SLAC – PUB – 4351
June 1987
(A)

LINEAR COLLIDER APPROACH TO A $B\overline{B}$ FACTORY*

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1. Introduction

In the following four sections we consider the basic design expressions and principal design constraints for a linear collider suitable for a $B\overline{B}$ factory: Energy ≈ 10 GeV, luminosity 10^{33}–10^{34} cm^{-2} s^{-1}, energy resolution $\lesssim 10^{-2}$. In Section 6 the design of room temperature linear colliders for a B factory is discussed. In such colliders, the rf energy stored in the linac structure is thrown away after each linac pulse. In Section 7 linear colliders using superconducting rf cavities are considered. Some brief conclusions are presented in the final section.

2. Scaling: Beam–Beam Effects

The basic expressions for luminosity, disruption and beamstrahlung are well-known.[1] They are listed for convenience in the Appendix. Some combinations of these basic parameters will prove useful. Using the notation in the Appendix, the luminosity can be written in terms of the beam power P_b, the disruption parameter $D = D_y$ and the bunch length σ_z as

$$\mathcal{L}(\mathrm{cm^{-2}\ s^{-1}}) = 3.45 \times 10^{31} \, \frac{D\, H_D\, P_b(\mathrm{MW})}{\sigma_z(\mathrm{mm})} \left\{ \frac{1+R}{2R} \right\} \quad . \tag{1}$$

Suppose we want $\mathcal{L} \approx 10^{34}$ cm^{-2} s^{-1} and $P_b \approx 1$ MW. The maximum disruption parameter for stable beam collisions is conventionally taken to be $D \approx 10$. For a round beam H_D $(D_y = 10) \approx 6$, while for a flat beam H_D $(D_y = 10) \approx \sqrt{6}$. The maximum bunch length for these two cases is, from Eq. (1),

$$\begin{aligned} \sigma_z \text{ (round beam)} &\approx 0.2 \text{ mm} \\ \sigma_z \text{ (flat beam)} &\approx 0.04 \text{ mm} \quad . \end{aligned} \tag{2}$$

Thus, for a high luminosity and reasonable beam power, one is forced to use rather short bunches, especially for flat-beam collisions.

It is well-known that the beamstrahlung depends on a scaling parameter Υ. For gaussian bunches it can be written (see the Appendix) in terms of the energy

* Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

Summary Talk Presented at the $B\overline{B}$ Factory Conceptual Design Workshop, Los Angeles, California, January 26–30, 1987 per beam E_0 in GeV, the lumonisity $\mathcal{L}$ in units of 10^{32} cm^{-2} s^{-1}, the bunch length σ_z in mm and the bunch collision rate f_b by

$$\Upsilon = 3.1 \times 10^{-3} \frac{E_0(\text{GeV}) \left[\mathcal{L}(10^{32})\right]^{1/2}}{\sigma_z(\text{mm}) \left[f_b(\text{Hz})\right]^{1/2}} \left\{ \frac{2\left[1+(R-1)H_{D0}^{1/2}\right]^{1/2}}{2+(R-1)H_{D0}^{1/2}} \right\} \quad . \tag{3}$$

The right-hand factor in brackets takes pinch into account, at least roughly, for bunches of any aspect ratio and disruption parameter.

For a beam energy in the range of interest ($E_0 \approx 10$ GeV) and for reasonable values of luminosity, bunch length and repetition rate, we will find that $\Upsilon \ll 1$. Thus, a linear collider for a B factory will operate in the classical beamstrahlung regime. The mean energy loss due to beamstrahlung is then

$$\delta_{c\ell} = 0.120 \frac{E_0(\text{GeV})\mathcal{L}(10^{32})}{\sigma_z(\text{mm}) f_b(\text{Hz})} \left\{ \frac{4\left[1+(R-1)H_{D0}^{1/2}\right]}{\left[2+(R-1)H_{D0}^{1/2}\right]^2} \right\} \quad . \tag{4}$$

The average number of photons emitted during the collision is[2]

$$\langle N_p \rangle = 2.2\, \delta_{c\ell} \Upsilon^{-1} \quad . \tag{5}$$

The rms center-of-mass energy spread is then[2]

$$\frac{\sigma_W}{W} = 0.32 \left[1 + \frac{10}{\langle N_p \rangle}\right]^{1/2} \delta_{c\ell} \quad . \tag{6}$$

