SLAC–PUB–4378
August 1987
T

Suppression of Nonrenormalizable Terms in the Effective Superpotential for (Blown-Up) Orbifold Compactification*

MIRJAM CVETIČ†

Stanford Linear Accelerator Center

Stanford University, Stanford, California, 94305

ABSTRACT

We show that *all* the nonrenormalizable terms in the effective superpotential for *any* Abelian symmetric orbifold with at least $(0,2)$ worldsheet supersymmetry as well as any blown-up $(2,2)$ orbifold are exponentially suppressed by the size R of the compactified space, *i.e.* $\propto \exp(-R^2/\alpha')$.

Submitted to *Physical Review Letters*

⋆ Work supported by the Department of Energy, contract DE–AC03–76SF00515.

† Address after September 15, 1987: Department of Physics, University of Pennsylvania, Philadelphia, PA 19104

Orbifold compactifications[1–5] of the superstring theories are especially attractive because interactions on orbifolds can be calculated *exactly* at the string tree-level.[6,7] Thus all the parameters of the tree-level effective superpotential can be determined exactly, *i.e.* including contributions which are perturbative as well as nonperturbative in the ratio $\sqrt{\alpha'}/R$, where α' is the string tension and R is the radius of the orbifold. (For example, the effects of worldsheet instantons are automatically incorporated.)

The left-right ((2,2)) symmetric orbifolds, *i.e.* those with spin and gauge connections identified, can also be blown-up[6–11] into the corresponding Calabi-Yau manifolds. This is achieved by giving nonzero vacuum expectation values (VEV's) to the massless scalar fields associated with the orbifold singularities, *i.e.* the so-called blowing-up modes, whose potential is flat.

Scattering amplitudes in the repaired Calabi-Yau background – and hence also parameters of the effective superpotential – can be calculated by inserting successively larger numbers of *background* blowing-up modes into orbifold amplitudes. Although perturbative in the blowing-up VEV's, the method enables one to obtain *explicit* values for parameters of the blown-up orbifolds, thus giving exact results at the string tree-level.

Calculations[10] for the mass spectrum and Yukawa couplings, *i.e.* the terms of dimension ≤ 4, for the blown-up orbifolds agree with the general results of the worldsheet instanton calculations.[8] In particular, *all* the matter singlets acquire masses which are proportional to $\exp(-R^2/\alpha')$ while $\mathbf{27}$ and $\overline{\mathbf{27}}$ of E_6 do not pair-up. Also all the "moduli" remain massless as expected. On the other hand, Yukawa couplings of the form $h_{ija}\mathbf{27}_i\overline{\mathbf{27}}_j\mathbf{1_a}$ for any pair (i,j) are nonvanishing for some a as well as Yukawa couplings of the type $h_{ijk}\mathbf{27}_i\mathbf{27}_j\mathbf{27}_k$ are nonzero in general. Some of these Yukawa couplings are nonzero already in the field theory limit, *i.e.* $\alpha'/R^2 \to 0$, while some become nonzero due to nonperturbative effects. The nonrenormalizable terms $(\mathbf{27}\overline{\mathbf{27}})^K$ (with $K \geq 2$) in the effective superpotential for Z_N orbifolds and their blown-up versions have been studied in Ref. 11. It was observed that for a large class of orbifolds and their blown-up versions *all* such terms are *absent*, thus questioning the mechanism[12] for generating an intermediate scale for such compactifications.

In this note we show that for *all* Abelian symmetric orbifolds with at least $(0,2)$ worldsheet supersymmetry as well as blown-up $(2,2)$ orbifolds, *all the non-renormalizable terms* in the effective potential are at most exponentially damped by the size of the compactified space, *i.e.* $\propto \exp(-R^2/\alpha')$. Here R is the radius of the compactified space and α' is the string tension.

All such orbifolds possess the local conformal invariance[13–15] in the right-moving (r) sector. One can thus use the picture-changing formalism, with vertices having different ghost numbers for the bosonized right-moving superconformal ghost in different "pictures".[15,16] Tree-level amplitudes involve collections of vertices such that the total ghost number, ϕ, equals -2.[15] The simplest form of the vertex operator for a space-time fermion is the $-1/2$ picture, while that for a space-time boson is the -1 picture. The picture-changing formalism enables one to obtain vertices in other pictures. For example, the vertex for a space-time boson in the 0 picture is obtained in the following way:[15]

$$(V_B(z))_0 = \lim_{w \to z} \exp(\phi) T_F(w) (V_B(z))_{-1} . \tag{1}$$

Here $(V_B(z))_{-1}$ is the corresponding vertex operator in the -1 picture and

$$T_F = T_F^{\rm int}(X^i, \bar{X}^{\bar{i}}, \psi^i, \bar{\psi}^{\bar{i}}) + \partial X^\mu \psi^\mu \tag{2}$$

is the worldsheet supersymmetry generator[15] – the stress-energy tensor. Here X and ψ are the string bosonic and fermionic coordinates, respectively; the indices $(i,\bar{i}) = (1,2,3)$ and $\mu = (1,2,3,4)$ denote the three complex internal and the four space-time dimensions, respectively. Partial derivatives are with respect to the right-moving worldsheet coordinate z. It is crucial that for an orbifold model, $T_F^{\rm int}$ takes the explicit form:

$$T_F^{\rm int} = \partial X^i \bar{\psi}^{\bar{i}} + \partial \bar{X}^{\bar{i}} \psi^i. \tag{3}$$

