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Cosmology with Decaying Vacuum Energy*

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ABSTRACT

Motivated by recent attempts to solve the cosmological constant problem, we examine the observational consequences of a vacuum energy density which decays in time. For all times later than $t \sim 1$ sec, the ratio of the vacuum to the total energy density of the universe must be small. Although the vacuum cannot provide the "missing mass" required to close the universe today, its presence earlier in the history of the universe could have important consequences. We discuss restrictions on the vacuum energy arising from primordial nucleosynthesis, the microwave and gamma ray background spectra, and galaxy formation. A small vacuum component at the era of nucleosynthesis, $0.01 < \rho_{vac}/\rho_{rad} < 0.1$, increases the number of allowed neutrino species to $N_\nu > 5$, but in some cases would severely distort the microwave spectrum.

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The cosmological constant has alternately been the most maligned and the most neglected of the constants of Nature. The seeds of this mistreatment lay in its early history: it is well known that Einstein introduced it purely to obtain steady state cosmological solutions to general relativity. When Hubble subsequently discovered the universe was expanding, the original motivation for the cosmological term was removed, and Einstein recommended with embarrassment that it be dropped as an ugly blemish on his theory. Except for occasional aberrations, as a rule, cosmologists have been happy to follow Einstein in forgetting about it. With the advent of unified gauge theories, this is no longer possible.

In the standard cosmological model, the early universe is thought to have passed through a series of symmetry breaking phase transitions at various energy scales M_x. As the temperature drops below M_x, the vacuum energy density associated with the order parameter (e.g., a Higgs field) changes by $O(M_x^4)$. It is therefore puzzling that the upper bound on the present value of the vacuum energy density, $\rho_{vac} < (0.004 \text{ eV})^4$, is much smaller than any of the energy scales associated with particle physics. Even if such a cancellation can be arranged classically, there is at present no known low energy symmetry which prevents quantum corrections from inducing a large value for ρ_{vac}. Since the stress energy of the vacuum is $T_{\mu\nu} = \rho_{vac} g_{\mu\nu}$, i.e., $p_{vac} = -\rho_{vac}$, it enters Einstein's equations precisely as a cosmological constant $\Lambda = 8\pi G \rho_{vac}$. The upper bound above, which is equivalent to $\Omega_{vac} = \rho_{vac}/\rho_{crit} \lesssim 3$, comes from the limit on the cosmological constant from measurements of the Hubble constant and the age of the universe[1], $\Lambda \lesssim 9H_0^2 \simeq 10^{-56}$ cm^2, or $\Lambda/m_{Pl}^2 \lesssim 10^{-119}$ in gravitational units. Thus, the problem: the cosmological constant is known to be tiny in any natural scale of units, but in the context of particle physics it does not appear to be a naturally small parameter.

The hope is sometimes voiced that a fundamental quantum theory of gravity will require $\rho_{vac} = 0$, but such a theory must in fact give rise to a cosmological term (say, at the Planck scale) which is precisely cancelled by all lower energy contributions (e.g., at the electroweak scale and below) to one part in 10^{119}!

Thus, the cosmological constant problem is essentially a difficulty of physics at very 'low' energies, suggesting that its solution will come instead from new physics which is manifest at low energies, or large distance- and timescales.

Along this line, several authors have recently discussed mechanisms for dynamically reducing ρ_{vac} to a very small value over cosmological timescales[2,3]. The simplest example[2] is a classical scalar field with a potential which depends on the spacetime curvature, $V(\phi) = V_0 - \xi R\phi^2$, where $\rho_{vac} = V_0 = \Lambda_0/8\pi G$ is the initial vacuum energy density and $\xi > 0$. Neglecting the scalar field kinetic energy, Einstein's equations give $R \sim 8\pi G V(\phi)$. If the field starts near the origin, then initially $R \sim \Lambda_0$ and the field begins to roll down the potential exponentially fast. This reduces both $V(\phi)$ and, by Einstein's equation, the Ricci scalar R over time. But this implies that the slope of the scalar field potential is also decreased, so the field slows down. Asymptotically, $\phi \sim t$ and the effective vacuum energy redshifts away, $\rho_{vac} \sim t^{-2}$. This simple 'feedback' model is indicative of the classical relaxation mechanisms which have been proposed[2]. It is also possible that the dynamical effects of *quantum* fields may render de Sitter space (the spacetime dominated by a cosmological constant) unstable to conformal perturbations [3]. At present, the significance for cosmology of such an instability is unclear, since it is not known how the system would evolve away from the initial de Sitter solution.

