quark can still be measured to some degree. For also the vector mesons T^* — the top analogue to ρ, K^*, D^* — decay weakly most of the time [14]: the mass splitting between the vector and pseudoscalar mesons is attributed to spin-spin forces of the constituents and therefore

$$M(T_q^*) - M(T_q) \sim \frac{m_c}{m_t}\frac{m_u}{m_q}\left[M(D^{0*}) - M(D^0)\right] < 10\,\mathrm{MeV} \quad . \tag{32}$$

Thus no strong decays $T^* \to T\pi$ are kinematically allowed. T^* can still decay electromagnetically into $T + \gamma$ via an M1 transition; yet its width depends strongly on the available phase space:

$$\Gamma_{M1}\left(T_q^* \to T_q + \gamma\right) \sim \frac{1}{3}\alpha k^3 \left(\frac{e_q}{m_q} + \frac{e_t}{m_t}\right)\delta_{rr'} \tag{33}$$

where e_q, e_t stand for the quark charges, k for the photon three momentum and $\delta_{rr'}$ for the spatial overlap of the vector and pseudoscalar wave functions. Putting everything together we conclude: since $\Gamma_{M1} \propto 1/m_t^3$ whereas $\Gamma(T^{(*)} \to bq_1\bar{q}_2)$ rises steeply with m_t, the weak process will dominate also T^* decays for sufficiently heavy top. More detailed calculations show this to happen for $m_t > 17$ GeV. In $T^* \to e\nu X$ decays one can measure the T^* polarization by looking for a correlation $\vec{p}_e \cdot \vec{s}(T^*)$. The degree to which the T^* polarization reflects the original t polarization depends very much on the relative weight of T and T^* production. Following the statistical prescription of just counting spin degrees of freedom, *i.e.*, $N(T^*) : N(T) = 3 : 1$, one finds a *dilution* factor of two in relating the measurable T^* polarization to the t quark polarization:

$$\mathrm{Pol}(T^*) \simeq \frac{1}{2}\,\mathrm{Pol}(t) \quad . \tag{34}$$

(γ) Toponium forms thus allowing a rather precise determination of m_t. If $m(t\bar{t}) \simeq m(Z^0)$, which is not completely ruled out by present data, a very intriguing interference pattern would emerge [5] . Independent of that, SQD represents a sizeable fraction of all (ortho-) toponium decays. This has an important consequence: by measuring the total toponium width in e^+e^- annihilation and the branching ratio for SQD one can extract $\Gamma(t \to bq_1\bar{q}_2)$ and thus $|V(tb)|$ — the only way to measure the short lifetime expected for top.

2. Beauty

Beauty quarks have been observed and studied for a few years now. They clearly hadronize before they decay. The masses of charged and neutral B mesons — $B_u^* = (\bar{b}u)^+$, $B_d = (\bar{b}d)$ — have been determined and the average lifetime of B_u, B_d, B_s mesons and the beauty baryons measured:

$$\langle\tau_b\rangle = (1.18 \pm 0.14)10^{-12} \quad sec \tag{35}$$

I — like most, but not all, authors — believe that the lifetimes of the various beauty hadrons will not vary by more than, say, 20 percent.

$$1 \le \frac{\tau(B^+)}{\tau(B^0)} \lesssim 1.2 \qquad \text{theor.} \tag{36}$$

Yet the data have not reached such a sensitivity level[10]

$$0.5 \lesssim \frac{\tau(B^+)}{\tau(B^0)} \lesssim 2 \qquad \text{exp.} \tag{37}$$

This has to improve.

– Beauty decays represent an immensely exciting field of study:

- One is analyzing the effects of strong interactions on the interface between the perturbative and nonperturbative regime.
- Since beauty quarks open up a new family, their KM parameters $V(cb), V(ub)$ are *a priori* unknown, but crucial parameters to be extracted from the data.
- $B^0 - \bar{B}^0$ mixing and CP violation is expected to be sizeable, if not even large — subjects that will be discussed in a detailed manner in the next two lectures.

Details on the first two points can be found in the literature [15]. Suffice it to say here that the impact of the strong forces is not yet under sufficient numerical control; therefore one has to take the values quoted for $V(cb), V(ub)$ with quite a grain of salt.

3. Charm

Charm was actually the first internal hadronic quantum number whose existence and approximate mass scale was predicted theoretically — though not universally believed — *before* it was observed.

Hadronization effects are clearly, and not surprisingly, crucial for understanding charm decays as exemplified by the large differences in lifetimes, e.g.,

$$\frac{\tau(D^+)}{\tau(D^0)} \sim 2.5 \quad . \tag{38}$$

Nevertheless I feel entitled to claim that we have developed a decent semiquantitative description of D decays where we have not encountered yet an acute theoretical embarrassment like the $\Delta I = 1/2$ rule in K decays.

Readers whose curiosity has been aroused, but not satisfied by these brief remarks can consult the rather extensive literature on this subject.[16]

Interlude: Summary of the First Two Chapters

(i) τ physics represents a fairly mature field

- since the theoretical technology available to us for treating τ transitions is rather well developed and understood; and
- almost all decay modes have been identified.

At the same time puzzling features emerge in the data — the (possibly) missing one-prong decays. Therefore, there exists a strong motivation for future experiments dedicated to τ studies ; hopefully they will be able to perform a general dynamical analysis as has been done for muon decays.

(ii) The decays of ultraheavy top — $t \to bW$ — are expected to be very similar to the decays of ultraheavy leptons: they decay as fermions instead of mesons thus exhibiting spin effects like leptons; toponia cannot form. The only difference is due to the coupling α_s of the gluons that are emitted perturbatively.

(iii) The decays of heavy top — $t \to bq_1\bar{q}_2$ — are still similar to heavy lepton decays, yet with some residual effects due to the confining forces: toponia form, yet SQD become more and more important; top hadronizes into mesons (and baryons) before it decays, yet spin effects can still be traced since also the vector mesons T^* decay weakly.

(iv) Hadronization effects become more important for beauty decays and even more so for charm decays, yet we seem to have developed a decent overall semiquantitative description. Thus we can attempt to extract KM parameters from the data with some (though not overwhelming) degree of confidence.

III. FLAVOR MIXING

A. $K^0 - \bar{K}^0$ Mixing

(1) Qualitative Introduction

Let us imagine a world where only protons, neutrons, pions, electrons, neutrinos and photons are known. After all, such a world does not appear so different from our real world. Physicists will quite naturally come up with the idea to collide protons with pions. Doing that they will observe the production of a neutral baryon at a rate P and its subsequent decay back to a proton plus a pion at a rate D:

$$\begin{array}{l} \pi p \xrightarrow[\text{rate } P]{} \Lambda^0 + X \\ \qquad\qquad \llcorner\!\!\xrightarrow[\text{rate } D]{} p\,\pi^- \quad . \end{array} \tag{39}$$

The most remarkable feature of this observation lies in the gross disparity between the decay and the production rate of this baryon state named the Λ hyperon:

$$\frac{D}{P} \sim 10^{-13} \quad . \tag{40}$$

Such a tiny ratio is very strange and highly unnatural — unless one postulates a new internal symmetry. This new symmetry is aptly called *"strangeness"* S; it is conserved by the strong forces responsible for the production ($\Delta S = 0$) while being violated by the weak forces driving the decay ($\Delta S = 1$). The Λ^0 baryon must then be produced in conjunction with another strange hadron, the K meson as in

$$\pi^- p \;\rightarrow\; \Lambda^0 K^+ \pi^-$$

Strangeness +1 is assigned to K^+ and -1 to Λ and K^-, the antiparticle of K^+. Neutral kaons are observed as well

$$\pi^- p \;\rightarrow\; \Lambda^0 K^{\text{neut}}$$

suggesting strangeness +1 for them.

Smart physicists everywhere are aware of the great virtues of e^+e^- annihilation; therefore they will turn to this process of choice to study K production. From the threshold behavior of

$$e^+e^- \rightarrow K^+K^-$$

they infer that K are spin-zero mesons. (*En passant* they also observe that K^+ decays both to $\pi^+\pi^0$ and to $\pi^+\pi^-\pi^+$ and conclude that parity is violated in these decays — the old $\theta - \tau$ puzzle.)

Next they observe the production of neutral kaons:

$$e^+e^- \rightarrow K^{\text{neut}}K^{\text{neut}} \quad . \tag{41}$$

Since the neutral K mesons have to be in a *p-wave* configuration (for a one-photon intermediate state), this reaction can occur only if there are two *distinct* neutral kaons. For otherwise the wave function describing the final state would necessarily be antisymmetric — in violation of Bose statistics. Thus there had to be four meson states with nontrivial strangeness: K^+ and K^0 with strangeness $+1$, K^- and $\bar{K}^0$ with -1.

This allows reaction (41) to proceed. Yet on closer scrutiny one discovers a puzzling feature: the two neutral kaons exhibit different decay modes with very different lifetimes:

$$\begin{aligned} \tau(K^{\text{neut}} &\rightarrow 2\pi) \sim 0.9 \times 10^{-10} \text{ sec} , \\ \tau(K^{\text{neut}} &\rightarrow 3\pi) \sim 5 \times 10^{-8} \text{ sec} . \end{aligned} \tag{42}$$

CPT invariance then tells us that the two states that have a definite *lifetime*, *i.e.*, are *mass* eigenstates, cannot be K^0 and $\bar{K}^0$, *i.e.*, states of definite strangeness. Instead the mass eigenstates K_1, K_2 are linear superpositions of K^0 and $\bar{K}^0$.

$$\begin{aligned} |K_1\rangle &= \cos\alpha \, |K^0\rangle + \sin\alpha \, |\bar{K}^0\rangle \\ |K_2\rangle &= -\sin\alpha \, |K^0\rangle + \cos\alpha \, |\bar{K}^0\rangle \quad . \end{aligned} \tag{43}$$

This in a nutshell characterizes the phenomenon of mixing: *the mass eigenstates are not equal to the flavor eigenstates.* It describes a rather common scenario in quantum mechanics: a certain quantum number leads to an energy degeneracy of two states; a (small) violation of this quantum number then lifts this degeneracy. In the case under study here this leads to $m(K_1) \neq m(K_2)$, $\Gamma(K_1) \neq \Gamma(K_2)$.

If one assumes CP invariance — as I will do throughout this chapter — one can say much more on very general grounds. For the mass eigenstates then have to be CP eigenstates as well, *i.e.*,

$$
\begin{aligned}
|K_+\rangle &= \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \\
|K_-\rangle &= \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)
\end{aligned}
\tag{44}
$$

where K_+ $[K_-]$ denotes the even [odd] CP eigenstate. We have used here the *phase convention*

$$CP\,|K^0\rangle = -|\bar{K}^0\rangle \; . \tag{45}$$

The relevance of this last statement will become clearer later on. Furthermore

$$
\begin{aligned}
K_+ &\rightarrow 2\pi \not\leftarrow K_- \\
K_- &\rightarrow 3\pi \not\leftarrow K_+
\end{aligned}
\tag{46}
$$

Since $m(K^{\mathrm{neut}})$ is barely above the three pion threshold — 498 MeV vs. 405 MeV —, the decay $K_- \rightarrow 3\pi$ is highly suppressed by phase space relative to $K_+ \rightarrow 2\pi$, which leads to the gross disparity in lifetimes, Eq. (42). This kinematical accident, namely $m(K) \gtrsim 3m(\pi)$, facilitated the observation of $K^0 - \bar{K}^0$ mixing greatly — yet we cannot expect another such stupendous present from Nature in the $B^0 - \bar{B}^0$ system, *etc.* In any case, $K_+ = K_S$, $K_- = K_L$ for now.

(2) Formal Description of Mixing

Combining K^0 and $\bar{K}^0$ (or B^0 and $\bar{B}^0$ *etc.*,) into one vector Ψ

$$\Psi = \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}$$

one writes down the free Schrödinger equation

$$i\,\frac{d}{dt}\,\Psi = H\Psi \quad , \tag{47}$$

where H denotes the generalized mass matrix

$$H = \underline{M} - \frac{i}{2}\underline{\Gamma} \quad .$$

$\underline{M}$ and $\underline{\Gamma}$ are Hermitian matrices

$$H = \begin{pmatrix} M - \frac{i}{2}\,\Gamma & M_{12} - \frac{i}{2}\,\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\,\Gamma_{12}^* & M - \frac{i}{2}\,\Gamma \end{pmatrix} \quad . \tag{48}$$

The equality of the two diagonal elements follows from CPT invariance. The off-diagonal matrix elements depend on $\Delta S = 2$ transitions.

To find the states with definite mass and width one has to diagonalize H. The result is

$$
\begin{aligned}
|K_{L,S}\rangle &= \frac{1}{\sqrt{1+|\alpha|^2}} \left(|K^0\rangle \pm \alpha\, |\bar{K}^0\rangle\right) \\
\alpha &= \sqrt{\frac{M_{12}^* - \frac{i}{2}\,\Gamma_{12}^*}{M_{12} - \frac{i}{2}\,\Gamma_{12}}} \ .
\end{aligned} \tag{49}
$$

$$
\begin{aligned}
\Delta m &= 2\mathrm{Re}\,\sqrt{\left(M_{12} - \frac{i}{2}\,\Gamma_{12}\right)\,\left(M_{12}^* - \frac{i}{2}\,\Gamma_{12}^*\right)} = m(K_L) - m(K_S) \\
\Delta\Gamma &= 4\mathrm{Im}\,\sqrt{\left(M_{12} - \frac{i}{2}\,\Gamma_{12}\right)\,\left(M_{12}^* - \frac{i}{2}\,\Gamma_{12}^*\right)} = \Gamma(K_L) - \Gamma(K_S) \ .
\end{aligned} \tag{50}
$$

If CP invariance is assumed, then M_{12} and Γ_{12} are real and thus

$$|\alpha| = 1 \ , \tag{51}$$

$$\Delta m = 2|M_{12}| \ , \tag{52}$$

$$\Delta\Gamma = -2|\Gamma_{12}| \ . \tag{53}$$

The minus sign in Eq. (53) is fixed by our definition of which of the two mass eigenstates is the longer lived K_L. Eq. (52) then reflects the *empirical* findings that K_L has the larger mass. This by the way shows that K_L and K_S can be claimed by the anglophone world as its rightful property: I have found no other language where the subscripts L and S carry the total fourfold information — L [S] longer/larger [shorter/smaller] for the lifetime/mass.

One observation on α might seem academic, yet will turn out to be highly relevant later on in discussions of CP violation: $|\alpha|$ represents a physical parameter, whereas α by itself does not. For example, changing the phase *convention* of $\bar{K}^0$ adopted in Eq. (3.7) by introducing a new phase ξ

$$|\bar{K}^0\rangle \to e^{i\xi}\, |\bar{K}^0\rangle \ , \tag{54}$$

leads to

$$
\begin{aligned}
M_{12} &\to e^{i\xi}\, M_{12} \\
\Gamma_{12} &\to e^{i\xi}\, \Gamma_{12} \ .
\end{aligned} \tag{55}
$$

and therefore

$$\alpha \to e^{-i\xi}\,\alpha \ . \tag{56}$$

This shows that α depends on the unphysical and thus arbitrary phase — but $|\alpha|$ does not!

