# DESIGN AND PERFORMANCE OF <br> - A DOUBLE-ARM 180 ${ }^{\circ}$ SPECTROMETER SYSTEM FOR MAGNETIC ELECTRON SCATTERING FROM DEUTERIUM* 

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#### Abstract

The design and operation of a double-arm $180^{\circ}$ spectrometer system for elastic and inelastic electron scattering from deuterium is described. The system consisted of a chicane of dipole magnets to transport an incident electron beam to a target, an upstream QQQDD spectrometer with a bend angle of $65^{\circ}$ to analyze backscattered electrons, a downstream QQQDDDD spectrometer with a bend angle of $66^{\circ}$ to detect recoil particles produced at $0^{\circ}$, and a moveable beam dump. The large acceptance of the system allowed measurements of the deuteron magnetic elastic $B\left(Q^{2}\right)$ and inelastic $W_{1}\left(\nu, Q^{2}\right)$ structure functions at high momentum transfers. The performance is compared to the predictions of a Monte Carlo simulation program.


- Submitted to Nuclear Instruments and Methods

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## 1. INTRODUCTION

This paper describes the design, operation and performance of a doublearm large acceptance spectrometer system for magnetic electron scattering from deuterium. The upstream arm was arranged to focus and momentum analyze electrons scattered near $180^{\circ}$ to a set of detectors. A portion of this arm also transmitted the primary beam to the target. The downstream arm transported nuclei recoiling near $0^{\circ}$ to another set of detectors, as well as directing the transmitted electron beam to a water-cooled dump. This was the first system ever built for measuring $180^{\circ}$ electron scattering in which the recoil particles are detected in coincidence with the scattered electrons.

The apparatus was installed in End Station A (ESA) at the Stanford Linear Accelerator Center (SLAC) for experiment NE4. ${ }^{[1]}$ The high intensity electron beam from the Nuclear Physics Injector (NPI) ${ }^{[2]}$ was used to make cross section measurements for elastic, threshold inelastic, and quasi-elastic electron scattering from deuterium. These measurements extended our knowledge of the deuteron magnetic structure functions $B\left(Q^{2}\right)$ and $W_{1}\left(\nu, Q^{2}\right)$ and allowed an important comparison with the previously measured forward angle structure functions $A\left(Q^{2}\right)$ and $W_{2}\left(\nu, Q^{2}\right)$ at high values of the four momentum transfer squared $Q^{2}$.

The need for a complicated spectrometer system was dictated by several factors. The principal factor was that the ratio $B\left(Q^{2}\right) / A\left(Q^{2}\right)$ was predicted to be small ( 0.05 to 0.005 ) in the $Q^{2}$ region of interest 1 to $5(\mathrm{GeV} / \mathrm{c})^{2}$. To enhance the contribution from the magnetic term $B\left(Q^{2}\right)$, the electron scattering angle needed to be close to $180^{\circ}$ where the contribution from $A\left(Q^{2}\right)$ vanishes. This can be seen from the equation for elastic scattering:

$$
\begin{equation*}
\quad \frac{d \sigma}{d \Omega}=\frac{\alpha^{2} E^{\prime}}{4 E^{3} \sin ^{4} \frac{\Theta}{2}}\left[A\left(Q^{2}\right) \cos ^{2} \frac{\Theta}{2}+B\left(Q^{2}\right) \sin ^{2} \frac{\Theta}{2}\right] \tag{1.1}
\end{equation*}
$$

where $\alpha$ is the fine structure constant, $\Theta$ is the electron scattering angle, and $E$ and $E^{\prime}$ are the incident and final electron energies. The selection of $180^{\circ}$ as
the electron scattering angle also resulted in measurements of $W_{1}$ only, with no contributions from $W_{2}$, as can be seen from the equation for inelastic scattering:

$$
\begin{equation*}
\frac{d^{2} \sigma}{d \Omega d E^{\prime}}=\frac{\alpha^{2}}{4 E^{2} \sin ^{4} \frac{\Theta}{2}}\left[W_{2}\left(\nu, Q^{2}\right) \cos ^{2} \frac{\Theta}{2}+2 W_{1}\left(\nu, Q^{2}\right) \sin ^{2} \frac{\Theta}{2}\right] \tag{1.2}
\end{equation*}
$$

where $\nu$ is the energy transfer $E-E^{\prime}$.
The elastic cross section falls rapidly with $Q^{2}$, and was predicted to be as small as $10^{-41} \mathrm{~cm}^{2} / \mathrm{sr}$ at $Q^{2}=5(\mathrm{GeV} / \mathrm{c})^{2}$. Therefore, an electron spectrometer with the largest practical solid angle and the longest possible targets were needed to maintain reasonable counting rates. The design goals were for a solid angle on the order of 20 msr using liquid deuterium targets between 10 and 40 cm in length.

The small binding energy of the deuteron ( 2.2 MeV ) and the poor resolution caused by the energy losses of the incident and back-scattered electrons in thick targets would have made it impossible to distinguish elastic electron-deuteron scattering events from inelastic scattering events by detecting electrons only. Therefore, the deuterons recoiling near $0^{\circ}$ were detected in coincidence using the downstream arm. This arm had to be capable of spatially separating the positively charged deuterons from the degraded and multiply scattered electron beam and directing the beam to a well-shielded dump. It also needed enough bending power to sweep away all the low energy charged particle background. Our design goal was to have the acceptance for the recoil spectrometer be large and completely overlap that of the electron spectrometer for elastic coincidence kinematics. The purpose of this requirement was to facilitate the calculation of radiative corrections and minimize the errors in the calculations of the doublearm solid angles.

A large momentum acceptance for the electron spectrometer was desirable to minimize the data taking time for the single-arm inelastic measurements. A final constraint on the overall design parameters came from the desire to use
double-arm electron-proton elastic scattering for momentum calibration and to check the calculations of double-arm solid angles. The allowed range in deflection angles in the primary beam transport and beam dump lines were increased to allow ep calibrations to be made over a large momentum range.

An overview of the entire system is given in Section 2. In Section 3 the hardware and beam optics design is described in detail. The calibration of the apparatus is detailed in Section 4 while the performance compared to the Monte Carlo predictions is made in Section 5. Concluding remarks are given in Section 6.

## 2. OVERVIEW OF THE SYSTEM

Figure 1 shows the entire spectrometer system. The electron arm and beam transport chicane are shown in more detail in Figure 2. As in other $180^{\circ}$ systems, ${ }^{[3,4]}$ there were two overlapping channels on the electron side: one for the incident beam and one for the back-scattered electrons. The beam, after exiting the SLAC beam switchyard into End Station A, traveled through the chicane of bending magnets $\mathrm{B} 1-\mathrm{B} 2-\mathrm{B} 3$ and along the magnetic axis of the quadrupole triplet Q1-Q2-Q3 to impinge on the target. The back-scattered electrons were focused by the same triplet and transported through the bending magnets B3 and B4 to a detection system. This quadrupole triplet provided the large gathering power for the relatively low energy back-scattered electrons needed to achieve a large solid angle while having little effect on the transported incident beam. For a thin target, the solid angle was 22.5 msr averaged over a momentum acceptance range of $\pm 4 \%$. The momentum and scattering angle resolutions were typically $\pm 0.4 \%$ and $\pm 8 \mathrm{mr}$.

The ratio of incident to scattered electron momenta changed with the desired kinematics for both elastic and inelastic scattering. Therefore the beam deflection angle in magnet B3 also changed. The bending magnets B 1 and B 2 served only as primary beam transport elements. Magnet B 2 was placed symmetrically along the beam direction with respect to B 1 and B 3 , and the B 2 deflection angle $\theta_{B 2}$ was
twice the deflection angle $\theta_{B 1}$ in magnets B 1 and B3. The horizontal position of B2 and the bend angles $\theta_{B 1}$ and $\theta_{B 2}$ were adjusted as necessary to accommodate specific kinematic settings.

On the recoil side, shown in more detail in Figure 3, there were also two overlapping channels: one for the recoil nuclei and one for the transmitted beam. Both the electrons and deuterons passed through the quadrupole triplet Q4-Q5-Q6 before entering the bending magnet B5, which played the crucial role of separating the recoil particles from the electrons. The transmitted electron beam (degraded by radiation, ionization, and multiple scattering) was deflected towards a water-cooled dump. The positively charged recoil particles were bent in the other direction and transported through three additional bending magnets B6-B7-B8 to a set of detectors.

The focusing strengths of the quadrupole triplet Q4-Q5-Q6 were chosen to maximize the transmission of nuclei through the recoil spectrometer, while simultaneously achieving an acceptable beam spot size on the dump, taking into account the angular and momentum spread in both channels from kinematics and interactions at the target. For radiationless elastic scattering above $Q^{2}=1$ $(\mathrm{GeV} / \mathrm{c})^{2}$, almost every deuteron associated with an electron detected in the electron spectrometer was transmitted to the recoil detectors for a wide range of target lengths. For a thin target, the solid angle of the recoil spectrometer was 5.9 msr averaged over a momentum acceptance range of $2.0 \%$. The momentum resolution was typically $\pm 0.3 \%$ and the resolution in the deuteron recoil angle was $\pm 9 \mathrm{mr}$.

The ratio of the incident electron momentum to the recoil deuteron or proton momentum varies with kinematics. Since the recoil angle was fixed, this caused the beam deflection in magnet B 5 to change. Fortunately, in the kinematic range considered, this variation was only a few degrees. This dictated the need for a movable dump placed as close to the magnet B 5 as the shielding requirements allowed. The large distances between the detectors and sources of direct
background at the target and the beam dump allowed sufficient space for protective layers of concrete and iron shielding blocks.

