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Mass Renormalization and BRST Anomaly in String Theories*

Ashoke Sen†

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

ABSTRACT

On shell two point functions generated due to string loop corrections are shown to give rise to BRST anomaly in the scattering amplitudes in the form of total derivative terms in the moduli space. This anomaly may be cancelled by modifying the vertex operators in a way that precisely corresponds to mass renormalization in the theory.

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† Address after March 1, 1988: Theoretical Physics Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India

1. Introduction

It has become clear through recent studies that the boundaries of the moduli space give rise to divergences[1,2] (as well as BRST anomaly[3,4]) in string theories whenever any of the massless fields in the theory develop a tadpole. These divergences (BRST anomaly) may be cancelled by introducing explicit divergences (BRST anomaly) at the tree level of the theory in the form of background fields which break conformal invariance on the sphere, and the requirement of the cancellation of divergences (BRST anomaly) gives rise to the loop corrected equations of motion for the background fields in the string theory[5-10]. There is another kind of divergence in string theory which also comes from the boundary of the moduli space. These are associated with the non-vanishing two point functions in the loop amplitudes. It was shown in ref.[11] using unitarity requirement that the presence of such divergences require the momenta carried by the vertex operators to be modified precisely in a way so as to correspond to mass renormalization. On the other hand, it was shown in ref.[12] that such divergences contribute to the anomalous dimensions of the vertex operators. Thus the vertex operators must be modified once the string loop effects are taken into account, so as to keep the net conformal dimension of the vertex operator fixed. This is again manifested as a mass renormalization in the theory. In this paper we give yet another way of understanding mass renormalization in the string theory, using BRST formalism. We shall show that the presence of non-vanishing, on-shell two point functions in the one loop string perturbation theory gives rise to BRST anomaly in the loop amplitude which, in general, prevents the zero normed states to decouple from the S-matrix. This anomaly may again be cancelled by modifying the vertex operators from their (string) tree level expression precisely in a way so as to correspond to a mass renormalization.

In sec.II of the paper we discuss this effect in detail by studying a one loop scattering amplitude. The analysis is generalized to higher loops in sec.III. In sec.IV we summarize our results.

2. One loop mass renormalization

Let $V(z,k)$ be the on-shell vertex operator for a given state with momentum k, and $\tilde{V}(z,-k)$ be its hermitian conjugate operator. Let us assume that the correlator of V with $\tilde{V}$ on a torus does not vanish, and define,

$$\delta m^2 = \int d^2\tau d^2 z_1 \langle V(z_1,k)c(z_2)\bar{c}(z_2)\tilde{V}(z_2,-k)(\eta \mid b)(\bar{\eta} \mid \bar{b})\rangle_T, \tag{2.1}$$

where b, c, $\bar{b}$, $\bar{c}$ are the standard ghost fields, τ is the Teichmuller parameter on the torus, $\eta(z)$, $\bar{\eta}(z)$ are the Beltrami differentials dual to τ, $\bar{\tau}$ respectively, $\langle\ \rangle_T$ denote correlation functions on the torus, and

$$(\eta \mid b) = \int d^2 w \eta(w) b(w). \tag{2.2}$$

For simplicity we have restricted ourselves to the bosonic string theory. We believe that the analysis may be generalized to the fermionic string theory by working with the vertex operators in the -1 picture, and carrying out the integration over the resulting supermoduli using the prescription of refs.[4,13,14], although we must first resolve the ambiguity associated with the choice of basis of the super-Beltrami differentials[14,15]. Also we shall assume, for simplicity, that $\tilde{V}$ is the only operator which has a non-vanishing correlator with V at one loop order, so that there is no mixing between various operators. We shall now show that the effect of non-vanishing δm^2 is to give rise to a BRST anomaly in the scattering amplitudes. For this let us consider a one loop scattering amplitude with $n+2$ external legs ($n \geq 1$) described by on-shell vertex operators $V_i(z_i, k_i)$ ($1 \leq i \leq n+2$). The corresponding amplitude is given by,

$$\int d^2\tau \Big(\prod_{i=1}^{n+1} d^2 z_i\Big) \Big\langle \Big(\prod_{i=1}^{n+1} V_i(z_i)\Big) c(z_{n+2})\bar{c}(z_{n+2}) V_{n+2}(z_{n+2})(\eta \mid b)(\bar{\eta} \mid \bar{b}) \Big\rangle_T. \tag{2.3}$$

Let us now assume that one of the vertex operators, say V_1, corresponds to the vertex operator V appearing in eq.(2.1), and another, say V_{n+2}, describes a null state, i.e.