3. Scaling: Damping Rings and Final Focus

The emittance of a damping ring is determined by contributions from quantum fluctuations (emittance increases with energy) and intrabeam scattering (emittance decreases with energy). The minimum emittance occurs at an energy such that the two contributions are approximately equal. The current in a damping ring is also limited by beam instabilities; in particular, bunch lengthening. In practical examples, the minimum horizontal invariant emittance that can be obtained using conservative assumptions for the lattice design and vacuum chamber impedance is about one-tenth that for the present SLC damping ring, or $\epsilon_{nx} \approx 3 \times 10^{-6}$ m, for $N \approx 2 \times 10^{10}$ particles per bunch.[3,4] The vertical emittance is determined by the coupling, which can be reduced to perhaps 1%, giving $\epsilon_{ny} \approx 3 \times 10^{-8}$ m. As a function of current, these emittances scale approximately as $N^{1/3}$. New concepts in damping ring design may, however, change this picture in the future.

The minimum β^* that can be obtained in a final focus system has been investigated under two operating conditions. For beams hitting head-on, a strong condition is imposed by the fact that the disrupted beams after colliding must pass through the apertures in the focusing quadrupoles closest to the interaction point. The β^* that can be achieved with this assumption for a given energy spread σ_p/p is[4]

$$\beta^*(\text{min}) \approx \left[\frac{c_1(\sigma_p/p)^2\gamma^{1/2}N}{B_p\,\epsilon_n^{1/2}}\right]^{2/3} \quad , \tag{7}$$

where $c_1 = 2.5 \times 10^{-15}$ T-m^2, B_p is the magnetic field at the pole face of the final quadrupole and N the number of particles in the bunch. If this expression is substituted into the usual expression for luminosity, we obtain an expression (in MKS units) originally due to K. Brown,[5]

$$\mathcal{L} = \frac{f_b H_D}{4\pi}\left[\frac{B_p\gamma N^2}{c_1(\sigma_p/p)^2\epsilon_n}\right]^{2/3} \quad . \tag{8}$$

For flat beams intersecting at an angle, the disrupted beams do not have to pass through the aperture in the final quadrupoles. For this case the β^* is limited by the acceptance of the final quadrupole, giving[4]

$$\beta^*(\text{min}) \approx \left[\frac{c_2(\sigma_p/p)^2\gamma^{1/2}\epsilon_n^{1/2}}{B_p}\right]^{2/3} \quad , \tag{9}$$

where $c_2 = 1.1$ T-m. For typical values of N and ϵ_n, this second condition gives considerably lower values for the attainable β^*.

4. Scaling: Energy Spread and Single Bunch Efficiency

For a given accelerating structure, the energy spread induced by longitudinal wakefields is a function only of the single bunch energy extraction efficiency, η_b, the ratios of bunch length to rf wavelength, σ_z/λ, and the bunch phase with respect to the crest of the rf wave. For a typical disk-loaded structure with a group velocity $v_g/c = 0.03$, the single bunch efficiency is given by

$$\eta_b = 11.3\frac{N(10^{10})}{G(\text{MV/m})\,[\lambda(\text{cm})]^2} \quad , \tag{10}$$

where G is the accelerating gradient.

For $v_g/c = 0.01$ the constant increases to 13.5, while for $v_g/c = 0.075$ it decreases to 8.3. By running the bunches off crest with respect to the peak of the rf accelerating wave, the wake-induced energy spread can be partially compensated by the slope of the rf wave. The exact degree of compensation that can be achieved is difficult to quantify precisely (see, for example, Refs. 4 and 6). As an approximation for scaling purposes, we can write that, in order to achieve a given σ_p/p, the following conditions are necessary:

$$\sigma_z/\lambda \lesssim 0.22(\sigma_p/p)^{1/2} \tag{11a}$$
$$\eta_b \lesssim 80(v_g/c)^{1/2}(\sigma_p/p) \quad . \tag{11b}$$

The dependence on group velocity in Eq. (11b) roughly takes into account the fact that the longitudinal wake potential depends on the disk aperature radius.