In summary, there are two complementary ways to describe neutral kaons:

- One employs the mass eigenstates $K_{L,S}$ which do not possess definite strangeness

$$
\begin{aligned}
|K_S(t)\rangle &= e^{-(1/2)\Gamma_S t}\, e^{im_S t}\, |K_S\rangle_0 \\
|K_L(t)\rangle &= e^{-(1/2)\Gamma_L t}\, e^{im_L t}\, |K_L\rangle_0 \ ,
\end{aligned}
\tag{57}
$$

  where $|K_{S,L}\rangle_0 = \frac{1}{\sqrt{2}}(|K^0\rangle_0 \mp |\bar{K}^0\rangle_0)$ in the phase convention (45).

- One uses the flavor eigenstates, whose time evolution is more complex:

$$
\begin{aligned}
|\overset{(-)}{K^0}(t)\rangle = \frac{1}{2}\, e^{-(1/2)\Gamma_S t}\, e^{im_S t} \Bigg\{ &\left(1 \underset{(-)}{+} e^{-(1/2)\Delta\Gamma t}\, e^{i\Delta m t}\, |K^0\rangle_0\right) \\
+ &\left(1 \underset{(+)}{-} e^{(1/2)\Delta\Gamma t}\, e^{i\Delta m t}\, |\bar{K}^0\rangle_0\right)\Bigg\}
\end{aligned}
\tag{58}
$$

  The two bases $\{K_S, K_L\}$ and $\{K^0, \bar{K}^0\}$ are completely equivalent; it is purely a matter of convenience which one to use in a specific problem.

(3) Theoretical Estimates on Δm_K

There are actually two basic parameters as read off from Eq. (58), Δm and $\Delta\Gamma$. Experimentally

$$
\frac{\Delta\Gamma}{\Gamma_S + \Gamma_L} = \frac{\Gamma_S - \Gamma_L}{\Gamma_S + \Gamma_L} \simeq 1 \ , \tag{59}
$$

$$
\frac{\Delta m_K}{m_K} \simeq 7 \times 10^{-15} \ . \tag{60}
$$

$\Delta\Gamma$ does not attract much theoretical interest since it is — as discussed before — dominated by the phase space suppression in $K_L \to 3\pi$. Δm_K is therefore at the center of theoretical scrutiny. Expressing it relative to m_K is intended mainly to impress or better still awe fellow scientists from other fields — like solid staters, molecular biologists, *etc.*, — who seem to doubt our professional seriousness. After all, some of us have not yet made up our mind on the number of space-time dimensions. Equation (60) is somewhat short on substance since there is no intrinsic connection between Δm_K and m_K.

It is much more meaningful to compare Δm, the mixing rate, with Γ, the decay rate of neutral mesons:

$$\frac{\Delta m_K}{\Gamma_S + \Gamma_L} \simeq \frac{\Delta m_K}{\Gamma_S} \simeq 0.4773 \pm 0.0023 \tag{61}$$

For mixing hardly matters if the mesons decay before they can turn into their antiparticle.

To calculate Δm_K one proceeds in two steps: first one derives the $\Delta S = 2$ transition operator and then one computes its matrix element.

(i) Since there are no direct $\Delta S = 2$ transitions in the Standard Model, one has to iterate $\Delta S = 1$ transitions. Since the celebrated paper of Gaillard and Lee [17] this is done by considering the quark box diagram, Fig. 4. Ignoring the top quark contributions — which is a safe procedure for Δm_K (though not for ε_K, as discussed later) — one finds:

$$\mathcal{L}_{\text{eff}}(\Delta S = 2) = \frac{1}{\Lambda^2}\,(\bar{s}_L\,\gamma_\mu\,d_L)(\bar{s}_L\,\gamma_\mu\,d_L) + \text{h.c.}\,, \tag{62}$$

$$\frac{1}{\Lambda^2} = \left(\frac{G_F}{4\pi}\right)^2 (m_c^2 - m_u^2)\sin^2\theta_c\cos^2\theta_c\,. \tag{63}$$

The coefficient $1/\Lambda^2$ obviously vanishes for $m_c^2 = m_u^2$, a direct consequence of the GIM ansatz; m_u^2 thus acts as a subtraction point which otherwise has little numerical weight in Eq. (63). The expression (62) can then be interpreted as follows: $\mathcal{L}_{\text{eff}}(\Delta S = 2)$ is obtained as a low-energy effective coupling by integrating out the heavy fields W and c while retaining the light fields d and s. Since the weak bosons couple only to left-handed fields, one retains purely left-handed fields in $\mathcal{L}(\Delta S = 2)$.

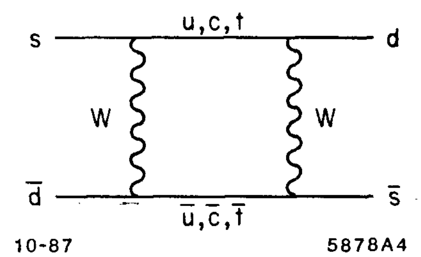

Fig. 4. The quark box diagram for $\Delta S = 2$ transitions.

Homework Problem 1

The coefficient $1/\Lambda^2$ which results from integrating out the internal loop does not contain any factor like $\log(M_W^2/m_c^2)$. What is the structural reason for this curious absence of a logarithmic cut-off?

So far we have completely ignored strong interactions; yet they enter already at this point via (hard) gluon corrections. However, their impact is completely overshadowed by other manifestations of the strong forces which will be addressed next; therefore I will ignore them here but re-address them when discussing $B^0-\bar{B}^0$ mixing.

(ii) When attempting to determine $\langle K^0|\,(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}\,|\bar{K}^0\rangle$ one has to face reality, however unpleasant that might be: calculating an on-shell matrix element is clearly well outside the realm of perturbative QCD and for any kind of estimate one has to rely on some kind of model describing hadronic wave functions. The following parametrization has become customary [18]:

$$\langle K^0|\,(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}\,|\bar{K}^0\rangle \equiv \frac{4}{3}\, B_K\, f_K^2\, m_K^2\,. \tag{64}$$

f_K is the kaon decay constant defined by

$$\langle 0|\,(\bar{s}d)_{V-A}\,|\bar{K}^0(p)\rangle \equiv i f_K\, p_\mu \tag{65}$$

and measured in $K^- \to \ell^- \nu_\ell$ (using the isospin invariance of the strong forces):

$$f_K \simeq 170 MeV \tag{66}$$

All our ignorance has been poured into the one factor B_K; a model is needed to compute it. There is actually no lack of volunteers willing to step forward and heed the summons — in particular since Turandot's razor (" you fail, you lose your head") is not brandished before potential candidates. Here is a short, yet typical list:

$$B_K \sim \begin{cases} 0.37 & \text{Chiral perturbation theory}^{[19]} \\ -0.4 & \text{MIT bag model}^{[20]} \\ 0.75 & 1/N_c{}^{[21]} \\ 0.84 & \text{QCD sum rules}^{[22]} \\ \equiv 1 & \text{Potential models}^{[23]} \\ \equiv 1 & \text{Vacuum saturation}^{[17]} \end{cases} \quad . \tag{67}$$

A few comments are in order here:

- The last line in Eq. (67) contains the definition of what is meant by vacuum saturation. Inserting the vacuum state $|0\rangle\langle 0|$ into $\langle K^0|(\bar{s}d)(\bar{s}d)|\bar{K}^0\rangle$ leads to products of matrix elements $\langle K^0|(\bar{s}d)|0\rangle\langle 0|(\bar{s}d)|\bar{K}^0\rangle \propto f_K^2 m_K^2$. There are actually two types of insertions, namely when the physical kaons are produced via a color singlet $(\bar{s}d)$ current and when $(\bar{s}d)$ is not automatically in the color singlet configuration. In the former case the color weight is 1; in the latter, $1/N_c$; thus

$$\langle K^0|(\bar{s}d)(\bar{s}d)|\bar{K}^0\rangle = \frac{4}{3}\, B_K\, f_K^2\, m_K^2 = \left(1+\frac{1}{N_c}\right) f_K^2\, m_K^2 \,, \tag{68}$$

  i.e., $B_K = 1$.
- It is easy to show that $B_K \equiv 1$ in any potential model ansatz [23].
- Equation (68) shows immediately that $B_K = 3/4$ in an approach where all terms nonleading in $1/N_c$ are dropped.
- B_K is indeed of order one (instead of ten or one tenth), yet there is easily a factor of three uncertainty in the size of the matrix element.

Homework Problem 2

Does $B_K(MIT) < 0$ matter, is it in conflict with $\Delta m_K = m(K_L) - m(K_S) > 0$? *Hint*: Look at Eq. (50) or re-read the remarks after Eqs. (51)-(53).

Putting everything together, one arrives at

$$\frac{\Delta m_K|_{\text{box}}}{\Delta m_K|_{\text{exp}}} \sim \frac{1}{3} - 1 \,. \tag{69}$$

(iii) Before passing judgment on Eq. (69), one has to ask whether all theoretical contributions, within the Standard Model, are contained in the box contribution, *i.e.*, $\Delta m_K|_{\text{theor}} = \Delta m_K|_{\text{box}}$? One can make the observation that by adjusting B_K one can always match $\Delta m_K|_{\text{box}}$ with $\Delta m_K|_{\text{exp}}$. As it turns out, $B_K = 1$, which is a reasonable value, is quite sufficient. The correctness of such a statement is not assailable; its profoundness however is, for it overlooks an important dynamical distinction: Contributions from the loop integration in the quark box are damped for momenta above m_c due to the GIM cancellation. The momentum range between m_c and m_K contributes rather uniformly. This means that the range between m_K and, say 1 GeV, is not singled out in a particular way as far as the quark box computation goes. On the other hand, that is the region where resonance effects are especially virulent; yet at the same time, only very few resonances enter here and thus one cannot invoke a duality concept to argue that such effects are properly included as an average in the quark box ansatz. Therefore it makes dynamical sense to write

$$\Delta m_K|_{\text{theor}} = B_K \, \Delta m_K|_{\text{box},B=1} + D \, \Delta m_K|_{\text{exp}} \; . \tag{70}$$

where D reflects the impact of ***purely long range*** dynamics operating between 0.5 and 1 GeV.

Two approaches have been tried to estimate at least the size of D:

(α) The virtual transition $K^0 \to$ "$\pi\pi$" $\to \bar{K}^0$ can be calculated quite reliably by employing a (once subtracted) dispersion relation and using the ***measured*** $\pi\pi$ phase shifts. This procedure yields[24,25]

$$D_{\pi\pi} \simeq 0.46 \pm 0.13 \tag{71}$$

The evaluation is much less reliable when the $K^0 - \bar{K}^0$ transition is mediated by a virtual π, η and η'. There are very large cancellation between the π and η terms, the size of which depends on the true amount of $SU(3)_{FL}$ breaking and on the $\eta - \eta'$ mixing angle. One typically obtains[24]

$$D = D_{\pi\pi} + D_{\pi,\eta,\eta'} \sim 0.2 \pm 0.6 \tag{72}$$

(β) In the $1/N$ approach one zeroes in on $K^0 \to \pi, \eta, \eta' \to \bar{K}^0$ as the dominant source of D and finds (after all, $SU(3)_{FL}$ breaking and the $\eta - \eta'$ mixing angle are at least in principle calculable in this ansatz)[21]

$$D \sim \frac{1}{3} \; . \tag{73}$$

There are clearly differences between these approaches, and their numerical reliability is not above every doubt. Yet even so, I feel entitled to draw the following semiquantitative conclusions

$$\begin{aligned} \Delta m_K|_{\text{box}} &\gtrsim \Delta m_K|_{\text{Long Dist}} \\ \Delta m_K|_{\text{exp}} &\sim \Delta m_K|_{\text{theor}} \end{aligned} \; , \tag{74}$$

and to claim success since

"success" = absence of proven failure

is a more reasonable definition in this situation than it might appear at first.

Lattice calculations will (hopefully) settle these questions once and for all some day.

B. General Expectations on Mixing

Within the Standard Model one can give rather reliable estimates on the pattern expected for $\Delta m/\Gamma$ when comparing mesons containing various up- and down-type quarks

$$\begin{pmatrix} H \\ L \end{pmatrix} = \begin{pmatrix} c \\ s \end{pmatrix} , \begin{pmatrix} t \\ b \end{pmatrix} , \begin{pmatrix} t' \\ b' \end{pmatrix} . \tag{75}$$

Without loss of generality, let us assume

$$m_H > m_L ; \tag{76}$$

then

(i) $\Gamma(H\bar{q}) \gg \Gamma(L\bar{q})$, since

- H decay has more phase space [Eq. (76)] and
- L decay is suppressed by presumably small KM angles, since the transition has to lead to a quark outside the same family — like $b \to c$.

(ii) According to the GIM ansatz, Δm is determined by the mass of the *internal* quark

$$\Delta m_H \propto m_L^2 , \qquad \Delta m_L \propto m_H^2 .$$

Both trends combine to yield

$$\frac{\Delta m}{\Gamma} (L\bar{q}) \gg \frac{\Delta m}{\Gamma} (H\bar{q}) . \tag{77}$$

Thus one expects large or at least sizeable $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing in contrast to small or even tiny $D^0 - \bar{D}^0$ and $T^0 - \bar{T}^0$ mixing. This qualitative statement will be made more specific in the next sections.

C. $B^0 - \bar{B}^0$ Mixing

(1) Phenomenology

As discussed extensively in Chapter III.A, mixing is driven by Δm and $\Delta\Gamma$, which are due to transitions where the flavor quantum numbers change by two units. The time evolution of neutral B mesons becomes then more complex (I follow the now standard treatment a la Pais and Treiman, ref.26):

$$\begin{aligned}
|B^0(t)\rangle &= g_+(t)\ |B^0\rangle_0 + \frac{q}{p}\, g_-(t)\ |\bar{B}^0\rangle_0 \ , \\
|\bar{B}^0(t)\rangle &= \frac{p}{q}\, g_-(t)\ |B^0\rangle_0 + g_+(t)\ |\bar{B}^0\rangle_0 \ , \\
g_\pm(t) &= \frac{1}{2}\exp\left[-\frac{1}{2}\Gamma_1 t\right]\exp\left[im_1 t\right]\left(1 \pm \exp\left[-\frac{1}{2}\Delta\Gamma t\right]\exp\left[i\Delta m t\right]\right) \ , \\
\Delta\Gamma &= \Gamma_2 - \Gamma_1 \ , \qquad \Delta m = m_2 - m_1 \ , \qquad \frac{q}{p} = \frac{1-\epsilon}{1+\epsilon} \ .
\end{aligned} \tag{78}$$

This is the most general expression (compatible with CPT invariance). For the present discussion, I will make two simplifying assumptions:

- I assume CP invariance implying $q/p = 1$ (in an appropriate phase convention).
- I ignore $\Delta\Gamma$. Later we will see that $\Delta\Gamma \leq \frac{1}{10}\Delta m$ is a fairly conservative estimate.

