# DETERMINATION OF $\alpha_{s}$ FROM ENERGY-ENERGY CORRELATIONS IN $e^{+} e^{-}$ANNIHILATION AT $29 \mathrm{GeV}^{*}$ 

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#### Abstract

We have studied the energy-energy correlation in $e^{+} e^{-}$annihilation into hadrons at $\sqrt{s}=29 \mathrm{GeV}$ using the Mark II detector at PEP. We find to $O\left(\alpha_{s}{ }^{2}\right)$ that $\alpha_{s}=0.158 \pm .003 \pm .008$ if hadronization is described by string fragmentation. Independent fragmentation schemes give $\alpha_{s}=.10-14$, and give poor agreement with the data. A leading-log shower fragmentation model is found to describe the data well.


## Submitted to Physical Review D

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## I. INTRODUCTION

The energy-energy correlation ${ }^{1}$ (EEC) and its asymmetry (EECA) were introduced in 1978 as powerful estimators of the strong coupling constant, $\alpha_{s}$. The EEC is an energy weighted angular correlation defined by

$$
\begin{equation*}
\operatorname{EEC}(\chi)=\frac{1}{N} \sum_{\text {events }}^{N} \sum_{i} \sum_{j} \frac{E_{i} E_{j}}{E_{c m}^{2}} \delta\left(\chi-\chi_{i j}\right) \tag{1}
\end{equation*}
$$

where $i$ and $j$ run over all particles (charged and neutral) in the event, and $\chi_{i j}$ is the angle between particles $i$ and $j$. The energy-energy correlation asymmetry (EECA) is conventionally defined as

$$
\begin{equation*}
\operatorname{EECA}(\chi)=\operatorname{EEC}\left(180^{\circ}-\chi\right)-\operatorname{EEC}(\chi) \tag{2}
\end{equation*}
$$

Several experiments ${ }^{2-8}$ have studied QCD processes by examining the EEC for hadronic events in $e^{+} e^{-}$annihilation. Simple $q \bar{q}$ events will produce back-to-back jets which will contribute to the EEC predominantly near $\chi=0^{\circ}$ and $\chi=180^{\circ}$. Events with hard gluon radiation, however, will populate the EEC at intermediate angles as well. In this way, the shape of the EEC is sensitive to $\alpha_{s}$.

The advantage of the EEC over jet counting methods is that all hadronic events are used in the measurement and no special algorithms are required to distinguish jets or clusters. The EECA has the additional advantage that many of the effects of fragmentation and experimental error contribute symmetrically to the EEC, and thus cancel in the EECA. This leads to the expectation that an $\alpha_{s}$ measurement from the EECA should be much less fragmentation dependent than other measurements. In simulations, however, even the EECA shows sensitivity to the way the gluon is imbedded in the fragmentation scheme and how energy and momentum are conserved in an event. ${ }^{9-11}$ Nonetheless, the EECA remains a useful tool for studying hadronic events in $e^{+} e^{-}$annihilation.

- We examine the EEC in $e^{+} e^{-}$collisions at a center-of-mass energy ( $E_{c m}$ ) of 29 GeV . We use data from the original Mark II experiment at the PEP storage ring and from a PEP run of the Mark II after its recent SLC Upgrade. We compare our measured EEC and EECA with the predictions of second-order quantum chromodynamics (QCD) plus fragmentation models and determine $\alpha_{s}$. We also compare our results with a leading log shower QCD model.

In 1982, the Mark II collaboration published a measurement of the EEC and EECA and made a first-order measurement of $\alpha_{s} .{ }^{12}$ Since that time the amount of data has increased four-fold and significant improvements have been made in QCD calculations and fragmentation models. The present results supersede the earlier ones.

## II. APPARATUS

The Mark II detector has operated in several different configurations. From Fall 1981 through Spring 1984, it accumulated $211 p^{-1}$ in a configuration to which we refer by its experiment number, PEP-5. This detector is described in detail elsewhere. ${ }^{13}$ Momenta of charged particles are measured with a sixteen-layer cylindrical drift chamber and a high-resolution vertex drift chamber immersed in a 2.3 kG axial magnetic field. The combined information provides a momentum resolution of $\left(\sigma_{p} / p\right)^{2}=(0.025)^{2}+(0.011 p)^{2}(p$ in $\mathrm{GeV} / \mathrm{c})$.

In preparation for its impending run at SLC, the Mark II was extensively upgraded. The detector was operated at PEP in the upgraded configuration during 1985-1986, and about $30 p^{-1}$ were logged. The general features of the Upgrade are described in the proposal. ${ }^{14}$ Several components of the Upgrade contribute to the present analysis. A new 72-layer drift chamber ${ }^{15}$ was installed together with a smaller trigger drift chamber. ${ }^{16}$ This configuration, along with a new coil operating at a field of 4.5 kG , provides an improved momentum resolution for charged particles of $\left(\sigma_{p} / p\right)^{2}=(0.014)^{2}+(0.0026 p)^{2}$. In addition, the acceptance for electromagnetic energy detection was increased by the addition of new end cap calorimeters ${ }^{17}$ which cover polar angles $\theta$ such that $0.70<|\cos \theta|<0.95$. The end caps are constructed of 36 layers of lead and proportional tubes and provide an energy resolution of $\sigma_{E} / E=0.2 / \sqrt{E}(E$ in GeV$)$ for photons and electrons.

The barrel calorimeter, common to both configurations, consists of eight modules of lead liquid argon shower counters and covers a range in polar angle of about $|\cos \theta|<0.7$. Electromagnetic energy is measured in this region with a resolution of about $0.14 / \sqrt{E}$.

Apart from the increased solid angle, the most important consequence of the upgrade is greatly improved two-track separation. The Upgrade drift chamber, with multiple hit readout capability and many more samples to aid in track identification, has much higher efficiency for sorting out tracks in the core of a jet.

## III. TRACK AND EVENT SELECTION

All tracks are required to pass fairly tight quality and solid-angle cuts. This ensures that the momenta and angles are well measured and that the detection efficiency for these tracks is reliably described by the Monte Carlo detector simulation. The cuts used for both the PEP-5 and Upgrade detectors are identical except for the solid angle and sphericity axis cuts.

We accept only those charged and neutral tracks whose polar angles at their production points satisfy $|\cos \theta| \leq 0.68$ (0.85) for PEP-5 (Upgrade) data. This guarantees that only the highest efficiency region of the detector is used. For neutral particles with $|\cos \theta| \leq 0.7$, we require in addition that the detected shower be at least $3^{\circ}$ from any of the eight cracks in $\phi$ between the barrel calorimeter modules.