The right-moving $N = 2$ superalgebra of $(0,2)$ as well as $(2,2)$ models incorporates a $U(1)_r$ current algebra, generated by $J_r = -i\sqrt{3}\partial H_r$. $H_r(z)$ is a free right-moving scalar field. Actually for orbifolds, $U(1)_r$ worldsheet symmetry of the r-sector is enlarged to $\left[U(1) \times U(1) \times U(1)\right]_r$. Thus instead of one conserved charge $H_r \equiv \sum_{i=1}^{3}(H_i)_r$, there are three conserved charges, $(H_1)_r$, $(H_2)_r$, and $(H_3)_r$ which are classified[1] for *all* the Z_N orbifolds and are related to the matrix of the discrete rotation θ acting on the three compactified coordinates. $(H_i)_r$ charges, along with the explicit form of the T_F (see eq. (2,3)), in turn uniquely determine the r-sector of the vertex operator for emission of massless states at the string tree-level. For example in the -1 and $-1/2$ pictures (emission of a massless boson and massless fermion, respectively and belonging to the space time chiral superfield with *positive chirality*) the r-sector of the vertex operators are the following:

$$(V_B)_{-1} = \exp(-\phi)\psi^{j}\exp(ik_\mu X^\mu) \qquad \text{untwisted sector} \tag{4.a}$$

$$(V_B)_{-1} = \exp(-\phi)\prod_i \sigma_i s_i \exp(ik_\mu X^\mu) \qquad \text{twisted sector} \tag{4.b}$$

$$(V_F)_{-1/2} = \exp(-\phi/2) u \prod_i \exp(-(\widehat{H}_i)_r/2) \psi^j \exp(ik_\mu X^\mu) \qquad \text{untwisted sector} \tag{5.a}$$

$$(V_F)_{-1/2} = \exp(-\phi/2) u \prod_i \sigma_i \exp(-(\widehat{H}_i)_r/2) s_i \exp(ik_\mu X^\mu) \qquad \text{twisted sector} \tag{5.b}$$

with $\mu, i, \bar{i}$ defined as before and u referring to the spinor of the four uncompactified dimensions. The bosonic twist fields σ_i and fermionic twist fields s_i take care of the emission of the massless state from the propagating string with the twisted boundary conditions for the bosonic X^i and the fermionic ψ^i coordinates, respectively.[7] Fermionic fields are presented in terms of the three bosonic $U(1)_r$ charges:

$$\psi^j = \exp\left[i(\widehat{H}_j)_r\right] \tag{6.a}$$

$$s_j = \exp\left[ik_j/N(\widehat{H}_j)_r\right]. \tag{6.b}$$

The three separate charges $(H_j)_r$ should satisfy the constraint that $H_r = \sum_j (H_j)_r$ $= \sum_j k_j/N = 1$.

The calculation of parameters of the effective superpotential in a particular theory reduces to the study of the corresponding amplitude of the massless states emitted from the string propagating in this particular background. It is most convenient to calculate[10,11] the following Yukawa-type n-point function in the orbifold background:

$$A = \left\langle V_{F_1} V_{F_2} V_{B_1} \dots V_{B_{(n-2)}} \right\rangle. \tag{7}$$

Here V_{F_i} and V_{B_i} denote the vertices for the emission of the massless fermionic and bosonic modes, respectively (see eqs. (4-6)). This amplitude enables one to probe the parameters of the superpotential directly, unlike the amplitude for n-bosons.[17]

With the explicit form of the vertices (4,5,1), one can then evaluate the amplitudes in the particular background which must obey selection rules that the total ϕ charge equals -2 and $(H_i)_r$ charges should be separately conserved.