The flavor quantum number of neutral B mesons can — within the Standard Model! — most conveniently be traced by studying semileptonic decays:

$$\begin{aligned}
\langle \ell^- X | \mathcal{L}(\Delta B = 1) | B^0\rangle_0 &= 0 \\
\langle \ell^+ X | \mathcal{L}(\Delta B = 1) | \bar{B}^0\rangle_0 &= 0 \ ,
\end{aligned} \tag{79}$$

where $B^0 = (\bar{b}q)$, $q = d, s$. Using the simplifications stated above one obtains

$$\begin{aligned}
\mathrm{rate}\big(B^0(t) \to \ell^- X\big) &\propto |\langle \ell^- X|\mathcal{L}|B^0(t)\rangle|^2 \\
&\propto |g_-(t)|^2 = \frac{1}{2}e^{-\Gamma t}(1 - \cos\Delta m t)
\end{aligned} \tag{80}$$

$$\mathrm{rate}\big(B^0(t) \to \ell^+ X\big) \propto |g_+(t)|^2 = \frac{1}{2}e^{-\Gamma t}(1 + \cos\Delta m t) \ . \tag{81}$$

It is this deviation from a simple exponential time evolution which is an unambiguous sign of mixing! Present experimental searches cannot resolve any time evolution and are sensitive to *time integrated* quantities only

$$r \equiv \frac{\Gamma(B^0 \to \ell^- X)}{\Gamma(B^0 \to \ell^+ X)} \simeq \frac{x^2}{2 + x^2} \tag{82}$$

$$\chi \equiv \frac{\Gamma(B^0 \to \ell^- X)}{\Gamma(B^0 \to \ell^\pm X)} = \frac{r}{1+r} \tag{83}$$

$$x \equiv \frac{\Delta m}{\Gamma} \quad . \tag{84}$$

It is not just academic to remember that an observed $r \neq 0$ *per se* does not prove the existence of mixing. It primarily establishes a violation of a global $\Delta B = \Delta Q_\ell$ rule. This would then be interpreted as either due to

- mixing or
- a violation of the $\Delta B = \Delta Q_\ell$ rule that is local in time, Eq. (79), *i.e.*, New Physics!

B Mesons are not produced in isolation since $\Delta B = 0$ holds for the strong and electromagnetic forces. Therefore one has to exercise a certain amount of care in interpreting data on, say, direct leptons attributed to semileptonic B decays.

(i) $B\bar{B}$ production well above beauty threshold can be treated in a simple probabilistic way: if the neutral B meson is produced together with a charged B (or a beauty baryon) which cannot mix one deals in effect with a situation where there is only a single state as far as mixing is concerned. When one encounters $B^0\bar{B}^0$ production like in

$$e^+e^- \to B^0\bar{B}^0 + X \to \ell\ell + X'$$

one can conclude directly, without doing an explicit calculation, for the ratio of such like-sign to opposite-sign dileptons

$$\frac{N(\ell^\pm\ell^\pm)}{N(\ell^+\ell^-)} = \frac{2\chi(1-\chi)}{(1-\chi)^2+\chi^2} = \frac{2r}{1+r^2} \quad . \tag{85}$$

(ii) Such a simple probabilistic prescription cannot be followed when one studies a near threshold process like

$$e^+e^- \to \Upsilon(4s) \to B\bar{B} \quad .$$

For the two B mesons now form a quantum mechanical state of definite orbital angular momentum, namely a p wave, which is odd under exchange. The requirement of Bose statistics then tells us that at no time can the original $B^0\bar{B}^0$ system evolve into two identical states $B^0(t)B^0(t)$ or $\bar{B}^0(t)\bar{B}^0(t)$. An equivalent statement is the following

$$e^+e^- \to \Upsilon(4s) \begin{array}{l} \nearrow B_1B_2 \\ \not\searrow B_1B_1, B_2B_2 \end{array}$$

where $B_{1,2}$ are the two mass eigenstates. Yet even so, there is a simple intuitive argument which immediately yields the correct ratio between like-sign and opposite-sign dileptons; it just goes beyond a purely probabilistic description. Let us visualize neutral B mesons as vectors in a plane where a B^0 [$\bar{B}^0$] is denoted by a vector that points perpendicular up [down]. This is exactly the configuration at production time $t = 0$, Fig. 5a. As times goes on, the two vector rotate around the origin; the important point here is that they always remain anti-parallel because of Bose statistics, Fig. 5b. When one of the mesons decays semileptonically, then the quantum coherence is destroyed and one knows immediately the identity of the other meson at that time, Fig. 5c — it is like an Einstein-Rosen-Podolsky scenario. This situation therefore corresponds to single B production as far as mixing is concerned:

$$\frac{N(\ell^\pm \ell^\pm)}{N(\ell^+ \ell^-)} \left[B^0 \bar{B}^0 \text{ in } p \text{ wave}\right] = r \quad . \tag{86}$$

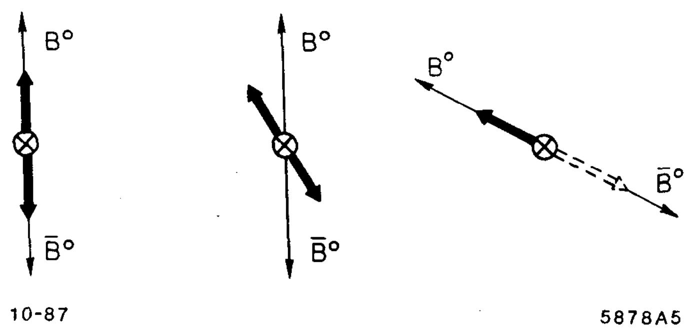

Fig. 5 (a,b,c). Schematic representation of the time evolution of a pair of neutral B mesons produced in a p wave.

(iii) $B^0 \bar{B}^0$ mixing affects also the forward-backward asymmetry of beauty jets in e^+e^- annihilation. This asymmetry is calculated for $e^+e^- \to b\,\bar{b}$; the b quarks then hadronize into beauty jets tracing more or less the direction of flight of the original quarks. The only remaining task then consists in identifying the flavor of the jet — is it B or $\bar{B}$? This can be achieved via semileptonic or any other flavor-specific decays — yet a fundamental problem cannot be circumvented. Any decay can reflect on the flavor of the decaying state only as it was at the time of decay! If mixing occurs, then the flavor at time of decay is not necessarily the flavor at time of production since

$$\bar{b} \;\to\; (\bar{b}d) \;\to\; (b\bar{d})$$

can occur. Thus one necessarily makes an accounting error and the observable forward-backward asymmetry is *smaller* than the one expected on the quark level. For the simple case where only B_d and B_u mesons are considered one finds

$$A_{FB}(B_u, B_d) = \frac{1}{1+r} A_{FB}(b \text{ quarks}) \quad . \tag{87}$$

The general case can be expressed in an analogous fashion:

$$A_{FB}\big(B_u, B_d, B_s, \Lambda_b\big) = \frac{1}{1+\bar{r}} A_{FB}(b \text{ quarks})$$

$$\bar{r} = \frac{2R\big[r_d + \rho_s r_s(1+r_d)/(1+r_s)\big]}{1 + r_d + R\big[\rho_\Lambda(1+r_d) + 1 - r_d + \rho_s(1-r_s)(1+r_d)/(1+r_s)\big]} \tag{88}$$

where

$$r_i = \frac{\Gamma(B_i \to \ell^- X)}{\Gamma(B_i \to \ell^+ X)} \ , \quad B_i = (\bar{b}i) \ , \quad i = d, s$$

$$R = \frac{b_{SL}(B_d)}{b_{SL}(B_u)} \le 1 \quad . \tag{89}$$

$\rho_\Lambda \, [\rho_s]$ denotes the $\Lambda_b \, [B_s]$ abundance relative to the number of B^+ mesons.

(2) Data

The experimental situation is highly intriguing and promising, yet not completely settled — which is hardly surprising considering the complexity involved: there are two neutral B mesons that can mix — B_d and B_s — and the relative abundance — B_u vs. B_d vs. B_s — is *a priori* and also actually quite unknown.

(i) UA1 was the first to report some positive evidence for mixing averaged over B_d and B_s mesons. Their most recent analysis yields[11]

$$\langle \chi \rangle = 0.158 \pm 0.059 \simeq \langle r \rangle = 0.188 \pm 0.07 \tag{90}$$

which is not in clear conflict with the upper bound reported by Mark II [27]

$$\langle \chi \rangle \le 0.12 \qquad (90\% \text{ C.L.}) \tag{91}$$

or by JADE which relies on its measurement of the forward-backward asymmetry

$$\langle \chi \rangle \le 0.13 \qquad (90\% \text{ C.L.}) \quad . \tag{92}$$

One should add that MAC has presented some (marginal) evidence for mixing[28]

$$\langle \chi \rangle = 0.21 \begin{array}{l} +\,0.29 \\ -\,0.15 \end{array} \tag{93}$$

which can help to reconcile signals from $p\bar{p}$ collisions and e^+e^- annihilation.

(ii) ARGUS has very recently created quite a stir by reporting intriguing evidence for rather sizable $B_d - \bar{B}_d$ mixing [29]

$$\chi_d = 0.17 \pm 0.05 \simeq r_d = 0.21 \pm 0.08 \quad . \tag{94}$$

Since $r = x^2/(2 + x^2)$ this implies

$$x_d = \frac{\Delta m}{\Gamma}(B_d) = 0.73^{+0.17}_{-0.19} \tag{95}$$

i.e., a mixing rate quite comparable to the decay rate. CLEO has published an upper bound on $B_d - \bar{B}_d$ mixing which (assuming $N(B^+B^-) : N(B_d\bar{B}_d) \simeq 0.6 : 0.4$) reads as follows[10]

$$r_d \leq 0.24 \qquad (90\%\ \text{C.L.}) \quad . \tag{96}$$

It should be kept in mind that some extra assumptions had to be made to extract (94) from the data.

- ◇ $b_{SL}(B^{\pm}) = b_{SL}(B_d)$
- ◇ $N(B^+B^-) : N(B_d\bar{B}_d) = 0.55 : 0.45$ as suggested by phase space $\left(m(B^{\pm}) < m(B_d)\right)$.

These are certainly reasonable assumptions — yet they are not established facts.

(iii) At present there is no unambiguous way to compare $\langle\chi\rangle$ with χ_d since the relative abundance of the various beauty hadrons is not known. Instead one can draw up different "reasonable" scenarios, for example

(α) scenario 1:

$$\text{Prob}(B_u) : \text{Prob}(B_d) : \text{Prob}(B_s) : \text{Prob}(\Lambda_b) \simeq 0.4 : 0.4 : 0.2 :\sim 0 \tag{97}$$

which leads to Fig. 6a.

(β) scenario 2:

$$\text{Prob}(B_u) : \text{Prob}(B_d) : \text{Prob}(B_s) : \text{Prob}(\Lambda_b) \simeq 0.375 : 0.375 : 0.15 : 0.10 \tag{98}$$

exhibited in Fig. 6b.

In scenario 1 one reads off $r_s \leq 0.6$ (90% C.L.) whereas in scenario 2 even $r_s = 1$ is allowed. A very detailled discussion can be found in ref.30.

Next we will discuss to which degree these mixing numbers are compatible with the Standard Model.

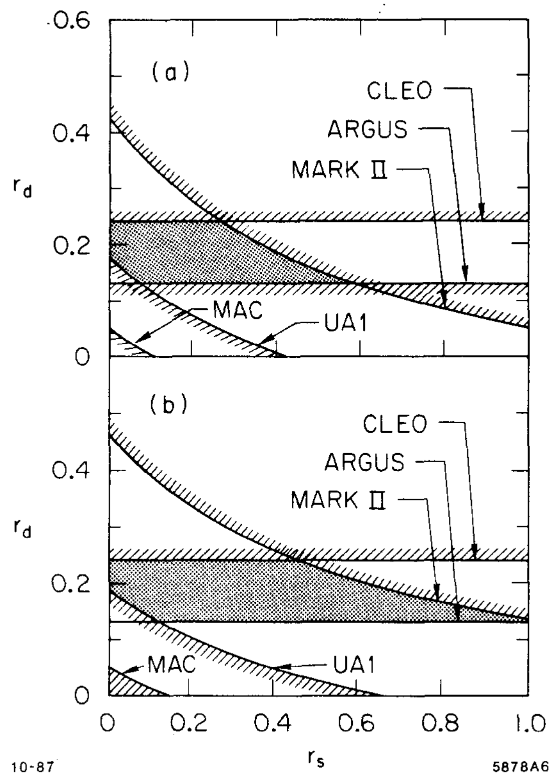

Fig. 6. (a) Experimental (90% C.L.) informtion on r_d, r_s for $N(B)_u) : N(B_d) : N(B_s) : N(\Lambda_b) = 0.4 : 0.4 : 0.2 : 0$. (b) As in (a), but with $N(B)_u) : N(B_d) : N(B_s) : N(\Lambda_b) = 0.375 : 0.375 : 0.15 : 0.1$ (Courtesy of R. Hurst).

(3) Theoretical Interpretation

It is fairly straightforward to convince oneself that within the Standard Model the quark box contribution is by far the most dominant term for Δm_B:

$$\Delta m_B|_{\text{theor}} \simeq \Delta m_B|_{\text{box}} \quad . \tag{99}$$

There are various lines of argument all leading to the same conclusion:

- There are no clear resonances anymore at high mass scales $\sim m_B$. It makes good sense then to invoke the duality argument that the quark description expressed in the box diagram represents an appropriate average over the contributing hadronic channels.
- The dominant mass scale for Δm_B is set by the top mass — $\Delta m_B \propto m_t^2$ to first approximation — which is much larger than the ≤ 1 GeV scale ruling long distance dynamics. Resonance effects will then have only a small impact on Δm_B (it could be somewhat different for $\Delta\Gamma_B$, see later) since its domain $\sim (1 \text{ GeV})^2$ is tiny compared to m_t^2 and small even relative

to m_B^2. The dynamical situation is thus quite different for the $K^0 - \bar{K}^0$ and the $B^0 - \bar{B}^0$ case.

$\Delta m_B|_{\text{box}}$ depends on three crucial input parameters as apparent from Fig. 7:

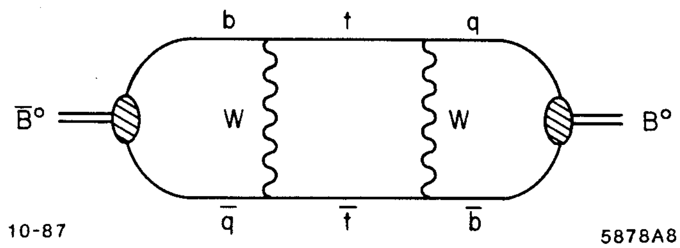

Fig. 7. Box diagram describing $B^\circ - \overline{B}^\circ$ transitions.