5. Transverse Emittance Growth

If the head of the bunch in a collider structure makes off-axis excursions in either the accelerating structure or focusing quadrupoles, transverse (deflecting) wakefields will be produced which can drive the betatron oscillation of the tail of the bunch to a large amplitude. If this amplitude is larger than the transverse bunch dimension, the emittance will be degraded. Fortunately, by introducing an appropriate energy spread between the head and the tail of the bunch (Landau damping), the emittance growth due to transverse wakefields can be suppressed. The required energy spread is[7]

$$\frac{\sigma_p}{p} \approx \frac{eN\, W_\perp(2\sigma_z)\beta^2}{8(E/e)} \quad . \tag{12}$$

Here β is the beta function provided by the linac focusing lattice, E is the energy and $W_\perp$ is the transverse wake potential (in MKS units). If the bunch is relatively short ($\sigma_z \lesssim .01\lambda$), the wake potential is roughly linear over the bunch length.

The slope of the wake potential for a SLAC-type, disk-loaded structure ($\lambda =$ 10.5 cm, $v_g/c = 0.012$) is $W'_\perp \approx 2 \times 10^{18}$ V/C-m^3. The slope scales approximately as

$$W'_\perp \sim \lambda^{-4}(v_g/c)^{-1} \quad . \tag{13}$$

The transverse wake is then estimated as $W_\perp \approx 2W'_\perp\sigma_z$. The energy spread required by Eq. (12) must be eliminated at the end of the linac by running at the rf zero crossing for an appropriate (short) distance.

Off-axis excursions by the head of the bunch can be caused by misalignments in both the acceleration sections and the quadrupoles. Landau damping will then be effective in preventing a growth in the tail oscillation amplitude. In the case of misaligned quadrupoles, the kicks at each quadrupole will cause a random walk in the bunch displacement away from the axis. For static misalignments, this displacement can be corrected by steering dipoles. For dynamic misalignments due to ground motion, the displacement jitter can in principle be corrected using feedback, up to a frequency comparable to the bunch repetition frequency. However, at higher frequencies the jitter cannot be suppressed by such pulse-to-pulse feedback. To avoid significant loss in luminosity the jitter amplitude (at the bunch repetition rate divided by 2π) must be less than

$$y_{\rm rms} \lesssim \sigma_y/N_Q^{1/2} \approx \beta \left(\frac{\epsilon_{ny}}{\gamma L}\right)^{1/2} \quad , \tag{14}$$

where N_Q is the number of quadrupoles, L is the total linac length, σ_y is the transverse beam dimension and $\beta = \beta_y$ is the linac beta function.

Uncorrected orbit distortions will lead to emittance growth due to dispersive effects. Following R. Ruth,[8] an approximate expression for the allowable orbit error is

$$y_{\rm rms} \lesssim \frac{\sigma_y(\beta/L)^{1/2}}{(\sigma_p/p)} \left\{1 + \left[\frac{L(\sigma_p/p)}{\beta}\right]^2\right\}^{-1/2} \quad . \tag{15}$$

In Eqs. (12), (14) and (15), β and σ_p/p will not necessarily be constant along the length of the linac. Precise expressions for the tolerances will therefore be somewhat more complex. However, the simplified equations above will give a rough idea of the required tolerances.

6. Room Temperature Collider Designs

The ac "wall plug" power required to drive a typical room temperature disk-loaded accelerating structure at rf wavelength λ is

$$P_{ac}(\text{MW}) \approx 5 \times 10^{-7}(f_b/b)E_0(\text{GeV})G(\text{MV/m})[\lambda(\text{cm})]^2 \quad . \tag{16a}$$

Here b is the number of bunches accelerated per rf fill, and $f_r = f_b/b$ is then the rf pulse repetition rate. In this expression it is assumed (perhaps optimistically) that 50% of the wall plug power is converted to rf power at the input of the accelerating sections, and that 58% of this power is useful for acceleration. This structure efficiency takes into account the effect of dissipation in the structure walls for a typical case ($v_g/c = 0.03$, attenuation parameter $= 0.58$). It is also useful to write the beam power as

$$P_b(\text{MW}) = 1.6 \times 10^{-6} f_b N(10^{10}) E_0(\text{GeV}) \quad . \tag{16b}$$

Let us now see if we can obtain a consistent set of design parameters for a B–factory collider with $E_0 = 10$ GeV, $\mathcal{L} = 10^{34}$ cm^{-2} s^{-1}, $D < 10$ and $\delta_{c\ell} < 10^{-2}$.