The detection system of the electron side consisted of a set of multi-wire proportional chambers (MWPCs) for track reconstruction, two planes of scintillation counters for triggering and fast timing, a gas threshold Čerenkov counter and a total absorption shower counter for particle identification. On the recoil side there was a set of multi-wire proportional chambers and two segmented planes of plastic scintillators with a separation of 8 meters for single- and double-arm time-of-flight measurements.

## 3. DESIGN OF THE SYSTEM

The detailed optimization of the spectrometers was carried out using several tools. The computer code TRANSPORT ${ }^{[5]}$ was used to evaluate first- and second-order matrix elements for specific configurations, while the computer code TURTLE ${ }^{[6]}$ was used to evaluate solid angles using first- and second-order matrix elements. For the optimization of specific design parameters (field strengths, drift distances, bend angles, etc.), the TRANSPORT and TURTLE programs were found to be too slow, so a simple thin lens model computer program was developed to evaluate first-order matrix elements and solid angles and determine resolutions. The details of this computer model are given below. The final optimization of some of the system parameters was made using a ray-tracing Monte Carlo program ${ }^{[7]}$ that used measured magnetic field maps to transport particles through the magnets and took into account the effects of multiple scattering and ionization and radiative energy loss.

The spectrometer system was designed to utilize existing surplus magnets due to budget constraints. Magnets were chosen to achieve the largest possible solid angle for the momenta ranges of the double-arm ed kinematics. To achieve this goal, quadrupoles of short length, high gradient, and large aperture were needed, as were bending magnets with vertical gaps comparable to the apertures
of the quadrupoles. As a rule, the acceptance of a spectrometer is inversely proportional to its length. For this reason the drift lengths were generally kept as short as possible consistent with the moderate resolution requirements.

The next sub-sections give details on the thin lens model and the detailed designs of the electron spectrometer, beam transport system, recoil spectrometer, and beam dump. Additional details can be found in Ref. 8.

### 3.1 THIN LENS SPECTROMETER MODEL

This section describes the formalism of the thin lens model for spectrometer systems. The transfer action of each magnet was decomposed into a drift length followed by a lens of power P followed by another drift length. This method resulted in a simple exact optical model for the study of first-order matrix elements and resolutions of the system. It was also used to study the effects of geometric and magnetic parameters on the angular and momentum acceptance of the spectrometers.

In the thin lens equivalence a focusing quadrupole may be replaced by a thin lens of power $P=-K \sin (K L)$, surrounded on each side by a drift space of length $K^{-1} \tan (K L / 2)$, where $L$ is the effective length of the quadrupole. The parameter $K=\sqrt{g_{\circ} / p_{\circ}}$ represents the focusing strength of the field where $g_{\circ}$ is the gradient of the quadrupole and $p_{\circ}$ is the momentum of the particle. A defocusing quadrupole may be replaced by a thin lens of power $P=K \sinh (K L)$, surrounded on each side by a drift space of length $K^{-1} \tanh (K L / 2)$. A rectangular dipole magnet with uniform field may be replaced in the momentum dispersive plane by a focusing thin lens of power $P=-\sin \theta / R$, surrounded on each side by a drift space of length $R \tan (\theta / 2)$, where $R$ is the radius of curvature of the central trajectory and $\theta$ is the deflection angle. A pole face rotation of angle $\gamma$ acts as - a lens of power $P=-\tan \gamma / R$ in the non-bend plane and $P=\tan \gamma / R$ in the bend plane.

The matrix elements that describe the transformation of a particle with initial coordinates (in the horizontal plane) of $x_{\circ}$ and $\theta_{\circ}$ passing through a series of N lenses are given by

$$
\begin{align*}
& \left(x \mid x_{\circ}\right)=1+\sum_{n=1}^{N} L_{n}^{h} A_{n}^{h} \\
& \left(x \mid \theta_{\circ}\right)=\sum_{n=0}^{N} L_{n}^{h} B_{n}^{h}  \tag{3.1}\\
& \left(\theta \mid x_{\circ}\right)=\sum_{n=1}^{N} C_{n-1}^{h} P_{n}^{h} \\
& \left(\theta \mid \theta_{\circ}\right)=1+\sum_{n=1}^{N} D_{n-1}^{h} P_{n}^{h}
\end{align*}
$$

where $P_{n}^{h}$ are the powers of each lens, $L_{0}^{h}$ is the distance between the origin and the first lens, $L_{N}^{h}$ is the distance between the last lens and the detectors, $L_{1}^{h}$ through $L_{N-1}^{h}$ are the distances between the lenses, $A_{0}^{h}=0, B_{0}^{h}=1, C_{0}^{h}=1$, $D_{0}^{h}=L_{0}^{h}$ and

$$
\begin{align*}
& A_{n}^{h}=A_{n-1}^{h}+C_{n-1}^{h} P_{n}^{h} \\
& B_{n}^{h}=B_{n-1}^{h}+D_{n-1}^{h} P_{n}^{h} \\
& C_{n}^{h}=C_{n-1}^{h}+L_{n}^{h} A_{n}^{h}  \tag{3.2}\\
& D_{n}^{h}=D_{n-1}^{h}+L_{n}^{h} B_{n}^{h}
\end{align*}
$$

We have used TRANSPORT ${ }^{[5]}$ notation for the matrix elements and coordinates, as illustrated in Figure 4.

Similar equations can be defined in the vertical plane for particles with initial coordinates $y_{\circ}$ and $\phi_{0}$. The momentum dispersion matrix elements ( $x \mid \delta_{0}$ ) and ( $\theta \mid \delta_{0}$ ) are calculated from simple matrix algebra using the individual matrix elements of the bending magnets involved. All the necessary formalism to derive the momentum dispersion matrix elements is given in a classical paper by S. Penner. ${ }^{[9]}$

Due to the large bend angles of the spectrometers, special care was taken in the evaluation of the effective entrance and exit pole face rotation angles. These angles are different from the geometric ones due to corrections arising from the presence of extended fringing fields and finite horizontal pole widths. The magnitude of these corrections are proportional to the deflection angle in the dipole magnet. The spatial extent of the fringing field causes a reduction of focusing in the non-bend plane and the finite pole width gives rise to change in focusing strength in both planes by curving the iso-induction lines. The measured shapes of the fringing fields of the dipole magnets were used in calculating the corrections ${ }^{[10,11]}$ to the geometric pole face rotations.

The maximum horizontal acceptance for a given set of $i$ lenses was determined by using equations similar to (3.1) to find the production angles $\theta_{\circ}^{i}$ for which rays would just hit apertures of half-width $X_{i}$ located at the middle of the $i^{\text {th }}$ lens, assuming an initial $x_{\circ}$ given by the typical beam size. This was done for values of $i$ ranging from 1 to the maximum number of lenses in the system $N$, plus an additional pass to take into account the width of the detectors. The horizontal acceptance $\theta$ was then given by

$$
\begin{equation*}
\theta=\min \left[\left|\theta_{\circ}^{1}\right|,\left|\theta_{\circ}^{2}\right|,\left|\theta_{\circ}^{3}\right|, \ldots,\left|\theta_{\circ}^{N+1}\right|\right] \tag{3.3}
\end{equation*}
$$

where $\theta_{\circ}^{i}=\left(X_{i}-x_{\circ}\right) / G_{i}^{h}$ with $G_{1}^{h}=L_{\circ}^{h}$ and

$$
\begin{equation*}
G_{i}^{h}=G_{i-1}^{h}+L_{i-1}^{h}\left(1+\sum_{j=1}^{i-1} G_{j}^{h} P_{j}^{h}\right) \tag{3.4}
\end{equation*}
$$

for $i>1$. The maximum vertical acceptance was found in a similar manner. The solid angle $\Delta \Omega$ for the system was then found from the product of the horizontal - and vertical acceptances: $\Delta \Omega=\pi \theta \phi$.

The computer program used to implement the above formulas had the ability to fit selected parameters (such as gradients, deflection angles, and drift
distances) to obtain desired matrix elements, minimize the momentum or angular resolutions, and maximize solid angle. Comparisons were made with both TRANSPORT and TURTLE to check that the program was working as expected.

### 3.2 THE ELECTRON SPECTROMETER

The electron spectrometer is shown in more detail in Figure 2. It was designed to analyze electrons with momenta from 0.25 to $0.64 \mathrm{GeV} / \mathrm{c}$ backscattering at $180^{\circ}$. Other design goals were to achieve the largest possible solid angle, moderate momentum and angular resolution, momentum acceptance on the order of $\pm 5 \%$, and a large bend angle to place the detectors well out of the path of unwanted background particles.

These goals were met using three powerful 10Q18 quadrupoles for the Q1-Q2Q3 triplet (see Table 1). They have a large radius and good field uniformity, so that a large solid angle could be obtained without introducing large second-order corrections. The good field uniformity and the large gap of two 29D36 rectangular dipole magnets justified their utilization as B3 and B4. These magnets have 'window-frame' type coils.