- m_t, the top mass (m_b, m_c are relevant for $\Delta\Gamma_B$);
- the KM parameter $V(tq)$ (assuming $|V(tb)| \simeq 1$);
- the hadron wave function $B_B f_B^2$ defined in complete analogy to the K^0 case:

$$\langle \bar{B}^0 | (b\bar{q})_{V-A}(b\bar{q})_{V-A} | B^0 \rangle \equiv \frac{4}{3} B_B f_B^2 m_B^2 \quad . \tag{100}$$

More specifically when ignoring m_c^2 and m_B^2 relative to m_t^2 — which amounts to a very good approximation of Δm_B — one finds

$$M_{12} = \eta_{\text{QCD}} \left(\frac{G_F}{4\pi}\right)^2 \frac{4}{3} B_B f_B^2 \xi_t^2 m_B E\left(\frac{m_t^2}{M_W^2}\right) \tag{101}$$

where $\xi_t = V(tb)V^*(tq)$ and[38]

$$E(x) = x\left(\frac{1}{4} + \frac{9}{4(1-x)} - \frac{3}{2(1-x)^2}\right) - \frac{3}{2}\left(\frac{x}{1-x}\right)^3 \log x \tag{102}$$

η_{QCD} contains the radiative QCD corrections. I will drop this factor in the following anticipating that it is not significant numerically considering the other uncertainties we are going to discuss. Nevertheless I want first to make two comments on it: the expression usually quoted for η_{QCD} in the literature as a function of (mainly) m_W, m_t, m_b has two shortcomings:

- It was derived assuming $m_t^2 \ll M_W^2$ — a quite popular expectation at the time. However for $m_t^2 \simeq M_W^2$ or $m_t^2 \gg M_W^2$ — at present seen as quite legitimate cases — different effective operators enter. They tend to lower η_{QCD} significantly (*i.e.*, by up to a factor of two) relative to the usual estimates [31]

- As usual, a low-energy scale μ is introduced which matches the scale at which the on-shell matrix element is evaluated. Almost all authors have used — without much discussion — $\mu^2 \simeq m_b^2$. While this is certainly a natural choice for B decays (to the extent that $m_b^2 \gg m_c^2$), I am unconvinced that this choice is appropriate for the coherent process $B^0 \to \bar{B}^0$. A more reasonable choice for the latter seems to me to be $\mu^2 \sim \frac{1}{4} - 1\ (\mathrm{GeV})^2$, *i.e.*, ordinary hadronic scales. Incorporating this would reduce η_{QCD} even further.

Finally (again for $m_c^2 \ll m_b^2 \ll m_t^2$)

$$\Delta m_B \simeq 2\mathrm{Re}\,|M_{12}| = 2|M_{12}| \tag{103}$$

in analogy to (50).

(i) $\Delta m(B_d)$ *vs.* $\Delta m(B_s)$

From (101) and (103) one reads off immediately

$$x_s = x_d \frac{|V(ts)|^2}{|V(td)|^2} \frac{(Bf_B[B_s])^2}{(Bf_B[B_d])^2} \geq x_d \frac{|V(ts)|^2}{|V(td)|^2} \tag{104}$$

where we have already anticipated $Bf_B[B_s] \geq Bf_B[B_d]$.

Since t quarks have not been observed yet, there exists no direct information on the KM parameters $V(td), V(ts)$ (or $V(tb)$ for that matter). However with three families only one can employ unitarity to constrain them quite considerably. I find the Wolfenstein parametrization [32] of the KM matrix most convenient for this and latter purposes:

$$V_{\mathrm{KM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{105}$$

plus terms of higher order in λ. As expected there are four independent parameters: λ, A, ρ, η.

The first one, λ, is basically the Cabibbo angles

$$\lambda \ \simeq\ 0.22 \quad . \tag{106}$$

A is estimated from the beauty lifetime[15]

$$A \ \simeq\ 1.0 \pm 0.3 \tag{107}$$

with considerable systematic uncertainties. Using[15]

$$0.1 \ \leq\ \left|\frac{V(ub)}{V(cb)}\right| \ \leq\ 0.25 \tag{108}$$

leads to

$$0.45 \leq \sqrt{\rho^2+\eta^2} \leq 1.14 \quad . \tag{109}$$

The unitarity of the 3×3 KM matrix then implies

$$|V(ts)| \simeq |V(cb)| = A\lambda^2 \tag{110}$$

The dependence on the KM parameters actually drops out from $x_s = \Delta m/\Gamma \propto |V(ts)|^2/|V(cb)|^2$.

Unfortunately there is no such simple relation between $|V(td)|$ and $|V(ub)|$, $|V(cb)|$; $|V(td)|$ in particular depends on the sign of ρ — in contrast to $|V(ub)|$ and becomes maximal for $\rho < 0$.

For $|\rho| \leq 1$ — it cannot be significantly larger and still satisfy (109) and reproduce the observed CP violation in K_L decays (see next chapter) — and $\rho < 0$ one finds

$$|V(td)| \lesssim 0.02 \tag{111}$$

$$\frac{|V(ts)|^2}{|V(td)|^2} = \frac{1}{\lambda^2\left((1-\rho)^2+\eta^2\right)} \gtrsim 5 \tag{112}$$

and therefore a quite *conservative* bound (see also ref.30)

$$x_s > 5x_d \gtrsim 2.2 \tag{113}$$

$$r_s > 0.71 \tag{114}$$

i.e., the mixing rate — Δm — is considerably larger than the decay rate — Γ — for B_s mesons. This has two consequences:

(α) $B_s - \bar{B}_s$ mixing is thus expected to approach its maximal value $r_s = 1$. An observation of slower mixing — say $r_s < 0.7$ — is thus a sign of New Physics — like a fourth family or an isoscalar quark or flavor changing neutral currents *etc.*, — that contributes *destructively* to $B_s - \bar{B}_s$ mixing. I have already mentioned that combining the lower bound χ_d from ARGUS with the upper bound on $\langle\chi\rangle$ from Mark II implies $r_s < 0.6$ — *if* the production probabilities of B_d, B_s, *etc.*, states hold as stated in Eq. (97).

(β) The real test of mixing consists of observing the special *time* evolution given in (80) and (81) where the exponential is modulated by a cos function, as shown in Fig. 8a,b for the two "typical" values $x = 0.75$ and $x = 5$. One realizes immediately that very good time resolution is required to observe very fast mixing.

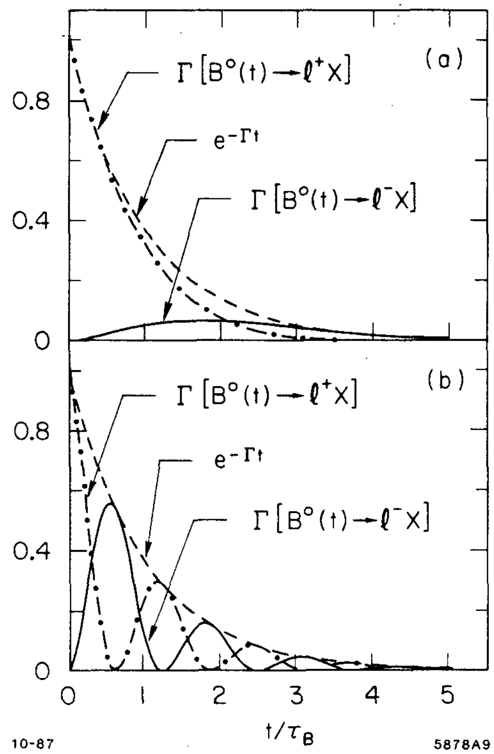

Fig. 8. Proper time evolution of semileptonic B° decays with (a) $\Delta_m/\Gamma = 0.75$ and (b) $\Delta_m/\Gamma = 5$.

(ii) $\Delta m(B_d)$ *and* m_t

Unfortunately it is much harder to make an absolute prediction on Δm as a function of m_t: there are the "hard" input parameter $V(tb)V^*(td)$ and the "soft" one $B_B f_B^2$. I have already stated that we have some nontrivial constraints on $|V(tb)V^*(td)|$ obtained via unitarity from $V(cb)$ and $V(ub)$. Since it is the ordinary strong interactions that are responsible for B and f_B we can conclude immediately

$$\begin{aligned} B_B &\simeq \mathcal{O}(1) \\ f_B &\simeq \mathcal{O}(f_\pi, f_K) \simeq \mathcal{O}(150\ \mathrm{MeV}) \quad . \end{aligned} \tag{115}$$

We find ourselves not in a position of complete ignorance concerning these parameters — the problem is that our understanding is numerically not sufficiently precise. For looking at Eqs. (101) and (103) we realize that Δm_B depends on the square both of f_B^2 and $|V(td)|^2$! Varying f_B by a factor of two which is perfectly consistent with (115) has a unpleasantly large impact. Δm_B changes by a factor of four and the real mixing observable $r = x^2/(2+x^2)$ by an order of magnitude! There are actually two sides to this coin, namely the strong dependence of $\Delta m_B|_{\mathrm{theor}}$ on certain input parameters:

- No precise prediction for or interpretation of Δm_B can be given as long as more than one of these inputs is unknown or only purely known.
- Numerically precise statements can however be made as soon as our ignorance has been narrowed down to only one (or better still, zero) input parameter.

Various theoretical models have been employed over the years to compute or at least estimate the relevant parts of the B meson wave function. The results are tabulated below: for B_d mesons

$$B_B f_B^2 \sim \begin{cases} (60-130 \text{ MeV})^2 & \text{MIT bag models}^{[33]} \\ (100-150 \text{ MeV})^2 & \text{Potential models}^{[34]} \\ (115 \pm 15 \text{ MeV})^2, (190 \pm 30 \text{ MeV})^2 & \text{QCD sum rules}^{[35,36]} \\ (120 \text{ MeV})^2/\alpha_s & B^* - B \text{ mass splitting}^{[37]} \\ \lesssim (220 \text{ MeV})^2 & \text{Scaling from } f_D \end{cases} \tag{116}$$

and for B_s mesons

$$B_B f_B^2 \sim \begin{cases} (140-200 \text{ MeV})^2 & \text{MIT bag models} \\ (140-200 \text{ MeV})^2 & \text{Potential models} \\ (140 \pm 20 \text{ MeV})^2, (210 \pm 30 \text{ MeV})^2 & \text{QCD sum rules} \end{cases} . \tag{117}$$

A few comments are in order.

- Comparing (116) with (117) exhibits the general feature

$$B_B f_B^2 [B_s] \geq B_B f_B^2 [B_d]$$

as expected intuitively: for in a nonrelativistic ansatz

$$f_B^2 = \frac{12|\varphi(0)|^2}{M_B} \tag{118}$$

where $\varphi(0)$ denotes the meson wave function at the origin.

This wave function is controlled by the reduced mass μ_B which is m_s $[m_d]$ for B_s $[B_d]$ mesons; the wave function is then more concentrated at the origin for B_s than for B_d mesons and despite $M(B_d) < M(B_s)$ one expects quite generally $f_B^2[B_s] > f_B^2[B_d]$. More explicitly — and therefore also in a more model dependent way — one finds

$$f_B \propto \mu_B \quad .$$

A fairly similar pattern holds also when relativistic effects are included [34].

- It is rather easy to prove that in potential models $B \equiv 1$ always holds — it amounts to a nice homework problem actually [23].
- B_B has not been calculated via the QCD sum rule approach yet. The values I have quoted there refer to f_B only. The two numbers for f_{B_d} are from an identical ansatz (namely that of Ref. 35) — the numerical difference is due completely to the usage of a different b quark mass. (One further remark can be made in passing: it is obviously highly dangerous and therefore inadvisable for a theorist to quote an error on his/her results. If they had not done that in this instance, one would be speaking of an uncertainty instead of a discrepancy.)
- The $B^* - B$ mass splitting yields at best on order of magnitude estimate on f_B (and nothing on B_B) since it is quite unclear which is the appropriate value for α_s: is it $\alpha_s \sim 1/2$ or $\alpha_s \sim 1$?
- The last line in (116) is obtained using the nonrelativistic expression (118) to relate f_B to the (90% C.L.) upper limit $f_D < 340$ MeV obtained by Mark III in its search for $D^+ \to \mu^+\nu_\mu$.

The uncertainties on the KM parameters and hadronic wave functions can be expressed quite conveniently in units of a calibration factor F

$$F = \frac{|V(td)|^2}{(0.01)^2} \, \frac{B\, f_B^2}{(150\ \text{MeV})^2} \quad . \tag{119}$$

Our preceding discussion leads to the range

$$F \sim 0.5 - 7 \tag{120}$$

as a realistic one, even with a certain touch of conservatism — nevertheless not one canonized by completely hard facts and/or calculations.

Figure 9 shows a comparison of x_d as a function of m_t with the ARGUS numbers; I conclude

$$m_t \gtrsim 50\ \text{GeV} \quad \text{if} \quad r_d \geq 0.1 \tag{121}$$

with m_t quite possibly much closer to 100 GeV!

A violation of (121) would indicate the presence of New Physics — a fourth family, a nonminimal Higgs sector *etc.*, — yet before such a conclusion would be finalized, one would of course re-analyze — with much more effort and impetus — whether the intrinsic theoretical uncertainties are truly reflected in (121)!

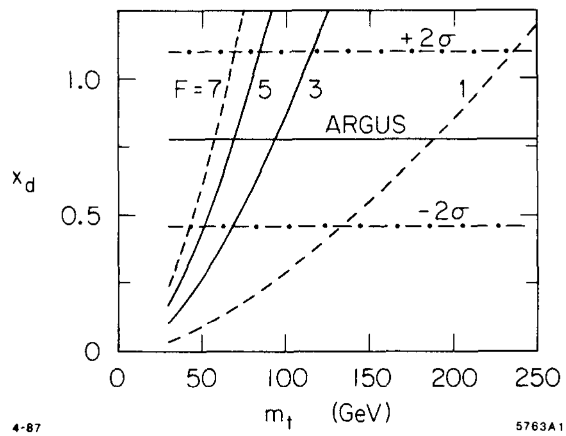

Fig. 9. (Δ_m/Γ) B_d as a function of m_t compared with the ARGUS findings.

(iii) $\Delta\Gamma_B$

In a box calculation it is the internal mass m_B and not the internal mass m_c that sets the scale:

$$\Delta\Gamma_B \propto m_B^2$$

More specifically one finds

$$\frac{\Delta\Gamma}{\Gamma}(B_d) \leq \mathcal{O}(0.01)$$

$$\frac{\Delta\Gamma}{\Gamma}(B_s) \leq \mathcal{O}(0.05)$$

D. $D^0 - \bar{D}^0$ Mixing

(1) Guesstimates on Δm_D

We know right away that $D^0 - \bar{D}^0$ mixing has to be small since it suffers from two quite efficient suppression mechanisms:

(α) *Cabibbo suppression:* $D^0 - \bar{D}^0$ like $K^0 - \bar{K}^0$ transitions are Cabibbo suppressed — yet the ordinary D in contrast to K decays are not! Therefore

$$\frac{\Delta m}{\Gamma}(D) \propto \frac{\sin^2\theta_c}{\cos^2\theta_c}\,, \qquad \frac{\Delta m}{\Gamma}(K) \propto \frac{\sin^2\theta_c}{\sin^2\theta_c} \quad . \tag{122}$$

(β) *GIM suppression:* Δm_D vanishes in the limits of SU(3)$_{FL}$ symmetry like $\Delta m_K\,[\Delta m_B]$ does for SU(4)$_{FL}$ [SU(6)$_{FL}$] symmetry. Yet SU(4)$_{FL}$ and in particular SU(6)$_{FL}$ breakings are considerably larger.

The still open question is "how small is small — is it actually tiny?"

