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STRING PERTURBATION THEORY AND EFFECTIVE LAGRANGIANS*

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ABSTRACT

We isolate logarithmic divergences from bosonic string amplitudes on a disc. These divergences are compared with 'tadpole' divergences in the effective field theory with a cosmological term, which also contains an effective potential for the dilaton. Also, corrections to β-functions are compared with variations of the effective action. In both cases we find an inconsistency between the two. This is a serious problem which could undermine our ability to remove divergences from the bosonic string.

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This talk is based on the work done in collaboration with W. Fischler and L. Susskind.[1]

Since the recent resurgence of interest in the theory of strings as the most fundamental objects in physics, a new approach to string dynamics has emerged. This approach, which did not figure during the earlier golden age of strings in the seventies, was pioneered in the papers by Callan, Martinec, Perry and Friedan,[2] Sen[3] and Lovelace.[4] These authors start by writing down a renormalizable two-dimensional field theory describing propagation of strings in classical backgrounds which can be viewed as condensates of massless string modes. If conformal invariance is considered to be a fundamental principle of string theory, then the vanishing of the β-functions of the 2-d field theory should insure consistent string propagation. Therefore we find that strings can propagate not only in flat 26-dimensional space, but also in curved space with some classical antisymmetric tensor and dilaton backgrounds. The β-functions can be computed in the loop expansion of the 2-d field theory. The resulting expressions are functions of background fields in spacetime with increasing number of derivatives, depending on how many field theory loops have been taken into account.

The amazing feature of the equations resulting from setting all the β-functions to zero is that they turn out to be variations of a generally covariant space-time action functional depending on the massless background fields. This equivalence was discovered and tested to low orders in the derivative expansion by a number of authors whose work relied primarily on the sigma-model background field method.[2,5,6] Although a general proof of equivalence between conformal invariance conditions of the world-sheet theory and variational equations of a spacetime effective action is yet to be constructed, recent work by a number of authors[7] has made important steps towards such a proof.

A crucial property of the effective action found via the sigma-model route is that it generates string scattering amplitudes for the massless modes. In other words, it is the effective action in the standard field theory sense. An immediate question that comes to mind is how to incorporate into our effective action the effects of string loop dynamics. Naively this appears to pose serious problems for the β-function method because of the following simple argument. The β-functions are sensitive only to the short distance effects on the string world sheet. Therefore they are independent of the world sheet topology: the standard field theory β-functions computed on a sphere (string tree level), a torus (one string loop), etc. are all going to be the same. If we believe that conformal invariance generates string equations of motion, this would imply that the effective action is not renormalized by string loop effects, which is in direct contradiction with non-vanishing of scattering amplitudes beyond tree level.

A natural resolution of this apparent paradox was proposed by Fischler and Susskind.[8] The basic observation is that beyond tree level the Polyakov functional integral prescription for S-matrix calculations contains more integrations than at the tree level. In order to calculate an n-particle amplitude one is instructed to integrate over the modular parameters of the surface with n punctures. For example, the fact that is crucial for this talk is that a disc with n punctures has more modular parameters than a sphere with n puncures. A more familiar statement is that on a sphere SL(2,C) invariance allows us to fix locations of 3 vertex operators, while on a disc SL(2,R) invariance allows for fixing only one closed string vertex, and the angular coordinate of the other. The extra integrations that need to be carried out on a disc give rise to extra logarithmic divergences beyond those encountered at string tree level. These divergences give rise to the loop corrections to β-functions. Therefore, for applications to strings, one is no longer interested in an ordinary two-dimensional field theory, but rather in a peculiar combination of 2-d field theories defined on world sheets with different topology and supplied with a prescription for integration over the moduli of these world sheets.

After having identified the source of the loop corrections to the string equations of motion it is important to check that the resulting equations follow the pattern discovered at the string tree level. Namely, they should be equivalent to variations of the loop corrected effective action whose form is dictated by general covariance and simple counting of the powers of the string coupling constant. In this talk I will present a calculation of the simplest consistency check, the one that concerns the cosmological term which is simultaneously the effective potential for the dilaton. The result of our calculation is rather perplexing: the loop corrections to β-functions do not turn out to be consistent with the expected form of the effective action! This may be a serious problem that could primarily affect our understanding of the behaviour of cosmological constant in string theory. I should mention that a conclusion opposite to ours have been reached in a recent preprint by Callan et. al.[9] Since the methods used there are quite different from ours, we do not fully understand the nature of this discrepancy.