Charged particles must have minimum transverse momenta with respect to the beam axis ( $p_{x y}$ ) greater than $0.1 \mathrm{GeV} / \mathrm{c}$. We cut on the distance of closest approach to the beam axis ( $r_{d c a}$ ) as follows:

$$
r_{d c a} \leq \begin{cases}2 m m, & p_{x y}>1 \mathrm{GeV} / c \\ \frac{2 m m G e V / c}{p_{x y}}, & p_{x y}<1 \mathrm{GeV} / \mathrm{c}\end{cases}
$$

where the momentum dependence allows for multiple scattering of low momentum tracks. At the point of closest approach, we also require that separation from the event vertex along the beam ( $z$ ) direction be less than 5 cm . Tracks with unphysically high measured momenta, $p>E_{b e a m} / c+3 \sigma_{p}$, are also removed. Since no particle identification is attempted, the pion mass is assigned to all charged tracks.

Accepted neutral tracks must deposit at least 0.5 GeV in the barrel or end cap calorimeters. In addition, each neutral shower must be separated by at least 30 cm from any charged track of momentum greater than the observed shower energy. This requirement helps to eliminate the fake photons that arise when charged hadrons interact in the coil.

Particles satisfying the above criteria are used in the selection of hadronic $e^{+} e^{-}$ annihilation events. Such events must have at least five charged tracks, and the
energies seen in charged particles ( $E_{c h}$ ) must exceed $30 \%$ of $E_{c m}$. Each event must have a reconstructed primary vertex consistent with the mean beam interaction point ( $\Delta r<2 \mathrm{~cm}, \Delta z<10 \mathrm{~cm}$ ). The sphericity axis ${ }^{18}$ is determined from the charged particles, and we require that $\left|\cos \theta_{s p h}\right| \leq 0.60$ ( 0.75 ) for PEP-5 (Upgrade) data, where $\theta_{s p h}$ is the angle between the sphericity axis and the beam axis. The following cuts are made on momentum balance of charged tracks: $|\Sigma \vec{p}| / E_{c h}<0.6$ and $\left|\Sigma p_{z}\right| / E_{c h}<0.25$. These requirements help to eliminate highly-boosted events such as those which arise from initial-state radiation and the two-photon production process. Since any direct photon radiation can alter the EEC, we also discard events in which hard isolated photons are detected. Such photons are defined as those with $E_{\text {shower }}>2.5 \mathrm{GeV}$ which are separated by more than 30 degrees from all charged tracks with $p_{c h}>0.5 \mathrm{GeV} / \mathrm{c}$.

These event cuts are chosen to remove backgrounds from QED interactions, two-photon collisions, and beam gas collisions, and also to select well-measured events which contain ample information about the energy flow structure.

Finally, a special cut is used to remove remaining tau pairs. The charged particles are separated into two hemispheres by a plane perpendicular to the sphericity axis. For plausible tau topologies the invariant mass in each hemisphere is calculated. If this mass is less than $1.8 \mathrm{GeV} / \mathrm{c}^{2}$ in both hemispheres, the event is rejected.

Only the highest quality data sets are used for this analysis. Notably, we omit PEP-5 runs in which the drift chamber was operated at reduced voltage. The samples which remain represent about $100 \mathrm{pb}^{-1}$ of PEP-5 data and $24 \mathrm{pb}^{-1}$ of Upgrade data. The cuts select 13,823 and 5,024 events, respectively. We estimate the contamination from two photon events to be about $1 \%$, with negligible contributions from tau pairs and beam gas events.

## IV. ENERGY-ENERGY CORRELATION MEASUREMENT

The EEC is accumulated from all accepted charged and neutral particles according to the formula

$$
\begin{equation*}
\operatorname{EEC}\left(\chi_{k}\right)=\frac{1}{N} \sum_{\text {events }}^{N} \sum_{i} \sum_{j} \frac{E_{i} E_{j}}{E_{v i s}^{2}}\left(\frac{1}{\Delta \chi} \int_{\chi_{k}-\Delta \chi / 2}^{\chi_{k}+\Delta x / 2} \delta\left(\chi-\chi_{i j}\right) d \chi\right), \tag{3}
\end{equation*}
$$

for 50 discrete bins in $\chi\left(\Delta \chi=3.6^{\circ}\right) .{ }^{19}$ Note that the detected charged plus neutral energy ( $E_{v i s}$ ) is used to normalize each weight rather than $E_{c m}$ so that undetected particles have less influence on the EEC.

The uncorrected EEC and EECA distributions for both detector configurations are shown in Fig. 1. The self-correlation contribution is responsible for the spike which appears in the lowest bin in Fig. 1(a). The large peaks near $0^{\circ}$ and $180^{\circ}$ show the predominance of two-jet events. The width of these peaks can be attributed to both fragmentation effects and the emission of soft and collinear gluons. At intermediate angles ( $30^{\circ}<\chi<150^{\circ}$ ), however, QCD predicts that major contributions come from three- and four-parton events produced by hard gluon radiation. The large difference between the two EEC measurements near $90^{\circ}$ is expected from the larger solid angle coverage of the Upgrade.

Before we draw conclusions from our data, we must take account of detector effects. This is accomplished by applying a simple multiplicative correction factor to the data:

$$
\begin{equation*}
\operatorname{EECA}_{c o r}(\chi)=C(\chi) \cdot \operatorname{EECA}_{d a t a}(\chi) \tag{4}
\end{equation*}
$$

The EEC itself is corrected separately in the same manner. The correction factors, $C$, are used to compensate for the effects of initial state radiation, detector acceptance, track and event selection bias, detection efficiency, and resolution.

The corrections are determined with a Monte Carlo simulation, and in principle they can depend on the parameters that go into the simulation, including the value of $\alpha_{s} .{ }^{20}$ Ideally, we would completely reevaluate the factors $C(\chi)$ for each value of $\alpha_{s}$ and each model that we consider. The computer time required is prohibitive,


Figure 1. Raw EECs (a) and EECAs (b). The data are from two detector configurations described in the text. No corrections have been made for acceptance, resolution, or efficiency.
hōwever, if we employ a complete detector simulation in each instance. Consequently, the correction factor $C$ is taken to be the product $C_{1} C_{2}$ of two separate factors whose precise definitions will be given below, following a more detailed description of our Monte Carlo simulation. Qualitatively, the factor $C_{2}$ takes account of initial state radiation and the gross geometry of the detector. It is sensitive to simulation model parameters and the value of $\alpha_{s}$. On the other hand, the factor $C_{1}$, which provides the relation between full detector simulation and the gross geometric corrections included in $C_{2}$, is close to unity and relatively insensitive to model assumptions. Thus the time-consuming calculation of $C_{1}$ need be done for only one set of model parameters, while the determination of $C_{2}$, which has to be repeated for many parameter and $\alpha_{s}$ choices, is relatively modest in its computer time requirements.