In order to maximize the solid angle of the electron spectrometer, the distance from the target center to Q3 was set at the minimum allowed by the mechanical constraints for the installation of the cryogenic target system. Since the momentum was dispersed in the horizontal plane, the focusing action of Q3 was in the vertical plane. This quadrupole provided a large acceptance collection for the vertical angle $\phi_{\circ}^{e}$ by running close to its maximum strength. The thin lens model was used to optimize this strength, as illustrated in Figure 5. Quadrupole Q2, with opposite polarity, was a limiting aperture for the horizontal angle $\theta_{\mathrm{o}}^{e}$. The overall solid angle was maximized by making the distance between Q2 and Q3 as short as possible. The field strength of Q1 and the distance from Q1 to Q2 were among the free parameters adjusted to give the desired first-order matrix elements for the system and to obtain the best possible resolutions.

It was desirable to the have the B 3 bend angle $\theta_{B 3}$ be as large as possible to preserve the $\phi_{\circ}^{e}$ acceptance. Although we could have chosen a value for $\theta_{B 3}$ as large as $25^{\circ}$ with no loss in momentum resolution or momentum acceptance, we were forced to choose a value of only $20^{\circ}$ to accommodate the angular range of available bellows for the chicane magnets. With the choice $\theta_{B 3}=20^{\circ}$, the values of $\theta_{B 1}$ ranged between $4^{\circ}$ and $11.5^{\circ}$ for the kinematic range of the experiment. The entrance pole face rotation for magnet B3 was fixed at $0^{\circ}$.

The distance between Q1 and B3 was kept as small as possible to preserve solid angle, but was made large enough to allow room for an insertable screen just in front of Q1. This screen was out of the fringe field of B3, and was used to periodically check the alignment of the incident electron beam.

The distance between B3 and B4 was made large enough to allow a straightahead beam pipe to be placed between B1 and B3 during set-up and testing of the recoil arm, where B2 was completely removed from the beam line and B1 and B3 were turned off. The bend angle of B4 was made as large as possible to maximize the vertical focusing from the entrance and exit pole face rotations, to obtain moderate momentum resolution, and to move the detectors as far away as possible from the path of background particles. A constraint was that too large a bending angle would cause degradation of resolution due to field non-uniformities and loss of acceptance from geometric limitations. The optimal value for $\theta_{B 4}$ was found to be $45^{\circ}$.

The final results for the parameters of the beam elements for the electron spectrometer are given in Table 2.

The absence of any first-order momentum dispersion in the vertical direction dictated a parallel-to-point optics for an optimum angular resolution: ( $y \mid y_{\circ}$ ) $=0$. All rays with the same $\phi_{o}^{e}$ at the target were brought to a focus at a focal plane located at a distance of 2.683 m from the end of B4. The angular dispersion was $-0.08 \mathrm{~cm} / \mathrm{mr}$, more than adequate to achieve the desired 8 mr resolution since the wire spacing of the wire chambers in the vertical direction was 0.2 cm .

The optics in the horizontal direction was close to point-to-parallel focusing: $\left(\theta \mid \theta_{0}\right) \approx 0$. The angle $\theta^{e}$ at the exit of B4 was almost proportional to the relative momentum deviation $\delta_{o}^{e}=\Delta p^{e} / p_{\mathrm{o}}^{e}$ of the particle and independent of $\theta_{\mathrm{o}}^{e}$. The momentum resolution was limited by the finite size of the beam spot and by the longitudinal extent of the target, even in the case of a perfect $\theta^{e}$ measurement. The $x^{e}$ coordinate carried all the $\theta_{\circ}^{e}$ information. Point-to-parallel optics was chosen instead of the traditional point-to-point, where $\left(x \mid \theta_{0}\right)=0$, for two reasons: it gave a significantly better resolution in the $\theta_{o}^{e}$ reconstruction and it gave a much bigger momentum acceptance through the vanishing of the $\left(\theta \mid \theta_{0}\right)$ term.

In this system second-order chromatic and geometric effects were expected to be significant. The first- and second-order matrix elements from the target to the $\phi_{o}$ focal plane were calculated using TRANSPORT ${ }^{[5]}$ and are listed in Table 3. Reverse first- and second-order matrix elements from the detectors back to the target were calculated by generating a set of test rays using the forward matrix elements from TRANSPORT and performing a least-squares fit. The results are listed in Table 4. The electron scattering angles $\Theta_{e}$ were calculated using

$$
\begin{equation*}
\pi-\Theta_{e}=\arctan \left[\sqrt{\tan ^{2} \theta_{\circ}^{e}+\tan ^{2} \phi_{\circ}^{e}}\right] \simeq \sqrt{\theta_{\circ}^{e^{2}+\phi_{\circ}^{e^{2}}}} \tag{3.5}
\end{equation*}
$$

The Monte Carlo program was also run to generate a different set of test rays. It used interpolations of field maps made of magnets B3 and B4 to transport the particles through these magnets, rather than relying on the TRANSPORT matrix elements. The field maps were made using a three-dimensional Hall probe positioned at 2.54 cm intervals in $\mathrm{x}, \mathrm{y}$ and z at three values of the magnet currents. The absolute calibration of the Hall probe measurements were checked using long coil measurements to measure the integral of the fields as a function of current. The Monte Carlo also took into account measured non-uniformities in the gradient of the quadrupoles. The reverse matrix elements found from a fit to these test rays did not differ significantly from those listed in Table 4, showing that a second-order description of the system was adequate.

The angular and momentum resolutions of the spectrometer were momentum dependent and depended on the beam spot size and the target length. They were further degraded by energy loss and multiple scattering in the targets, vacuum windows, and wire chambers, and the uncertainties in track measurements from the finite wire spacing in the MWPCs. Table 5 gives, for a thin target, the expected resolutions in $\theta_{\mathrm{o}}^{e}, \phi_{\mathrm{o}}^{e}$, and $\delta_{\mathrm{o}}^{e}$ for 0.3 and $0.6 \mathrm{GeV} / \mathrm{c}$ scattered electron momenta, calculated using the estimated uncertainties in $x^{e}, y^{e}, \theta^{e}$ and $\phi^{e}$ at the MWPCs, also given in the Table.

The solid angle $\Delta \Omega_{e}$ depended strongly upon the target length and was determined by the magnet apertures and the size of the detectors. Figure 6 shows the maximum horizontal and vertical acceptance angles $\left(\theta_{o}^{e}\right)_{\max }$ and $\left(\phi_{o}^{e}\right)_{\max }$ of the electron spectrometer as a function of target position $z_{0}^{e}$. The average solid angles were found to be $22.7,22.4,21.5$, and 19.4 msr for target lengths of 5,10 , 20 , and 40 cm respectively, averaged over a momentum acceptance of $\pm 1 \%$. The momentum acceptance was limited by the size of the detectors rather than the magnet apertures. The variation of the solid angle with relative momentum is shown for a thin target in Figure 7.

The optical properties of this spectrometer are illustrated in Figures 8 and 9. In the horizontal plane, rays with same momentum emerged parallel at the exit of the system. In the vertical plane parallel rays from the target ended up at the same spot on the $\phi_{o}^{e}$ focal plane. Figure 8 also illustrates the effect of including second-order matrix elements for typical rays. Second-order effects were much larger for extreme rays, as illustrated in Figure 9.

A configuration using a quadrupole doublet rather than the triplet Q1-Q2Q3 was also investigated. The triplet solution was chosen because it gave a bigger angular acceptance in the horizontal direction and a larger solid angle for long targets.

### 3.3 THE BEAM TRANSPORT SYSTEM

Two 18D36 rectangular dipole magnets with very uniform fields were selected for the beam transport system to serve as B1 and B2. Their physical properties are listed in Table 1. The distance $L_{c}$ between the centers of B1-B2 and B2-B3 is 5.09 m on the beam center-line, the minimum allowed by the B 4 location, as shown in Figure 2. A remotely insertable screen was installed before the entrance of magnet B1 for periodic alignment of the electron beam entering the transport system. Two 'roller screens' located upstream of B1 are part of the permanent ESA facilities and were also used for beam alignment. All screens were removed for normal data taking.

The magnet B2 was mounted on a carriage that moved on rails and was positioned remotely from the experimental control area. Constraints from the flexibility of bellows in the movable vacuum system between B1 and B3 limited the $\theta_{B 1}$ angular range to be $4^{\circ}$ to $11.5^{\circ}$.

The target center was located 24.8 m downstream of the End Station A target pivot. The symmetric arrangement and operation of the chicane magnets B1, B2 and B3 preserved the initial achromaticity of the electron beam; its final position and divergence after B3 were independent of the momentum spread. The matrix elements $\left(x \mid \delta_{0}\right)$ and $\left(\theta \mid \delta_{o}\right)$ of the total transformation from the beginning of the switchyard to the target were identically zero.

The A-Line ${ }^{[12]}$ quadrupoles in the beam switchyard were used for imaging the beam from the end of the accelerator to the new target position. The TRANSPORT model ${ }^{[13]}$ for the A-line was used to obtain quadrupole gradients that both optimized beam transmission through the narrow apertures of the switchyard and gave horizontal and vertical magnifications of nearly $1: 1$ at the target when Q1, Q2, and Q3 were turned off. The effect of the quadrupole triplet Q1-Q2-Q3 varied with kinematics, resulting in a typical beam spot size of a few mm and a typical divergence of 1 mr at the target. These spatial and angular spreads were
well within the resolution tolerances for the experiment, being much smaller than the spreads caused by multiple scattering in the targets.

### 3.4 THE RECOIL SPECTROMETER

The recoil spectrometer is shown in Figure 3. The quadrupole triplet Q4-Q5Q6 was carefully chosen and tuned to ensure a good overlap of the acceptances of the two arms for a range of target lengths. The long targets and the relatively high momentum of the recoiling deuterons made that task difficult.