These ideas suggest the intriguing possibility that the universe evolves to a state in which the effective cosmological term (ρ_{vac}) is small and continues to decrease with time. In this talk, I summarize the consequences for observational cosmology of such a continuously *decaying* vacuum energy density[4,5]. Such scenarios are interesting because a redshifting vacuum energy can have effects over many expansion times, while a constant vacuum density (the usual case) could only have become dynamically important at very recent epochs. Our major conclusion is that dynamical models of decaying vacuum energy of a rather general variety are consistent with observational cosmology; however, the deviation from the standard model must be small.

Suppose that in addition to ordinary matter and radiation, there is an energy density associated with the vacuum which is time dependent, $\rho_v(t)$. This additional component enters the Einstein equation for the Robertson-Walker scale factor $a(t)$,

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_v), \tag{1}$$

where we have assumed flat spatial sections ($k = 0$). The pressure of the vacuum is assumed to have the usual form $p_v = -\rho_v$, while $p_m = 0$, $p_r = \rho_r/3$.

The conservation equation for energy-momentum takes the form

$$\dot{\rho}_v + \frac{1}{a^3}\frac{d}{dt}(\rho_m a^3) + \frac{1}{a^4}\frac{d}{dt}(\rho_r a^4) = 0 \tag{2}$$

We notice immediately that if $\dot{\rho}_v \neq 0$, at least one of the ordinary adiabatic relations $\rho_r \sim a^{-4}$, $\rho_m \sim a^{-3}$ ceases to be valid. If $\dot{\rho}_v < 0$ then entropy or matter *must* be generated in the expansion. In this talk, I focus on the radiation-dominated epoch ($\rho_m < \rho_r$), in which case the second term in Eqn.(2) is small and can be neglected. That is, we are only concerned with the coupling of the vacuum to massless radiation, and shall further assume the coupling to massive particles is suppressed, e.g. due to threshold effects. (In the dynamical decay scenario of refs.2,3, the only lengthscale in the problem appears to be the Hubble radius or possibly the Compton wavelength associated with a very small mass, so we expect the radiation emitted to be peaked at very long wavelengths or ultralow energies. For a discussion of vacuum decay to massive particles, see ref.[4].)

It is useful to define a new parameter which characterizes these models,

$$x = \rho_v/((\rho_r + \rho_v) \tag{3}$$

From Eqn.(2), we obtain[5] the evolution equation

$$\frac{\dot{x}}{x} = \frac{\dot{\rho}_v}{\rho_v} + 4\frac{\dot{a}}{a}(1-x) \tag{4}$$

There are three possibilities for the behavior of $x(t)$ at large t: (i) for $x \rightarrow 1$, the vacuum term dominates, and the universe becomes de-Sitter-like as the radiation is redshifted away. This case is ruled out at the level of the bounds of Ref. 1; (ii) the vacuum density falls more rapidly than the radiation density, *i.e.*, $x \rightarrow 0$, and we recover the standard cosmological model; (iii) the only genuinely new cosmology is obtained if x approaches a non-zero constant betwen 0 and 1, which corresponds to the vacuum and radiation densities redshifting at the same rate. If the vacuum and the radiation are coupled by particle creation or if the relaxation mechanism is of the 'feedback' form discussed above (i.e., the scalar field responds to the total curvature, which gets a contribution from the radiation), one may expect behavior of the form (iii) to be generic. We consider only this case in the following.

From Eqn.(4), we find

$$\rho_v(t) = \left(\frac{x}{1-x}\right)\rho_r \sim a^{-4(1-x)}. \tag{5}$$

As expected, the radiation density drops more slowly as a function of the scale factor than in the standard cosmology, whereas the matter density approximately redshifts in the usual way, $\rho_m \sim a^{-3}$, if matter creation is negligble. With this scaling of the two components, the constraint that an early radiation epoch be followed by a matter dominated era requires that $x < \frac{1}{4}$ in both the matter and radiation epochs. Eqns.(1) and (5) are easily solved to yield

$$a \sim t^{\frac{1}{2(1-x)}} \ , \ \rho_v = 3x/32\pi G(1-x)^2 t^2 \tag{6}$$

Increasing x towards unity speeds up the expansion rate of the universe in the radiation era.

As long as the created radiation reaches thermal equlibrium, it can be characterized by its temperature, with $\rho_r = \frac{\pi^2}{30} g_{\rm eff} T^4$; here $g_{\rm eff}$ is the number of relativistic degrees of freedom. In this case, from Eqns.(5,6), we find

$$T(t) = \left[\frac{16\pi^3 G\ g_{\rm eff}(1-x)}{45}\right]^{-\frac{1}{4}} t^{-\frac{1}{2}}, \tag{7}$$

for the radiation temperature as a function of time in the radiation-dominated epoch. The electromagnetic radiation created by vacuum decay thermalizes completely up to at least a time $t_T \approx 10^5$ sec (and, in particular, throughout primordial nucleosynthesis); thus it makes sense to describe the radiation by a Planck spectrum with a temperature given by (7). The assumption of thermal equilibrium determines how the radiation number density and energy per particle change with time. From Eqn.(5), as long as the photons remain in thermal equilibrium, we have $T \sim a^{x-1}$. To maintain a Planck distribution, the energy per particle must redshift like the temperature, so $E_\gamma \sim a^{x-1}$ as well. From Eqn.(5), this implies the photon number density scales as $n_\gamma \sim a^{-3(1-x)}$.