The Monte Carlo simulation is used in three modes: the event generator alone (GEN), the generator with gross geometric acceptance corrections and initial state radiation (AC), and a detailed full detector simulation (FS). The event generator produces a list of four-vectors for the final state particles (including neutrinos) and is completely independent of the detector configuration. It includes the effects of QCD, fragmentation, and decays of short-lived particles. When the FS is included, the trajectory of each of the particles produced by the event generator is traced and the interactions with the active and passive material in the detector are simulated in detail. A simulated raw data image is produced which is subsequently processed by the same event reconstruction program as is used for the real data. This simulation has been extensively studied and tuned to reproduce reliably the observed detector performance.

The AC accounts for the detector effects in a simpler but more approximate manner. It uses the particle four-vectors directly from the event generator, but accepts only the detectable, stable particles ( $e^{ \pm}, \mu^{ \pm}, \pi^{ \pm}, K^{ \pm}, p, \bar{p}, \gamma$ ) that are pointed into the acceptance region of the detector. Momenta and energies are not smeared, the detection efficiency is assumed to be $100 \%$ within the specified solid angle, and the pion mass is assigned to all charged particles. Track and event selection cuts,
based on quantities determined from these accepted particles, are applied subsequently. The effects of initial state radiation are included as well. ${ }^{21}$ For many studies, the AC would be grossly inadequate, but for the EEC it incorporates the most important experimental effects (solid angle, radiative corrections, and event selection bias) without requiring the time-consuming full simulation.


Figure 2. Correction factors for the EEC and EECA. The two factors $C_{1}$ and $C_{2}$ (described in the text) are shown separately with solid and dashed curves respectively. The hashed regions show the errors assigned to these factors.

We define the correction factors $C_{1}$ and $C_{2}$ for the EECA as follows:

$$
\begin{equation*}
C_{1}(\chi)=\operatorname{EECA}_{A C}\left(\chi, \alpha_{s}^{0}\right) / \operatorname{EECA}_{F S}\left(\chi, \alpha_{s}^{0}\right) \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
C_{2}\left(\chi, \alpha_{s}\right)=\operatorname{EECA}_{G E N}\left(\chi, \alpha_{s}\right) / \operatorname{EECA}_{A C}\left(\chi, \alpha_{s}\right) \tag{6}
\end{equation*}
$$

and similarly for the EEC. The correction factors $C_{1}$ are determined from large hadronic Monte Carlo samples which are carried through the full detector simulation. For these samples, there is reasonably good agreement with the data for most observables, including the EEC and EECA. Figure 2 shows the calculated $C_{1}$ and $C_{2}$ for the two detector configurations. Bands are used to indicate the systematic uncertainties on these factors. The bin to bin fluctuations are smoothed out in the central region of the EEC corrections ( $14.4^{\circ} \leq \chi \leq 165.6^{\circ}$ ) by convolution with a Gaussian. For the asymmetry correction, Gaussian smoothing is used for $\chi \geq 10.8^{\circ}$. The large corrections to the EECA near $90^{\circ}$ are of little consequence because the asymmetry itself is vanishing in this region. Note that, aside from this, the corrections made with $C_{1}$ are $\$ 10 \%$ within the regions used for $\alpha_{s}$ studies.

In order to estimate the systematic errors on $C_{1}$, we separate the Monte Carlo events into three sub-samples according to the number of charged particles generated: low multiplicity ( $n_{c h} \leq 10$ ), medium multiplicity ( $n_{c h}=12,14$ ), and high multiplicity ( $n_{c h} \geq 16$ ). The combined sample approximately reproduces the measured average multiplicity of $12.9 \pm 0.6,{ }^{22}$ and this decomposition divides the sample into roughly equal thirds. The quantity $C_{1}$ is calculated separately for the high and low multiplicity sub-samples, and the deviation between the two is used as an estimate of the systematic error. This should be considered a realistic estimate of the systematic error because the largest contribution to deviations from unity in $C_{1}$ is the loss of detected tracks in crowded environments. The contributions to the systematic error from Monte Carlo statistics are also included where they are appreciable. The widths of the bands in Fig. 2 indicate the sizes of the total systematic errors.

In addition, $C_{1}$ is checked for model dependence. Figure 3 shows a comparison between two determinations of $C_{1}$ for the PEP-5 detector. One is obtained from a sample of Lund string ${ }^{23}$ Monte Carlo and is shown with the errors discussed above. The other is determined from a comparable sample of independent fragmentation ${ }^{24}$ Monte Carlo. The two calculations of $C_{1}$ are consistent within errors. Similar checks for the Upgrade detector give very good agreement between calculations of


Figure 3. Comparison of PEP-5 $C_{1}$ from Lund and IF samples. The dashed band shows $C_{1}$ as determined from the Lund sample. The width of the band indicates the size of the systematic errors assigned to $C_{1}$. The points show $C_{1}$ from the independent fragmentation sample, and the error bars are from the IF Monte Carlo statistics.
$C_{1}$ with string fragmentation and shower models.
For the PEP-5 detector, the tracking efficiency has been studied in detail. In hadronic events, the Monte Carlo has been found to overestimate the true single track efficiency by $1.5 \% \pm 3.0 \% .^{25}$ The effects of overestimating the efficiency are evaluated by analyzing a large block of data (not used elsewhere in our analysis) for which the drift chamber was operated at reduced voltage, resulting in a $10 \%$ degradation in efficiency. From a comparison between this and the higher quality data, we conclude that the efficiency uncertainties can be neglected in the EEC and

EECA measurements.
For the Upgrade data, we study the effect of the two-track separation on the efficiency. The two-hit resolution is altered in the detector Monte Carlo to be slightly worse than what is observed in the data, and this is found to have a negligible effect on $C_{1}$.

We make an explicit check for any bias remaining from a dependence of $C_{1}$ upon $\alpha_{s}$. We calculate $C_{1}$ for Monte Carlo samples in which the two-, three-, and four-parton components are reweighted to simulate values of $\alpha_{s}$ from . 11 to .20 . $C_{1}$ (EECA) changes by less than $1 \%$ for $\chi>30^{\circ}$ over this entire range of $\alpha_{s}$ for both PEP-5 and Upgrade configurations.

For the purpose of determining the best detector-independent measures of the EEC and EECA, the corrections $C_{2}$ are calculated from a large AC Monte Carlo sample generated with a value of $\alpha_{s}=0.158$ with the Lund String Monte Carlo ${ }^{23}$ and the Gottschalk and Shatz matrix element. ${ }^{26}$ This value of $\alpha_{s}$ corresponds to our measurement described in the next section. To establish the errors on $C_{2}$ due to $\alpha_{s}$ uncertainty and model dependence, we recalculate $C_{2}$ with four different Monte Carlo samples: Lund String with $\alpha_{s}=0.141$, Lund String with $\alpha_{s}=0.173$, Hoyer Independent Fragmentation ${ }^{27}$ with $\alpha_{s}=0.105$, and Lund Shower ${ }^{23}$ with $\Lambda_{L L A}=400$ MeV . The two string Monte Carlo samples represent roughly the two-sigma limits (statistical and systematic) of our measured value of $\alpha_{s}$. The comparison of the four calculations yields an estimated uncertainty in $C_{2}$ for each bin in $\chi$, and this is used to assign the systematic errors which appear in Fig. 2.