Choose also an rf wavelength of λ =3 cm, a gradient $G = 100$ MV/m, $B_p = 1.5$ T, a wall plug power per linac of 50 MW, and a round beam with $\epsilon_n = 3.0 \times 10^{-6}$ m. Then from Eqs. (1), (4), (8), (10), (11) and (16), assuming $H_D = 6$ (large D), we have

$$
\begin{aligned}
(16a):&\quad f_r = f_b/b = 1.11 \times 10^4 && (17a)\\
(10):&\quad \eta_b = 1.26 \times 10^{-2} N(10^{10}) && (17b)\\
(4):&\quad f_b\sigma_z(\mathrm{mm}) > 1.2 \times 10^4 && (17c)\\
(1),(16b):&\quad \sigma_z(\mathrm{mm}) < 3.3 \times 10^{-6} f_b N(10^{10}) && (17d)\\
(11a):&\quad \sigma_z(\mathrm{mm}) < 6.6(\sigma_p/p)^{1/2} && (17e)\\
(11b):&\quad \eta_b < 14(\sigma_p/p) && (17f)\\
(8):&\quad f_b^{3/4} N(10^{10})/(\sigma_p/p) > 2.8 \times 10^6 && (17g)
\end{aligned}
$$

Trial and error will quickly show that a consistent solution to the above equations is impossible for $b = 1$. The lowest value of b for consistent parameters is $b = 4$, giving the beam parameters shown in Table 1.

From Eq. (13) the slope of the transverse wake is for ($v_g/c = 0.03$) $W'_\perp = 1 \times 10^{20}$ V/C-m^3, giving $W_\perp = 6 \times 10^{16}$ V/C-m^2 at $z = 2\sigma_z$. From Eq. (12) the energy spread for Landau damping ($\beta = 2$ m) is then 1.1%. The quadrupole jitter tolerance, Eq. (14), and the orbit tolerance, Eq. (15), are quite acceptable.

The rf design of this accelerator will be discussed only briefly. A peak power of about 150 MW/m would be required to achieve a gradient of 100 MV/m. Conceivably this peak power could be produced directly by microwave tubes at the 10 GHz frequency, for example by gyroklystrons with a pulse length equal to the structure filling time ($\approx$ 125 ns), or by tubes with a lower peak power and a longer pulse length together with rf pulse compression. Using two stages of pulse compression, for example, a tube with a pulse length of 0.5 μs and a peak power of 150 MW could drive 4 m of accelerating structure. Twenty-five such tubes would then be required for each 100 m in the long linac.

Finally, we should note that it is still not certain that more than one bunch per rf fill can be successfully collided with the opposite beam. New accelerating structures with reduced long-range transverse wakes may be needed. The single-bunch luminosity in the design of Table 1 is 2.5×10^{33} cm^{-2} s^{-1}. Also, a lower beamstrahlung parameter (higher resolution) may be desired. From Eq. (4) we see that there can be a direct trade-off between the luminosity and the beamstrahlung energy spread. For example, if the luminosity is lowered to 10^{33}, the beamstrahlung spread can be reduced to 10^{-3} (about 5×10^{-4} sigma in the center-of-mass). The number of particles per bunch, the beam power and the disruption parameter are decreased by a factor of three in this case.

Table 1. Parameters for a 10 GeV
B–factory linear collider.

E_0	$= 10$ GeV
$\mathcal{L}$	$= 10^{34}$ cm^{-2} s^{-1}
L	$= 100$ m
b	$= 4$
σ_p/p	$= 2.0 \times 10^{-3}$
σ_z	$= 0.30$ mm
η_b	$= 0.028$
N	$= 2.2 \times 10^{10}$
$\sigma_\perp^*$	$= 0.32$ μm
D	$= 9.0$, $H_D = 6$
Υ	$= 5 \times 10^{-3}$
$\delta_{c\ell}$	$= 9 \times 10^{-3}$
f_r	$= 11.1$ kHz
f_b	$= 44.4$ kHz
P_b	$= 1.6$ MW
ϵ_n	$= 3.0 \times 10^{-6}$ rad
β^*	$= 0.7$ mm
$\langle N_p \rangle$	$= 4.0$
σ_W/W	$= 5 \times 10^{-3}$

Wurtele and Sessler[9] have also proposed a B–factory collider, powered by a parallel driving beam (two-beam accelerator). In this scheme, FEL modules periodically extract rf energy from the driving beam, and induction accelerator modules add the energy back. The rf wavelength is 1.0 cm, the accelerating gradient is 250 MV/m, and the wall plug power for the rf system is 34 MW per linac. The beam parameters are given in Table 2 for $b = 1$.