The assumption that photons are created in vacuum decay also implies that the baryon to photon ratio, n_B/n_γ, decreases as the universe expands. Since baryons are not created, $n_B \sim a^{-3}$, and the baryon to photon ratio thus scales as

$$\eta = \frac{n_B}{n_\gamma} \sim a^{-3x} \sim T^{\frac{3x}{1-x}} \tag{8}$$

at least up to t_T. (We have not considered the case where the vacuum decays, in whole or in part, into noninteracting, nonthermal particles such as gravitons, or shadow photons but our treatment does include massless neutrino production.)

i) Nucleosynthesis

Since a non-zero vacuum component changes both the expansion rate through Eqn.(6), the temperature-time relation, Eqn.(7), and the baryon to photon ratio (8), it can alter the delicate balance with nuclear reaction rates at the time of helium and deuterium synthesis that holds in the standard cosmology. (For a review of standard big bang nucleosynthesis, see the talk by Steigman in this volume and references therein.) At the high temperatures in the early universe, the ratio of neutrons to protons is determined by its thermal equilibrium value,

$$n/p = e^{-Q/kT}, \quad T \geq T_F \tag{9}$$

where the neutron-proton mass difference $Q = 1.293$ MeV and k is Boltzmann's constant. Neutrons drop out of equilibrium below a freeze-out temperature T_F, where the weak interaction rates can no longer keep up with the expansion of the universe. Below T_F the n/p ratio continues to fall due to β-decay on the time scale of the neutron half-life τ_n. In the standard model, nucleosynthesis begins at a temperature approximately given by

$$T_D = \frac{2.2 \text{ MeV}}{-\ln\ \eta}. \tag{10}$$

Once T_D is reached, deuterium becomes stable against photodissociation and nucleosynthesis takes place very rapidly, efficiently converting essentially all of the available neutrons into ^{4}He. In this approximation, the primordial helium abundance Y_p is given by

$$Y_p = \left(\frac{2n}{n+p}\right)_D = \left(\frac{2n}{n+p}\right)_F \exp\left[-\Gamma(t_D - t_F)\right] \approx \frac{2e^{-\Gamma t_D}}{1+\exp\left[Q/kT_{\rm F}\right]} \tag{11}$$

where the final approximation is valid since $\Gamma^{-1} = \tau_n / \ln\ 2 \gg t_F \sim 1$ sec.

In the presence of a small vacuum component $x \ll 1$ (we will see from the numerical results that x must be less than 0.1), we can illustrate heuristically the deviation from the standard model. Recall that freeze-out occurs when a typical $n \Leftrightarrow p$ weak interaction rate $\Gamma \sim G_F^2 T_F^5$ is equal to the expansion rate $H \sim [G(\rho_r + \rho_v)]^{\frac{1}{2}}$. Since $\rho_v + \rho_r = \rho_r/(1-x)$ and $\rho_r \sim T^4$, we have $H \sim T^2/(1-x)^{-1/2}$. Equating $\Gamma \simeq H$, we obtain

$$T_F = \bar{T}_F (1-x)^{-\frac{1}{6}} \tag{12}$$

where an overbar indicates the standard model value ($x = 0$). By itself this would tend to increase Y_p by increasing the n/p ratio at freeze out. However, Eqns.(7)

and (10) indicate that

$$t_D = \bar{t}_D(1-x)^{-\frac{1}{2}}, \tag{13}$$

which increases the available time for neutrons to β-decay. This turns out to be the larger effect in the domain of interest, so that Y_p is a *decreasing* function of x.

The numerical analysis, using Wagoner's[6] nucleosynthesis code modified to include a nonzero vacuum term, does indeed follow this general trend, although the results differ quantitatively. One of the most dramatic changes from the standard model (not included in the discussion above) is the behavior of the entropy, which in the standard model is constant throughout and after nucleosynthesis (except for the infusion of e^+e^- pairs). In the decaying vacuum model the entropy per baryon can change drastically through nucleosynthesis and continues to change afterward according to Eqn.(8).