$$c(z)\bar{c}(z)V_{n+2}(z) = \{Q_B, \bar{c}(z)\hat{V}(z)\} = \oint J_B(w)dw\bar{c}(z)\hat{V}(z), \qquad (2.4)$$

for some operator $\hat{V}$. Here J_B is the right handed BRST current and Q_B is the corresponding BRST charge. The w contour is taken around the point z. (We could also have chosen the left-handed BRST current and replaced $\bar{c}(z)$ by $c(z)$ and w by $\bar{w}$ on the right hand side of eq.(2.4)). In order to get a unitary and gauge invariant theory, the null states must decouple, hence the amplitude (2.3) must vanish. We may try to prove this using eq.(2.4), deforming the BRST contour away from z_{n+2}, and shrinking it to zero. During this deformation the BRST contour will pick up residues from the locations of the ghost field b, as well as the locations of the vertex operators $V_i(z_i)$. Using the fact that the commutator of Q_B with b is the stress tensor, the residue at the location of b may be shown to generate a total detivative in τ[16,17]. In the bosonic string theory, this picks up a divergent boundary contribution from $\tau = \infty$ after integration over τ. This divergence is associated purely with the presence of the tachyon in the theory, and we do not have anything more to say about this in this paper. This boundary contribution is absent in the heterotic and the superstring theories. The sum of the residues at the location of the vertex operators takes the form,

$$-\int d^2\tau(\prod_{i=1}^{n+1} d^2 z_i) \sum_{j=1}^{n+1} \frac{\partial}{\partial z_j} \left\langle c(z_j)V_j(z_j)(\prod_{i\neq j} V_i(z_i))\bar{c}(z_{n+2})\hat{V}(z_{n+2})(\eta \mid b)(\bar{\eta} \mid \bar{b}) \right\rangle_T, \qquad (2.5)$$

using,

$$[Q_B, V_j(z_j)] = \frac{\partial}{\partial z_j}(c(z_j)V_j(z_j)). \qquad (2.6)$$

Thus the contribution is a total derivative in z_j, and hence may be expressed as boundary terms. There are two types of boundaries which may give non-vanishing contribution to (2.5). Firstly, we may consider boundary contribution where all the points z_i approach each other. For reasons which will become clear later, we shall pictorially represent this configuration as in fig.1(a), where S and T denote a sphere and a torus respectively. The contributions to the BRST anomalies from such boundaries are associated with one loop tadpoles, and, if present, must be removed by the Fischler-Susskind mechanism[5]. We shall focus our attention on the contribution from the boundaries shown in fig.1(b), where all but one of the z_i's approach each other. As we shall see, anomalies coming from these graphs may be cancelled by modifying the tree level vertex operators in a way which corresponds to mass renormalization. Finally, there are boundaries of the form shown in fig.1(c), where all but two (or more) external lines approach each other. These boundaries are harmless due to the following reason. Near any boundary, we may introduce a complex parameter t parametrizing the configuration of the vertex operators such the t approaches zero as we approach the specific boundary. ($| t |$ measures the distance scale between the points which approach each other relative to the size of the original torus). It can then be shown that the contribution from the boundary term has a factor of $| t |^{\ell^2}$, where ℓ is the total momentum carried by the vertex operators which approach each other. For example, in fig.1(c) $\ell^2 = (k_1 + k_2)^2$, and we may always analytically continue this to sufficiently large positive value by adjusting the external momenta, so that the boundary contributions vanish. This freedom is absent in fig.1(a), where ℓ vanishes identically, and also in fig.1(b), where $\ell^2 = k_1^2$ is fixed by the tree level mass-shell condition.

We may now proceed to evaluate the boundary contribution corresponding to fig.1(b). Since in this configuration $z_2, \ldots z_{n+1}$ approach z_{n+2}, we shall change variables as,*

* For $n \geq 2$, there is an ambiguity in the choice of the set of independent variables near the boundary. This ambiguity may be resolved by regarding the n-point amplitude as an integral over the moduli space of a Riemann surface with n punctures, as explained in the next section. For the time being we may restrict ourselves to the case $n = 1$ where there is no ambiguity, and the analysis given in this section is completely rigorous for such amplitudes.

$$
\begin{aligned}
&z_2 - z_{n+2} = t, \\
&z_i - z_{n+2} = t x_i \quad (3 \leq i \leq n+1).
\end{aligned}
\tag{2.7}
$$

Then,

$$
\begin{aligned}
&\prod_{i=2}^{n+1} d^2 z_i = t^{n-1}\bar{t}^{n-1} d^2 t \prod_{i=3}^{n+1} d^2 x_i \equiv J d^2 t \prod_{i=3}^{n+1} d^2 x_i \\
&\frac{\partial t}{\partial z_2} = 1, \quad \frac{\partial t}{\partial z_i} = 0, \quad (3 \leq i \leq n+1).
\end{aligned}
\tag{2.8}
$$