The long bunch length ($\sigma_z/\lambda = 0.03$) in this collider design implies that it will be difficult to reduce the energy spread below about 2% [see Eq. (11a)]. From Eq. (8), the normalized emittance required to reach the design luminosity will be about 1.0×10^{-6} m. This is somewhat lower than present conservative estimates for the emittance that can be attained for a round beam in a damping ring (see Section 3). The required β^* for this emittance is 12 mm.

S. Yu[10] has also proposed a B–factory collider design based on the two-beam accelerator approach. In his design the FEL units are replaced by klystron-type output cavities (relativistic klystron) producing high peak power rf at a wavelength of 2.6 cm. The principle beam parameters are the same as those listed in Table 2 for the FEL–based, two–beam accelerator. However, because of the longer wavelength used in the relativistic klystron approach, the rf repetition rate must be reduced by a factor of ten to 7 kHz and the bunch number per fill increased to $b = 10$. The gradient in this design is also lower (80 MV/m), the single bunch efficiency is reduced to about 3%, and the total wall plug power for both linacs is also reduced (to about 20 MW).

Table 2. Parameters for a linear collider $B\overline{B}$ factory, proposed by Wurtele and Sessler.[9]

	A	B	C
E_+ (GeV)	6.0	12.0	12.0
E_- (GeV)	6.0	2.0	2.0
$\mathcal{L}$ (cm^{-2} sec^{-1})	1.0×10^{33}	1.0×10^{33}	1.0×10^{33}
N_+ (10^{10})	2.0	2.0	0.2
N_- (10^{10})	2.0	2.0	10.0
σ_{z+} (mm)	0.3	0.6	0.7
σ_{z-} (mm)	0.3	0.1	0.1
σ_t (μm)	1.00	1.01	0.86
D	1.35	1.35	1.35
$H(D)$	4.6	4.6	4.6
$\overline{P}_b$ (MW)	2.7	3.1	3.6
f_b (kHz)	70.0	70.0	100.0
δ_+ (%)	0.03	0.03	8.4
δ_- (%)	0.03	0.03	<0.01
e^+ (10^{15}/sec)	1.4	1.4	0.2
η_b (%)	8.0		

7. Superconducting Colliders

A conceptual design for a B–factory collider using a superconducting linac structure has been proposed by Amaldi and Coignet.[11]. In this design the electron and positron beams are recirculated several times through the superconducting linac sections, operating at a gradient of 7 MV/m and 350 MHz. By using superconducting rf cavities, the constraint imposed on the repetition rate for a room temperature collider by Eq. (16a) is removed. The wall plug power is now determined by the refrigeration requirements and the rf power into the beam, for a total of $P_{ac} \approx 11$ MW in the Amaldi–Coignet design. The positron source will require an additional 10 MW or so. The long rf wavelength also leads to a low value of η_b, a correspondingly low σ_p/p, and hence the constraint on β^* in Eq. (7) is not a problem. The principal beam parameters are listed in Table 3.

Table 3. Parameters for a superconducting B–factory collider, proposed by Amaldi and Coignet.[11]

	High–Resolution Mode	Low–Resolution Mode
Energy (GeV)	5	10
Luminosity ($cm^{-2}\ s^{-1}$)	10^{33}	10^{34}
Power per beam (MW)	0.5	1.5
Bunch frequency (kHz)	12	12
Emittance ϵ_n (m-rad)	2×10^{-6}	2×10^{-6}
Disruption parameter D	16	13
Pinch factor H_D	6	6
Particles per bunch	5×10^{10}	8×10^{10}
Bunch length (mm)	1.3	0.4
β^* (mm)	6	4
Bunch radius (μm)	1.1	0.6
Quantum parameter Υ	4.5×10^{-4}	8.5×10^{-4}
Beamstrahlung $\delta_{c\ell}$	4.5×10^{-4}	2.5×10^{-2}
$\langle N_\gamma \rangle$	2.5	7.5
σ_W/W	3.5×10^{-4}	1.3×10^{-2}

In this collider design, the high bunch repetition rate requires multiple damping rings (4+4 rings). The high repetition rates for the room temperature collider designs of the previous section are similarly faced with this complexity. We note also that the relatively large number of particles per bunch in the low-resolution mode may lead to emittance growth in the damping ring due to turbulent bunch lengthening.