Let me now proceed to a more formal explanation of the problem. The propagation of the closed bosonic string in gravitational and dilaton backgrounds is governed by the following 2-d action:[10,2]

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{\gamma}(\gamma^{\mu\nu}g_{ij}(X)\partial_\mu X^i\partial_\nu X^j - \frac{\alpha'}{2}\phi(X)R^{(2)}) - \frac{1}{4\pi}\oint ds\phi(X)\kappa \quad (1)$$

where $\gamma_{\mu\nu}(\sigma_1,\sigma_2)$ is the world sheet metric, $g_{ij}(X)$ is the 26-dimensional metric, and $\phi(X)$ is the dilaton field which couples to the world sheet and boundary curvatures. The fields $g_{ij}(X)$ and $\phi(X)$ can be thought of as an infinite collection of couplings in the 2-d field theory. These couplings become renormalized and satisfy renormalization group equations

$$\frac{\partial g_{ij}}{\partial \log\lambda} = \beta_{ij}(g_{ij},\phi) \quad (2)$$

$$\frac{\partial \phi}{\partial \log\lambda} = \beta_{\phi}(g_{ij},\phi) \quad (3)$$

Bypassing certain technicalities associated with the choice of renormalization scheme, the conformal invariance conditions are

$$\beta_{ij} = \beta_\phi = 0 \quad (4)$$

Remarkably, these equations can be derived from a generally covariant action[2]

$$I = \int d^{26}X\sqrt{g}e^{\phi}(R + (\partial\phi)^2 + O(\alpha')) \tag{5}$$

The $O(\alpha')$ corrections in (5) are higher derivative terms where each additional pair of derivatives introduces a factor of α'.[2,5]

The processes leading to the effective action of (5) are string tree graphs calculated on a spherical world sheet. The factor $\exp(\phi)$ is understood as follows: the path integral for the string has a factor

$$\exp(\frac{\phi_0}{8\pi}\int d^2\sigma\sqrt{\gamma}R^{(2)}) = \exp(\phi_0(1-g)) \tag{6}$$

where ϕ_0 is the zero momentum part of ϕ and g is the genus of the world sheet. For a sphere ($g = 0$) the effective action must be weighted by $\exp(\phi_0)$, and locality requires that this be replaced by $\exp(\phi)$.

Let us now consider corrections to the action (5) due to processes with a small hole in the world sheet. Such processes occur in the theory of coupled open and closed strings. Since the disc has genus 1/2, the action term with the least possible number of derivatives is

$$\delta I = \int d^{26}X\sqrt{g}\exp(\phi/2)J \tag{7}$$

where J is a constant.

To understand the leading corrections to closed string β-functions we must consider closed-string scattering amplitudes on a disc. We will represent the disc by a sphere with a hole and for simplicity look at three-particle scattering since it illustrates the basic point. As usual, the sphere is stereographically projected onto the complex plane. We may integrate over positions of two of the vertex operators. Alternatively we can hold the vertices fixed and integrate over the radius a and location z of the hole. Then the tadpole divergences occur in the

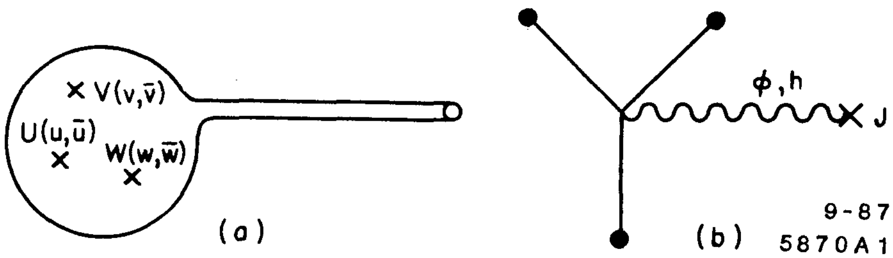

Figure 1. The tadpole configurations: a) in string theory, b) in field theory.