We now use the integration rule,

$$
\int d^n x \frac{\partial f^\mu}{\partial x^\mu} = \int d^n y \frac{\partial}{\partial y^\nu}(J f^\mu \frac{\partial y^\nu}{\partial x^\mu})
\tag{2.9}
$$

under a change in the integration variables from $\{x^\mu\}$ to $\{y^\nu\}$. Here J is the Jacobian of the transformation. Using eqs.(2.7-2.9) we may write down the term in (2.5) which involves $\frac{\partial}{\partial t}$ after change of variables; this is the only relevant term which contributes to the boundary at $t = 0$. This gives the following contribution to the BRST anomaly,

$$
\begin{aligned}
A = - \int d^2\tau \prod_{i=3}^{n+1} d^2 x_i d^2 t \frac{\partial}{\partial t} \Big\{ & t^{n-1}\bar{t}^{n-1} \Big\langle c(z_{n+2}+t) V_2(z_{n+2}+t) \\
& (\prod_{i=3}^{n+1} V_i(z_{n+2}+t x_i)) \bar{c}(z_{n+2}) \hat{V}(z_{n+2}) V_1(z_1) (\eta \mid b)(\bar{\eta} \mid \bar{b}) \Big\rangle_T \Big\}.
\end{aligned}
\tag{2.10}
$$

Let us now define x_∞ to be a large but fixed number. We may now evaluate (2.10) by introducing a complete set of states at the boundary of a disk of radius $t x_\infty$ with center at the point z_{n+2}. Denoting by ϕ the complete set of operators, we may express the correlator appearing in (2.10) in the $t \to 0$ limit as,

$$\sum_{\phi} \langle \tilde{\phi}(z_{n+2}) V_1(z_1)(\eta \mid b)(\bar{\eta} \mid \bar{b}) \rangle_T (t x_\infty)^{2h_\phi} (\bar{t}\bar{x}_\infty)^{2\bar{h}_\phi}$$
$$\left\langle c(z_{n+2}+t) V_2(z_{n+2}+t) (\prod_{i=3}^{n+1} V_i(z_{n+2}+t x_i)) \bar{c}(z_{n+2}) \hat{V}(z_{n+2}) \phi(z_{n+2}+t x_\infty) \right\rangle_S , \tag{2.11}$$

where $\tilde{\phi}$ is the operator conjugate to ϕ, $(h_\phi, \bar{h}_\phi)$ are the conformal weights of the operator ϕ, and $\langle\ \rangle_S$ denotes the correlation function on a sphere. By our original assumption, the only operator $\tilde{\phi}$ that contributes to the correlator on the torus is $\bar{c}c\tilde{V}$ appearing in eq.(2.1), since we have taken V_1 to be the operator V. Consequently we get $\phi = c\partial c\bar{c}\bar{\partial}\bar{c}V$, since $c\partial c$ is the operator conjugate to c. The correlator on the sphere may be simplified by using $SL(2,C)$ invariance[18]. Finally, noting that,

$$\left\langle \bar{c}(0)\bar{c}(x_\infty)\bar{\partial}\bar{c}(x_\infty) \right\rangle_S = -x_\infty^2 = \left\langle \bar{c}(0)\bar{c}(1)\bar{c}(x_\infty) \right\rangle_S, \tag{2.12}$$

we may express (2.11) as,

$$-\left\langle \bar{c}(0)\hat{V}(0)c(1)\bar{c}(1)V_2(1)(\prod_{i=3}^{n+1} V_i(x_i)) c(x_\infty)\partial c(x_\infty)\bar{c}(x_\infty)V(x_\infty) \right\rangle_S$$
$$\langle c(z_{n+2})\bar{c}(z_{n+2})\tilde{V}(z_{n+2})V(z_1)(\eta \mid b)(\bar{\eta} \mid \bar{b}) \rangle_T t^{-n+1}\bar{t}^{-n}. \tag{2.13}$$

Substituting this in (2.10) we see that the BRST anomaly A may be written as,

$$A = \int d^2 t \frac{\partial}{\partial t}(F(t,\bar{t})), \tag{2.14}$$

where,

$$
\begin{aligned}
&\lim_{t\to 0} F(t,\bar{t}) \\
&= -\delta m^2 \bar{t}^{-1} \int (\prod_{i=3}^{n+1} d^2 x_i) \Big\langle \bar{c}(0)\hat{V}(0)c(1)\bar{c}(1)V_2(1) \\
&\qquad (\prod_{i=3}^{n+1} V_i(x_i)) c(x_\infty)\partial c(x_\infty)\bar{c}(x_\infty)V(x_\infty) \Big\rangle_S \\
&\equiv \delta m^2 \bar{t}^{-1} A_S .
\end{aligned}
\tag{2.15}
$$