We present our results graphically for ^{4}He and D abundances in Figs.1-3 for $N_\nu = 3, 4$, and 5 neutrino species. For comparison with observation we have chosen a temperature $T_N = 10^8$ K to signal the end of nucleosynthesis; after this time, element abundances from the code no longer change significantly even for nonzero x, although the entropy continues to drop according to Eqn.(8). Requiring that $0.22 \leq Y_p \leq 0.26$ and $10^{-5} \leq D/H \leq 10^{-4}$[7], we find the (η, x) plane at T_N is restricted as shown. For fixed number of light neutrino species N_ν, the constraints on the vacuum component are: $x < 0.08(N_\nu = 3)$, $x < 0.09(N_\nu = 4)$, and $x < 0.10(N_\nu = 5)$. Although at most four neutrino (or equivalent numbers of light) species can be accommodated in the standard model, for $x \geq 0.01$ five neutrinos (or more) are consistent with the observed element abundances (see Fig.3).

We have also confirmed consistency with observation of the ^{7}Li abundance obtained from the code for this range of parameters. The abundances of ^{4}He, D, and ^{7}Li are all lower than in the standard model, whereas the H abundance is slightly higher. The constraints on $\eta(T_N)$ become more restrictive in the presence of a nonzero x, but remain within the same range as in the standard model ($10^{-10} \leq \eta(T_N) \leq 10^{-9}$). Thus we reach these conclusions regarding nucleosynthesis:

1) Primordial nucleosynthesis in the presence of a vacuum component with $x \leq 0.1$ is consistent with observations of abundances of ^{4}He, D and the other light nuclei.

2) If $x > 0$, the preferred values of η at nucleosynthesis lie within the same range as in the standard model.

3) If $x > 0$, Y_p decreases. Observational estimates of the ^{4}He abundance in HII regions have decreased recently, and may make even 3 light neutrinos uncomfortable for the standard nucleosynthesis model (See the contribution by Pagel in these proceedings.) If these observations hold up and if $N_\nu > 3$ neutrinos are found at the SLC (or if $N_\nu > 4$ neutrinos are found, regardless), the standard model would be in difficulty. This could be resolved by the presence of a small vacuum component at nucleosynthesis. We also note that non-standard scenarios (with baryon diffusion or late hadronic decays) with a large baryon density, $\Omega_B \simeq 1$, rely on reducing an initial overproduction of ^{4}He. Since our model underproduces ^{4}He, the constraint on the vacuum component cannot be evaded by invoking such a scenario.

ii) Microwave Background Distortions

An interesting feature of models with x =const. and $\rho_v \sim 1/t^2$ is that some fraction of the microwave background photons in the present universe was created by the decay of the vacuum. The spectrum of radiation emitted by the decaying ρ_v is model-dependent: in general it may be quite different from the Planck distribution appropriate for fully equilibrated radiation. If this is the case, and if the processes involved in the relaxation of the injected photon spectrum toward equilibrium are not 100% efficient, then distortions of the Planck spectrum may arise[8].

In this section we explicitly assume that the vacuum does not decay into photons fully equilibrated to a Planck spectrum. As mentioned above, the most likely possibility seems to be that the emission is peaked at long wavelengths ($E_\gamma < kT$). In that case the photons would be efficiently absorbed by the free electron plasma via inverse bremsstrahlung, since the cross section for this process rises like $1/\omega^3$. At frequencies lower than the plasma frequency, any electromagnetic radiation produced by the decaying vacuum is rapidly damped and its energy transferred to the plasma by ohmic heating. (However, a zero-frequency magnetic component may survive; one can speculate that vacuum decay may produce large-scale primordial magnetic fields.) The result in either case would be to increase the electron energy density relative to the radiation density. However, at very early times, *i.e.*, for redshifts greater than $z_{\rm T} = 6.3 \times 10^4 (\Omega_B h^2)^{-\frac{6}{5}}$, the injected energy is completely thermalized by double Compton and bremsstrahlung process: no distortions survive. [In this and all the following we take $H_0 = 100h$ km/sec Mpc^{-1} for the present value of the Hubble parameter, $T_0 = 2.7\,^\circ$K for the present radiation temperature and we assume 3 massless neutrino species;Ω_B is the density parameter for the ionized gas. We also neglect the small x-dependent factors in all redshifts defined in this section.]