Extensive calculations with a thin lens acceptance model for the recoil spectrometer, similar to that used for the electron spectrometer, resulted in the selection of an 8Q32 quadrupole for Q4 and a 10Q36 quadrupole for Q5. Both have high gradients similar to the quadrupoles used in the electron spectrometer, but are essentially twice as long, allowing much higher momentum particles to be focused (see Table 1). As in the electron spectrometer, Q4 is focusing in the vertical direction and Q5 in the horizontal. The field strengths of Q4 and Q5, the drift distance between them, and the drift length from the target to Q4 were the crucial parameters in achieving the desired solid angle for this spectrometer.

A third quadrupole was found to be necessary on this side for the transmission of the recoil particles through the long path and narrow apertures of the bending magnets of the recoil system. The length and strength of Q6 and its distance from Q5 were free parameters in the spectrometer optimization. Although a 10Q36 quadrupole would have provided better focusing, a shortage of power supplies dictated the use of a 10Q18 quadrupole as Q6. The current required for this quadrupole was slightly less than that of Q5, permitting the two quads to be run in series from a single power supply. A small current bypass of variable impedance was coupled in parallel to Q6 to fine tune the current flowing in that -quadrupole.

The split magnet B5 had to be wide enough to both bend the deuterons by a reasonably large angle (to minimize the distance to B 6 ) and at the same
time transmit the electron beam onto the dump. The separation between B5 and B 6 was determined by the need to accommodate a 90 cm diameter beam pipe behind B 5 to transmit the photons produced in the target to a shielded downstream photon beam dump. The only magnet available that could both bend the deuterons by a large angle and simultaneously bend the electrons to the dump without having them hit the magnet iron was a 26D72 rectangular dipole magnet with a total horizontal aperture of 100 cm (see Table 1). This large width permitted a bend angle $\theta_{B 5}$ for the recoil nuclei of $15^{\circ}$, but resulted in relatively poor field uniformity. For this reason we made extensive field measurements of this magnet, similar to the ones made of B3 and B4. The separation between B5 and Q6 was chosen to provide enough space for another beam viewing screen.

The anticipation of large background rates in the deuteron detectors determined the need for the three bending magnets B6-B7-B8. This background was expected to be low energy, multi-bounce charged particles or photons originating mainly from the region of the split magnet B5. These magnets give many kG-m of sweeping power for charged background particles and put B5 out of line-ofsight of the detectors to protect them from direct photons. Originally the recoil spectrometer was designed and built without magnet B8. A background test-run showed that this magnet was necessary to keep the counting rates in the recoil detectors at tolerable levels.

Three 18D72 rectangular bending magnets with window-frame type coils were utilized for B6, B7 and B8. They were the most powerful, high quality, long magnets available. The separation between them was kept at the minimum required for mechanical installation. All three magnets were run in series from one power supply. The bend angles were $17^{\circ}$ for B 6 and B 7 , and $16.45^{\circ}$ for B 8 . This choice of bending angles put the detectors far away from the straight-ahead -beam line without running the magnets in the saturation region.

The beam elements of the recoil spectrometer are listed in Table 6 for the maximum $2.603 \mathrm{GeV} / \mathrm{c}$ tune. The first- and second-order forward transport
coefficients, from the target center to the center of the recoil side wire chambers, are given in Table 7. The optics in the horizontal direction was close to point-toparallel. In the vertical plane it was found impossible to achieve parallel-to-point optics and at the same time to match the solid angles of the two arms, so a compromise optics solution was chosen. The optical properties of this spectrometer are pictured in Figures 10 and 11. These figures show some extreme rays originating from the target, plotted as a function of longitudinal position through the spectrometer up to the center of the wire chambers. The reverse first- and second-order matrix elements are tabulated in Table 8. They were obtained in a method similar to that used for the reverse matrix coefficients of the electron spectrometer.

In order to facilitate the calculation of the double-arm acceptance, a design requirement was that the solid angle of the recoil spectrometer subtend that of the electron spectrometer for elastic scattering kinematics. In terms of the maximum acceptance angles of the electron spectrometer, the angles required for the recoil spectrometer were given by:

$$
\begin{align*}
& \left(\phi_{\circ}^{r}\right)_{\max }=\arcsin \left[\frac{p_{e}^{\prime}}{p_{r}} \sin \left(\phi_{\circ}^{e}\right)_{\max }\right]  \tag{3.6}\\
& \left(\theta_{\circ}^{r}\right)_{\max }=\arcsin \left[\frac{p_{e}^{\prime}}{p_{r}} \sin \left(\theta_{\circ}^{e}\right)_{\max }\right] \tag{3.7}
\end{align*}
$$

where $p_{r}$ is the momentum of the recoil nuclei. The magnitudes of $\left(\phi_{o}^{r}\right)_{\max }$ and $\left(\theta_{o}^{r}\right)_{\max }$ were further increased to take into account multiple scattering of the recoil particles in the targets. Figure 12 shows the maximum acceptance angles of the recoil spectrometer (solid lines) as well as the design requirements (dashed curves). It can be seen the recoil acceptance angles were everywhere larger than -that required to subtend those of the electron spectrometer for elastic kinematics.

The recoil momentum acceptance was limited by the apertures of the bending magnets. It was small compared to the momentum acceptance of the electron
spectrometer because of the large momentum dispersion of the dipole magnets B5-B6-B7-B8, but sufficient to cover the momentum spread of the deuterons from elastic kinematics and from energy loss in the targets. The variation of the recoil spectrometer solid angle with relative momentum is shown for a thin target in Figure 13.

The angular and momentum resolutions were determined by the optics of the recoil spectrometer, energy loss and multiple scattering in the target, vacuum windows, and detectors, and by the finite spatial resolution of the MWPCs. When the spectrometer was designed it was not considered important to have good angular and momentum resolution because elastic events were to be simply identified from electron-deuteron time-of-flight measurements. Subsequently it was found that measurements of the deuteron momentum were needed to eliminate an unanticipated source of background. ${ }^{[1]}$ Table 9 shows typical values for resolutions in the reconstructed kinematic quantities for a thin target calculated using the estimated uncertainties in track position and angles measured at the MWPCs.

### 3.5 THE BEAM DUMP

The beam dump was a water-cooled copper block contained in a stainless steel jacket approximately 56 cm high, 18 cm wide, and 70 cm long. Except for the side facing the beam, it was surrounded by a layer of lead approximately 20 cm thick. A 60 cm diameter toroid charge monitor was mounted in front on the dump to measure the transmission of the beam through the spectrometer system. High intensity beam could only be delivered into the End Station when the transmission was above a predetermined threshold.

Magnet B5 deflected the electron beam towards the dump by an angle $\theta_{d}$ -given by

$$
\begin{equation*}
\theta_{d}=\arcsin \left(\frac{p_{r}}{p_{e}} \sin \theta_{B 5}\right) \tag{3.8}
\end{equation*}
$$

Its value ranged from $19^{\circ}$ to $25^{\circ}$ for electron-deuteron elastic scattering from $Q^{2}=1$ to $5(\mathrm{GeV} / \mathrm{c})^{2}$ and electron-proton elastic scattering from $Q^{2}=0.25$ to $1.25(\mathrm{GeV} / \mathrm{c})^{2}$. The beam dump, lead shielding, and toroid monitor were mounted on a remotely controlled platform that could be moved to accommodate this range of deflection angles. A ZnS screen viewed by a TV camera was mounted on the front face of the dump. It was used to verify the alignment of the dump for each new recoil spectrometer setting and each new beam energy.

The dump location was chosen as a compromise between the desire to have it close to the exit of B5, where the beam spot was smaller, and the need for adequate shielding. For radiation protection it was necessary to surround the dump area with 1.5 m of iron and 2 m of concrete. The straight-ahead vacuum pipe from B5 to the photon dump needed to be large enough to prevent the intense photon beam produced in the target from striking the vacuum chamber and producing background particles near the exit of B5. These requirements forced the dump to be located 7.5 m from B 5 .

The beam spot size on the dump was estimated using the DECAY TURTLE program, taking into account the energy spread from ionization losses and the angular spread from multiple scattering in the targets. Figure 14 shows the beam spatial distributions on the dump for the two limiting cases of electrondeuteron elastic scattering: $Q^{2}=1$ and $5(\mathrm{GeV} / \mathrm{c})^{2}$, with 10 cm and 40 cm long deuterium targets respectively. The shape of the beam envelope on the dump changed with kinematics since the ratios of the gradients of the quadrupoles Q4-Q5-Q6 to the beam momentum varied with the recoil particle momentum.

## 4. CALIBRATION

The momentum calibration of the electron spectrometer was performed using electron-proton elastic scattering. At several known scattered electron momenta, the fields of the bending magnets B3 and B4 were adjusted to center the electron elastic peak distributions in $x^{e}$ and $\theta^{e}$ measured at the center of the wire chambers. The initial settings were calculated using effective magnet lengths and assuming sharp cutoff fringing fields for B3 and B4 without taking into account the displacement of the central ray due to the extent of the fringing field as discussed by Enge. ${ }^{[10,11]}$ When this displacement was taken into account, the calculated current settings of the two magnets agreed with those required to center the elastic peak distributions measured at the center of the wire chambers, as shown in Figure 15.