Before proceeding further, we shall fix some normalization convention. We set $\alpha' = 2$ and the string coupling constant to be unity. Then a suitable normalization convention consistent with tree level unitarity is to take,*

$$
V(z,k)\tilde{V}(w,-k) = \frac{1}{(z-w)^2(\bar{z}-\bar{w})^2} + \text{non} - \text{leading terms}, \tag{2.16}
$$

and define,

$$
d^2 t \equiv \frac{1}{4\pi i} dt d\bar{t} = \frac{1}{2\pi} dx dy, \tag{2.17}
$$

where $t = x + iy$. We may now define the integral (2.14) by cutting out a small hole of radius ϵ around the origin $t = 0$, carrying out the integration, and take the $\epsilon \to 0$ limit at the end. This gives the boundary contribution at $t = 0$ as,

$$
A = -\frac{1}{2} \delta m^2 A_S \tag{2.18}
$$

with A_S as defined in eq.(2.15).

⋆ In our convention the propagator of a field with mass m is givan by $(k^2 + m^2)^{-1}$.

We shall now show that this BRST anomaly may be cancelled by modifying the tree level vertex operator. If,

$$V(z,k) = V_0(z) : e^{ik\cdot X(z)} :, \tag{2.19}$$

then let us define,

$$\begin{aligned}\delta V(z,k,\delta k) &= V_0(z)(: e^{i(k+\delta k)\cdot X(z)} : - : e^{ik\cdot X(z)} :) \\ &\equiv iV_0(z) : e^{ik\cdot X(z)}\delta k \cdot X : + O((\delta k)^2),\end{aligned} \tag{2.20}$$

for some δk. It can be easily seen that,

$$[Q_B, c(z)\bar{c}(z)\delta V(z,k,\delta k)] = -\frac{1}{2}\Delta k^2 c(z)\partial c(z)\bar{c}(z)V(z,k+\delta k), \tag{2.21}$$

where,

$$\Delta k^2 = (k+\delta k)^2 - k^2. \tag{2.22}$$

Let us now consider the tree level scattering amplitude involving the same set of vertex operators $V_1, \ldots V_{n+2}$, except that we now use the vertex $V + \delta V$ for V_1. This amplitude may be written as,

$$\prod_{i=3}^{n+1} d^2x_i \Big\langle c(1)\bar{c}(1)V_2(1)(\prod_{i=3}^{n+1} V_i(x_i))c(0)\bar{c}(0)V_{n+2}(0)c(x_\infty)\bar{c}(x_\infty)V_1(x_\infty)\Big\rangle_S . \tag{2.23}$$

We now use eq.(2.4), and deform the BRST contour away from the point 0, shrinking it to a point on the sphere. During this process we pick up residues from the poles at x_i, 1 and x_∞. All the total derivative terms integrate out to zero assuming that there are no tadpoles or two point functions on the sphere. The only non-vanishing contribution comes from the commutator (2.21) and gives,

$$\frac{1}{2}\Delta k^2 A_S. \tag{2.24}$$

This can be made to cancel the anomaly given in (2.18) if,

$$\Delta k^2 = \delta m^2 \tag{2.25}$$

which corresponds to a mass shift of $-\delta m^2$. This agrees with the result of ref.[11] based on unitarity arguments or of ref.[12] based on anomalous dimension calculation.

3. Higher genus

Before discussing the generalization of the above analysis to the higher genus case, let us note that a more systematic way to calculate string loop amplitudes is to express an n-point amplitude as an integral over the moduli space of a Riemann surface with n-punctures. In this formalism we insert a factor of $c\bar{c}$ at each vertex operator, trade in the integration over the z_i for integration over the moduli of the punctured surface, and insert a factor of $(\eta_i \mid b)(\bar{\eta}_i \mid \bar{b})$ in the correlator for each moduli, η_i, $\bar{\eta}_i$ being the Beltrami differentials dual to the moduli[16,4,19-21]. This way all the vertex operators commute with Q_B, and in an amplitude involving a null state, all the total derivative terms come from the anti-commutator $\{Q_B, (\eta_i \mid b)\}$. A typical boundary of the moduli space of the punctured surface consists of two punctured surfaces described by the coordinate system w and y (say), glued together near the origins $w = 0$ and $y = 0$ through the transition function $y = t/w$, t being the particular moduli that vanishes at the boundary[22]. In terms of the variables appearing in (2.10), the coordinate y on the punctured sphere (appearing on the left hand side of fig.1) is $\frac{t}{z-z_{n+2}}$, while the coordinate w on the punctured torus (appearing on the right hand side of fig.1) is $z - z_{n+2}$, where z denotes the coordinate on the original torus. If we define a third coordinate $v = -\ln y = \ln w - \ln t$ to describe the region $\mid w \mid << 1$, $\mid y \mid << 1$, then we get the picture shown in fig.1, the long neck (of length $\sim -\ln \mid t \mid$) describing this region in the v-coordinate system. The behavior of the integrand near the boundary may be written down using the factorization hypothesis of ref.[23], by introducing a complete set of states at the two boundaries of the long neck. The integration over the moduli splits into the integration over t, $\bar{t}$, and integration over the moduli of the two separate punctured Riemann surfaces. Correspondingly, the set of beltrami differentials near the boundary splits into the beltrami differentials η_t, $\bar{\eta}_t$ dual to t, $\bar{t}$, and the beltrami differentials dual to the moduli associated with the separate punctured surfaces. In the calculation we are interested in, the $(\eta_t \mid b)$ factor is removed from the correlator, since $\{Q_B, (\eta_t \mid b)\}$ is used to generate the $\frac{\partial}{\partial t}$ operator. The