small hole limit which is conformally equivalent to a sphere with a long tube attached (fig. 1a)). After this change of integration variables, the amplitude is

$$\exp(\phi_0/2)\int daa^{-3}\int d^2z\mid v-w\mid^2\mid v-u\mid^2\mid u-w\mid^2 < V(v,\bar{v})U(u,\bar{u})W(w,\bar{w}) > \tag{8}$$

where the expectation value is computed on a sphere with a hole of radius a centered at z, and V,U,W are vertex operators for emission of arbitrary closed string physical states. The Green's function needed to compute (8) is given by

$$< X(u,\bar{u})X(w,\bar{w}) >= -\log\mid u-w\mid^2 - \log\mid 1-\frac{a^2}{(z-u)(\bar{z}-\bar{w})}\mid^2 \tag{9}$$

Expanding (9) for small a gives

$$< X(u,\bar{u})X(w,\bar{w}) >= -\log\mid u-w\mid^2 - \frac{a^2}{(z-u)(\bar{z}-\bar{w})} - \frac{a^2}{(z-w)(\bar{z}-\bar{u})} \tag{10}$$

The $O(a^2)$ contribution to the propagator is identical to the effect of the operator $:\partial X_i \bar{\partial} X^i:$ inserted at the point z (normal ordering means that we drop the self-contraction). Thus, to obtain the coefficient of the logarithmic divergence in the amplitude (8), we expand the integrand to order a^2 and integrate over a:[11]

$$e^{\phi_0/2}\int_\lambda \frac{da}{a}\int d^2z\mid v-w\mid^2\mid v-u\mid^2\mid u-w\mid^2 \left\langle V(v,\bar{v})U(u,\bar{u})W(w,\bar{w}){:}\partial X_i\bar{\partial}X^i{:}(z,\bar{z})\right\rangle \tag{11}$$

where the expectation is now evaluated on a sphere. This logarithmic divergence in the disc amplitudes provides the leading correction to the tree level β-functions for the closed string modes. To find these corrections, we need to decompose the operator insertion that replaces a small hole on a sphere in terms of the operators that enter the two-dimensional action (1). Using dimensional regularization, de Alwis has found that[12]

$$:\partial X_i \bar{\partial} X^i := \partial X_i \bar{\partial} X^i + \frac{d}{8}\sqrt{\gamma} R^{(2)} \tag{12}$$

where we have set $\alpha' = 2$. Recalling the logarithmically divergent counterterms that are necessary on a sphere, we find that the β-functions corrected by small holes are:

$$\beta_{ij} = R_{ij} - \nabla_i \nabla_j \phi + J g_{ij} \exp(-\phi/2) + \ldots$$

$$\beta_\phi = -1/2 \nabla^2 \phi - 1/2(\partial\phi)^2 - \frac{d}{2} J \exp(-\phi/2) + \ldots \tag{13}$$

J is the normalization factor of the insertion that we did not bother to fix. A subtle point in the formulae (13) is that the β-function terms $-\nabla_i \nabla_j \phi$ and $-1/2(\partial\phi)^2$ are absent unless we implement a divergent shift of the sigma model target space variables: $X^i \to X^i + \log\lambda \partial^i \phi$.[5] This particular choice of renormalization scheme is necessary even at tree level to insure that the β-functions are equivalent to variations of the effective action. However, it is easy to check that the leading corrections arising from small holes cannot be shifted by different choices of renormalization scheme on a sphere (they do not contain any spacetime derivatives).

We observe now that, even though the first two terms in (13) are equivalent to variations of a spacetime action, the small hole corrections violate this equivalence. They cannot be obtained by adding to the action the 'cosmological term', which is fixed up to normalization by general covariance and counting of string coupling constants. This is the basic result of our work. To make our discussion of this important point as clear as possible, let us enumerate all the zero-momentum vertex operators relevant for our problem. To find the correct soft graviton and soft dilaton operators, it is convenient to carry out a rescaling on the couplings of the two-dimensional theory, $g_{ij} \to g_{ij} \exp(-2\phi/d-2)$ which, as will be shown later, is necessary to identify the physical dilaton.[10] Then we can read off the vertex operators as the coefficients of terms linear in dilaton ϕ and graviton h_{ij} in the 2-d action (1) (we will state all operators up to overall normalization constants). Soft graviton emission is given by an insertion of $\partial X^i \bar{\partial} X^j$ while the soft dilaton emission is produced by

$$\partial X_i \bar{\partial} X^i + \frac{d-2}{8}\sqrt{\gamma}R^{(2)} =: \partial X_i \bar{\partial} X^i : -\frac{1}{4}\sqrt{\gamma}R^{(2)} \tag{14}$$