It has been shown[10,11] that in the amplitudes (7) which probe the terms of the superpotential, only the terms of $(V_B)_0$ with $H_r = 0$ contribute, *i.e.* only terms proportional to $\partial X^i \bar{\psi}^{\bar{i}}$ survive in such amplitudes in order to conserve the total H_r charge.[18] Note that for $V_{-1/2}, V_{-1}$; $H_r = -1/2, 1$, respectively. Then A assumes the following form:

$$A = \left\langle V_{-1/2} V_{-1/2} V_{-1} V_0 \ldots V_0 \right\rangle \propto \left\langle \partial_z X^i, \ldots \right\rangle_{\sigma^{J_1} \ldots \sigma^{J_n}} . \tag{8}$$

For the nonrenormalizable terms, *i.e.* $n \geq 4$, the amplitude (8) has $n - 3 \geq 1$ vertices in the 0-picture and it is thus proportional to at *least one power of* $\partial_z X$ evaluated in the presence of the twist fields σ^J.[19] This part of the amplitude can be evaluated[7] by separating the classical and the quantum part of the solution for ∂X's. Namely:

$$\langle \partial_z X \ldots \rangle_{\sigma^{J_1} \ldots \sigma^{J_n}} = Z_{qu} \sum_{\partial_z X_{cl}} \partial_z X_{cl} \ldots e^{-S_{cl}}, \tag{9}$$

where $\partial_z X_{cl}$ denotes the classical solution for $\partial_z X$ in the presence of the twist fields σ^J, $S_{cl} = \int d^2 z \left(\partial_z X_{cl} \partial_{\bar{z}} \bar{X}_{cl} + \partial_{\bar{z}} X_{cl} \partial_z \bar{X}_{cl} \right)$ and Z_{qu} is the quantum part of the twist correlation function *independent* of the size of the compactified space. Note that in (9) there are *no* factors proportional to $\partial_z X_{qu}$, since $\langle \partial_z X_{qu} \rangle = 0$.

The form of ∂X_{cl}'s is determined[7] as follows:[20]

$$
\begin{aligned}
\partial_z X_{cl}(z) &= \sum_{i=1}^{L-M-1} a^i \omega_K^i(z) \qquad & \partial_z \bar{X}_{cl}(z) &= \sum_{j=1}^{M-1} \bar{b}^j \omega_{N-K}^j(z) \\
\partial_{\bar{z}} \bar{X}_{cl}(\bar{z}) &= \sum_{i=1}^{L-M-1} \bar{a}^i \bar{\omega}_K^i(\bar{z}) \qquad & \partial_{\bar{z}} X_{cl}(\bar{z}) &= \sum_{j=1}^{M-1} b^j \bar{\omega}_{N-K}^j(\bar{z}).
\end{aligned}
\tag{10}
$$

Here L is the number of twist fields σ^J, ω_K and ω_{N-K} are determined by the operator product expansion[7] of ∂X with the twist fields. *E.g.* $\omega_K^i(z) = z^{i-1}\prod_{j=1}^{L}(z-z_j)^{-(1-k_j/N)}$ with $\sum_{j=1}^{L} k_j/N = M$ and $M = 1,\ldots,L-1$, and similarly for $\omega_{N-K}^i(z)$. The condition on k_j's arises from the point group selection rules, *i.e.* Z_N symmetry of interactions. The coefficients a^i ($\bar{a}^i$) and b^j ($\bar{b}^j$) are a particular linear combination of the coset vectors $\underset{\sim}{v}$ ($\underset{\sim}{\bar{v}}$) which belong to a class of lattice vectors. They are determined by the global monodromy conditions;[7,21] *i.e.* by choosing $L-2$ independent "closed loops"[21] γ_i around which X ($\bar{X}$) acquires no phase, but it may be translated by particular coset vectors which depend on the type of twist fields which are encircled by the independent closed loops γ_i. Note that for the symmetric orbifold, *i.e.* when the right- and the left-moving internal coordinates are rotated in the same way, one sees that ω_K^i (ω_{N-K}^j) $= \bar{\omega}_K^{i*}$ ($\bar{\omega}_{N-K}^{j*}$) as well as a^i (b^j) $= \bar{a}^{i*}$ ($\bar{b}^{j*}$). So S_{cl} assumes the form, $\sum_{ii'} a^i \Omega_K^{ii'} \bar{a}^{i'} + \sum_{jj'} b^j \Omega_{N-K}^{jj'} \bar{b}^{j'}$ where $\Omega_K^{ii'} \equiv \int d^2 z\, \omega_K^i \bar{\omega}_K^{i'}$ and $\Omega_{N-K}^{jj'} \equiv \int d^2 z\, \omega_{N-K}^j \bar{\omega}_{N-K}^{j'}$ are entries of strictly positive definite matrices $\underset{\sim}{\Omega}_K$ and $\underset{\sim}{\Omega}_{N-K}$, respectively.[22] This in turn implies:

$$
S_{cl} = 0 \Longleftrightarrow a^i \equiv 0 \text{ and } b^j \equiv 0. \tag{11}
$$

In this case also all ∂X_{cl}'s are identically equal to zero (see eq. (10)).