$(\bar{\eta}_t \mid \bar{b})$ factor, on the other hand, inserts a $\bar{b}$ zero mode on the neck.

Using this formalism we may analyze the amplitude described in eq.(2.3), and get back expression for the BRST anomaly given in eqs.(2.14-2.15). In this derivation we never have to use the rearrangement of the $\bar{c}$'s given in eq.(2.12), since V_2 automatically comes with a factor of $c\bar{c}$; at the same time the $\bar{b}$ zero mode on the neck coming from $(\bar{\eta}_t \mid \bar{b})$ tells us that corresponding to the insertion of a $\bar{c}(z_{n+2})$ on the torus, we need the insertion of a $\bar{c}(x_\infty)$ (and not $\bar{c}(x_\infty)\bar{\partial}\bar{c}(x_\infty)$) on the sphere. This is esthetically pleasing for the bosonic string theory, but absolutely essential for the fermionic string theories, since there the picture changing operators coming from integration over the supermoduli involve ghost fields, and the rearrangement of the ghost fields such as in eq.(2.13) is not possible in the presence of these operators.

This formalism also lets us tackle the higher loop effects of mass renormalization in the same way as in the one loop case. Let us, for definiteness, consider the case of two loop corrections to the amplitude involving the same set of vertex operators $V_1, \ldots V_{n+2}$. The two loop BRST anomaly associated with the two point function $\langle \tilde{V} V \rangle$ comes from two different boundaries, as shown in figs.2(a) and (b). In order to compute the total BRST anomaly to this order, we must also include the BRST anomaly in the one loop graph where the vertex V_1 is replaced by δV. This comes from two sources, one, due to the non-commutativity of Q_{BRST} with $c\bar{c}\delta V$, this contribution is shown in fig.3(a). The other source is the boundary contribution shown in fig.3(b). Of these, fig.3(a) exactly cancels fig.2(a), the cancellation mechanism being the same as in the one loop case. Fig.2(b) gives a new contribution to the BRST anomaly. This contribution, however, is divergent, the divergence coming from the boundary region where the genus two surface breaks up into two genus one surfaces, as shown in fig.4. The operator insertions responsible for the divergent contribution are shown in fig.4. This contribution is given by,

$$-\frac{1}{2}A_S(\delta m^2)^2 \int \frac{d^2t}{t\bar{t}} = \frac{1}{2}(\delta m^2)^2(\ln\epsilon)A_S, \tag{3.1}$$

where, as before, ϵ is the lower cut-off of the t integration. The physical origin of this divergence is the fact that the two loop two point string amplitude automatically includes the one particle reducible graph which has an on-shell internal propagator. From our experience in field theory we know that such graphs are cancelled by lower order counterterms, and hence should not be included in the two loop mass correction. We shall now show that a similar mechanism operates in string theory, namely, the graph of fig.3(b) has a divergence which precisely cancels the divergence of fig.2(b).

In order to do this we express δV in fig.3(b) as a difference of two terms, $V_0 e^{i(k+\delta k)\cdot X}$ and $V_0 e^{ik\cdot X}$. The contribution from each of these terms has the form given in eqs.(2.14) and (2.15), except that for the $V_0 e^{i(k+\delta k)\cdot X}$ term, $\bar{t}^{-1}$ in eq.(2.15) is replaced by $\bar{t}^{-1}(t\bar{t})^{\frac{1}{2}\Delta k^2} = \bar{t}^{-1}(1 + \frac{1}{2}\Delta k^2 \ln(t\bar{t}) + \ldots)$. Thus the leading contribution to fig.3(b) is given by,

$$\frac{1}{2}\Delta k^2(\delta m^2)A_S \int d^2t \frac{\partial}{\partial t}\left(\frac{\ln(t\bar{t})}{\bar{t}}\right) = -\frac{1}{2}(\delta m^2)^2 A_S(\ln\epsilon), \tag{3.2}$$

using eq.(2.25). Thus we see that the divergent contribution to the BRST anomaly from eqs.(3.2) and (3.1) cancel each other.