We found that the operator insertion that must replace a small hole to satisfy the desired equivalence with the effective action is

$$: \partial X_i \bar{\partial} X^i : +\frac{1}{4}\sqrt{\gamma}R^{(2)} \tag{15}$$

Please note that, although an accidental conspiracy of factors makes the operator (15) appear as a soft dilaton operator with a flipped sign, their origin is very different. Actually, the operator (15) must be a linear combination of a physical operator (dilaton) and an unphysical operator (the trace of graviton). This is so because the cosmological constant term creates a tadpole for the trace of graviton. If one thinks of tadpoles as injecting the operators onto the sphere and substitutes correct tadpole and propagator factors, one recovers the precise mixture of the dilaton and graviton operators that must replace the hole for consistency with the effective action. This procedure is an alternative to finding precisely which combinations of the variations of the effective action should be equal to the β-functions and it gives identical results. Now it should be clear to you why the old statement of Ademollo et al.[13] that the logarithmic divergences should be proportional to soft dilaton emission amplitudes cannot be compatible with the idea of effective action.

As elucidated in the previous discussion, the precise insertion we are finding by a direct string-theoretic argument is : $\partial X_i \bar{\partial} X^i$:. It disagrees with what is necessary for consistency of the effective action as well as with the Ademollo et al. theorem.[13] Thus, in order to find agreement with the effective action, we need to find an additional renormalization of the curvature coupling, which is topological when integrated over the sphere. Since identification of such terms is notoriously subtle, we have carried out a test of our results.

We calculated the logarithmically divergent part of a three-graviton amplitude on a disc and compared it with tadpole diagrams of the effective field theory, specified by the action $I + \delta I$. The string calculation amounts to carrying out the integration over the position z and radius a of the hole in (8). For each radius a, the z integration covers the whole plane excluding only those regions which would cause one of the vertex operators to be inside the hole. This exercise turns out to have a simple answer: the logarithmically divergent part of the amplitude is proportional to $\sqrt{\alpha'}\partial/\partial\sqrt{\alpha'}A_{tree}(\rho_i, k_i)$ where $\sqrt{\alpha'}\partial/\partial\sqrt{\alpha'}$ effectively counts the power of momenta in a given term of the corresponding tree amplitude for three gravitons.

This answer is to be compared with the effective field theory divergences of the form $1/k^2|_{k^2=0}$ which arise from the tadpole graphs of fig. 1b). We identify this divergence with the logarithmic divergence in the world sheet cut-off:

$$\frac{1}{k^2} = \int_{\lambda}^{1} da a^{k^2-1}$$

A very important feature of the field theory calculation is the presence of a tadpole for an unphysical state, the trace of graviton. Contrary to what is sometimes said in the literature, propagation of this state into vacuum provides an additional source of divergence.

In order to calculate the dilaton contribution to the graphs of fig. 1b), we recall the soft dilaton theorem. The emission amplitude for a zero-momentum dilaton is given by[13]

$$A_\phi = -\frac{1}{d-2}(\sqrt{\alpha'}\frac{\partial}{\partial\sqrt{\alpha'}} - 2)A \tag{16}$$

where A is the amplitude without the soft dilaton. Multiplying this by the propagator and tadpole factors from the conveniently rescaled action ($g_{ij} \to g_{ij}\exp(-2\phi/d-2)$)

$$I = \int d^{26}X\sqrt{g}(R - \frac{(\partial\phi)^2}{d-2} + ...) + \int d^{26}X\sqrt{g}\exp(\phi(-\frac{2}{d-2} - \frac{1}{2}))J \tag{17}$$

we obtain the dilaton contribution to the divergence. Note that in (17), as opposed to (5), the dilaton has a standard kinetic term (up to a factor). The mixing between dilaton and graviton has been eliminated. This identifies the dilaton field in (17) as proportional to the physical dilaton.[2,10]

Since the gravitational couplings are determined by general covariance, we can derive a similar theorem for the graviton contribution to the divergence (we found it convenient to work in the standard harmonic gauge). Adding the two, we find that the net divergence in the amplitude is proportional to $(\sqrt{\alpha'}\partial/\partial\sqrt{\alpha'} + 2)A_{tree}$. The string and field theory answers disagree! As expected, the missing β-function insertion $\sim \sqrt{\gamma}R^{(2)}$ translates into a missing logarithmic divergence proportional to the tree amplitude. The reason is that an integrated curvature insertion on a sphere produces an answer proportional to the tree amplitude. Therefore, our results, although inconsistent with what we expected to find, possess some internal consistency.