The momentum dispersion matrix elements $\left(x \mid \delta_{0}\right)$ and $\left(\theta \mid \delta_{0}\right)$ were extracted by sweeping the electron-proton elastic peak distributions across the acceptance of the spectrometer in special calibration runs using the hydrogen target. This was done by lowering and raising the central momentum of the spectrometer while keeping the beam energy fixed. The values of $\left(x \mid \delta_{0}\right)$ and $\left(\theta \mid \delta_{0}\right)$ measured at the center of the wire chambers were found to be in excellent agreement with the predictions.

The angular dispersion in the vertical direction was verified in a special calibration measurement by placing a tungsten shield between a thin Al target and Q3. The shield was pierced by holes placed along the vertical axis. In this way, electrons with well-defined vertical angles $\phi_{o}^{e}$ were imaged at specific $y^{e}$ locations in the wire chambers as shown in Figure 16. Figure 17 shows the data for the $y^{e}$ measurement at the center of the MWPC and the prediction of the optics model of the spectrometer. The measurements were found to be in good agreement - with the model.

The recoil spectrometer was calibrated in the momentum range of 1.5 to 2.5 $\mathrm{GeV} / \mathrm{c}$ with an electron beam and in the range 0.7 to $1.5 \mathrm{GeV} / \mathrm{c}$ with double-arm
ep elastic scattering. For the high momentum calibration, the field direction of bending magnets $\mathrm{B} 5-\mathrm{B} 6-\mathrm{B} 7-\mathrm{B} 8$ was reversed and a low intensity electron beam was run through the spectrometer. The fields of these magnets were adjusted to center the beam on two fluorescent screens placed 3.8 m apart on the spectrometer axis after B8. The momentum dispersion matrix elements and momentum acceptance were also measured in the course of this procedure. For momenta below $1.5 \mathrm{GeV} / \mathrm{c}$, the magnet currents were adjusted to center elastic recoil proton peak distributions in a manner similar to that used to calibrate the electron spectrometer. The measured dispersions agreed very well with the optics calculations, and the calibration data from the two momentum ranges joined smoothly.

## 5. PERFORMANCE

- The performance of the system was evaluated using ep elastic scattering, for which the cross sections are well-known. Both single-arm (electron spectrometer only) and double-arm (both spectrometers) cross sections were measured for a variety of target lengths. Effective solid angles (the convolution of geometric solid angles with radiative and ionization energy losses, multiple scattering, etc.) were calculated with the Monte Carlo program. ${ }^{[7]}$ The program was also used to generate momentum and angle spectra which could be compared to the measured distributions.

The Monte Carlo program was needed in the evaluation of the performance so that the effects of energy loss, multiple scattering, nuclear absorption, apertures, finite target length, and elastic kinematics could all be taken into account simultaneously. It could also take into account the detailed field maps that were made of magnets B3, B4, and B5, as well as the measured non-uniformity in the gradients of the quadrupoles. Electron events were generated by randomly choosing initial values for $z_{o}^{e}=-z_{o}^{r}$ within the given target length and for $\theta_{o}^{e}$ and $\phi_{\mathrm{o}}^{e}$ within ranges larger than the spectrometer acceptance. Values for $x_{\mathrm{o}}^{e}$ and $y_{\mathrm{o}}^{e}$ were chosen within the given beam spot size. A value for $E$ was chosen within
the assumed energy defining slits using a Gaussian distribution. The value of $E$ was corrected for ionization energy loss using Landau distributed values around the most probable energy loss, which was evaluated with the density effect properly taken into account. ${ }^{[14]}$ It was also corrected for radiative energy loss using the formula of G. Miller ${ }^{[15]}$ to approximate the radiation probability distribution given by Mo and Tsai. ${ }^{[16]}$ The variation of the cross section with $Q^{2}$ was taken into account in the selection of initial events. The angle of the incident electron was corrected for multiple scattering and the values of $p_{e}^{\prime}, p_{r}, \theta_{o}^{r}$, and $\phi_{o}^{r}$ were then calculated using elastic kinematics. The final electron momentum was corrected for ionization and radiative energy loss, while the recoil particle energy was corrected for ionization energy loss only. Multiple scattering was taken into account for both particles. The nuclear absorption probability for the recoil particle was evaluated. Particles were made to pass through the spectrometers by stepping through the measured magnetic fields and checking to see if they hit any apertures. Field maps were not made of magnets B6-B8, so first-order TRANSPORT coefficients were used in this case. Spectra were made of events which successfully made it to the detectors. Effective single-arm and double-arm solid angles were calculated using the ratio of successful events to the number of trials.

The agreement between observed elastic $e p$ scattering and that predicted by the Monte Carlo was in general good. Figures 18 through 21 show the comparison between data and Monte Carlo predictions for the most important distributions at the detectors for $e p$ elastic scattering from a 10 cm hydrogen target at 0.937 GeV incident beam energy. It can be seen that very good agreement is found on the electron side, while on the recoil side the measured elastic peaks are slightly broader than those predicted by the model. Agreement with distributions not shown (i.e. $x^{e}, \phi^{e}, y^{r}$ and $\phi^{r}$ ) were generally good.

The Monte Carlo program was used to calculate the effective single- and double-arm solid angles for each of the ep data runs. The results were found to vary slowly with beam energy for a given target length, and are shown as a
function of target length for a beam energy of 1.0 GeV in Figure 22. Also shown are the geometric solid angles. The principal reason that the effective solid angles were smaller than the geometric ones was the loss of electrons and recoil particles due to radiative energy loss by the incident and scattered electrons.

The calculated solid angles were used to evaluate the double-arm absolute cross sections for all of the ep elastic measurements. The results are compared to previous data ${ }^{[17]}$ in Figure 23, where it can be seen that the agreement is quite good. Cross sections were measured at each beam energy using several of the four target lengths available. No dependence on target length was observed at any of the beam energies; the results shown in Figure 23 are for the average over all targets used. The good agreement between different target lengths shows that the Monte Carlo program correctly predicts the differences in effective solid angles due to radiative effects, energy loss, multiple scattering, and absorption.

As a further check, cross sections were calculated using only the electron arm data, not requiring a coincidence with the recoil proton. In this case, a subtraction was made for electrons back-scattering from the aluminium endcaps of the targets. The ratios of single-arm to double-arm cross sections (averaged over target length) are shown in Figure 24. The ratios are all close to 1, showing that the Monte Carlo program correctly predicts the loss of protons due to radiative corrections, multiple scattering, momentum acceptance, nuclear absorption, and apertures.

## 6. SUMMARY

A double-arm $180^{\circ}$ spectrometer system for electron scattering from deuterium has been described. Lumped quadrupole elements on the beam line provided a large acceptance collection for the back-scattered electrons and the recoil nuclei while having little effect on the incident electron beam. The large solid angle of this system made possible elastic and inelastic cross section measurements from deuterium at high momentum transfers. It was the first facility for $180^{\circ}$ electron scattering in which recoiling nuclei were detected in coincidence at $0^{\circ}$. The analysis results found the performance of the system in very good agreement with the design specifications and with the predictions of the second-order models. With minor modifications, the system could be used for elastic and inelastic electron scattering from Tritium.

We acknowledge the work of $Z$. Vassilian in designing and supervising the modifications to the magnets and of D. R. Jensen in performing the magnetic measurements of all the quadrupoles. J. Lambert, A. Lung, and G. Peterson helped in measuring the field maps of the dipole magnets. C. Hudspeth was of great help in the assembly of the spectrometers. The support of M. M. Berndt, J. F. Brown, H. Harvey, L. Otts and the rest of the SLAC staff is greatly appreciated. This work was supported by the Department of Energy Contract DOE-AC0376SF00515 and National Science Foundation Grant PHY85-10549.

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## TABLE CAPTIONS

1. Physical properties of the magnets used in the system.
2. The beam elements of the electron spectrometer for the maximum 0.635 $\mathrm{GeV} / \mathrm{c}$ tune. The values of $L_{e f f}$ are the effective lengths of the central trajectory. The first drift length is from the target center. The last drift length is to the $\phi_{0}$ focal plane. The center of the MWPCs is 0.325 m upstream of the $\phi_{o}^{e}$ focal plane. The parameters $\alpha^{h}$ and $\alpha^{\nu}$ are the effective entrance pole face rotation angles in the horizontal and vertical direction. The parameters $\beta^{h}$ and $\beta^{v}$ are the effective exit pole face rotation angles in the horizontal and vertical direction.
3. The first- and second-order forward matrix elements of the electron spectrometer from the target center to the $\phi_{\circ}$ focal plane, determined from TRANSPORT. The units of $x_{\circ}$ and $y_{\circ}$ are cm and the units of $\theta_{\circ}$ and $\phi_{o}$ are mr . The momentum deviation from the central trajectory $\delta_{0}$ is in percent.
4. The first- and second-order reverse matrix elements of the electron spectrometer from the $\phi_{0}$ focal plane to the target center. The units of $x$ and $y$ are cm and the units of $\theta$ and $\phi$ are mr.
5. Resolutions in reconstructed quantities and in positions and angles measured at the MWPCs for a thin target at two values of the scattered electron momentum.
6. The beam elements of the recoil spectrometer for the maximum 2.603 $\mathrm{GeV} / \mathrm{c}$ tune. The values of $L_{e f f}$ are the effective lengths of the central trajectory. The first drift is from the target center. The last drift is to the center of the MWPCs. The parameters $\alpha^{h}$ and $\alpha^{v}$ are the effective entrance pole face rotation angles in the horizontal and vertical direction. The parameters $\beta^{h}$ and $\beta^{v}$ are the effective exit pole face rotation angles in the horizontal and vertical direction.
7. The first- and second-order forward matrix elements of the recoil spectrometer from the target center to the center of the wire chambers, determined from TRANSPORT. The units of $x_{0}$ and $y_{0}$ are cm and the units of $\theta_{0}$ and $\phi_{0}$ are mr . The momentum deviation $\delta_{0}$ from the central trajectory is in percent.
8. The first- and second-order reverse matrix coefficients of the recoil spectrometer from the center of the wire chambers to the target. The units of $x$ and $y$ are cm and the units of $\theta$ and $\phi$ are mr .
9. Resolutions in reconstructed quantities and in positions and angles measured at the MWPCs for a thin target at two values of the recoil particle momentum.