Although the logarithmic divergences in fig.2(b) and 3(b) cancel each other, extracting the finite part after the cancellation is not entirely straightforward. The problem is to compare the parameter t, or more specifically, the lower limits of integration of this parameter, appearing in the two different diagrams, fig.2(b) and 3(b). It can be seen following the analysis of ref.[12] that this lower limit must depend on the conformal factor of the metric if we want to maintain two dimensional reparametrization invariance. A related problem is that since δV is not an operator of conformal dimension (1,1), matrix elements involving δV on a given Riemann surface will also depend on the choice of the metric on that Riemann surface. As a result, in order to carry out any computation, we must specify some standard metric on each Riemann surface. We propose the following scheme for this purpose (see also ref.[10]). To start with we specify some standard metric on the sphere for each configuration of punctures on the sphere in some standard coordinate chart. The standard metric on the torus is then chosen so as to satisfy the following consistency condition. In the degeneration limit shown in fig.1, the configuration should look like a punctured sphere with the standard metric in the standard coordinate system (denoted by z), and a punctured torus with the standard metric in the standard coordinate system (denoted by w), glued together near the origin through the transition function $z = t/w$[22]. This procedure may be continued to higher genus surfaces, – the metric on the genus g surface has to satisfy the requirement that in the degeneration limit the surface decomposes into two punctured surfaces of genus g_1 and $g - g_1$ respectively, with the standard metric on each of them in the standard coordinate charts denoted by z and w, glued together near the origin through the transition function $z = t/w$. This scheme gives an unambiguous definition of t, and hence enables us to compare the degeneration parameters t for two different Riemann surfaces, e.g. fig.3(b) and fig.4. The final answer for the mass shift should be independent of the initial choice of the standard metric. After all, it was shown in ref.[12] that after taking into account all the corrections, the modified vertex operator, rather than the original one, has the right conformal weights to give rise to metric independent amplitudes. It will be interesting to see if these qualitative ideas may be developed into a concrete form to calculate some explicit two loop mass corrections in some specific string theory.

4. Conclusion

To summarize, in this paper we have shown that the effect of mass renormalization due to string loop effects may be interpreted as a cancellation of BRST anomaly between string tree and loop graphs. This has the advantage over the anomalous dimension calculation that this formalism checks the cancellation of the full BRST anomaly, and hence the anomaly in all the generators of the Virasoro algebra, whereas the anomalous dimension calculation only checks the cancellation of the anomaly in the subgroup of the Virasoro algebra generated by L_0[12]. Furthermore, the calculation we have presented at one loop is free from divergences at all stages. At higher loop order, although the individual contribution to the BRST anomaly suffers from divergences, they cancel when we sum up the graphs, and again gives us a finite answer for the BRST anomaly. Finally, we should emphasis that although we have used the bosonic string theory to illustrate the general ideas, the analysis is based on the general properties of the BRST operator, and hence is expected to be valid for all theories. This includes fermionic string theories (once the ambiguities pointed out in refs.[14,15] are resolved), as well as theories based on general two dimensional (super-)conformal field theories, rather that just free field theories.

Note added: Effects of wave-function and coupling constant renormalization in string theory have been discussed in a recent paper by Minahan[24]. Some issues involving tachyon mass generation at one loop have been discussed in ref.[25].