Since there is nothing we can do about Cabibbo suppression, our discussion will center on our understanding of SU(3)$_{FL}$ breaking (or on the lack thereof). Like in the kaon case, Eq. (70), we distinguish between box and long-distance contributions

$$\Delta m_D = \Delta m_D|_{\text{box}} + \Delta m_D|_{\text{L.D.}} \quad . \tag{123}$$

$\Delta m_D|_{\text{box}}$ renders tiny values that are presumably forever unmeasurable: $\Delta m_D/\Gamma < 0.01$, $r_D < 10^{-4}$ (r_D is defined in analogy to (82)). The reason for that is easy to understand: SU(3)$_{FL}$ breaking in the box computation is represented by the difference between the s and d quark mass, calibrated by the W mass:

$$\frac{m_s^2 - m_d^2}{M_W^2} \ll 1 \quad . \tag{124}$$

Nevertheless I find it instructive to ask the following admittedly academic question:

Homework Problem 3

Is the box operator $\mathcal{L}_{\text{eff}}(\Delta C = 2)$ a *local* operator?

—

There is a second, considerably less academic question: does (124) represent a realistic treatment of SU(3)$_{FL}$ breaking in D decays? The answer is clearly negative! The detailed argument can be found in Ref. 16; here I want only to state one illustrative and relevant example:

$$\left.\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)}\right|_{\text{exp}} \sim 3-4 \quad . \tag{125}$$

I am not saying we do have a clear understanding why SU(3)$_{FL}$ symmetry is such a misleading guide in some D decays — only that this appears to be an observational fact. This has to be taken into account when calculating or at least estimating $\Delta m_D|_{\text{L.D.}}$.

Homework Problem 4

Show that there are three classes of diagrams for Δm_D when quark, gluon and W fields are used and that each of these classes contains four diagrams.

The $D^0 - \bar{D}^0$ transition amplitude is thus given by twelve(!) quark-gluon-W diagrams; those have to have alternating signs since they must — as already stated — cancel in the limit of $SU(3)_{FL}$ invariance. Keeping all the resonances in mind that could be and therefore will be relevant in this region, it is obviously and utterly beyond our capabilities to perform a computation based on first principles.

At this point one either folds up the tents and moves on to greener pastures or attempts to rough it out. I will try the latter.

Let us consider particularly simple intermediate states namely

$$D^0 \to PP \to \bar{D}^0 \tag{126}$$

where P denotes a pseudoscalar meson, π or K. There are four such transition amplitudes:

$$A(D^0 \to PP \to \bar{D}^0) = \sin^2\theta_c \cos^2\theta_c \left\{[K^+K^-] + [\pi^+\pi^-] - [\pi^+K^-] - [K^+\pi^-]\right\} \tag{127}$$

and four more for neutrals π's and K's. The relative signs between the different intermediate states are fixed by the GIM ansatz: the K^+K^- $[\pi^+\pi^-]$ pair couples to both D^0 and $\bar{D}^0$ with strength $\sin\theta_c\cos\theta_c\,[-\sin\theta_c\cos\theta_c]$; $K^+\pi^-$ $[\pi^+K^-]$ on the other hand couples to D^0 with $-\sin^2\theta_c\,[\cos^2\theta_c]$ and to $\bar{D}^0$ with $\cos^2\theta_c\,[-\sin^2\theta_c]$.

To obtain a rough guesstimate one proceeds as follows: starting from the experimental numbers $BR(D^0 \to K^+K^-) \sim 0.6\%$, $BR(D^0 \to \pi^+\pi^-) \sim 0.2\%$ to which one adds one half their value to include the $K^0\bar{K}^0$ and $\pi^0\pi^0$ modes one arrives at

$$BR(D_+ \to K\bar{K} + \pi\bar{\pi}) \simeq 2BR(D^0 \to K\bar{K} + \pi\bar{\pi}) \sim 0.025 \tag{128}$$

(with $|D_+\rangle = CP\,|D_+\rangle$) where we have used the coherent nature of mixing. Thus very roughly

$$\Delta m_D \sim 0.03\,\Gamma_D\,, \qquad \Delta\Gamma_D \sim 0.03\,\Gamma_D \qquad r_D \sim \mathcal{L}(10^{-3})\,. \tag{129}$$

Needless to say the real value of r_D due to long distance dynamics could be considerably smaller. The point of this exercise was to show that values like those in (129) are not necessarily ruled out in the Standard Model.

There is a well-known strategy for improving the quality of our estimate:

- derive a dispersion relation like for $K^0 \to \pi\pi \to \bar{K}^0$;
- evaluate it using measured $\pi\pi$, $K\bar{K}$, $K\pi$ phase shifts.

The actual execution of this program is however rather nontrivial and is therefore unlikely to be undertaken unless $D^0 - \bar{D}^0$ mixing is actually found with $r_D \sim 10^{-3}$.

(2) Phenomenology and Data

The most general expression describing $D^0 \to K^+\pi^-$ for small mixing (and assuming CP invariance) reads

$$\Gamma(D^0(t) \to K^+\pi^-) \propto e^{-\Gamma t}\left\{(\Gamma t)^2(x^2+y^2) + 4tg^4\theta_c|\hat{\rho}_f|^2 + 4y(\Gamma t)tg^2\theta_c\hat{\rho}_f\right\} \quad (130)$$

with

$$x = \frac{\Delta m}{\Gamma} \ll 1\,, \qquad y = \frac{\Delta\Gamma}{2\Gamma} \ll 1 \quad (131)$$

$$tg^2\,\theta_c\,\hat{\rho}_f = \frac{A(D^0 \to K^+\pi^-)}{A(D^0 \to K^-\pi^+)} \quad . \quad (132)$$

It is easy to see that the ratio of amplitudes for the doubly Cabibbo suppressed and the Cabbibo allowed mode, Eq. (132), has to be negative, $\hat{\rho}_f < 0$:

- in the absence of strong interaction $\hat{\rho}_f = -1$ since

$$\frac{V(cd)\,V^*(us)}{V(cs)\,V^*(ud)} = -\frac{\sin^2\theta_c}{\cos^2\theta_c}$$

- no *new* phase is introduced into this *ratio* by "switching on" the strong forces since $K^+\pi^-$ is the CP conjugate of $K^-\pi^+$ and the strong forces obey CP invariance. At the same time one can identify $\Delta\Gamma$ with $\Gamma_- - \Gamma_+$ where Γ_- [Γ_+] is the width of the CP odd [even] mass eigenstate. Thus

$$\begin{aligned}\Gamma(D^0(t) \to K^+\pi^-) \propto e^{-\Gamma t}\{&(\Gamma t)^2(x^2+y^2) + 4tg^4\theta_c|\hat{\rho}_f|^2 \\ &+ 2((\Gamma_+ - \Gamma_-)t)tg^2\theta_c|\hat{\rho}_f|\} \quad .\end{aligned} \quad (133)$$

As emphasized before, mixing is primarily characterized by its non-exponential time evolution — as exhibited by the first term in (133); the second term is purely exponential — not surprisingly, since it represents the doubly Cabibbo suppressed decays; the third term finally represents interference between doubly Cabibbo suppressed transitions and $\Delta\Gamma$ mixing.

The E691 Collaboration has presented a preliminary analysis using only the first two terms, they find

$$r_D \simeq \frac{1}{2}(x^2+y^2) \lesssim 0.5\% \quad . \tag{134}$$

The third term, if present at all, is likely to contribute with a positive sign. This expectation is based on the observation that more (two-body) decay modes are open to D_+ than to D_- decays: $\Gamma_+ > \Gamma_-$. This would even strengthen or lower the upper limit (134). Unfortunately one cannot rule out completely that $\Gamma_+ < \Gamma_-$ due to sizeable destructive interferences in D_+ decays.

There exists a complementary way to establish $D^0 - \bar{D}^0$ mixing, namely by observing

$$e^+e^- \to \psi''(3770) \to D^0\bar{D}^0 \to (K^\pm\pi^\mp)_D(K^\pm\pi^\mp)_D \tag{135}$$

i.e., special $S = \pm 2$ final states. For there is one by now (hopefully) familiar argument: in the ***absence*** of mixing the process (135) cannot proceed since Bose statistics requires the final state to be symmetric under exchange of the two kaons or the two pions. This is not possible however since $D^0\bar{D}^0$ and thus also the two identical $(K\pi)$ clusters form a p wave configuration which is antisymmetric under exchange!

By now the reader should have sharpened his/her skills of reasoning to proof the general result:

Homework Problem 5

Consider the reactions

$$e^+e^- \to \psi''(3770) \to D^0\bar{D}^0 \to f_a\, f_b\, ,\, f_a\, \bar{f}_b$$

where strangeness $S[f_a] = S[f_b] = \pm 1$, $S[\bar{f}_b] = -S[f_b]$

- Then in the ***absence*** of mixing, *i.e.*, $x = y = 0$

$$\frac{N(f_a, f_b)}{N(f_a, \bar{f}_b)} = tg^4\theta_c \left|\hat{\rho}_{f_a} - \hat{\rho}_{f_b}\right|^2 \quad . \tag{136}$$

For $a = b$, e.g. $f_a = K^-\pi^+$

$$\frac{N(f_a, f_a)}{N(f_a, \bar{f}_a)} = 0 \quad . \tag{137}$$

- How does (137) change when mixing is present?

Mark III has observed two events of

$$e^+e^- \to \psi'' \to D^0\bar{D}^0 \to (K^\pm\pi^\mp\pi^0)_D(K^\pm\pi^\mp\pi^0)_D \tag{138}$$

where one event is consistent with

$$\begin{array}{lll} D^0\bar{D}^0 \to (K^\pm\,\rho^\mp) & (K^\pm\,\rho^\mp) & \\ \quad\quad\quad\quad\quad\quad\ \ \llcorner\!\!\to \pi^\mp\pi^0 & \llcorner\!\!\to \pi^\mp\pi^0 & \end{array}$$

the other with

$$D^0\bar{D}^0 \;\to\; (K^*\pi)(K^*\pi) \quad .$$

If taken at face value this would correspond to

$$r_D \;\sim\; 0.5 - 1\% \tag{139}$$

which is not in real conflict with the E691 bound (134). Yet considering the very small number of events on one hand and the large width of K^* and of ρ mesons on the other hand it seems wise to reserve judgment for the moment and ask instead for the vigorous pursuit of more data.

IV. CP VIOLATION

In case you had been around university campuses in the late 1960's with an open eye and an open ear, you were bound to encounter passionate debates going on about the topic "what distinguishes the Left from the Right?" Listening to them you would have noticed that sooner or later "Left" and "Right" would be defined in terms of "Good" and "Evil". This sounds fine, were it not for a minor problem: there was no universal agreement on who the good and the bad guys are.

There is a close analogy in physics to this question in the political debate (like there are many other similarities and correspondences between physics and politics — which probably comes as a sobering thought to physicists rather than politicians). Let us assume that we want to communicate to a civilization in outer space what we mean by "left" and "right" in science. (Do not ask why we want to do that — it is like politics again, once a certain goal has been set.) And we do not want to achieve this noble goal by sending them a copy of a left hand or a more sophisticated version of it, namely polarized light; after all that would amount to an exchange of a convention only. At first thought it sounds like a fairly easy problem since parity symmetry is clearly violated: tell them to study $\pi \to \mu\nu$ decays — the emerging neutrino is left-handed. Thinking just a little bit harder, we realize the fallacy of this suggestion: we had of course the decays of *positive* pions in mind,

$$\pi^+ \rightarrow \mu^+ \nu_L$$

yet the decays of *negative* pions lead to *right*-handed (anti-)neutrinos! Hence we realize that even maximal parity violation does not allow to distinguish "left" from "right" in an absolute fashion — it only relates it to a *convention* on "positive" and "negative," like in the political debate cited above.

1964 marks a real revolution in physics: the violation of CP invariance was observed. One of its signals (though historically not the first one) is

$$\frac{\Gamma(K_L \rightarrow \ell^+ \nu \pi^-)}{\Gamma(K_L \rightarrow \ell^- \nu \pi^+)} \simeq 1.006 \neq 1 \quad . \tag{140}$$

Our message to outer space consists then of three parts:

- Find K_L mesons.
- Study $K_L \rightarrow \ell^{\mp} \nu \pi^{\pm}$ transitions; the one with the slightly larger rate produces *positively* charged leptons.
- With this definition of "positive" look at π^+ decays — they lead to *left*-handed neutrinos.

Thus we have learned a fundamental lesson: because and only because CP invariance has been found to be violated, can we say Nature makes an absolute, physical distinction between "left" and "right" which can be defined *without* taking recourse to any convention.

This describes to me the most basic importance of CP violation. Other aspects — that it is necessarily an integral part of any attempt to explain the baryon number of the Universe or that it has the highest sensitivity level to New Physics (see later), *etc.*, — are not quite as profound.

1. Theoretical Implementation of CP Violation

CPT invariance is assumed throughout these lectures. CP violation can then enter only via a complex relative phase between two coupling constants, g_1 and g_2, *i.e.*,

$$\mathrm{Im}\, g_1 g_2^* \neq 0 \quad . \tag{141}$$

As pointed out almost exactly fifteen years ago in the classic paper by Kobayashi and Maskawa [39] there is one and only one way to achieve this in the Standard Model (I ignore the "Strong CP Problem"):

- Neutral currents are flavor diagonal — therefore they cannot exhibit CP violating couplings.

- The charged currents on the other hand can — if there are enough families. Their couplings (in units of G_F) are combined in the unitary KM matrix. The latter has N^2 real parameters if there are N quark families; not all of those are physical parameters: having N up-type and N down-type quarks allows for $(2N-1)$ of those parameters to be changed at will by the arbitrariness in the phase convention for these fermions (the $2N^{\text{th}}$ phase transformation for the $2N$ fermions does not reduce the number of parameters since it corresponds to a *universal* phase change for all fermions — an operation that has no impact on quark bilinears like currents). Therefore we are left with $(N-1)^2$ real parameters.
- For $N = 2$ there is just one real parameter — the Cabibbo angle; therefore, CP invariance has to hold when there are two families only.
- For $N = 3$ there are four real parameters. Since there are only three Euler angles for 3×3 matrices, there is room for one phase δ, which controls the intrinsic strength of CP violation.
- For $N = 4$ these are nine real parameters — namely, six Euler angles and three complex phases.

The KM matrix emerges when the mass matrix for up- and down-quarks is diagonalized. The quark masses in turn are produced via the Yukawa couplings of Higgs fields to quark bilinears. A complex KM phase thus requires a complex Yukawa coupling. Since this enters in a dimension four operator, CP violation is described as a "hard" symmetry breaking which is uncalculable in the Standard Model as a matter of principle.

By the way, Kobayashi and Maskawa made their observation on having three families as the minimal requirement for accommodating CP violation at a time when charm had not been discovered yet, *i.e.*, when even the second family was still incomplete! They also specified two generic cases for implementing CP violation by going beyond the Standard Model: right-handed currents and/or additional Higgs doublets! In those scenarios CP invariance is typically broken in a spontaneous fashion.

On derives an equally simple and general prediction from the observation that the interplay of three families is required for CP violation to become observable: in B decays there are bound to be CP asymmetries that are much larger than in K decays (or D decays). For the latter decays in contrast to the former do not involve the third family directly. Thus as far as the Standard Model is concerned there can be no doubt: rather large CP asymmetries are bound to exist in some beauty decays — however, much more detailed considerations have to be made to predict the exact size of these asymmetries and in which modes they are most readily found. We are thus in a situation very similar to that of a detective who knows that the perpetrator of a crime is hiding in a certain building, but not on which floor, let alone which room. We are of course confident that we will not follow in the footsteps of Inspector Clouzot.