Our fully corrected EEC and EECA distributions with separate statistical and systematic errors are given in Table 1 and Table 2. Note that when summing bins in $\chi$, the statistical errors may be added in quadrature, but the systematic errors are strongly correlated. To allow simple comparisons with models and other experiments, we give here the integrals over the conventional intervals: ${ }^{29}$

$$
\begin{aligned}
& \int_{57.6^{\circ}}^{122.4^{\circ}} \operatorname{EEC}(\chi) d \chi= \begin{cases}.1486 \pm .0005 \pm .0018 \pm .0014, & \text { PEP-5; } \\
.1458 \pm .0007 \pm .0006 \pm .0010, & \text { Upgrade }\end{cases} \\
& \int_{28.8^{\circ}}^{90^{\circ}} \operatorname{EECA}(\chi) d \chi= \begin{cases}.0297 \pm .0008 \pm .0010 \pm .0016, & \text { PEP-5 } \\
.0306 \pm .0010 \pm .0006 \pm .0010, & \text { Upgrade }\end{cases}
\end{aligned}
$$

where the first error is statistical and the second and third are the systematic errors which result from the uncertainties on $C_{1}$ and $C_{2}$, respectively.

The fully corrected data are shown with combined errors in Fig. 4. The agreement between the two detector configurations is quite good. In Fig. 5, we compare our EECA directly to those of MAC, ${ }^{6}$ JADE, ${ }^{4}$ CELLO, ${ }^{2}$ and PLUTO ${ }^{8}$ who correct their data in a similar fashion. Note that only the MAC results were obtained at the same energy.


Figure 4. Fully corrected data. The fully corrected EEC and EECA are shown separately for the PEP-5 and Upgrade detectors. The errors shown are the sum in quadrature of the statistical and systematic errors.


Figure 5. Comparison to other experiments. The fully corrected EEC and EECA (PEP-5 and Upgrade combined) are compared with fully corrected data from other $e^{+} e^{-}$experiments. The Mark II and MAC results are at $\sqrt{s}=29 \mathrm{GeV}$ while the others are at $\sqrt{s}=34 \mathrm{GeV}$. The region of the EECA above the dotted line in (b) is used to measure $\alpha_{s}$.

Table 1. Fully corrected EEC, $\left(\operatorname{rad}^{-1}\right) \times 10^{3}$. Statistical errors are followed by systematic errors.

| $\chi$ (degrees) | PEP-5 EEC | Upgrade EEC | $\chi$ (degrees) | PEP-5 EEC | Upgrade EEC |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0-3.6 | $1633 \pm 9 \pm 103$ | $1645 \pm 15 \pm 37$ | 90.0-93.6 | $122 \pm 2 \pm 2$ | $117 \pm 3 \pm 1$ |
| 3.6-7.2 | $451 \pm 6 \pm 15$ | $468 \pm 8 \pm 3$ | 93.6-97.2 | $121 \pm 2 \pm 2$ | $121 \pm 3 \pm 1$ |
| 7.2-10.8 | $563 \pm 6 \pm 20$ | $576 \pm 8 \pm 7$ | 97.2-100.8 | $122 \pm 2 \pm 2$ | $121 \pm 3 \pm 1$ |
| 10.8-14.4 | $583 \pm 5 \pm 16$ | $589 \pm 7 \pm 7$ | 100.8-104.4 | $125 \pm 2 \pm 2$ | $119 \pm 2 \pm 1$ |
| 14.4-18.0 | $556 \pm 5 \pm 11$ | $557 \pm 7 \pm 4$ | 104.4-108.0 | $133 \pm 2 \pm 2$ | $130 \pm 3 \pm 1$ |
| 18.0-21.6 | $486 \pm 4 \pm 9$ | $489 \pm 6 \pm 3$ | 108.0-111.6 | $140 \pm 2 \pm 2$ | $137 \pm 3 \pm 1$ |
| 21.6-25.2 | $415 \pm 4 \pm 8$ | $405 \pm 5 \pm 3$ | 111.6-115.2 | $144 \pm 2 \pm 2$ | $147 \pm 3 \pm 1$ |
| 25.2-28.8 | $345 \pm 3 \pm 6$ | $347 \pm 5 \pm 2$ | 115.2-118.8 | $154 \pm 2 \pm 2$ | $152 \pm 3 \pm 1$ |
| 28.8-32.4 | $306 \pm 3 \pm 6$ | $299 \pm 4 \pm 2$ | 118.8-122.4 | $163 \pm 2 \pm 2$ | $159 \pm 3 \pm 1$ |
| 32.4-36.0 | $268 \pm 3 \pm 5$ | $260 \pm 4 \pm 2$ | 122.4-126.0 | $174 \pm 2 \pm 3$ | $175 \pm 3 \pm 1$ |
| 36.0-39.6 | $241 \pm 3 \pm 4$ | $228 \pm 3 \pm 2$ | 126.0-129.6 | $192 \pm 2 \pm 3$ | $188 \pm 3 \pm 1$ |
| 39.6-43.2 | $215 \pm 2 \pm 4$ | $209 \pm 3 \pm 1$ | 129.6-133.2 | $208 \pm 2 \pm 4$ | $210 \pm 4 \pm 1$ |
| 43.2-46.8 | $194 \pm 2 \pm 3$ | $190 \pm 3 \pm 1$ | 133.2-136.8 | $243 \pm 3 \pm 5$ | $226 \pm 4 \pm 2$ |
| 46.8-50.4 | $178 \pm 2 \pm 3$ | $167 \pm 3 \pm 1$ | 136.8-140.4 | $268 \pm 3 \pm 5$ | $256 \pm 4 \pm 2$ |
| 50.4-54.0 | $165 \pm 2 \pm 3$ | $156 \pm 3 \pm 1$ | 140.4-144.0 | $300 \pm 3 \pm 6$ | $296 \pm 5 \pm 2$ |
| 54.0-57.6 | $156 \pm 2 \pm 3$ | $149 \pm 3 \pm 1$ | 144.0-147.6 | $340 \pm 4 \pm 7$ | $333 \pm 5 \pm 3$ |
| 57.6-61.2 | $144 \pm 2 \pm 2$ | $142 \pm 3 \pm 1$ | 147.6-151.2 | $393 \pm 4 \pm 8$ | $384 \pm 6 \pm 3$ |
| 61.2-64.8 | $137 \pm 2 \pm 2$ | $132 \pm 3 \pm 1$ | 151.2-154.8 | $457 \pm 4 \pm 10$ | $463 \pm 7 \pm 4$ |
| 64.8-68.4 | $134 \pm 2 \pm 2$ | $128 \pm 2 \pm 1$ | 154.8-158.4 | $526 \pm 5 \pm 11$ | $531 \pm 7 \pm 5$ |
| 68.4-72.0 | $130 \pm 2 \pm 2$ | $123 \pm 2 \pm 1$ | 158.4-162.0 | $611 \pm 5 \pm 13$ | $633 \pm 8 \pm 6$ |
| 72.0-75.6 | $124 \pm 2 \pm 2$ | $119 \pm 2 \pm 1$ | 162.0-165.6 | $700 \pm 6 \pm 15$ | $723 \pm 9 \pm 7$ |
| 75.6-79.2 | $121 \pm 2 \pm 2$ | $121 \pm 3 \pm 1$ | 165.6-169.2 | $781 \pm 7 \pm 16$ | $783 \pm 11 \pm 4$ |
| 79.2-82.8 | $118 \pm 2 \pm 2$ | $116 \pm 2 \pm 1$ | 169.2-172.8 | $760 \pm 8 \pm 24$ | $790 \pm 12 \pm 5$ |
| 82.8-86.4 | $119 \pm 2 \pm 2$ | $118 \pm 2 \pm 1$ | 172.8-176.4 | $627 \pm 8 \pm 14$ | $626 \pm 11 \pm 10$ |
| 86.4-90.0 | $116 \pm 2 \pm 2$ | $117 \pm 2 \pm 1$ | 176.4-180.0 | $243 \pm 5 \pm 9$ | $243 \pm 7 \pm 6$ |