TABLE 1

Dipole Magnets

| Name | Length | Width | Gap | Max. Field |
| :---: | :---: | :---: | :---: | :---: |
| B1, B2 | 91.4 cm | 45.7 cm | 15.2 cm | 18 kG |
| B3 | 91.4 cm | 73.7 cm | 25.6 cm | 18 kG |
| B4 | 91.4 cm | 73.7 cm | 28.0 cm | 16 kG |
| B5 | 182.9 cm | 100 cm | 20.0 cm | 13 kG |
| B6, B7, B8 | 182.9 cm | 46.2 cm | 25.4 cm | 15 kG |

Quadrupoles

| Name | Length | Radius | Max. Field |
| :---: | :---: | :---: | :---: |
| Q1, Q2, Q3, Q6 | 45.7 cm | 12.86 cm | 12 kG |
| Q4 | 81.3 cm | 10.47 cm | 15 kG |
| Q5 | 91.4 cm | 12.86 cm | 13 kG |

TABLE 2

| Beam element | Action | $\begin{gathered} L_{e f f} \\ (\mathrm{~m}) \end{gathered}$ | Strength ( kG or $\mathrm{kG} / \mathrm{m}$ ) |
| :---: | :---: | :---: | :---: |
| Drift |  | 0.600 |  |
| Quadrupole Q3 | Vertical focus | 0.490 | 90.19 |
| Drift |  | 0.354 |  |
| Quadrupole Q2 | Vertical defocus | 0.490 | 69.83 |
| Drift |  | 0.354 |  |
| Quadrupole Q1 | Vertical focus | 0.490 | 38.28 |
| Drift |  | 1.204 |  |
| Pole Rotation | $\alpha^{h}=0.0^{\circ} \alpha^{v}=0.0^{\circ}$ |  |  |
| Dipole B3 | $20.01^{\circ}$ horizontal bend | 1.209 | 6.117 |
| Pole Rotation | $\beta^{h}=18.81^{\circ} \beta^{v}=16.93^{\circ}$ |  |  |
| Drift |  | 2.145 |  |
| Pole Rotation | $\alpha^{h}=20.12^{\circ} \alpha^{v}=15.50^{\circ}$ |  |  |
| Dipole B4 | $45.33^{\circ}$ horizontal bend | 1.237 | 13.548 |
| Pole Rotation | $\beta^{h}=20.12^{\circ} \beta^{v}=15.50^{\circ}$ |  |  |
| Drift |  | 2.683 |  |

TABLE 3
\(\left.$$
\begin{array}{ccccc}\hline & x & \theta & y & \begin{array}{c}\phi \\
(\mathrm{cm})\end{array}
$$ <br>
\& \& (\mathrm{mr}) \& \& <br>

(\mathrm{cm})\end{array}\right]\)| $(\mathrm{mr})$ |
| :---: |
| $x_{\circ}$ |

TABLE 4

|  | $\begin{gathered} x_{\circ} \\ (\mathrm{cm}) \end{gathered}$ | $\begin{gathered} \theta_{0} \\ (\mathrm{mr}) \end{gathered}$ | $\begin{gathered} y_{\circ} \\ (\mathrm{cm}) \\ \hline \end{gathered}$ | $\begin{gathered} \phi_{0} \\ (\mathrm{mr}) \end{gathered}$ | $\delta_{0}$ <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $x$ | $-6.1 \times 10^{-3}$ | 4.79 | $-1.2 \times 10^{-5}$ | $1.6 \times 10^{-3}$ | $-3.6 \times 10^{-2}$ |
| $\theta$ | $1.0 \times 10^{-3}$ | -2.06 | $-3.7 \times 10^{-6}$ | $1.6 \times 10^{-4}$ | $-7.5 \times 10^{-2}$ |
| $y$ | $2.2 \times 10^{-4}$ | $2.9 \times 10^{-3}$ | $-6.0 \times 10^{-3}$ | -12.00 | $-1.3 \times 10^{-4}$ |
| $\phi$ | $1.2 \times 10^{-4}$ | $-1.3 \times 10^{-2}$ | $8.2 \times 10^{-2}$ | $-4.4 \times 10^{-3}$ | $3.1 \times 10^{-4}$ |
| $x^{2}$ | $-2.1 \times 10^{-4}$ | $9.9 \times 10^{-3}$ | $1.3 \times 10^{-5}$ | $-4.4 \times 10^{-4}$ | $-2.1 \times 10^{-4}$ |
| $x \theta$ | $-5.6 \times 10^{-6}$ | $3.6 \times 10^{-2}$ | $-6.5 \times 10^{-6}$ | $3.7 \times 10^{-4}$ | $-1.2 \times 10^{-3}$ |
| $x y$ | $-3.6 \times 10^{-4}$ | $-3.9 \times 10^{-4}$ | $-9.3 \times 10^{-4}$ | $2.7 \times 10^{-2}$ | $1.2 \times 10^{-4}$ |
| $x \phi$ | $-2.6 \times 10^{-4}$ | $3.8 \times 10^{-4}$ | $7.0 \times 10^{-5}$ | $-6.9 \times 10^{-3}$ | $6.3 \times 10^{-5}$ |
| $\theta^{2}$ | $4.4 \times 10^{-5}$ | $-1.7 \times 10^{-2}$ | $-6.5 \times 10^{-7}$ | $-6.3 \times 10^{-5}$ | $6.5 \times 10^{-4}$ |
| $\theta y$ | $2.0 \times 10^{-4}$ | $4.5 \times 10^{-5}$ | $2.1 \times 10^{-3}$ | $-3.8 \times 10^{-2}$ | $-6.5 \times 10^{-5}$ |
| $\theta \phi$ | $8.4 \times 10^{-5}$ | $3.9 \times 10^{-5}$ | $-1.8 \times 10^{-4}$ | $1.2 \times 10^{-2}$ | -2.2 $\times 10^{-5}$ |
| $y^{2}$ | $3.3 \times 10^{-4}$ | $-1.4 \times 10^{-3}$ | $4.8 \times 10^{-7}$ | $-3.5 \times 10^{-4}$ | $2.8 \times 10^{-4}$ |
| $y \phi$ | $-5.5 \times 10^{-4}$ | $1.9 \times 10^{-3}$ | $-3.8 \times 10^{-5}$ | $5.7 \times 10^{-4}$ | $2.5 \times 10^{-5}$ |
| $-\phi^{2}$ | $-2.2 \times 10^{-4}$ | $1.7 \times 10^{-3}$ | $1.7 \times 10^{-5}$ | $1.2 \times 10^{-4}$ | $4.9 \times 10^{-5}$ |

TABLE 5

$$
p_{e}^{\prime}=0.3 \mathrm{GeV} / \mathrm{c} \quad p_{e}^{\prime}=0.6 \mathrm{GeV} / \mathrm{c}
$$

Resolutions in reconstructed quantities

| $\Delta \theta_{\circ}(\mathrm{mr})$ | $\pm 8.6$ | $\pm 5.8$ |
| :---: | :---: | :---: |
| $\Delta \phi_{\circ}(\mathrm{mr})$ | $\pm 5.6$ | $\pm 3.2$ |
| $\Delta \delta_{\circ}(\%)$ | $\pm 0.60$ | $\pm 0.32$ |
|  | Resolutions at MWPCs |  |
| $\Delta x(\mathrm{~cm})$ | $\pm 0.4$ | $\pm 0.2$ |
| $\Delta \theta(\mathrm{mr})$ | $\pm 5.3$ | $\pm 2.8$ |
| $\Delta y(\mathrm{~cm})$ | $\pm 0.4$ | $\pm 0.2$ |
| $\Delta \phi(\mathrm{mr})$ | $\pm 5.0$ | $\pm 2.3$ |