2. Phenomenology of CP Violation in K_L Decays

(i) $K_L \to \ell\nu\pi$ Decays

In (49) I have already given the general expression for the mass eigenstates $K_{L,S}$ in terms of the flavor eigenstates:

$$
\begin{aligned}
|K_{L,S}\rangle &= \frac{1}{\sqrt{1+|\alpha|^2}}\left(|K^0\rangle \pm \alpha\,|\bar{K}^0\rangle\right) \\
\alpha &= \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \quad .
\end{aligned}
\tag{142}
$$

It was also mentioned that only the modulus of α is physical, the phase is not:

$$
|\bar{K}^0\rangle \;\to\; e^{i\xi}\,|\bar{K}^0\rangle \tag{143}
$$

leads to

$$
(M_{12}, \Gamma_{12}) \;\to\; e^{i\xi}\,(M_{12}, \Gamma_{12}) \tag{144}
$$

and therefore

$$
\alpha \;\to\; e^{-i\xi}\,\alpha \tag{145}
$$

yet

$$
\begin{aligned}
|\alpha| &= 1 + \frac{\frac{1}{2}|\Gamma_{12}|^2}{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2}\,\mathrm{Im}\,\frac{M_{12}}{\Gamma_{12}} \to |\alpha| \\
|K_{L,S}\rangle &\;\to\; |K_{L,S}\rangle \quad .
\end{aligned}
\tag{146}
$$

The data tell us with very good accuracy[40]

$$
\Delta m_K \simeq 2|M_{12}|\;, \quad \Delta\Gamma_K \simeq -2|\Gamma_{12}|\;, \quad x = \frac{\Delta m}{\Gamma_s} \simeq 0.477\;. \tag{147}
$$

Then we can write

$$
|\alpha| \simeq 1 + \frac{1}{2}\,\phi(\Delta S = 2) \tag{148}
$$

with

$$
\phi(\Delta S = 2) \equiv \arg\frac{M_{12}}{\Gamma_{12}} \tag{149}
$$

due to some numerical coincidences like $\Delta\Gamma \simeq \Gamma_s$, $x \simeq 0.5$.

$\phi(\Delta S = 2)$ denotes a physical phase emerging from $\Delta S = 2$ transitions — thus it measures CP violation in these reactions. It can be determined most directly in semileptonic K_L decays, due to the experimentally confirmed $\Delta S = \Delta Q$ rule $K^0 \not\to \ell^- X$, $\bar{K}^0 \not\to \ell^+ X$:

$$\delta_\ell \equiv \frac{\Gamma(K_L \to \ell^+ \nu \pi^-) - \Gamma(K_L \to \ell^- \nu \pi^+)}{\Gamma(K_L \to \ell^+ \nu \pi^-) + \Gamma(K_L \to \ell^- \nu \pi^+)} = \frac{1 - |\alpha|^2}{1 + |\alpha|^2} \simeq -\frac{1}{2}\phi(\Delta S = 2) \quad (150)$$

Experimentally one finds after averaging over $\ell = e, \mu$

$$\delta_\ell = (3.30 \pm 0.12) \times 10^{-3} \quad (151)$$

and therefore

$$\frac{M_{12}}{\Gamma_{12}} = -(0.4773 \pm 0.0023)[1 - i\phi(\Delta S = 2)] \quad (152)$$

$$\phi(\Delta S = 2) = (6.58 \pm 0.26) \times 10^{-3} \quad (153)$$

with $\phi(\Delta S = 2)$ representing CP violation in the $K^0 - \bar{K}^0$ mass mixing.

(ii) $K_L \to \pi\pi$ Decays

Historically CP violation was first observed in a different way, namely by detecting[41]

$$K_L \to \pi^+ \pi^- \quad . \quad (154)$$

For it showed that K_L could not be a CP eigenstate since it decayed both to CP odd 3π as well as CP even 2π final states — a reincarnation of the old $\theta - \tau$ puzzle. Defining a ratio of *amplitudes* for $K_L \to \pi\pi$ and $K_S \to \pi\pi$

$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+ \pi^-)}{A(K_S \to \pi^+ \pi^-)} \quad (155)$$

one deduces after a little bit of work

$$\eta_{+-} \simeq -\frac{ix}{2x + i}\,\phi(\Delta S = 2) \quad . \quad (156)$$

Inserting the measured numbers for x and $\phi(\Delta S = 2)$ one expects

$$\eta_{+-}|_{x,\phi} \simeq (2.27 \pm 0.08) \times 10^{-3}\, e^{i(43.7 \pm 0.2)^\circ} \quad (157)$$

in excellent agreement with direct measurements of both its modulus and its phase

$$\eta_{+-}|_{\exp} = (2.279 \pm 0.026) \times 10^{-3}\, e^{i(44.6 \pm 1.2)^\circ} \quad . \quad (158)$$

Could one have observed $\eta_{+-}|_{x,\phi} \neq \eta_{+-}|_{\exp}$? Certainly, for I have made an implicit assumption in deriving (156): The final state in $K_L \to \pi\pi$ is in general a mixture of isospin zero and two configurations. Therefore one defines two different isospin amplitudes:

$$A\left(K_L \to (\pi\pi)_{I=0,2}\right) \equiv A_0, A_2 \quad . \tag{159}$$

A priori there is no reason why these two amplitudes have to be real relative to each other; thus it makes perfect sense to define a second physical phase which characterizes decay, *i.e.*, $\Delta S = 1$ processes:

$$\phi(\Delta S = 1) \equiv \arg \frac{A_2}{A_0} \quad . \tag{160}$$

This phase was ignored in deriving (156) and one has to write more generally

$$\eta_{+-} = -\frac{ix}{2x+i}\left(\phi(\Delta S = 2) - 0.1\phi(\Delta S = 1)\right) \tag{161}$$

where the small coefficient in front of $\phi(\Delta S = 1)$ is due to the observed relation

$$\mathrm{Re}\,\frac{A_2}{A_0} \simeq 0.05 \tag{162}$$

i.e., the $\Delta I = 1/2$ rule. With

$$\eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \tag{163}$$

one deduces with the help of the isospin algebra

$$\frac{\eta_{+-}}{\eta_{00}} = 1 - 0.3\,\frac{\phi(\Delta S = 1)}{\phi(\Delta S = 2)} \tag{164}$$

and thus (anticipating $\phi(\Delta S = 1) \ll \phi(\Delta S = 2)$)

$$-\frac{BR(K_L \to \pi^+\pi^-)}{BR(K_L \to \pi^0\pi^0)}\,\frac{BR(K_S \to \pi^0\pi^0)}{BR(K_S \to \pi^+\pi-)} = \left|\frac{\eta_{+-}}{\eta_{00}}\right|^2 \simeq 1 - 0.6\,\frac{\phi(\Delta S = 1)}{\phi(\Delta S = 2)} \quad . \tag{165}$$

Special linear combinations of η_{+-}, η_{00} are quite often used to parametrize CP violation

$$\begin{aligned} \eta_{+-} &= \epsilon_K + \epsilon' \\ \eta_{00} &= \epsilon_K - 2\epsilon' \quad . \end{aligned} \tag{166}$$

Therefore

$$\begin{aligned} \frac{\eta_{+-}}{\eta_{00}} &= \frac{\epsilon_K + \epsilon'}{\epsilon_K - 2\epsilon'} \simeq 1 + 3\,\frac{\epsilon'}{\epsilon_K} + \mathcal{O}\left[\left(\frac{\epsilon'}{\epsilon_K}\right)^2\right] \\ \frac{\epsilon'}{\epsilon_K} &\simeq \frac{1}{10}\,\frac{\phi(\Delta S = 1)}{\phi(\Delta S = 2)} \quad . \end{aligned} \tag{167}$$

The physical interpretation of ϵ_K and ϵ' can directly be read off from (161), (166) and (167)

- ϵ_K characterizes the decaying state; it is given by $\phi(\Delta S = 2)$ which measures CP violation in the $K^0 - \bar{K}^0$ mass mixing.
- ϵ' differentiates between different decay modes; it is therefore determined by $\phi(\Delta S = 1)$ which parametrizes direct CP violation. If there were more nonleptonic decay channels for K_L, there would be more than one ϵ'; one encounters such a situation in B decays.

The two most recent measurements of ϵ'/ϵ are still preliminary; they read[42]

$$\frac{\epsilon'}{\epsilon_K} = \left\{ \begin{array}{c} 3.5 \pm 0.7 \pm 0.4 \pm 1.2 \\ 3.5 \pm 3.0 \pm 2.0 \end{array} \right. \times 10^{-3} \quad . \tag{168}$$

A nonzero value for direct CP violation has still not been established; yet somewhat optimistically one can say that a tantalizing trend into that direction has emerged. From (168) one obtains

$$\phi(\Delta S = 1) \;\simeq\; (2.3 \pm 1.0) \times 10^{-4} \tag{169}$$

a tiny number indeed.

A caveat:

Quite often a different ansatz is used to describe CP violation. Introducing a complex parameter $\bar{\epsilon}$ one writes down

$$\begin{aligned} |K_L\rangle &= \frac{1}{\sqrt{2(1+|\bar{\epsilon}|^2)}} \left\{ (1+\bar{\epsilon}) \left|K^0\right\rangle + (1-\bar{\epsilon}) \left|\bar{K}^0\right\rangle \right\} \\ &= \frac{1}{\sqrt{1+|\bar{\epsilon}|^2}} \left\{ |K_-\rangle + \bar{\epsilon}\, |K_+\rangle \right\} \quad , \qquad \mathrm{CP}\, |K_\pm\rangle = \pm\, |K_\pm\rangle \end{aligned} \tag{170}$$

where the following definition has been used

$$\mathrm{CP} \left|K^0\right\rangle = - \left|\bar{K}^0\right\rangle \tag{171}$$

leading to

$$|K_\pm\rangle = \frac{1}{\sqrt{2}} \left(\left|K^0\right\rangle \mp \left|\bar{K}^0\right\rangle \right) \quad . \tag{172}$$

Equation (170) is interpretated rather easily by saying that $\bar{\epsilon}$ parametrizes the admixture of the even, *i.e.*, "wrong" CP state in K_L. Yet one has to exercise some caution in dealing with the quantity $\bar{\epsilon}$: it is not automatically a physical parameter as is seen from the identity

$$\alpha = \frac{1-\bar{\epsilon}}{1+\bar{\epsilon}} \quad . \tag{173}$$

Only $|\alpha|$ is independent of phase conventions, Eqs. (145) and (146)

$$|\alpha|^2 = 1 - \frac{4\,\mathrm{Re}\,\bar{\epsilon}}{|1+\bar{\epsilon}|^2} \tag{174}$$

implying that $\mathrm{Re}\,\bar{\epsilon}$ is a fully physical parameter

$$\mathrm{Re}\,\bar{\epsilon} = -\frac{1}{4}\,\phi(\Delta S = 2) \tag{175}$$

whereas $\mathrm{Im}\,\bar{\epsilon}$ per se is not. The latter does not mean that $\mathrm{Im}\,\bar{\epsilon}$ is devoid of meaning — only that proper care has to be taken in adopting consistent phase conventions. I will address this issue again in discussing CP violation in B decays.

The question of phase conventions affects also the decay amplitudes A_0, A_2. In the phase conventions of Wu and Yang one chooses A_0 to be real:

$$\mathrm{Im}\,A_0|_{WY} = 0 \quad . \tag{176}$$

One then finds[43]

$$\epsilon' = \frac{i}{\sqrt{2}}\, e^{i(\delta_2-\delta_0)}\, \frac{\mathrm{Im}\,A_2}{A_0}\bigg|_{WY} \tag{177}$$

where δ_2, δ_0 denote the $\pi\pi$ phase shifts in the two isospin channels.

With due respect to history and precedent I find describing CP violation in terms of $\phi(\Delta S = 1, 2)$ much more transparent.

(iii) CPT Invariance

I had mentioned before, Eqs. (156) and (157), that the phase of η_{+-} is to a very good approximation determined by mass mixing. This holds also for η_{00}:

$$\eta_{+-,00} = |\eta_{+-,00}|\; e^{i\varphi_{+-,00}} \tag{178}$$

$$\varphi_{+-} \simeq \varphi_{00} = \mathrm{arctg}\; 2x = (43.7 \pm 2)^\circ \quad . \tag{179}$$

The measured value of φ_{+-}, Eq. (158), is in good agreement with this expectation. Yet a two σ deviation is observed for φ_{00}:

$$\phi_{00}|_{\mathrm{exp}} = (54.5 \pm 5.3)^\circ \quad . \tag{180}$$

There is no cause for alarm yet or even concern, but our curiosity should be aroused. In the coming years the phase difference $\varphi_{+-} - \varphi_{00}$ will be measured to within 1° hopefully. These data will in all likelyhood confirm our expectations — if not, it would indicate that CPT invariance is violated in $K^0 \rightarrow \pi\pi$ decays.

(iv) Time Reversal Invariance

There is no experimental evidence against CPT invariance, even less theoretical evidence. The observation of CP violation therefore means that T has to be violated as well, *i.e.,* that Nature distinguishes between future and past on the microscopic level — another reason why CP violation is bound to affect you on the guts (and not merely the GUT) level.

Yet no T violation has been observed directly. This is hardly surprising since, strictly speaking, one has to rely on analyzing static quantities like electric dipole moments (see below). For in scattering or decay processes it is for practical reasons impossible to study time reversal invariance directly, without further assumptions: while the decay $A \to B + C$ occurs spontaneously one is unable to prepare the corresponding recombination process $B + C \to A$.

The electric dipole moment d is defined via a term in the Hamiltonian

$$H = d\vec{\sigma} \cdot \vec{E} \tag{181}$$

where $\vec{\sigma}$ denotes the spin of the particle and $\vec{E}$ the electric field. Since $\vec{\sigma} \to -\vec{\sigma}$ and $\vec{E} \to \vec{E}$ under T, d has to vanish if T invariance holds. There is another quantity sometimes referred to as induced electric dipole moment d_{ind} which is defined by $H = d_{ind}\, \vec{r} \cdot \vec{E}$; $\vec{r}$ denotes the spatial separation between two opposite charges. It is clearly allowed by T invariance. The highest sensitivity level has been achieved for the neutron electronic dipole moment d_N where two different groups found

$$d_N = \begin{cases} (-2.0 \pm 1.0) \\ (-1.8 \pm 2.9) \end{cases} \times 10^{-25} \text{ ecm} \quad . \tag{182}$$

These numbers have to be seen as establishing an upper bound on d_N of a few $\times 10^{-25}$ ecm — an impressively tiny number when compared to the neutron's radius, $r_N \sim 10^{-13}$ cm.

It is true that the KM ansatz predicts numbers that are even much tinier:

$$d_N^{KM} \; < \; 10^{-30} \text{ ecm} \quad . \tag{183}$$

Yet other models of CP violation — invoking right-handed currents or a nonminimal Higgs sector — lead to predictions ranging from 10^{-24} ecm down to 10^{-27} ecm. Experimentalists expect to reach a sensitivity level of 10^{-26} ecm in the near future — they should be both admired for and encouraged in their tenacity.