When $z < z_T$, the injected radiation energy heats the electron plasma, and photon production by the electron gas continues in the far Rayleigh-Jeans region ($\hbar\omega \ll kT$). At higher energies, however, photon production by the hot electrons becomes inefficient, and Compton scattering cannot redistribute the excess low energy radiation toward the peak. Thus, except at the very low end of the spectrum, we have $T_e > T_r$, and multiple scattering off the electrons shifts the background radiation to higher frequencies without changing the total number of photons. The spectrum then takes on a Bose-Einstein form, with a nonzero chemical potential μ. If $\frac{T_e - T_r}{T_r} \ll 1$, the resulting value of μ (also small compared to kT_r) depends only on the total amount of energy injected into photons, $\Delta\rho_r$, and is *independent* of their initial frequency distribution:

$$\mu = 1.4\ kT_r \frac{\Delta\rho_r}{\rho_r} \tag{14}$$

For a continuous injection of photons by vacuum decay, $\frac{\Delta\rho_r}{\rho_r}$ is the time integral of $-\frac{\dot{\rho}_v}{\rho_r}$ (the contribution of neutrinos to ρ_r cancels from the integral),

$$\begin{aligned}\frac{\Delta\rho_r}{\rho_r} &= -\int_{z_i}^{z_f} \frac{\dot{\rho}_v}{\rho_r}\frac{dt}{dz}dz = \frac{x}{1-x}\ln\left(\frac{\rho_r(z_i)}{\rho_r(z_f)}\right) \\ &= 4x\ln\left(\frac{z_i}{z_f}\right)\end{aligned} \tag{15}$$

by Eqn.(5). Thermalization to a Bose-Einstein spectrum requires multiple photon scattering, but the average number of scatterings per photon decreases as the universe expands. For times later than $t_1 \cong 10^{11}$ sec (or redshifts less than $z_1 = 8.5\times10^3(\Omega_B h^2)^{-\frac{1}{2}}$), multiple scattering becomes inefficient, and the background subsequently evolves too slowly to relax to a Bose-Einstein spectrum. So we take $z_i = z_T$ and $z_f = z_1$ for these μ distortions. Since observations of the microwave background spectrum require $\mu < 0.01kT_r$ [9] we obtain the bound on x:

$$x < 4\times10^{-4} \tag{16}$$

where we have taken $\Omega_B h^2 = 2.5\times10^{-2}$ here and below.

At later times $t > t_1$, energy injected and efficiently absorbed by the electron plasma produces a different distortion of the microwave background spectrum. Compton scattering shifts the photons to higher energies, creating an excess in the Wien region and a shortage in the Rayleigh-Jeans part of the spectrum. The resulting spectrum is parametrized by a new variable y, which can again be related to the total energy injected:

$$y = \frac{1}{4}\frac{\Delta\rho_r}{\rho_r} = x\ \ln\left(\frac{z_i}{z_f}\right) \tag{17}$$

by Eqn.(15).

If we assume that the injected photons are too low in energy to reionize the gas after it has recombined, then z_f for this distortion should be taken to be of the order of the redshift of recombination, $z_2 \cong 10^3$. For $t > t_2$, energy injection will continue to raise the temperature of the residual ionized gas and heat the intergalactic medium, without distorting the microwave spectrum.

Taking $z_i = z_1$ and $z_f = z_2$ in Eqn.(17) and using the observational bound $y < 0.02$ [9] yields the following bound on x:

$$x < 5 \times 10^{-3} \tag{18}$$

so that the μ bound is the most stringent constraint on x we have obtained. These constraints are so severe because the background radiation is being subjected to the injection of energy over many expansion times, when the processes responsible for restoring equilibrium are inefficient. We reiterate that the key assumptions used in deriving these bounds are (i) that the vacuum produces photons which are out of equilibrium with the pre-existing radiation and (ii) that essentially all of the energy injected by the vacuum decay goes into heating the electron gas to $T_e > T_r$. If the vacuum decays into some non-interacting form of dark matter instead, then (ii) need not be true and we would again lose the very stringent bounds of Eqns.(16,18).

iii) Entropy Generation

Since entropy is produced by the decay of the vacuum, η decreases with temperature according to Eqn.(8). The vacuum energy density can be constrained by the evolution of η after nucleosynthesis. The most stringent bound is obtained if the vacuum decays to a thermal spectrum of radiation for all time. In this case, Eqn.(7) applies through the present epoch,

$$\eta(2.7\ \mathrm{K}) = \eta(T_N) \left(\frac{2.7\ \mathrm{K}}{T_N} \right)^{\frac{3x}{1-x}} . \tag{19}$$

Taking $\eta(2.7\ \mathrm{K}) \gtrsim 2\times10^{-11}$ [7] for a conservative lower bound today and $\eta(T_N) \leq$

10^{-9} at nucleosynthesis, we find

$$x \leq 0.07 \tag{20}$$

On the other hand, if, as considered above, the radiation produced by vacuum decay only thermalizes up to a time t_T, then η remains constant for $T < T_T \simeq \left(3.3 \times 10^5 (\Omega h^2)^{-\frac{6}{5}}\right)^{1-x} 2.7$ K. In this case, we obtain the less stringent bound

$$x < 0.15. \tag{21}$$

Additional model-dependent bounds on x from the evolution of η arise from consideration of big bang baryogenesis; the limits are comparable to those above.