8. Conclusions

We have seen that at least three linear collider approaches can lead to consistent B–factory designs with an energy of 5–10 GeV, a luminosity of 10^{33} and a beamstrahlung energy spread in the range 3×10^{-4} to 10^{-3}, or to a higher luminosity design (10^{34}) with a beamstrahlung energy spread of 10^{-2} to 2.5×10^{-2}. All of the collider designs examined here have a high bunch repetition frequency (12–70 kHz). Because of limitations on the damping time in a damping ring, multiple rings with multiple bunches per ring will be required. The required invariant emittances range from $1\text{–}3 \times 10^{-6}$ m, and the number of particles per bunch from $2\text{–}8 \times 10^{10}$. The emittance at the lower end of this range, and the particles per bunch at the upper end, may be difficult to achieve.

The suggested rf wavelengths for the room temperature collider examples are in the range 1–3 cm. Rf sources which can produce the required peak power are currently under development: gyroklystrons, FEL's driven by induction linac modules and the relativistic klystron powered by induction linac modules. The collider design using superconducting rf cavities is also based on presently existing technology. The ac wall plug power required to power the rf systems for the designs discussed here is in the range 10–100 MW.

Perhaps the toughest problem posed by all of these linear collider designs is the high positron production rate, $\approx 10^{15}$/sec. To produce positrons at this rate, an electron beam power of about 5 MW on the conversion target will be needed. This implies an additional wall plug power of at least 10 MW. The storage ring approach to a B factory is not faced with this positron production problem. However, the attainable luminosity for a linear collider seems to be about an order of magnitude higher, at least in the low-resolution mode. If high energy resolution is required ($\lesssim 10^{-3}$), the relative advantage of a linear collider over a storage ring tends to disappear, at least in the round beam case.

Finally, we note that all of the collider options presented here have been based on round beams. Both damping rings and quadrupole focusing systems are more naturally adapted to flat beams. By optimizing the design of the damping ring and the final focus system for a beam with a high aspect ratio, a gain in luminosity is possible (see, for example, Ref. 4). From Eq. (4), the beamstrahlung energy spread will also be reduced. As a future effort, it is definitely worthwhile to study design parameters for a B–factory collider using flat beams.

Appendix

Summary of Expressions for Luminosity, Disruption and Beamstrahlung

The following expressions use the notation in Ref. 1.

Luminosity

$$\mathcal{L} = \frac{N^2 f_b H_D}{4\pi A} \quad . \tag{A1}$$

Here N is the number of particles per bunch, H_D is the pinch enhancement factor, f_b is the bunch collision rate and $A = \sigma_x \sigma_y$ is the beam area. The area in turn is related to the vertical and horizontal invariant emittances ϵ_{nx} and ϵ_{ny} by

$$A = (\epsilon_{nx}\epsilon_{ny}\beta_x^* \beta_y^*)^{1/2}/\gamma \quad , \tag{A2}$$

where γ is the electron energy in units of the rest energy and β_x^* and β_y^* are the beta functions at the collision point. If a single bunch is accelerated during each linac pulse, then f_b is also the linac repetition rate, f_r. If a train of b branches is accelerated during each linac pulse, then $f_b = bf_r$. The luminosity can also be written

$$\mathcal{L}\left(10^{32}/\text{cm}^2/\text{s}\right) = 8.0 \times 10^{-6} \left\{ \frac{\left[N(10^{10})\right]^2 f_b H_D}{A(\mu\text{m})^2} \right\} \quad . \tag{A3}$$