Table 2. Fully corrected EECA, $\left(\mathrm{rad}^{-1}\right) \times 10^{3}$. Statistical errors are followed by systematic errors.

| $\chi$ (degrees) | PEP-5 EECA | Upgrade EECA |
| :---: | :---: | :---: |
| 0.0-3.6 | $-1389 \pm 9 \pm 97$ | $-1399 \pm 15 \pm 31$ |
| 3.6-7.2 | $180 \pm 6 \pm 17$ | $161 \pm 10 \pm 11$ |
| 7.2-10.8 | 199土 9土 14 | $214 \pm 13 \pm 11$ |
| 10.8-14.4 | $183 \pm 10 \pm 12$ | $186 \pm 14 \pm 8$ |
| 14.4-18.0 | $154 \pm 8 \pm 9$ | $173 \pm 12 \pm 7$ |
| 18.0-21.6 | $128 \pm 6 \pm 7$ | $146 \pm 10 \pm 6$ |
| 21.6-25.2 | $112 \pm 5 \pm 6$ | $126 \pm 8 \pm 5$ |
| 25.2-28.8 | $112 \pm 5 \pm 6$ | $115 \pm 7 \pm 5$ |
| 28.8-32.4 | $88 \pm 4 \pm 4$ | $85 \pm 6 \pm 3$ |
| 32.4-36.0 | $73 \pm 3 \pm 3$ | $73 \pm 5 \pm 3$ |
| 36.0-39.6 | $59 \pm 3 \pm 3$ | $66 \pm 5 \pm 3$ |
| 39.6-43.2 | $52 \pm 3 \pm 3$ | $47 \pm 4 \pm 2$ |
| 43.2-46.8 | $47 \pm 3 \pm 2$ | $37 \pm 4 \pm 1$ |
| 46.8-50.4 | $30 \pm 3 \pm 2$ | $40 \pm 4 \pm 2$ |
| 50.4-54.0 | $26 \pm 2 \pm 2$ | $30 \pm 3 \pm 1$ |
| 54.0-57.6 | $18 \pm 2 \pm 1$ | $25 \pm 3 \pm 1$ |
| 57.6-61.2 | $19 \pm 2 \pm 1$ | $17 \pm 3 \pm 1$ |
| 61.2-64.8 | $16 \pm 2 \pm 1$ | $19 \pm 3 \pm 1$ |
| 64.8-68.4 | $11 \pm 2 \pm 1$ | $18 \pm 3 \pm 1$ |
| 68.4-72.0 | $10 \pm 3 \pm 1$ | $13 \pm 3 \pm 1$ |
| 72.0-75.6 | $9 \pm 3 \pm 2$ | $10 \pm 3 \pm 1$ |
| 75.6-79.2 | $2 \pm 3 \pm 1$ | $0 \pm 3 \pm 0$ |
| 79.2-82.8 | $3 \pm 4 \pm 1$ | $5 \pm 3 \pm 1$ |
| 82.8-86.4 | $1 \pm 5 \pm 0$ | $3 \pm 3 \pm 1$ |
| 86.4-90.0 | $9 \pm 4 \pm 8$ | $0 \pm 3 \pm 0$ |

## V. $\alpha_{s}$ DETERMINATION

To measure $\alpha_{s}$, we compare our data with the $O\left(\alpha_{s}{ }^{2}\right)$ perturbative QCD predictions for $e^{+} e^{-} \rightarrow$ quarks and gluons. We use the recent dressed matrix element calculation of Gottschalk and Shatz. ${ }^{26}$ Previous measurements used either the ERT ${ }^{30}$ or FKSS/GKS ${ }^{31}$ matrix element calculations. The differences among these are discussed in detail in Ref. 32, and the new calculation incorporates significant terms that are neglected in the GKS matrix element. The calculation assumes massless partons, and quark masses are inserted a posteriori. The individual two-, three-, and four-parton cross sections are separated by employing a $y_{\text {min }}$ cutoff of 0.015 , where $y_{i j}=\left(p_{i}+p_{j}\right)^{2} / s$ is the scaled invariant mass of a pair of partons. We verify that the predicted EECA is stable at small values of this infrared cutoff, as shown in Fig. 6.


Figure 6. Cutoff stability of the EECA. The integrated asymmetry is shown from Monte Carlo samples generated with the Gottschalk and Schatz matrix element and Lund string fragmentation for several values of the infrared cutoff parameter $y_{\text {min }}$.

To account for fragmentation effects, we use the Lund string model with the Lund symmetric fragmentation function. This model is quite successful in describ-
ing the general features of our data, ${ }^{28}$ and, in particular, it favorably reproduces the distribution of particles in three-jet events. ${ }^{33,34}$ We comment on the effects of fragmentation models more fully in the next section. The parameters of the model have been initially chosen to describe the global features of our data, including distributions of multiplicity, momentum, and sphericity. ${ }^{28,35}$

To determine $\alpha_{s}$, we compare our data with high-statistics samples of Monte Carlo events generated with five different values of $\alpha_{s}$. Only the detailed detector corrections represented by the factor $C_{1}(\chi)$ are applied to the data, and the radiative and gross acceptance effects are included in the Monte Carlo simulations to which the data are compared. Thus the effects of $\alpha_{s}$ on the properties of the generated events and the geometric acceptance are properly included.