We have investigated the cosmological constraints on and consequences of a vacuum energy density which dynamically decays in time. We conclude that such a scenario can be consistent, but the universe cannot be vacuum-dominated for times later than about $t \sim 1 sec$. For vacuum decay to a non-thermal radiation distribution, the microwave background spectrum provides the strongest constraint, $x < 4 \times 10^{-4}$. On the other hand, if the radiation produced by the vacuum retains a Planck spectrum for all time, the requirement that the baryon-to-photon ratio not drop too low after nucleosynthesis gives the strongest bound, $x < 0.07$. Vacuum decay appears to be a promising framework for solving the cosmological constant problem, but more work needs to be done in constructing realistic particle physics models. For example, the simplest 'feedback' relaxation models suggest values of $x \sim 1$. It would be interesting to see if more sophisticated models can naturally generate the requisite smaller values of x reported here, without fine tuning.

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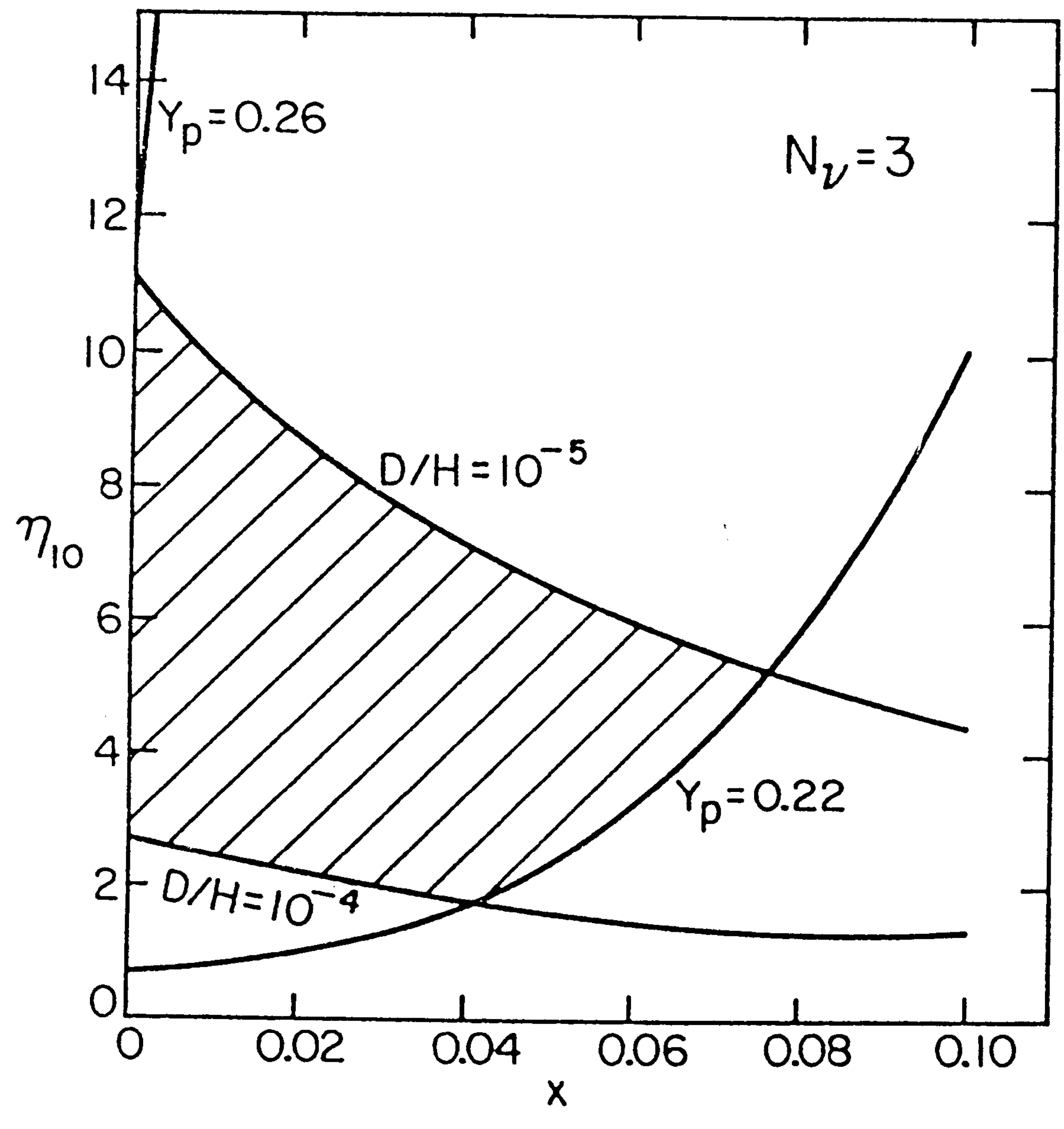
FIGURE CAPTIONS

1) Element abundances as a function of the vacuum energy density parameter x and the baryon-to-photon ratio at T_N, $\eta = \eta_{-10}10^{-10}$ for $N_\nu = 3$ neutrino species. The primordial ^{4}He abundance satisfies $0.22 < Y_p < 0.26$ and the ratio $10^{-5} < D/H < 10^{-4}$. Cross-hatching indicates the allowed region.