TABLE 6

| Beam element | Action | $\begin{gathered} L_{e f f} \\ (\mathrm{~m}) \end{gathered}$ | $\begin{gathered} \text { Strength } \\ (\mathrm{kG} \text { or } \mathrm{kG} / \mathrm{m}) \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| Drift |  | 0.881 |  |
| Quadrupole Q4 | Vertical focus | 0.905 | 122.25 |
| Drift |  | 0.341 |  |
| Quadrupole Q5 | Vertical defocus | 0.959 | 96.81 |
| Drift |  | 0.354 |  |
| Quadrupole Q6 | Vertical focus | 0.490 | 91.37 |
| Drift |  | 0.740 |  |
| Pole Rotation | $\alpha^{h}=0.00^{\circ} \alpha^{v}=0.00^{\circ}$ |  |  |
| Dipole B5 | $15.00^{\circ}$ horizontal bend | 2.061 | 11.027 |
| Pole Rotation | $\beta^{h}=14.75^{\circ} \beta^{v}=14.21^{\circ}$ |  |  |
| Drift |  | 4.879 |  |
| Pole Rotation | $\alpha^{h}=7.75^{\circ} \alpha^{v}=6.99^{\circ}$ |  |  |
| Dipole B6 | $17.00^{\circ}$ horizontal bend | 2.099 | 12.275 |
| Pole Rotation | $\beta^{h}=7.75^{\circ} \beta^{v}=6.99^{\circ}$ |  |  |
| Drift |  | 0.965 |  |
| Pole Rotation | $\alpha^{h}=7.75^{\circ} \alpha^{v}=6.99^{\circ}$ |  |  |
| Dipole B7 | $17.00^{\circ}$ horizontal bend | 2.099 | 12.275 |
| Pole Rotation | $\beta^{h}=7.75^{\circ} \beta^{v}=6.99^{\circ}$ |  |  |
| Drift |  | 0.960 |  |
| Pole Rotation | $\alpha^{h}=7.51^{\circ} \alpha^{v}=6.78^{\circ}$ |  |  |
| Dipole B8 | $16.45^{\circ}$ horizontal bend | 2.098 | 11.881 |
| Pole Rotation | $\beta^{h}=7.51^{\circ} \beta^{v}=6.78^{\circ}$ |  |  |
| Drift |  | 3.869 |  |

TABLE 7

|  | $x$ | $\theta$ | $y$ | $\phi$ <br> $(\mathrm{cm})$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $(\mathrm{mr})$ | $(\mathrm{cm})$ | $(\mathrm{mr})$ |
|  |  |  |  |  |
| $x_{\circ}$ | -2.92 | -2.22 | 0.0 | 0.0 |
| $\theta_{\circ}$ | 0.58 | 0.10 | 0.0 | 0.0 |
| $y_{\circ}$ | 0.0 | 0.0 | -5.64 | 2.26 |
| $\phi_{\circ}$ | 0.0 | 0.0 | -0.14 | -0.12 |
| $\delta_{\circ}$ | 11.56 | 11.11 | 0.0 | 0.0 |
| $x_{\circ}^{2}$ | $-1.6 \times 10^{-3}$ | $-3.0 \times 10^{-3}$ | 0.0 | 0.0 |
| $x_{\circ} \theta_{\circ}$ | $-8.6 \times 10^{-4}$ | $4.2 \times 10^{-4}$ | 0.0 | 0.0 |
| $x_{0} y_{\circ}$ | 0.0 | 0.0 | $-3.9 \times 10^{-3}$ | $8.6 \times 10^{-3}$ |
| $x_{\circ} \phi_{\circ}$ | 0.0 | 0.0 | $3.0 \times 10^{-4}$ | $1.8 \times 10^{-4}$ |
| $x_{0} \delta_{\circ}$ | 0.35 | 0.18 | 0.0 | 0.0 |
| $\theta_{\circ}^{2}$ | $5.4 \times 10^{-5}$ | $-1.5 \times 10^{-5}$ | 0.0 | 0.0 |
| $\theta_{\circ} y_{\circ}$ | 0.0 | 0.0 | $4.3 \times 10^{-4}$ | $1.1 \times 10^{-3}$ |
| $\theta_{\circ} \phi_{\circ}$ | 0.0 | 0.0 | $-5.7 \times 10^{-5}$ | $4.0 \times 10^{-5}$ |
| $\theta_{\circ} \delta_{\circ}$ | $6.7 \times 10^{-2}$ | $2.8 \times 10^{-2}$ | 0.0 | 0.0 |
| $y_{\circ}^{2}$ | $-1.7 \times 10^{-2}$ | $-6.6 \times 10^{-3}$ | 0.0 | 0.0 |
| $y_{\circ} \phi_{\circ}$ | $-1.1 \times 10^{-3}$ | $-2.6 \times 10^{-4}$ | 0.0 | 0.0 |
| $y_{\circ} \delta_{\circ}$ | 0.0 | 0.0 | -0.11 | -0.17 |
| $\phi_{\circ}^{2}$ | $-1.5 \times 10^{-5}$ | $-1.3 \times 10^{-5}$ | 0.0 | 0.0 |
| $\phi_{\circ} \delta_{\circ}$ | 0.0 | 0.0 | $8.5 \times 10^{-3}$ | $-5.2 \times 10^{-3}$ |
| $\delta_{\circ}^{2}$ | -0.10 | -0.13 | 0.0 | 0.0 |
|  |  |  |  |  |

TABLE 8

|  | $x_{0}$ | $\theta_{0}$ | $y_{0}$ | $\phi_{0}$ | $\delta_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(\mathrm{~cm})$ | $(\mathrm{mr})$ | $(\mathrm{cm})$ | $(\mathrm{mr})$ | $(\%)$ |
|  |  |  |  |  |  |
| $x$ | $-4.1 \times 10^{-3}$ | 2.10 | $3.4 \times 10^{-6}$ | $1.6 \times 10^{-4}$ | $-1.9 \times 10^{-2}$ |
| $\theta$ | $-1.2 \times 10^{-3}$ | -2.19 | $-4.4 \times 10^{-6}$ | $-3.0 \times 10^{-4}$ | 0.11 |
| $y$ | $1.8 \times 10^{-4}$ | $1.9 \times 10^{-3}$ | -0.12 | -2.25 | $6.7 \times 10^{-5}$ |
| $\phi$ | $-4.7 \times 10^{-4}$ | $-1.5 \times 10^{-3}$ | 0.14 | -5.63 | $-1.1 \times 10^{-4}$ |
| $x^{2}$ | $4.8 \times 10^{-4}$ | $3.3 \times 10^{-3}$ | $-2.9 \times 10^{-6}$ | $-1.5 \times 10^{-4}$ | $1.7 \times 10^{-4}$ |
| $x \theta$ | $-9.4 \times 10^{-4}$ | $-2.1 \times 10^{-2}$ | $1.0 \times 10^{-5}$ | $3.3 \times 10^{-4}$ | $-7.3 \times 10^{-4}$ |
| $x y$ | $-1.8 \times 10^{-6}$ | $1.8 \times 10^{-4}$ | $1.9 \times 10^{-4}$ | $-5.5 \times 10^{-3}$ | $4.7 \times 10^{-6}$ |
| $x \phi$ | $-4.2 \times 10^{-4}$ | $-2.5 \times 10^{-4}$ | $3.0 \times 10^{-4}$ | $3.1 \times 10^{-3}$ | $-7.4 \times 10^{-5}$ |
| $\theta^{2}$ | $-4.6 \times 10^{-4}$ | $1.9 \times 10^{-2}$ | $-5.4 \times 10^{-6}$ | $-1.7 \times 10^{-4}$ | $6.9 \times 10^{-4}$ |
| $\theta y$ | $-3.2 \times 10^{-4}$ | $-5.0 \times 10^{-4}$ | $-6.6 \times 10^{-4}$ | $2.1 \times 10^{-2}$ | $-6.4 \times 10^{-5}$ |
| $\theta \phi$ | $6.7 \times 10^{-4}$ | $7.5 \times 10^{-4}$ | $-1.1 \times 10^{-3}$ | $-1.3 \times 10^{-2}$ | $1.3 \times 10^{-4}$ |
| $y^{2}$ | $-1.1 \times 10^{-3}$ | $-1.5 \times 10^{-3}$ | $-1.8 \times 10^{-5}$ | $2.5 \times 10^{-4}$ | $-2.1 \times 10^{-4}$ |
| $y \phi$ | $2.9 \times 10^{-3}$ | $5.1 \times 10^{-3}$ | $3.1 \times 10^{-5}$ | $-3.3 \times 10^{-4}$ | $6.0 \times 10^{-4}$ |
| $\phi^{2}$ | $-1.2 \times 10^{-3}$ | $-3.5 \times 10^{-3}$ | $-9.0 \times 10^{-6}$ | $4.6 \times 10^{-6}$ | $-2.1 \times 10^{-4}$ |
|  |  |  |  |  |  |

$$
p_{r}=1.0 \mathrm{GeV} / \mathrm{c} \quad p_{r}=2.6 \mathrm{GeV} / \mathrm{c}
$$

Resolutions in reconstructed quantities

| $\Delta \theta_{\circ}(\mathrm{mr})$ | $\pm 8.3$ | $\pm 4.0$ |
| :---: | :---: | :---: |
| $\Delta \phi_{\circ}(\mathrm{mr})$ | $\pm 17.4$ | $\pm 5.6$ |
| $\Delta \delta_{\circ}(\%)$ | $\pm 0.41$ | $\pm 0.23$ |
|  | Resolutions at MWPCs |  |
| $\Delta x(\mathrm{~cm})$ | $\pm 0.8$ | $\pm 0.2$ |
| $\Delta \theta(\mathrm{mr})$ | $\pm 3.2$ | $\pm 1.4$ |
| $\Delta y(\mathrm{~cm})$ | $\pm 0.8$ | $\pm 0.2$ |
| $\Delta \phi(\mathrm{mr})$ | $\pm 3.1$ | $\pm 1.0$ |