(v) Standard Model Expectation for ϵ and ϵ'

In Chapter III I have discussed why one cannot equate Δm_K with $\Delta m_K|_{\text{box}}$: there are virtual low-energy transitions $K^0 \to \pi\pi, \pi, \eta, \eta' \to \bar{K}^0$ which are not included appropriately in the quark-box description. The situation is quite different for ϵ: according to the KM ansatz a quantity can exhibit CP violation only if it is sensitive to the presence of three families. The hadronic transitions $K^0 \to \pi\pi, \pi, \eta \to \bar{K}^0$ are insensitive to charm and top — therefore they do not contribute to ϵ while being significant for Δm_K. (Once the $\Delta S = 1$ "Penguin" operators are included which are crucial for ϵ' this ceases to be strictly correct, in particular for $K^0 \to \eta' \to \bar{K}^0$; yet since $\epsilon'/\epsilon \ll 0.05$ such effects represent small corrections only.) Therefore for practical purposes one can rely on the quark box ansatz [44]:

$$\epsilon|_{\text{theor}} \simeq \epsilon|_{\text{box}} \quad . \tag{184}$$

(It should be noted that Eq. (184) does not necessarily hold for other mechanisms of CP violation: e.g. when CP violation is produced via a nonminimal Higgs sector it is long distance dynamics that dominates ϵ.[45]

First one determines the effective $\Delta S = 2$ Lagrangian where it is now essential to include all three families:

$$\begin{aligned}\mathcal{L}_{\text{eff}}(\Delta S = 2) = \left(\frac{G_F}{4\pi}\right)^2 M_W^2 \left\{\lambda_c^2 E(x_c, x_c) f_1 + \lambda_t^2 E(x_t, x_t) f_2 + 2\lambda_c \lambda_t E(x_c, x_t) f_3\right\} \\ \times \left[\alpha_s(\mu^2)\right]^{-6/27} (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma_\mu d_L)\end{aligned} \tag{185}$$

with λ_i denoting the KM parameters

$$\lambda_i = V(is)\, V^*(id) \quad , \qquad i = c, t \quad . \tag{186}$$

To describe CP violation in the K^0 system one has to use the Wolfenstein expansion of the KM matrix to order λ^4:

$$V_{KM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3\left(\rho - i\eta + i\eta\frac{1}{2}\lambda^2\right) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 - i\eta A^2\lambda^4 & A\lambda^2\left(1 + i\eta\lambda^2\right) \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{187}$$

$E(x_i, x_j)$ reflecting the box loops with internal quarks, i, j. Since the relevant momenta in the loop are between m_c and m_t (this is the same argument as given above (184), only made more explicit) the strong forces enter only via QCD radiative corrections f_1, f_2 and f_3. The factor containing $\alpha_s(\mu^2)$ explicitly reflects the renormalization of $\mathcal{L}_{\text{eff}}(\Delta S = 2)$ when changing the scale μ at which $\langle \bar{K}^0 | (\bar{s}_L d_L)(\bar{s}_L d_L) | K^0 \rangle$ is calculated. This factor has to drop out from the full expression for physical quantities. Putting everything together one obtains

$$
\begin{aligned}
|\epsilon_K| &= 1.9\times 10^4 B_K \left|\mathrm{Im}\,(\lambda_c)^2 f_1 E(x_c,x_c) + \mathrm{Im}\,(\lambda_t)^2 f_2 E(x_t,x_t)\right. \\
&\qquad\qquad \left. + \,\mathrm{Im}\,(\lambda_c\lambda_t) f_3 E(x_c,x_t)\right| \\
&\simeq 4.33 A^2 \eta B_K \left|-f_1 E(x_c,x_c) + 2.3\times 10^{-3} A^2(1-\rho) f_2 E(x_t,x_t)\right. \\
&\qquad\qquad \left. + \, f_3 E(x_c,x_t)\right|
\end{aligned}
\tag{188}
$$

with B_K being defined as in (64).

Strictly speaking the KM ansatz does not predict CP violation — it only allows for it as expressed by the parameter η: from the observed value of ϵ_K one extracts η as a function of m_t (and m_c) and of B_K. For 30 GeV $\leq m_t \leq$ 180 GeV one can employ a numerical approximation that is much easier to handle

$$
B_K \eta \simeq \frac{0.53}{A^2}\left\{0.94 x_t^{0.1105} - 0.3 + A^2(1-\rho) x_t^{0.8363}\right\}^{-1} \tag{189}
$$

where I have used $m_c = 1.5$ GeV; as usual $x_t = m_t/M_w$. $B_K\eta$ thus drops rather quickly with increasing m_t; for $B_K = 2/3$, $\rho = -0.7$ — which are perfectly reasonable values with our present level of understanding — one finds

$$
\eta \simeq 0.5\ [0.2] \quad \text{if} \quad m_t = 60\ [130]\ \text{GeV} \tag{190}
$$

which agrees with the bounds obtained on $b \to u$ transitions, Eq. (109).

However *if* $B_K = 1/3$, then $\eta = 1$ for $m_t = 60$ GeV and thus $\rho^2 + \eta^2 \simeq 1.5$ — in violation of the bound (109). Statements like "the KM ansatz is insufficient to accommodate the observed strength of CP violation" which were made a few years ago were based on these three "pillars": a small value of B_K — $B_K \sim 1/3$ as it once was very popular in the community; a small top mass — $m_t \sim 40$ GeV, as it appeared established; a small upper bound on $b \to u$ transitions — $\rho^2 + \eta^2 < 0.5$ as it was once widely accepted. However at present we cannot claim any insufficiency of the KM ansatz.

Homework Problem 6

Equation (4.49) shows that a nonvanishing ϵ_K can be produced from a quark box with only charm quarks as virtual states, *i.e.*, when one ignores top quarks in the internal lines. How does this observation match up with the general statement that a quantity can exhibit CP violation only if it is sensitive to the presence of three families?

Homework Problem 7

It was emphasized that the essence of ϵ_K is contained in $\phi(\Delta S = 2)$ as defined in (149). Using $\arg(M_{12}/\Gamma_{12}) = \arg M_{12} - \arg \Gamma_{12} \simeq \arg M_{12}$ and inserting $(M_{12})_{\text{box}}$ would lead to the observation that $\phi(\Delta S = 2)$ depends only on V_t, V_c, m_t and m_c, but *not* on B_K — in conflict with (188). Where is the loop hole in this argument?

Two general observations can be made on ϵ':

- ϵ' suffers from an "accidental" suppression: it requires the interference between A_2 and A_0 amplitudes which is strongly suppressed: $\mathrm{Re}\,(A_2/A_0) \simeq 0.05$ — the $\Delta I = 1/2$ rule. This is another reason why $\phi(\Delta S = 1)$ — as defined in (160) — is a truer representation of direct CP violation.
- There could not be any direct CP violation in K_L decays, *i.e.*, $\epsilon' = 0$, in the KM scheme — were it not for radiative corrections. It is through those that charm and top make their presence felt.

There is in particular one class of QCD radiative corrections which does not suffer from "strong" GIM suppression — m_c^2/M_w^2 or m_t^2/M_w^2 — but only from "mild" GIM suppression — $\log(m_t^2/m_c^2)$ — which actually amounts to an enhancement factor since $m_t \gg m_c$: the "Penguins", as shown in Fig. 10.

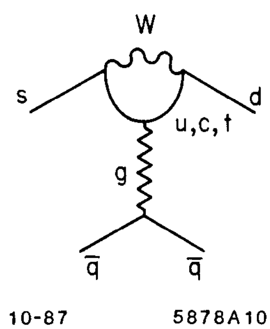

Fig. 10. Penguin contribution for K° decays.

To actually calculate ϵ' one has to grapple with some technicalities I have briefly referred to before: the conventional formulae for ϵ' are expressed using the Wu-Yang phase convention where $\mathrm{Im}\,A_0 = 0$. When using a description based on quark fields, one has implicitly used a different phase convention: A_2 is described by the tree-level diagram $s\bar{d} \to d\bar{u}\ u\bar{d}$ alone, *i.e.*, $\mathrm{Im}\,A_2 = 0$. A_0 on the other hand receives contributions both from the tree and the loop diagram; the latter contains the weak phases due to the charm and top couplings; thus

$$A_0^{\rm quark} = |A_0|\, e^{i\xi} \quad , \qquad \xi \neq 0 \quad . \tag{191}$$

One then either uses the general expressions

$$\epsilon \simeq \frac{1-D}{2\sqrt{2}}\, e^{i\pi/4} \left\{ \epsilon_m + 2\xi + \frac{D}{1-D}\,\chi \right\} \tag{192}$$

$$\frac{\epsilon'}{\epsilon} \simeq -\exp\left\{ i\left(\frac{\pi}{4} + \delta_2 - \delta_0\right)\right\} \frac{2\xi}{20(1-D)\left[\epsilon_m + 2\xi + \frac{D}{1-D}\chi\right]} \tag{193}$$

where

$$M_{12} = (M_{12})_{SD} + (M_{12})_{LD} \equiv (M_{12})_{\rm box} + DM_{12}$$

$$\epsilon_m = \frac{\mathrm{Im}\,(M_{12})_{\rm box}}{\mathrm{Re}\,(M_{12})_{\rm box}} \quad , \qquad \xi \simeq \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0} \ll 1 \tag{194}$$

$$\mathrm{Im}\,(M_{12})_{LD} = (-2\xi + \chi)\mathrm{Re}\,(M_{12})_{LD} \quad .$$

Or one applies a phase transformation

$$\left|K^0\right\rangle \rightarrow e^{-i\xi}\left|K^0\right\rangle$$

to recover the Wu-Yang phase convention. Either way one obtains [46]

$$\frac{\epsilon'}{\epsilon} \simeq 15.6\xi = 0.014 A^2 \eta \left(\frac{\mathrm{Im}\,\tilde{c}_6}{-0.1}\right)\left(\frac{\langle \pi\pi |\, Q_6 \left|K^0\right\rangle}{1.0\ (\mathrm{GeV})^3}\right) \tag{195}$$

where Q_6 denotes the Penguin operator emerging in the renormalization of $\mathcal{L}_{\rm eff}(\Delta S = 1)$ with coefficient $\tilde{c}_6$ (the KM parameters have been factored out). As emphasized before, when CP violation is concerned, only the momentum scales from $\sim m_c$ to $\sim m_t$ contribute: the perturbative calculation of $\mathrm{Im}\,\tilde{c}_6$ which yields ~ -0.1 should then be fairly reliable. The value of the matrix element of Q_6 is much more uncertain: different models yield values between 0.3 and 1 $(\mathrm{GeV})^3$. The parameter η appears since I have inserted the measured value for ϵ_K. As discussed above for $B_K \sim 2/3$ one finds $\eta \simeq 0.5$ [0.2] for $m_t \simeq 60$ [130] GeV and thus

$$\frac{\epsilon'}{\epsilon} \sim 7[3] \times 10^{-3} \left(\frac{\mathrm{Im}\,\tilde{c}_6}{-0.1}\right)\left(\frac{\langle \pi\pi |\, Q_6 \left|K^0\right\rangle}{1.0\ (\mathrm{GeV})^3}\right) \sim \mathcal{O}(\mathrm{few} \times 10^{-3}) \quad . \tag{196}$$

It should be kept in mind that (195) contains a significant dependence on m_t — as exemplified by (196): since ϵ_K is known, an increase in m_t implies a decrease in η, as exhibited in (190).

All these considerations contain a lot of fuzziness since B_K is not known for certain and neither is $\langle \pi\pi | Q_6 | K^0 \rangle$. There is certainly no justification for despair: we can expect lattice Monte Carlo computations to yield reasonably accurate results. On the other hand:

- I do not expect reliable numerical results to emerge in the near future;
- analytical methods, despite their shortcomings, will continue to play a role be it only to offer physical insights into the numbers obtained by a Monte Carlo ansatz.

3. CP Violation in B Decays

I have already presented general though only qualitative arguments why CP asymmetries in B decays will be much larger than in K decays — if one relies on the KM implementation of CP violation. This does not mean that experimental searches will be easy — far from it! Yet there are two facts working in our favor:

- we are hunting big, though elusive game;
- we can design well defined strategies for going after it.

(i) On the Observability of CP Asymmetries

Assuming CPT invariance implies that CP violation can enter only via relative phases between (effective) coupling constants. These phases can be observed only if two *different* amplitudes contribute *coherently* to the same process; the asymmetry is produced by their interference. In principle there are just two ways to realize such a scenario:

- via final state interactions = FSI, like ϵ';
- via mixing, like ϵ_K.

(ii) CP Violation and FSI

When two different amplitudes contribute to the decay of a bottom hadron B into a final state f, one writes for the matrix element

$$\begin{aligned} M_f &= \langle f | \mathcal{L}(\Delta B = 1) | B \rangle \\ &= \langle f | \mathcal{L}_1 | B \rangle + \langle f | \mathcal{L}_2 | B \rangle \\ &= g_1 \, M_1 \, e^{i\alpha_1} + g_2 \, M_2 \, e^{i\alpha_2} \end{aligned} \tag{197}$$

where M_1, M_2 denote the matrix elements for the weak transition operators $\mathcal{L}_1, \mathcal{L}_2$ with the KM parameters g_1, g_2 and the strong (or electromagnetic) phase shifts α_1, α_2 factored out. For the CP conjugate decay $\bar{B} \to \bar{f}$ one then finds

$$\begin{aligned} \overline{M}_f &= \langle \bar{f} | \, \mathcal{L}(\Delta B = 1) \, | \bar{B} \rangle \\ &= g_1^* \, M_1 \, e^{i\alpha_1} + g_2^* \, M_2 \, e^{i\alpha_2} \quad . \end{aligned} \tag{198}$$

The same phase shifts α_1, α_2 (instead of $-\alpha_1, -\alpha_2$) have been written down in (198) since CP invariance is obeyed by the strong and electromagnetic forces. Comparing (197) with (198) one obtains

$$\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f}) \propto \mathrm{Im}\, g_1^* g_2 \sin(\alpha_1 - \alpha_2)\, M_1 M_2 \quad . \tag{199}$$

Thus two conditions have to be met simultaneously for such an asymmetry to show up:

(α) The weak couplings g_1 and g_2 have to possess a relative complex phase; therefore small KM angles have to be involved.

(β) Nontrivial phase shifts $\alpha_1 \neq \alpha_2$ have to be generated from the strong (or electromagnetic) forces.

This is in complete analogy to direct CP violation in K^0 decays:

$$\Gamma(K^0 \to \pi\pi) - \Gamma(\bar{K}^0 \to \pi\pi) \propto \mathrm{Im}\, g_0 g_2^* \sin(\delta_0 - \delta_2)\, A_0 A_2$$

where the subscripts refer to the isospin of the $\pi\pi$ system.