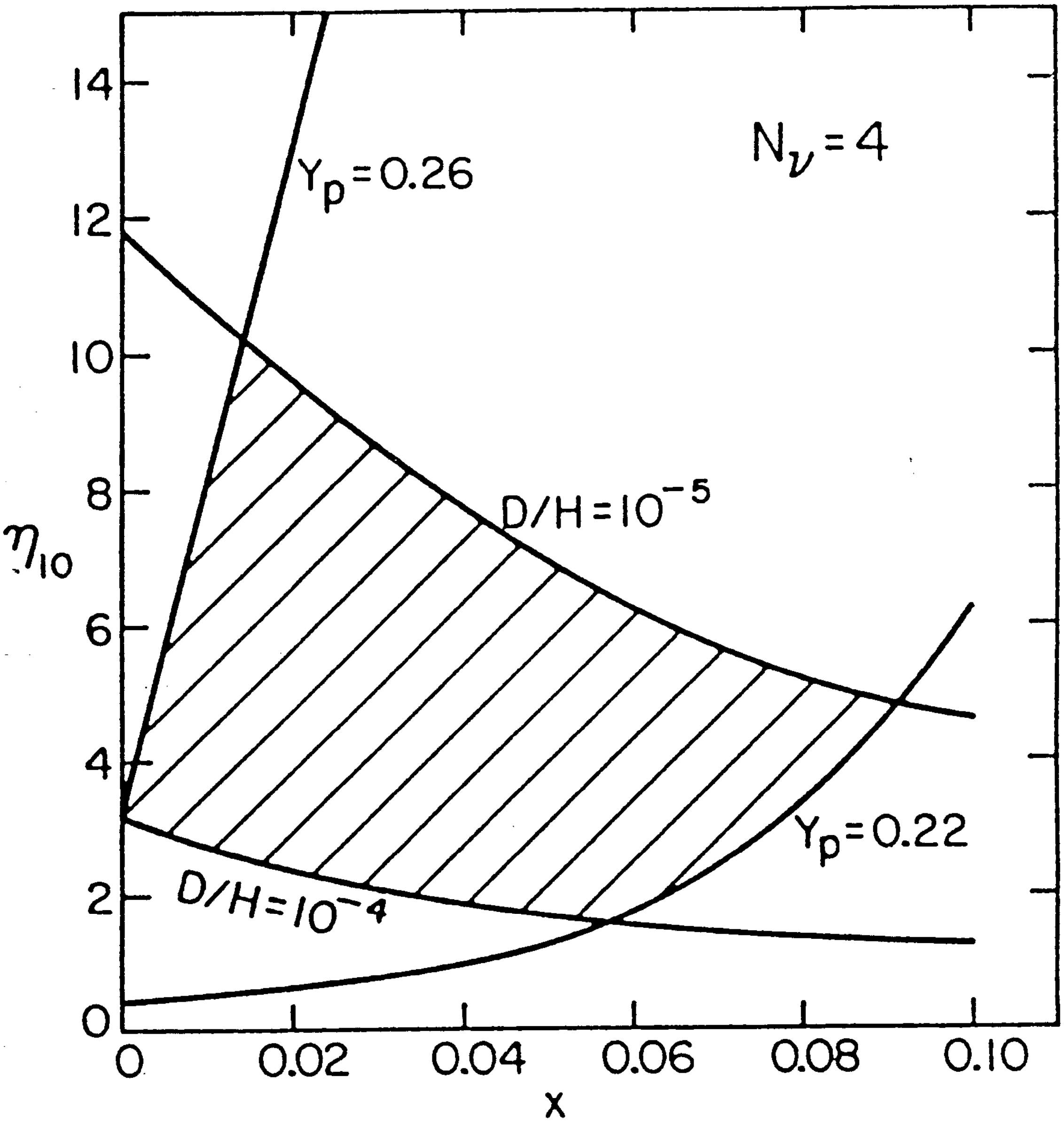
2) Same as Fig.1 for $N_\nu = 4$ neutrino species.

3) Same as Fig.1 for $N_\nu = 5$ neutrino species. Note that models with $x > 0$ can accomodate $N_\nu > 4$.