I have no time to dwell on quite a few attempts we made to find the missing term. None of them have led to clear-cut results. I would like to add a note of caution, however. The amplitudes that we considered are divergent. At this time there is no universally applicable prescription for regularizing divergences in string theory. Although the 'small fixture' regularization that we introduced is physically plausible and is easy to carry out, it is not inconceivable that some other scheme will identify the extra term needed for consistency. Such a scheme may be available only in the framework of closed string field theory. Alternatively one could look at string amplitudes directly in non-flat backgrounds. By adjusting the background to eliminate all divergences we would then find the loop-corrected equations of motion. At this point both of these approaches appear to be difficult.

I would like to conclude my talk by mentioning a possibly interesting extension of our results. The problem we have found occurs on a disc when all the closed string vertex operators approach each other. Let us consider an equivalent calculation on a surface of arbitrary topology. The situation when all the vertices approach each other can be conformally mapped into a sphere with a small fixture of any genus. Using multipole expansion for the Green's function on an arbitrary surface we can show that the term quadratic in the size of the fixture is proportional to an insertion of the same operator $:\partial X \cdot \bar{\partial} X:$ replacing the fixture on a sphere. As we argued previously, the effective action requires that the operator replacing the fixture depend on its genus. This argument would generalize the inconsistency we are finding to a surface of arbitrary genus. Unfortunately it is hard to make this argument precise due to existence of overlapping divergences in all higher-order calculations. For example, if we calculate closed string amplitudes on an annulus, there are going to be $(\log \lambda)^2$ divergences due to the fact that each hole gives rise to a tadpole. Thus, a new apparatus of stringy renormalization is needed for subtracting higher order divergences and exposing the β-functions. We know of at least one case of a higher genus calculation, however, where this is not necessary. This is the recently investigated case of D-term supersymmetry breaking in string theory.[14] The first place where the dilaton tadpole shows up is a double torus. Due to the arguments stated above, we expect a careful identification of the insertion on a sphere to reveal a mismatch with the effective action.

The only topology where the insertion turns out to be consistent with the effective action considerations is a torus. This may have to do with the fact that only in this case the integrated curvature vanishes (the tadpole for a state that couples to $R^{(2)}$ is zero). We should remark here that, in a theory of closed bosonic strings only, the torus provides the leading contribution to tadpoles. We have carried out an explicit calculation on a torus and convinced ourselves that the insertion that must replace a small handle is $: \partial X \cdot \bar{\partial} X :$, the same as for a small hole. For the case of genus 1 this insertion turns out to be consistent with the effective action considerations which proceed in complete analogy with our calculations for genus 1/2. However, as explained above, we expect trouble once we move on to genus 2.

Let me point out that the small hole limit in the sense of the above paragraph is quite different from the small hole limit of the annulus diagram of the open string theory. If one inserts open string vertex operators on the outer boundary of an annulus, there is a logarithmic divergence that occurs in the limit when the radius of the inner boundary vanishes. In this case, however, this variable is a modular parameter of an empty annulus.[15] Therefore the ratio of the determinants needed to calculate the zero-point function depends on the radius of the hole. This brings in an extra term into the operator that must replace the hole. It is not easy, however, to identify this extra term as a curvature insertion. A reasonable thing to do is to carry out a comparison of the logarithmically divergent part of the amplitude for three gauge bosons with the effective field theory. This work is in progress now (the issue is complicated by the presence of open string tachyons in the bosonic model).[16] However, a similar comparison of the four gauge boson amplitude in open superstring theory (for a gauge group not equal to SO(32)) appears to yield agreement with the effective action including the tadpole terms. Therefore, we conjecture that the problem we found does not afflict open string amplitudes but is only present for pure closed string amplitudes. It is suggestive to compare the situation with A. Strominger's associativity anomaly which occurs only in the tadpole limit in pure closed string amplitudes.[17] Perhaps, clarifying the relation between our calculation and the associativity anomaly could shed some further light on the problem.

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