Our best estimates of $\alpha_{s}$ are obtained from a $\chi^{2}$ comparison between the data and Monte Carlo EECA distributions as just described. We limit the sensitivity to fragmentation effects in $q \bar{q}$ events by utilizing the EECA information only for a limited region in $\chi$, namely $\chi \geq 28.8^{\circ}$ ( 17 bins). Only statistical errors are considered in the $\chi^{2}$ calculations. The results are shown in Fig. 7. Parabolas are fitted to the $\chi^{2}$ points, and from the positions of the minima and the curvatures we obtain the values and errors of $\alpha_{s}$ :

$$
\alpha_{s}= \begin{cases}0.155 \pm .004, & \text { PEP-5 } \\ 0.159 \pm .004, & \text { Upgrade }\end{cases}
$$

where the errors are statistical only. These values each correspond to $\chi^{2} \approx 20$ for 16 degrees of freedom.

The statistical error on the Upgrade measurement is comparable to that from the PEP-5 measurement in spite of the smaller number of events. This is a consequence of the larger solid angle and higher efficiency of the Upgrade detector, since the statistical precision of the EECA measurement improves not only with the number of events but also with the number of particles detected in each event.

The details of the fragmentation introduce additional systematic uncertainties into the $\alpha_{s}$ determination. Hadronization in the string model is governed largely by


Figure 7. $\chi^{2}$ comparison between data and Monte Carlo. The points represent a $\chi^{2}$ (for 17 data points) calculated from comparing the EECA with Monte Carlo data generated at several values of $\alpha_{s}$. The errors represent the expected variation of this quantity with the Monte Carlo statistics. The curves are parabolas fitted through the points, and the locations of the minima indicate the best values of $\alpha_{s}$. The vertical lines show the one-sigma statistical errors on $\alpha_{s}$.
the parameters $\sigma_{q}, A$ and $B$. The momenta of hadrons along the string direction is obtained from the symmetric Lund fragmentation function ${ }^{36}$

$$
\begin{equation*}
f(z)=\frac{1}{z}(1-z)^{A} \exp -\left(B m_{\perp}^{2} / z\right) \tag{7}
\end{equation*}
$$

where $m_{\perp}^{2}=\left(m^{2}+p_{\perp}^{2}\right)$ and $z$ is the fraction of $\left(E+p_{\|}\right)$acquired by the hadron. The transverse momenta are distributed according to a gaussian of width $\sigma_{q}$. The fragmentation parameters $A$ and $B$ are strongly correlated, and therefore $B$ is left fixed at $0.7 \mathrm{GeV}^{-2}$ while $A$ is varied over a range that agrees with the observed charged particle multiplicity, namely $.6 \leq A \leq 1.2$. A small correlation


Figure 8. Sensitivity of the EECA to model parameters. We show the integrated EECA from Monte Carlo samples at the generator level. The EECA is integrated over the range $28.8^{\circ}<\chi<90^{\circ}$ and plotted vs. (a) the parameter $A$ in the Lund symmetric fragmentation function, and (b) $\sigma_{q}$. The one-sigma limits of these parameters are indicated with the dotted lines. The right-hand scales show the changes in the measured value of $\alpha_{s}$ which result from different choices of the parameter values.
exists between the multiplicity and the input value of $\alpha_{s}$ which is accounted for in the systematic errors. If both $A$ and $B$ are varied so as to maintain a constant multiplicity, the variations in the EEC are negligible. The range of $\sigma_{q}$ is confined to be between .240 GeV and .290 GeV in order to give reasonable agreement with the distribution of particle momenta normal to the sphericity plane ( $p_{\perp}^{o u t}$ ). ${ }^{28}$ The detailed shape of the EECA for $\chi>30^{\circ}$ is insensitive to small changes in these parameters, and therefore the integrated EECA is used to investigate the systematic errors. Figure 8 shows the changes introduced by varying $A$ and $\sigma_{q}$.

We have also tried using Peterson ${ }^{37}$ fragmentation functions for heavy quarks. The measured spectra of $D^{*}$ mesons provide strong limits on the fragmentation function parameter $\epsilon_{c},{ }^{38}$ and the fractional uncertainty on $\alpha_{s}$ introduced by the allowed variations is less than $1 \%$.

The contributions from tau pair and two photon backgrounds are estimated with Monte Carlo simulations. They are found to have negligible effects on the $\alpha_{s}$ measurement.

Table 3. Systematic errors on $\alpha_{s}$ measurement.

| Source | PEP-5 EECA | Upgrade EECA |
| :---: | :---: | :---: |
| Data correction | $3.3 \%$ | $1.8 \%$ |
| $\sigma_{q}$ | $1.3 \%$ | $1.1 \%$ |
| Frag. param and mult. | $4.6 \%$ | $4.6 \%$ |

It has been shown recently that the second order QCD matrix elements underestimate the ratio of four-jet to three-jet events. ${ }^{39}$ The deficiency in the four-jet rate presumably results from the lack of higher order contributions. Thus we can roughly estimate the size of higher order effects by artificially increasing the hard four-parton cross section accordingly. We carry out this procedure by doubling the four-parton rate ${ }^{40}$ in the Monte Carlo and then determining $\alpha_{s}$. This results in a decrease of .005 in the measured value of $\alpha_{s}$. We do not include this effect in our systematic errors, however, because we are quoting $\alpha_{s}$ at $O\left(\alpha_{s}{ }^{2}\right)$.

The sources and their estimated contribution to the uncertainty in $\alpha_{s}$ are summarized in Table 3. The data correction errors are derived from the uncertainties on $C_{1}$. The total systematic errors (combined in quadrature) are .009 (.008) for PEP-5 (Upgrade). We have not included the effects of different fragmentation models in the systematic errors; these are discussed separately in the following section. We quote errors for the Lund model alone because it is the only $O\left(\alpha_{s}{ }^{2}\right)$ model that adequately describes our data.

The results from the two configurations are now combined to give $\alpha_{s}(29 \mathrm{GeV})=0.158 \pm 0.003 \pm 0.008$. This $\alpha_{s}$ value is used to generate Monte Carlo events which are compared with the data in Fig. 9 and Fig. 10. The agreement is very good for both the EEC and the EECA. The data in these figures are fully corrected and are identical to the Mark II data shown in Fig. 5 except that the self-correlation contributions are removed from the lowest bin for clarity.


Figure 9. Comparisons of EEC with Monte Carlo. The predictions of several models are compared to the corrected EEC. The self-correlation contributions are removed from the lowest bin to make the figure more clear. The values of $\alpha_{s}$ or $\Lambda_{L L A}$ are chosen from fits of the Monte Carlos to the EECA. The curve for IF (Hoyer) $\alpha_{s}=.102$ is not drawn because it coincides with the Ali curve.