14
12
10
8
6
4
2
0
$Y_p = 0.26$
$N_\nu = 3$
$D/H = 10^{-5}$
$Y_p = 0.22$
$D/H = 10^{-4}$
η_{10}
0
0.02
0.04
0.06
0.08
0.10
x

Fig. 1



14
12
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η_{10}
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0.02
0.04
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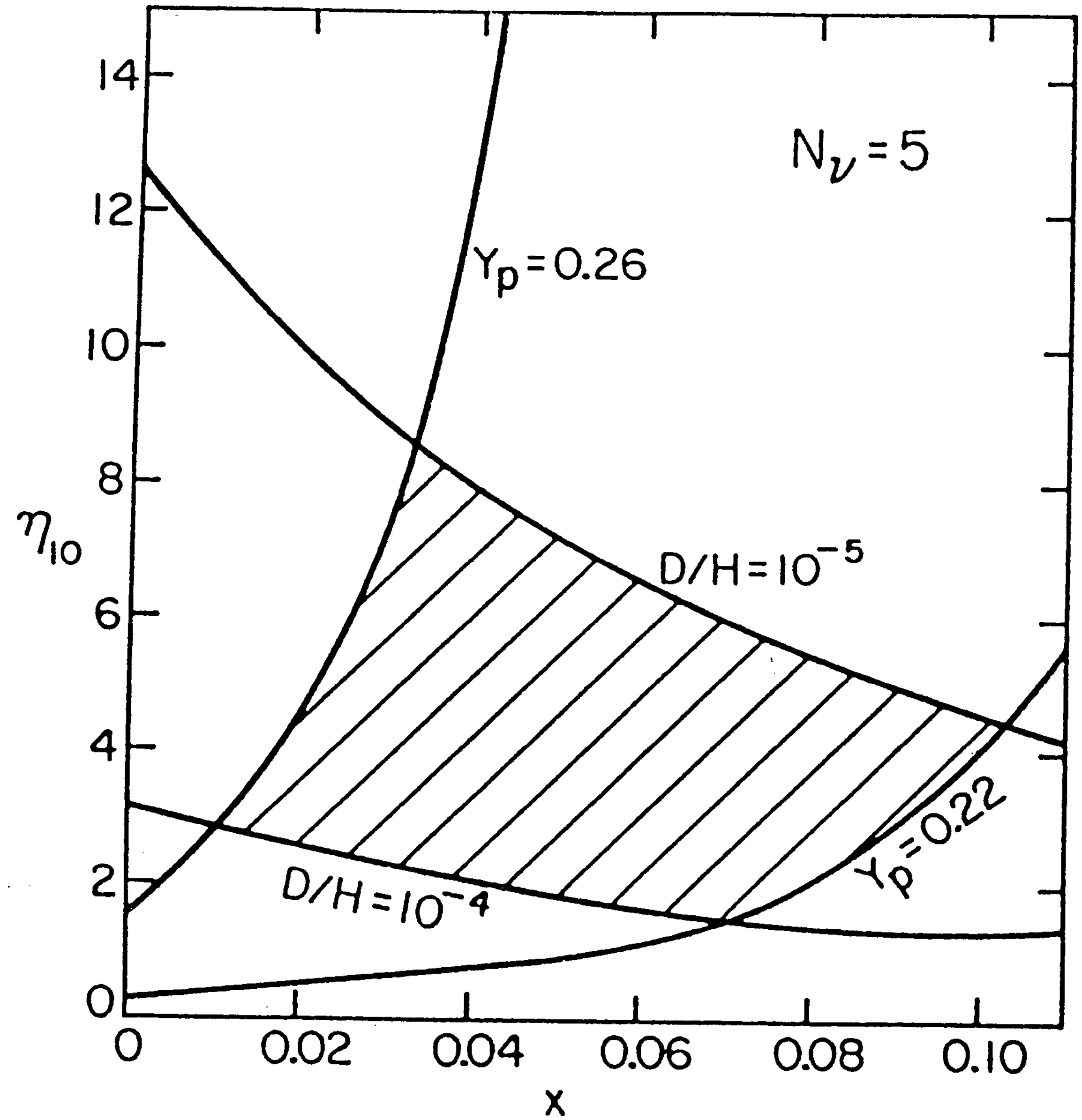
0.10
X
$N_\nu = 4$
$Y_p = 0.26$
$D/H = 10^{-5}$
$D/H = 10^{-4}$
$Y_p = 0.22$

Fig. 2



14
12
10
8
6
4
2
0
η_{10}
0
0.02
0.04
0.06
0.08
0.10
x
$N_\nu = 5$
$Y_p = 0.26$
$D/H = 10^{-5}$
$D/H = 10^{-4}$
$Y_p = 0.22$

Fig. 3