## FIGURE CAPTIONS

1. The $180^{\circ}$ double-arm spectrometer system, placed between the SLAC 8 $\mathrm{GeV} / \mathrm{c}$ and $20 \mathrm{GeV} / \mathrm{c}$ spectrometers. The elements B 1 to B 8 are dipole magnets and Q1 to Q6 are quadrupoles. Shown also are the detector systems, the target chamber, the beam dump, and the concrete and iron shielding.
2. The electron spectrometer and the incident beam transport system. Shown also are the vacuum chambers, the target scattering chamber, the electron detector system, and the B1 and B3 screens. The deflection angles $\theta_{B 1}$ through $\theta_{B 4}$ and $X_{c}$, the lateral displacement of B2, are discussed in the text.
3. The recoil spectrometer and the beam dump. Also shown are the vacuum system, the target scattering chamber, the recoil detection system, the dump toroid and the B5 and dump screens.
4. The coordinate systems $\left(x^{e}, y^{e}, z^{e}\right)$ and $\left(x^{r}, y^{r}, z^{r}\right)$ of the electron and recoil spectrometers. The origin of both systems is the target center. Shown are the initial horizontal angles $\theta_{\circ}^{e}$ and $\theta_{\circ}^{r}$ and initial vertical angles $\phi_{\circ}^{e}$ and $\phi_{\circ}^{r}$ for a typical electron and recoil particle. The total scattering angles $\Theta_{e}$ and $\Theta_{r}$ are also shown.
5. The maximum acceptance angles $\left(\theta_{\circ}^{e}\right)_{\max }$ and $\left(\phi_{\circ}^{e}\right)_{\max }$ of the electron spectrometer for rays originating at the center of the target versus the pole-tip field of quadrupole Q3. The curves were calculated using the thin lens model described in the text. For a given value of $B_{Q 3}$, the focusing strengths of Q1 and Q2 were adjusted to obtain parallel-to-point optics in the vertical direction and point-to-parallel optics in the horizontal direction. The arrow indicates the value chosen for $B_{Q 3}$ to maximize the spectrometer acceptance.
6. The maximum acceptance angles $\left(\theta_{o}^{e}\right)_{\max }$ and $\left(\phi_{\circ}^{e}\right)_{\max }$ of the electron spectrometer for rays originating along the beam axis as a function of position
$z_{o}^{e}$ near the target center. The results were computed using the thin lens model after the magnet positions and field strengths were fixed to maximize the acceptance at $z_{o}^{e}=0$.
7. The solid angle of the electron spectrometer as a function of relative momentum $\delta_{o}^{e}$ for a thin target as calculated from the Monte Carlo model.
8. The optical properties of the electron spectrometer for some typical rays: a) the point-to-parallel focusing in the horizontal plane; two rays are shown with different $\theta_{\circ}^{e}$, each with $y_{\circ}^{e}=\phi_{\circ}^{e}=x_{\circ}^{e}=0$ and $\delta_{\circ}^{e}=-1 \%$. b) the parallel-to-point focusing in the vertical plane; two rays are shown with different $y_{\circ}^{e}$, each with $x_{\circ}^{e}=\theta_{\circ}^{e}=\delta_{\circ}^{e}=0$ and $\phi_{\circ}^{e}=80 \mathrm{mr}$. The rays were traced using the TRANSPORT model. The dashed curves (indistinguishable from the solid lines in plot b ) were obtained using only first-order matrix elements, while for the solid curves both first- and second-order matrix elements were used. The horizontal solid lines indicate, from left to right, the apertures in magnets Q3, Q2, Q1, B3 and B4.
9. Optical properties of the electron spectrometer for two extreme rays showing the large chromatic second-order effects: a) horizontal plane with $y_{o}^{e}=$ $\phi_{\circ}^{e}=0$ and large values for $x_{o}^{e}, \theta_{o}^{e}$ and $\delta_{\circ}^{e}$ and b) vertical plane with $x_{o}^{e}=\theta_{o}^{e}=0$ and large values for $y_{\circ}^{e}, \phi_{\circ}^{e}$ and $\delta_{o}^{e}$. The rays were traced using the TRANSPORT model. The solid curves were obtained using both first- and second-order matrix elements, while for the dashed curves only first-order matrix elements were used. The horizontal solid lines indicate, from left to right, the apertures in magnets Q3, Q2, Q1, B3 and B4.
10. Optical properties of the recoil spectrometer for two extreme rays in the horizontal plane: a) medium value for $\theta_{\mathrm{o}}^{r}$ and large value for $\delta_{\mathrm{o}}^{r}$ and b) large value for $\theta_{\circ}^{r}$ and $\delta_{\circ}^{r}=0$. In both cases $y_{o}^{r}=\phi_{o}^{r}=0$, and $x_{o}^{r}=0.25 \mathrm{~cm}$. The curves were calculated with the TRANSPORT model. The solid curves include both first- and second-order matrix elements, while the dashed curves (indistinguishable from the solid curve in plot b) include only first-
order matrix elements. The horizontal solid lines indicate, from left to right, the apertures of magnets Q4, Q5, Q6, B5, B6, B7 and B8.
11. Same as Figure 10 except for rays in the vertical plane: a) medium $y_{0}^{r}$ value and large $\phi_{\circ}^{r}$ and b ) large $y_{\circ}^{r}$ value medium $\phi_{\circ}^{r}$. In both cases $x_{\circ}^{r}=$ $\theta_{0}^{r}=\delta_{o}^{r}=0$.
12. The maximum acceptance angles $\left(\phi_{\circ}^{r}\right)_{\max }$ and $\left(\theta_{\circ}^{r}\right)_{\max }$ of the recoil spectrometer as a function of position along the beam axis $z_{0}^{r}$. The target center is at $z_{\mathrm{o}}^{r}=0$. The solid lines were calculated using the thin lens model described in the text. The dashed lines show the largest recoil angles dictated by electron-deuteron elastic kinematics for the design values of the electron arm acceptance angles $\left(\theta_{\mathrm{o}}^{e}\right)_{\max }$ and $\left(\phi_{\mathrm{o}}^{e}\right)_{\max }$. These calculations used formulas (3.6) and (3.7) convoluted with multiple scattering effects.
13. The variation of the recoil spectrometer solid angle with relative momentum $\delta_{0}^{r}$ for a thin target calculated with the Monte Carlo model.
14. Electron beam intensity profiles at the beam dump: a) projected onto the horizontal axis, and b) projected onto the vertical axis. Two extreme cases are shown corresponding to electron-deuteron kinematics at $Q^{2}=1$ $(\mathrm{GeV} / \mathrm{c})^{2}$ with a 10 cm long target (solid curves) and $Q^{2}=5(\mathrm{GeV} / \mathrm{c})^{2}$ with a 40 cm target (dashed curves). The profiles were generated using the program DECAY TURTLE and include the effects of energy loss and multiple scattering in the targets.
15. a) The ratio of the nominal current $I_{e p}$ of magnet B 3 as derived from the electron-proton calibration data to the calculated current $I_{\text {cal }}$ versus the momentum $p_{e}^{\prime}$ of the electron spectrometer. b) The same ratio for magnet B4. The calculation used the absolute measurement of the integrated magnetic field from the long coil measurements and the shape of the field from the Hall probe measurements.
16. Setup used to measure the vertical focusing of the electron spectrometer. A 1.1 cm thick tungsten shield with circular apertures along the vertical
axis was placed between Q3 and a 0.64 cm thick Al target. Electrons with well defined vertical angles $\phi_{\circ}^{e}$ passed through the holes to be imaged at specific $y^{e}$ locations in the wire chambers.
17. Comparison between the optics model of the electron spectrometer and data from the special calibration measurement run with the tungsten shield and the Al target (see Figure 16). The arrows show the predicted positions for the centroids of the peaks.
18. Comparison between data and Monte Carlo prediction for the $\theta^{e}$ electron distribution at the center of the electron arm wire chambers for $e p$ elastic scattering using a 0.937 GeV beam incident on a 10 cm long target. The coordinate $\theta^{e}$ is directly proportional to $\delta_{o}^{e}$.
19. Same as Figure 18 except for $y^{e}$, which is proportional to $\phi_{\circ}^{e}$.
20. Comparison between data and Monte Carlo prediction for the $x^{r}$ recoil proton distribution at the center of the recoil arm wire chambers for ep elastic scattering using a 0.937 GeV beam incident on a 10 cm long target.
21. Same as Figure 20 except for $\theta^{r}$.
22. Effective single-arm (squares) and double-arm (circles) solid angles for $e p$ elastic scattering versus target length for a beam energy of 1 GeV calculated with the Monte Carlo program. The geometric single-arm solid angles for $\delta_{\circ}= \pm 1 \%$ are also shown (diamonds).
23. Double-arm electron-proton elastic cross sections divided by the prediction of the dipole law, compared with backward angle data from previous experiments (CEA, BONN, DESY, Stanford) and forward angle data from SLAC, as compiled in Ref. 17. The NE4 cross sections are averaged over the different target lengths used at each kinematic point.
24. Ratio of single-arm to double-arm electron-proton elastic cross sections as a function of $Q^{2}$, averaged over the different target lengths used at each kinematic point.


Fig. 1


Fig. 2


Fig. 3


Fig. 4


Fig. 5


Fig. 6


Fig. 7


Fig. 8


Fig. 9


Fig. 10


Fig. 11


Fig. 12


Fig. 13


Fig. 14


Fig. 15


Fig. 16


Fig. 17


Fig. 18


Fig. 19


Fig. 20


Fig. 21


Fig. 22


Fig. 23


Fig. 24


[^0]:    * Work supported in part by the Department of Energy, contract DOE-AG03-76SF00515 (SLAC) and by National Science Foundation, grant PHY85-10549 (AU).