The QCD scale parameter $\Lambda$ is related to $\alpha_{s}$ by

$$
\begin{equation*}
\alpha_{s}=\frac{2 \pi}{\frac{\left(33-2 N_{f}\right)}{6} \ln \left(\frac{Q^{2}}{\Lambda^{2}}\right)+\frac{\left(153-19 N_{f}\right)}{\left(33-2 N_{f}\right)} \ln \left(\ln \left(\frac{Q^{2}}{\Lambda^{2}}\right)\right)}, \tag{8}
\end{equation*}
$$



Figure 10. Comparisons of EECA with Monte Carlo. The results of the best fits to the EECA are shown for four different Monte Carlo generators: $\alpha_{s}=0.158$ for Lund String, $\alpha_{s}=0.131$ for Ali, $\alpha_{s}=0.102$ for Hoyer and $\Lambda_{L L A}=390 \mathrm{MeV}$ for Lund Shower. The fits are performed over the region above the dotted line.
in the $\overline{M S}$ renormalization scheme, ${ }^{41}$ where $N_{f}$ is the number of flavors open. At $Q=29 \mathrm{GeV}$ with $N_{f}=5$, our $\alpha_{s}$ value corresponds to $\Lambda_{\overline{M S}}=330 \pm 40 \pm 70 \mathrm{MeV}$.

Our result is compared with other EECA measurements of $\alpha_{s}$ in Fig. 11. The present measurement is, as expected, in better agreement with the ERT values than with those obtained from the FKSS/GKS matrix element. ${ }^{32}$ For the sake of comparison, we repeat our analysis using the GKS matrix element, and we obtain $\alpha_{s}(29 \mathrm{GeV})=0.174 \pm 0.004 \pm 0.009$. Both results appear in the figure, where they are scaled to $Q=34 \mathrm{GeV}$ according to Eqn. 8.


Figure 11. Comparison of $\alpha_{s}$ measurements. Our valuc of $\alpha_{s}$ is compared to those from similar experiments, taken from Ref. 42. The horizontal bars represent the statistical and systematic errors (where available) added in quadrature. The vertical bars indicate the size of the statistical errors alone. All values were obtained by comparing the EECA with an $O\left(\alpha_{s}{ }^{2}\right)$ matrix element plus Lund string fragmentation. The results are grouped according to the matrix element calculations used, which are indicated at the left. All measurements are at $\sqrt{s}=34 \mathrm{GeV}$, except for Mark II and MAC which are rescaled from 29 GeV to 34 GeV according to Eqn. 8 ( $\Delta \alpha_{s} \approx-0.005$ ). Where two points appear for the same experiment, they are not statistically independent.

## VI. MODEL COMPARISONS

Several alternatives exist to the string fragmentation model which enjoy varying degrees of success in describing hadronic events at these energies. We examine some of these briefly in regard to the EEC and EECA.

Independent fragmentation (IF) models are the most common alternative to string fragmentation. Since IF models do not automatically conserve momentum and energy, a particular method must be chosen to accomplish this, and this appears to be the dominant source of uncertainty in measuring $\alpha_{s}$. The two cases we examine here are the Ali scheme, ${ }^{43}$ where jet angles are adjusted and energies are preserved, and the Hoyer scheme, ${ }^{27}$ where the opposite prescription is imposed. A fit to the EECA using the Ali scheme gives an $\alpha_{s}$ value of $0.131 \pm 0.003$ (statistical). Concurrent agreement with the EEC, however, cannot be achieved with any reasonable value of $\sigma_{q}$. The Hoyer scheme represents an even more extreme departure from the string model. It yields $\alpha_{s}=0.102 \pm 0.003$ (statistical) and similar disagreement with the EEC. The results of a best fits to the EECA are shown in Fig. 9 and Fig. 10. In each case, the model parameters $A, B$, and $\sigma_{q}$ are tuned to give agreement with the average multiplicity and $p_{\perp}^{o u t}$ from the data. These results concur with other experiments ${ }^{3-8}$ which found that IF models tend to give lower values of $\alpha_{s}$.

Finally, we compare our data with a leading-log QCD shower Monte Carlo. As an example, we show the EEC from the Lund shower model, Version 6.3. ${ }^{23}$ This model includes a matrix-element weighting of the first branching, and coherence effects are included by angular ordering of subsequent parton emission. As for the string model, the parameters have been adjusted to reproduce a variety of distributions. ${ }^{28}$ The agreement of this model with the EEC and EECA data is quite good, as shown in Fig. 9 and Fig. 10.

In the shower model, the amount of gluon emission is determined by the QCD scale parameter $\Lambda_{L L A}$. We determine this parameter from the EECA just as we measure $\alpha_{s}$. We find $\Lambda_{L L A}=390 \pm 30 \mathrm{MeV}$ (statistical). The definitions of $\Lambda_{\overline{M S}}$ and $\Lambda_{L L A}$ are sufficiently different that the agreement should be viewed as fortuitous.

The best agreement between the global features of the data and the shower model is obtained at a very low shower cutoff value $\left(Q_{0}=1 \mathrm{GeV}\right) .{ }^{28}$ The EECA, however, shows little sensitivity to this cutoff for $Q_{0} \lesssim 4 \mathrm{GeV}$, as shown in Fig. 12. In contrast to the results of PLUTO, ${ }^{8}$ who showed that an earlier shower model was unable to describe their EECA, this good agreement reflects recent improvements in leading-log models.


Figure 12. Cutoff sensitivity in the shower model. The EECA predicted by the Lund Shower Monte Carlo is shown for three different values of the shower cut off mass, $Q_{0}$. The fragmentation parameters $A, B$, and $\sigma_{q}$ are adjusted for each $Q_{0}$ value to maintain a constant multiplicity and $p_{\perp}^{o u t}$ spectrum, but $\Lambda_{L L A}$ is fixed at 400 MeV .

## VII. SUMMARY

We have studied the energy-energy correlation in $e^{+} e^{-}$annihilation into hadrons at 29 GeV . We have used data from the Mark II detector both before and after its upgrade for the SLC, and we find good agreement between the two data sets. We also compare our data to the published results of other experiments. We find reasonable agreement with the EEC and EECA distribution from MAC, which has also operated at 29 GeV . The agreement is best in the perturbative region of the EECA ( $\chi \gtrsim 30^{\circ}$ ). PETRA experiments at 34 GeV also compare well in this region.

We determine $\alpha_{s}$ from our EECA measurement. The results from the PEP-5 and Upgrade data agree well, and give a combined value of $\alpha_{s}=0.158 \pm 0.003 \pm 0.008$ when we use the matrix element calculation of Gottschalk and Shatz and string fragmentation. This result is in reasonable agreement with similar measurements made with the ERT matrix elements, and is about $10 \%$ lower than FKSS/GKS determinations. Independent fragmentation models yield considerably lower values of $\alpha_{s}$ (0.11-0.14).