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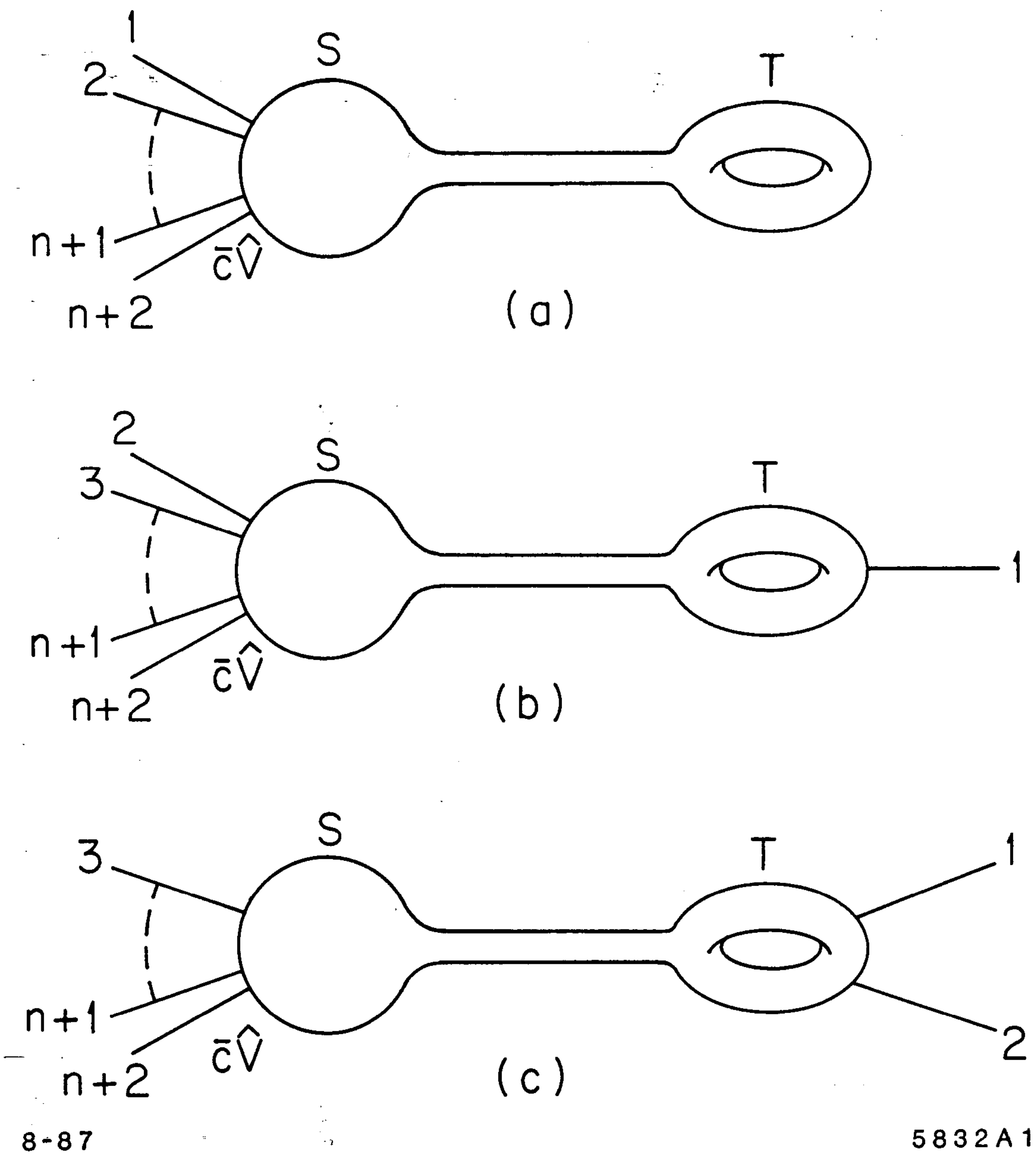

1
2
S
T
n+1
n+2
$\bar{c}\hat{V}$
(a)
2
3
S
T
1
n+1
n+2
$\bar{c}\hat{V}$
(b)
3
S
T
1
n+1
n+2
$\bar{c}\hat{V}$
2
(c)
8-87
5832A1