Disruption

The disruption parameter is

$$D = D_y = \frac{r_0 N \sigma_z}{\gamma A} \left(\frac{2R}{1+R} \right) \quad , \tag{A4}$$

when $R = \sigma_x/\sigma_y \geq 1$ is the aspect ratio, σ_z is the bunch length and $r_0 = 2.82 \times 10^{-13}$ cm is the classical electron radius. The pinch enhancement factor for a round beam (plotted, for example, in Ref. 1) is in the range 5 to 6 for $1.5 \lesssim D \lesssim 20$. For a flat beam with aspect ratio R it can be expressed, at least approximately, in terms of the round beam enhancement factor H_{D0} by

$$H_D(R, D_y) = \frac{R H_{D0}}{1 + (R-1) H_{D0}^{1/2}} \quad , \tag{A5}$$

where H_{D0} is taken at D (round beam) $= D_y$ (flat beam). The disruption parameter can also be calculated using

$$D_y = 14.4 \, \frac{N(10^{10}) \sigma_z(\mathrm{mm})}{E_0(\mathrm{GeV}) A(\mu\mathrm{m})^2} \quad . \tag{A6}$$

For a flat beam $D_x = D_y/R$. The maximum disruption angles are also of interest, given by

$$\theta_{Dx} \approx \theta_{Dy} = \frac{2 F_{x,y} r_0 N}{\gamma(\sigma_x + \sigma_y)} \quad . \tag{A7}$$

Here, $F_{x,y}$ are form factors on the order of unity, which vary only slightly with R.

Beamstrahlung

The effective beamstrahlung scaling parameter $\overline{\Upsilon}$ for a gaussian bunch is

$$\overline{\Upsilon} = \frac{F_\Upsilon r_0 \lambda\!\!\!^{-}_c \gamma N}{\sigma_z A^{1/2}} \left\{ \frac{2 \left(H_{D0} R \right)^{1/2}}{2 + (R-1) H_{D0}^{1/2}} \right\} \quad . \tag{A8}$$

Here $F_\Upsilon \approx 0.41$ is a form factor and $\lambda\!\!\!^{-} = 3.86 \times 10^{-11}$ cm is the reduced electron Compton wavelength. The factor in brackets takes pinch into account, at least approximately. In practical units Eq. (A8) becomes

$$\overline{\Upsilon} = 8.8 \times 10^{-6} \frac{N(10^{10}) E_0(\mathrm{GeV})}{\sigma_z(\mathrm{mm}) \left[A \left(\mu\mathrm{m} \right)^2 \right]^{1/2}} \left\{ \frac{2 \left(H_{D0} R \right)^{1/2}}{2 + (R-1) H_{D0}^{1/2}} \right\} \quad . \tag{A9}$$

The classical beamstrahlung parameter (average electron energy loss after collision normalized to the initial electron energy) is

$$\delta_{c\ell} = \frac{F_\delta r_0^3 \gamma N^2}{\sigma_z A} \left\{ \frac{4 H_{D0} R}{\left[2 + (R-1) H_{D0}^{1/2} \right]^2} \right\} \quad , \tag{A10}$$

where $F_\delta \approx 0.22$ is a form factor. In practical units

$$\delta_{c\ell} = 9.9 \times 10^{-7} \frac{\left[N(10^{10}) \right]^2 E_0(\mathrm{GeV})}{\sigma_z(\mathrm{mm}) A(\mu\mathrm{m})^2} \left\{ \frac{4 H_{D0} R}{\left[2 + (R-1) H_{D0}^{1/2} \right]^2} \right\} \quad . \tag{A11}$$

References

1. See, for example, P. B. Wilson, SLAC–PUB–4310 (1987).
2. Robert J. Noble, SLAC–PUB–3871 (1986).
3. L. Rivkin, private communication.
4. R. Palmer, AAS Notes 30 and 31, SLAC (1987) [unpublished].
5. K. Brown, presented at this Workshop.
6. P. B. Wilson, SLAC–PUB–2884 (1982).
7. K. Bane, SLAC–PUB–4169 (1986).
8. R. Ruth, AAS Note 4, SLAC (1985) [unpublished].
9. J. S. Wurtele and A. M. Sessler, TBA Note 33, Lawrence Berkeley Laboratory (1986) [unpublished].
10. S. Yu, presented at this Workshop.
11. U. Amaldi and G. Coignet, CERN–EP/86–211 (1986). Submitted to *Nucl. Instrum. Methods.*