Both the EECA and EEC are described well by the Lund string model, but cannot be simultaneously fit with independent fragmentation models. The recent Lund leading-log shower model also describes both distributions well with a QCD scale parameter of $\Lambda_{L L A}=390 \pm 30 \mathrm{MeV}$.

## Acknowledgements

The authors are grateful to S. Bethke for many useful discussions and for providing computer code for the Gottschalk and Shatz matrix element. This work was supported in part by Department of Energy contracts DE-AC03-81ER40050 (CIT), DE-AA03-76SF00010 (UCSC), DE-AC02-86ER40253 (Colorado), DE-AC0276 ER03064 (Harvard), DE-AC03-83ER40103 (Hawaii), DE-AC02-84ER40125 (Indiana), DE-AC03-76SF00098 (LBL), DE-AC02-84ER40125 (Michigan), and DE-AC03-76SF00515 (SLAC), and by the National Science Foundation (Johns Hopkins).

## REFERENCES

1. C.L.Basham, et al., Phys. Rev. D17, 2298 (1978).
2. H.-J.Behrend et al., Z. Phys. C14, 95 (1982).
3. H.-J.Behrend et al., Phys. Lett. B138, 31 (1984).
4. W.Bartel et al., Z. Phys. C25, 231 (1984).
5. M. Althoff et al., Z. Phys. C26, 157 (1984).
6. E.Fernandez et al., Phys. Rev. D31, 2724 (1985).
7. B. Adeva et al., Phys. Rev. Lett. 54, 1750 (1985).
8. Ch. Berger et al., Z. Phys. C28, 365 (1985).
9. S. D. Ellis, Phys. Lett. B117, 333 (1982).
10. A. Ali and F. Barreiro, Phys. Lett. B118, 155 (1982).
11. T. Sjöstrand, Z. Phys. C26, 93 (1984).
12. D. Schlatter et al., Phys. Rev. Lett. 49, 521 (1982).
13. R.H.Schindler et al., Phys. Rev. D24, 78 (1981).
14. Proposal for the Mark II at SLC, CALT-68-1015 (April, 1983), unpublished.
15. G. Hanson, Nucl. Instr. Meth. A252, 343 (1986).
16. W.T.Ford et al., Nucl. Inst. Meth. A255, 480 (1987).
17. R.C.Jared et al., I.E.E.E. Trans. Nucl. Sci. N-S 33, \#1, 916 (1986).
18. J.D.Bjorken and S.J.Brodsky, Phys.Rev. D1, 1416 (1970); G. Hanson et al., Phys.Rev.Lett 35, 1609 (1975); we used the definition from C. Berger et al., Phys.Lett 82B, 449 (1979).
19. Our choice of bin intervals follows a convention established by CELLO ${ }^{2}$ and followed by JADE ${ }^{4}$ and MAC ${ }^{6}$.
20. M.Chen and L.Garrido, Phys. Lett. B180, 409 (1986).
21. F.A.Berends and R.Kleiss, Nucl. Phys. B178, 141 (1981).
22. P.C.Rowson et al., Phys. Rev. Lett. 54, 2580 (1985).
23. T. Sjöstrand, Computer Phys. Comm. 39 (1986); T. Sjöstrand, M. Bengtsson, LU TP 86-22 (1986).
24. This is an improved version with baryon production based on A.Ali et al.,

Phys.Lett. 93B, 155 (1980).
25. H.M.Schellman, Ph.D. Thesis, LBL-18699 (1984), unpublished.
26. T.D. Gottschalk and M.P. Shatz, Calt.-68-1172,-1173,-1199 (1985), unpublished, and T.D.Gottschalk, private communication.
27. P. Hoyer et al., Nucl. Phys. B161, 349 (1979). We implement the Hoyer scheme with the Lund symmetric fragmentation function with parameters $A=1.1, B=.7, \sigma_{q}=.295 \mathrm{GeV}$.
28. A.Petersen et al., Submitted to Phys. Rev. D.
29. These integrals are shown for several experiments and models in Ref. 28.
30. R.K. Ellis, D.A. Ross and A.E. Terrano, Nucl. Phys. B178, 421 (1981).
31. F. Gutbrod, G. Schierholz and G. Kramer, Z. Phys. C21, 235 (1984); K. Frabricius, G. Kramer, G. Schierholz and I. Schmitt, Phys. Lett. 97B, 43 (1980).
32. T.D. Gottschalk and M.P. Schatz, Phys. Lett. B150, 451 (1985).
33. H. Aihara et al., Z. Phys. C28, 31 (1985).
34. M. Althoff et al., Z. Phys. C29, 29 (1985);H. Aihara et al., Phys. Rev. Lett. 57, 945 (1986); P.D. Sheldon et al., Phys. Rev. Lett. 57, 1398 (1986).
35. The parameters used for the Lund string model are $y_{\min }=0.015, A=0.9$, $B=0.7$, and $\sigma_{q}=0.265 \mathrm{GeV} / \mathrm{c}$. Parameters for the Lund Shower model are $Q_{0}=1.0 \mathrm{GeV}, A=0.45, B=0.9$, and $\sigma_{q}=0.230 \mathrm{GeV} / \mathrm{c}$.
36. B. Andersson et al., Phys. Rep. 97, 31 (1983).
37. C. Peterson et al., Phys. Rev. D27, 105 (1983).
38. S.Bethke, Z. Phys. C29, 175 (1985).
39. W. Bartel et al., Z. Phys. C33, 23 (1986).
40. In this case, we define the individual $n$-parton rates by a $y_{\min }$ cut of 0.04 .
41. M. Dine and S. Sapirstein, Phys. Rev. Lett. 43, 668 (1979); W. Marciano, Phys. Rev. D29, 580 (1984).
42. B. Naroska, Phys. Rep. 148, 67 (1987), and references therein.
43. A.Ali et al., Phys.Lett. 93B, 155 (1980). We implement the Ali scheme with the Lund symmetric fragmentation function with parameters $A=.75, B=.7$, $\sigma_{q}=.285 \mathrm{GeV}$.


[^0]:    * This work was supported in part by Department of Energy contracts DE-AC03-81ER40050 (CIT), DE-AA03-76SF00010 (UCSC), DE-AC02-86ER40253 (Colorado), DE-AC0276ER03064 (Harvard), DE-AC03-83ER40103 (Hawaii), DE-AC02-84ER40125 (Indiana), DE-AC03-76SF00098 (LBL), DE-AC02-84ER40125 (Michigan), and DE-AC03-76SF00515 (SLAC), and by the National Science Foundation (Johns Hopkins).