Using (10) one sees that in the amplitude (9) the only terms that survive *are* exponentially damped, *i.e.* $\propto |\underset{\sim}{v}|^{n-3} e^{-|\underset{\sim}{v}|^2 O(1)} \leq (R/\sqrt{\alpha'})^{n-3} e^{-R^2/\alpha' O(1)}$ with $n > 3$.

For this *exact* string tree level result we needed only the $(0,2)$ worldsheet supersymmetry, thus the result is valid not only for all the $(0,2)$ Abelian as well as $(2,2)$ Abelian orbifolds but also for the Calabi-Yau manifolds obtained by blowing-up the $(2,2)$ Abelian orbifolds. However, the above conclusion need not apply to the asymmetric orbifolds.[3,5] In this case the global monodromy condition may be different for the l- and the r-moving sectors. Therefore one can in principle satisfy the constraint $S_{cl} = 0$, but $\partial_z X_{cl} \neq 0$ thus making the amplitude (9) for the nonrenormalizable terms *not* to be exponentially damped.

I am grateful to C. Kounnas, P. Nilles, A. Schellekens, and especially L. Dixon for discussions. I would also like to thank ICTP at Trieste for their hospitality, where part of this work has been done.

REFERENCES

1. L. Dixon, J. Harvey, C. Vafa, and E. Witten, Nucl. Phys. **B261**, 678 (1986).; **B274**, 285 (1986).
2. L. Dixon and J. Harvey, Nucl. Phys. **B274**, 93 (1986).
3. M. Mueller and E. Witten, Phys. Lett. **182B**, 28 (1985).
4. L. Ibañez, H. Nilles, and F. Quevedo, Phys. Lett. **187B**, 25 (1987).
5. K. S. Narain, M. H. Sarmadi, and C. Vafa, Nucl. Phys. **B288**, 551 (1987).
6. S. Hamidi and C. Vafa , Nucl. Phys. **B279**, 465 (1987).
7. L. Dixon, D. Friedan, E. Martinec, and S. H. Shenker, Nucl. Phys. **B282**, 13 (1987).

8. M. Dine, N. Seiberg, X. G. Wen, and E. Witten, Nucl. Phys. **B278**, 769 (1986).; and **B289**, 319 (1987).

9. L. Dixon, to appear in *Proceedings of the Workshop in High Energy Physics and Cosmology*, June 29–August 7, 1987, Trieste, Italy.

10. M. Cvetič, SLAC-PUB-4325, to appear in *Proceedings International Workshop on Superstrings, Composite Structures, and Cosmology*, March 11–18, 1987, University of Maryland, College Park.

11. M. Cvetič, SLAC-PUB-4324, to appear in *Proceedings Eighth Workshop on Grand Unification*, April 16–18, 1987, Syracuse, New York.

12. M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi, and N. Seiberg, Nucl. Phys. **B259**, 549 (1985).

13. P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985).; and in *Proceedings of the Argonne Symposium on Anomalies*, W. A. Bardeen and A. R. White, editors (World Scientific, 1985).

14. C. Lovelace, Phys. Lett. **135B**, 75 (1984).; D. Friedan, and S. H. Shenker, unpublished talk at the Aspen Summer Institute (1984); D. Friedan, Z. Qiu, and S. H. Shenker, in *Proceedings of the 1984 Santa Fe Meeting of the APS Division of Particles and Fields*, T. Goldman and M. Nieto, editors (World Scientific, 1985); E. Fradkin and A.Tseytlin, Phys. Lett. **158B**, 316 (1985).; C. Callan, D. Friedan, E. Martinec, and M. Perry , Nucl. Phys. **B262**, 593 (1985).; A. Sen, Phys. Rev. Lett. **55**, 1846 (1985).; and , Phys. Rev. **D32**, 279 (1985).

15. D. Friedan, E. Martinec, and S. H. Shenker, Nucl. Phys. **B271**, 93 (1986).

16. J. Cohn, D. Friedan, Z. Qiu, and S. H. Shenker, Nucl. Phys. **B278**, 577 (1986).

17. Note that in this case one is probing the scalar potential, which is the mixture of the F- *and* D-terms.

18. This in turn implies that the effective superpotential calculated in this way cannot be mimicked by a massless exchange of gauge or gravitational particles; the amplitudes of such exchanges would be proportional to k^2 which are absent in the amplitude (7).

19. From now on we shall suppress the index i for internal coordinates.

20. We suppress the dependence on z_i, the location of the vertices.

21. M. Bershadskii and A. Radul, Int. J. Mod. Phys. **A2**, 165 (1987).

22. One can show that by using the Schwartz inequality: $|\Omega_K^{ii'}|^2 \leq \Omega_K^{ii}\Omega_K^{i'i'}$ and similarly for Ω_{N-K}'s. The equality sign applies only when $i = i'$.