Condition (β) does not, in principle, pose a severe restriction; in practice it introduces considerable uncertainties into numerical predictions. The most interesting scenario — in my judgment — is provided by invoking Penguin contributions [47], Fig. 11. The phase shift $\alpha_1 - \alpha_2 \neq 0$ is produced by the loop diagram with charm as the internal quark — which does *not* yield a local, though hopefully a short-distance operator. Doing detailed calculation one finds[48]

$$\begin{aligned} BR(B \to K^{\pm}\pi^{\mp}) &\sim \mathcal{O}(10^{-5}) \\ \frac{\Gamma(B^0 \to K^+\pi^-) - \Gamma(\bar{B}^0 \to K^-\pi^+)}{\Gamma(B^0 \to K^+\pi^-) + \Gamma(\bar{B}^0 \to K^-\pi^+)} &\sim 1 - 10\% \quad . \end{aligned} \tag{200}$$

The nice feature of this decay mode is that it is flavor specific: $K^+\pi^-$ can come only from a B^0 whereas $K^-\pi^+$ were necessarily produced in a $\bar{B}^0$ decay. The draw-backs are that the predictions both on the branching ratio and the CP asymmetry are rather uncertain and that the decay distributions in proper time are given by a single exponential, the time evolution does therefore not provide a striking signature — in contrast to the scenario I sketch next.

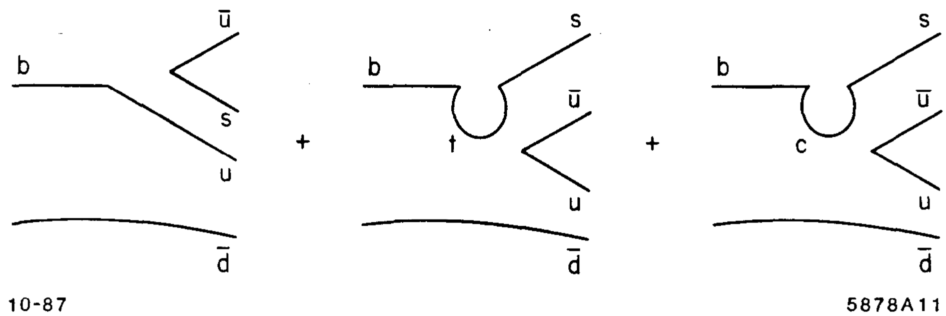

Fig. 11. Quark diagrams for $\overline{B}^0 \to K^- \pi^+$.

(iii) CP Violation and Mixing

(a) Semileptonic decays

As discussed in Chapter III, $B^0 - \bar{B}^0$ mixing is conveniently searched for via like-sign di-leptons. It then appears quite natural to search for a CP asymmetry there:

$$a_{SL} = \frac{\sigma(B^0\bar{B}^0 \to \ell^+\ell^+X) - \sigma(B^0\bar{B}^0 \to \ell^-\ell^-X)}{\sigma(B^0\bar{B}^0 \to \ell^+\ell^+X) + \sigma(B^0\bar{B}^0 \to \ell^+\ell^-X)} \quad . \tag{201}$$

Unfortunately detailed calculations yield a rather sobering result [48]:

$$a_{SL} = \frac{\mathrm{Im}\,(\Gamma_{12}/M_{12})}{1 + \frac{1}{4}|\Gamma_{12}/M_{12}|^2} \sim \begin{cases} \lesssim 10^{-3} \\ \lesssim 10^{-4} \end{cases} \text{for} \begin{array}{c} B_d \\ B_s \end{array} \text{mesons} \quad . \tag{202}$$

The smallness of CP asymmetries in semileptonic B decays is a rather general result in a short-distance ansatz like a quark box description since then $|\Gamma_{12}| \ll |M_{12}|$. In K_L decays this feature is vitiated by long distance effects leading to $|\Gamma_{12}| \sim |M_{12}|$. This is however not the end of the story. For (202) can be re-expressed as

$$a_{SL} = \frac{1 - |p/q|^4}{1 + |p/q|^4} \simeq \frac{4\mathrm{Re}\,\bar{\epsilon}}{|1 + \bar{\epsilon}|^2} = 1 - |\alpha|^2 \tag{203}$$

(202) therefore tells us only that $\mathrm{Re}\,\bar{\epsilon}$ is extremely tiny. I have already discussed that $\mathrm{Im}\,\bar{\epsilon}$ *per se* is not an observable — yet there is a loophole in this apparent impasse.

(b) Nonleptonic decays

The basic idea is quite simple [48]: find a final state f that is common to both B^0 and $\bar{B}^0$ decays

$$\begin{array}{ccc} B^0 & & \\ & \searrow & \\ & & f \\ & \nearrow & \\ \bar{B}^0 & & \end{array} \tag{204}$$

f can be either a CP eigenstate, for example

◇ $\overset{(-)}{B^0} \rightarrow D\bar{D}, \psi K_S, \pi\pi$ like $\overset{(-)}{K^0} \rightarrow \pi\pi$

or not, for example

◇ $\overset{(-)}{B^0} \rightarrow D^-\pi^+, D^+\pi^-$.

I will discuss only the first case for simplicity; the second case can be analyzed along rather similar lines (though beset with more uncertainties) as can be found in the literature.

A little theorem can help to illustrate the situation: let $B_{\rm neut}$ denote any combination of B^0 and $\bar{B}^0$ mesons, be f a CP eigenstate of definite CP parity. CP violation has then be observed in B decays if the time dependence for the decay rate $B_{\rm neut} \rightarrow f$ is found to be different from a ***single pure*** exponential, *i.e.*,

$$\frac{d}{dt} e^{\Gamma t} \mathrm{rate}(B_{\rm neut}(t) \rightarrow f) \neq 0 \qquad \text{for all } \Gamma \quad . \tag{205}$$

The proof is very elementary: assume CP to be conserved. Then the mass eigenstates are the CP eigenstates $\mathrm{CP}|B_\pm\rangle = \pm|B_\pm\rangle$; furthermore $B_+ \rightarrow f_+$, $B_- \not\rightarrow f_+$ for $\mathrm{CP}|f_\pm\rangle = \pm|f_\pm\rangle$. Thus

$$\mathrm{rate}\,(B_{\rm neut}(t) \rightarrow f_+) = \mathrm{rate}\,(B_+(t) \rightarrow f_+) \propto e^{-\Gamma_+ t}\ \text{constant} \quad . \tag{206}$$

q.e.d.

One can be even more specific and show that the most general time evolution can be expressed by four terms:

$$\begin{aligned} \mathrm{rate}\,(B_{\rm neut}(t) \rightarrow f) \propto e^{-\Gamma_1 t} \Big\{ & 1 + Ae^{-\Delta\Gamma t} + Be^{-\frac{1}{2}\Delta\Gamma t}\cos(\Delta m t) \\ & + Ce^{-\frac{1}{2}\Delta\Gamma t}\sin(\Delta m t) \Big\} \quad . \end{aligned} \tag{207}$$

Let me just state (the details can be found in Ref. 48) that in the KM ansatz one estimates quite generally

$$A \simeq 0 \simeq B \quad , \qquad \Delta\Gamma \ll \Delta m \tag{208}$$

and accordingly

$$\begin{aligned} \text{rate}\left(B^0(t) \to f\right) &\propto e^{-\Gamma t}\left(1 + \sin \Delta m t \, \text{Im} \frac{q}{p} \bar{\rho}_f\right) \\ \text{rate}\left(\bar{B}^0(t) \to f\right) &\propto e^{-\Gamma t}\left(1 - \sin \Delta m t \, \text{Im} \frac{q}{p} \bar{\rho}_f\right) \end{aligned} \tag{209}$$

using the definitions

$$\begin{aligned} &\left|B^0(t)\right\rangle = g_+(t)\left|B^0\right\rangle_0 + \frac{q}{p} g_-(t)\left|\bar{B}^0\right\rangle_0 \\ &\left|\bar{B}^0(t)\right\rangle = \frac{p}{q} g_-(t)\left|B^0\right\rangle_0 + g_+(t)\left|\bar{B}^0\right\rangle_0 \\ &\frac{p}{q} \equiv \frac{1+\bar{\epsilon}}{1-\bar{\epsilon}} = \frac{1}{\alpha} \quad ; \qquad g_\pm(t) = \frac{1}{2} e^{-\Gamma t} e^{i m_1 t}\left(1 \pm e^{i \Delta m t}\right) \\ &\bar{\rho}_f = \frac{A(\bar{B}^0 \to f)}{A(B^0 \to f)} \quad . \end{aligned} \tag{210}$$

Homework Problem 8

— What is the intuitive reason for the CP asymmetry being proportional to $\sin \Delta m t$, Eq. (209)?

Equation (209) contains two crucial features:

(i) No asymmetry can be *observed* for $\Delta m \to 0$, *i.e.*, when there is no mixing.

(ii) It is the factor $\text{Im}\,\frac{q}{p}\bar{\rho}_f$ that is *intrinsically* connected with CP violation. Actually only the combination $\frac{q}{p}\bar{\rho}_f$ can be an observable: for a change in the phase convention

$$\left|\bar{B}^0\right\rangle \;\to\; e^{i\delta}\left|\bar{B}^0\right\rangle \tag{211}$$

leads to

$$\begin{aligned} \frac{q}{p} &\to \frac{q}{p} e^{-i\delta} \\ \bar{\rho}_f &\to e^{i\delta}\,\bar{\rho}_f \end{aligned}$$

yet

$$\frac{q}{p}\,\bar{\rho}_f \;\rightarrow\; \frac{q}{p}\,\bar{\rho}_f \quad . \tag{212}$$

The relation

$$\mathrm{Im}\,\frac{q}{p}\,\bar{\rho}_f = \mathrm{Im}\,\frac{q}{p}\,\mathrm{Re}\,\bar{\rho}_f + \mathrm{Re}\,\frac{q}{p}\,\mathrm{Im}\,\bar{\rho}_f$$

together with

$$\mathrm{Im}\,\alpha = \mathrm{Im}\,\frac{q}{p} \simeq -\frac{2\mathrm{Im}\,\bar{\epsilon}}{1+\mathrm{Im}^2\bar{\epsilon}}$$

for $\mathrm{Re}\,\bar{\epsilon} \ll 1$ shows that $\mathrm{Im}\,\bar{\epsilon}$ is not completely devoid of physical meaning if embedded properly.

From it one learns also another subtlety: as long as CP violation is observed in *just one* of these nonleptonic B decays — blessed be the day when it arrives — it does not make any sense at all to distinguish between CP violation in mixing — $\mathrm{Im}\,\frac{q}{p} \neq 0$ — or in the decay — $\mathrm{Im}\,\bar{\rho}_f \neq 0$: a simple change in the phase convention for $\bar{B}^0$ will take you from one case to the other. Only when at least two decay modes have been analyzed with sufficient sensitivity, can one make a meaningful statement: if

$$\mathrm{Im}\,\frac{q}{p}\,\bar{\rho}\bigg|_{f_1} \neq \mathrm{Im}\,\frac{q}{p}\,\bar{\rho}_f\bigg|_{f_2} \tag{213}$$

then one has necessarily observed direct CP violation — $\mathrm{Im}\,\bar{\rho}_f \neq 0$ — as well. If on the other hand

$$\mathrm{Im}\,\frac{q}{p}\,\bar{\rho}\bigg|_{f_1} = \mathrm{Im}\,\frac{q}{p}\,\bar{\rho}_f\bigg|_{f_2} \tag{214}$$

then the super-weak ansatz based on $\mathrm{Im}\,\bar{\rho}_f = 0$ is at least consistent with the data.

I have already stated that $|q/p| \simeq 1$, Eqs. (202) and (203). Furthermore one can show that typically $|\rho_f| \simeq 1$ as well and thus $|\frac{q}{p}\bar{\rho}_f| \simeq 1$.

Homework Problem 9

Show that

$$\left|\frac{A(\bar{B}^0 \rightarrow \psi K_S)}{A(B^0 \rightarrow \psi K_S)}\right| = 1 \tag{215}$$

holds.

Therefore $\frac{q}{p}\bar{\rho}_f$ is — to a very good approximation — given by a unit vector in the complex plane:

$$\mathrm{Im}\,\frac{q}{p}\,\bar{\rho}_f = \sin(\beta+\gamma)\ , \quad \beta = \arg\left(\frac{q}{p}\right)\ , \quad \gamma = \arg\bar{\rho}_f$$

which can be expressed purely as a ratio of KM parameters. For $B \to \psi K_S$, *e.g.*, one finds

$$\frac{q}{p}\,\bar{\rho}_{\psi K_S} \simeq \frac{(V(td)V^*(tb))^2}{|V(td)V^*(tb)|^2}\,\frac{(V^*(cs)V(cb))^2}{|V^*(cs)V(cb)|^2} \tag{216}$$

$$\mathrm{Im}\left(\frac{q}{p}\,\bar{\rho}\right)_{\psi K_S} \simeq 0.2 - 0.5 \tag{217}$$

i.e., asymmetries of order 10 percent that could even range as high as 50 percent!

Homework Problem 10

The expression in (216) is apparently not invariant when changing the phase of the s and d quark fields. How can this paradox be resolved? (Hint: it can be shown that the CP asymmetries disappear if one sums over $B \to \psi K_S$ and $B \to \psi K_L$.)

There are strong advantages to this method to search for CP violation:

(α) The CP asymmetries are large, *i.e.*, $\sim \mathcal{O}(10\%)$, and — in contrast to the branching ratios — quite reliably predicted in terms of the basic quantities, namely the KM parameters and m(top). If a search fails despite having reached the predicted sensitivity level, there exists no "plausible deniability".

(β) The very special dependence of the decay rate on the (proper) time provides a quite spectacular signature, not easily faked by background sources.

Unfortunately there exist clear draw-backs as well.

(γ) The branching ratios of the interesting decay modes are nothing to brag about, for they do not exceed the 10^{-3} level.

(δ) Since the final state is fundamentally not flavor specific, it does not tell us whether it came from a B^0 or $\bar{B}^0$ decay. To be able to define a CP asymmetry one has to obtain some independent information on the flavor of the decaying neutral B meson. This need for "flavor tagging" is bound to cost us dearly in statistics.

One is then tempted to increase statistics by summing over different final states. This can be achieved — but only when considerable care and caution is applied [48].

Homework Problem 11

Show that the ***sign*** of the asymmetry depends on the CP parity of the final state f.

Let us conclude this chapter by some general remarks:

- large CP asymmetries ($\sim \mathcal{O}(10\%)$) are expected for certain B^0 decays, if the KM ansatz represents the dominant source of CP violation.
- A high premium has to be attributed to a good proper time resolution.
- It represents a highly challenging experimental program which appears to go clearly beyond existing or approved machines. In any case new capabilities in detector technologies are required.

Epilogue

Our study of heavy flavour decays has reached a very promising stage: in the language of mountaineering, which is quite appropriate to the Santa Fe environment, one can say that we have established our base camp. The arrival of more supplies and equipment has been promised for the near future. Scouting parties — like the Conquistadores of the past — have returned with intriguing reports on riches they have glanced: They have found access routes to the beauty mountain standing out in the clear air. They have gotten close to the peaks of the charm and τ mountains before descending fog forced them to turn back for now. They have found another thick wall of fog starting at a slightly higher altitude than the camp: echos provide clear evidence for top hiding in there.

None of the climbs to be undertaken in the future will be quick or easy; they will require painful displays of stamina , as emphasized by Max Weber. He was referring to politics, yet are physics and politics really that different?

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