Fig. 1

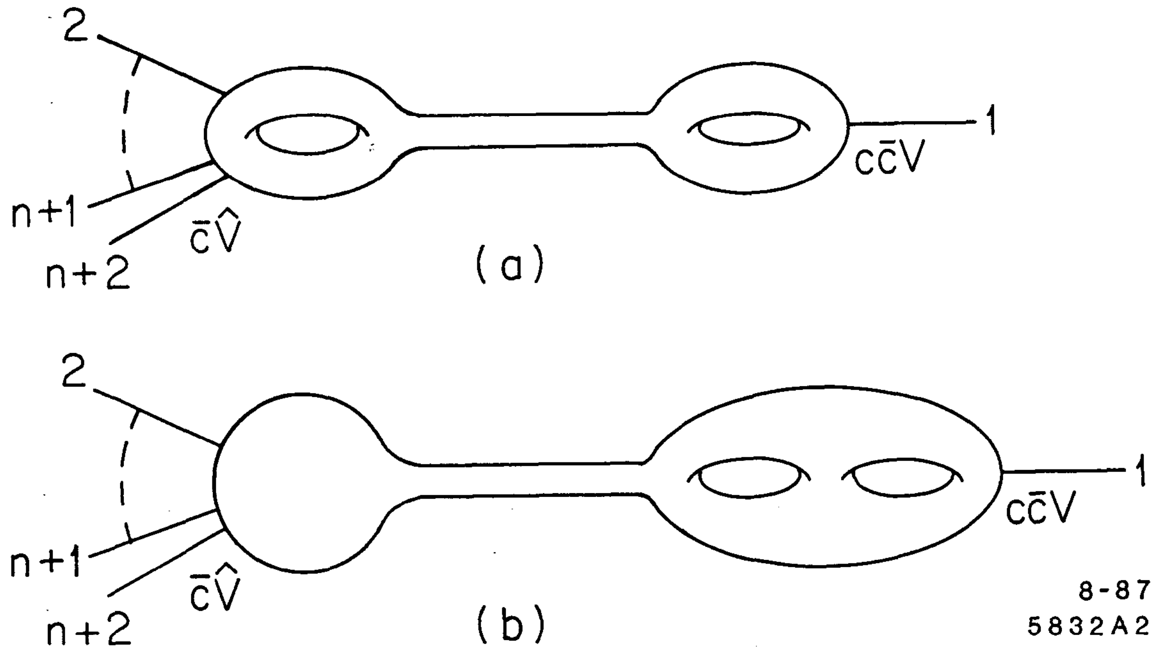
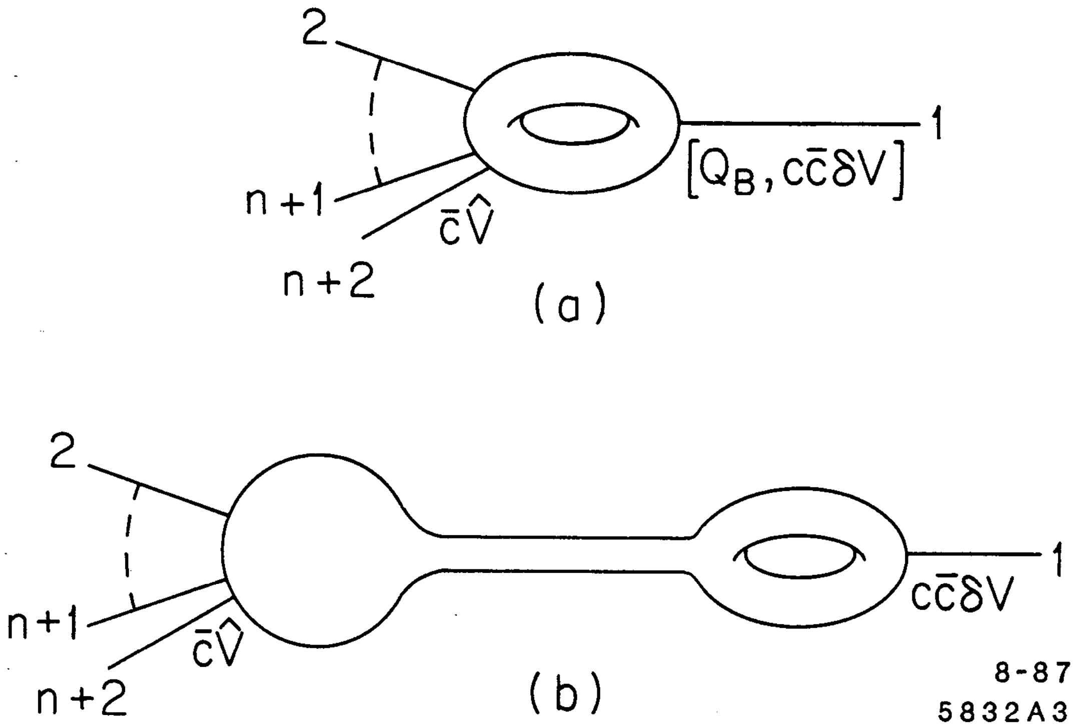

2
n+1
n+2
$\bar{c}\hat{V}$
1
$c\bar{c}V$
(a)
2
n+1
n+2
$\bar{c}\hat{V}$
1
$c\bar{c}V$
(b)
8-87
5832A2

Fig. 2


2
n+1
n+2
$\bar{c}\hat{V}$
$[Q_B, c\bar{c}\delta V]$
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(a)
2
n+1
n+2
$\bar{c}\hat{V}$
$c\bar{c}\delta V$
1
(b)
8-87
5832A3

Fig. 3

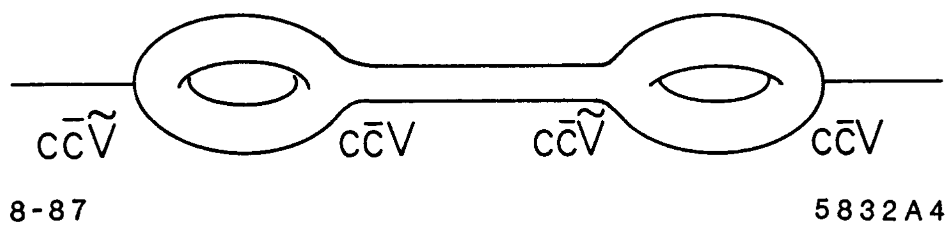

Fig. 4