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ON CHARM DECAYS — PRESENT STATUS AND FUTURE GOALS*

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ABSTRACT

After a qualitative introduction into the dynamics underlying charm decays I describe in some detail three different theoretical treatments: the Stech et al. description based on factorization, the 1/N approach and an ansatz employing QCD sum rules. The overall agreement of the emerging theoretical picture with the data is rather encouraging and indicates that the effects of hadronization on these decays are under reasonable control. Yet more and more detailled data are needed to confirm (hopefully) this simple picture. I list the processes most relevant in this respect and emphasize the need for increasing our theoretical sophistication. Once this is achieved we have on one hand acquired the theoretical tools to deal with B physics; on the other hand we will then be ready to exploit charm physics to the fullest in searching for exotic D decays, $D^0 - \bar{D}^0$ mixing and CP violation.

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1. Introduction: Motivation

The decays of hadrons carrying the quantum number charm have been studied for quite some time now, both on the experimental as well as the theoretical side. Large amounts of data have been accumulated in the last few years and their analysis has reached the maturity level. It is then tempting to argue that after these *existing* data have been analyzed one should move on to newer, greener pastures, i.e. topics of study.

I believe such a conclusion would be quite erroneous – there is actually still a strong motivation for further dedicated charm studies. The answer to the question, "What is the physics interest in charm decays?", consists of two parts and goes quite clearly beyond the mountain climber's reply who when asked, "Why do you climb mountains?", might just state, "Because they are there!".

A. The first part of the answer notes that according to the Standard Model the electroweak forces in charm decays are of a rather uninspiring, if not even dull, kind:

(*i*) The charged current couplings of charm quarks—$V(cs)$ and $V(cd)$—are known to be given by the Cabibbo angle θ_c to a high accuracy (the main ingredient here is the "long" bottom life time of roughly 1 psec).

(*ii*) Very tiny $D^0 - \overline{D}^0$ mixing, no observable CP violation and other flavor changing neutral currents are expected (as explained in more detail later on).

This apparent vice can however be swiftly turned into a virtue by noting that charm decays therefore offer a "clean" laboratory to study hadronization effects in quark transitions. The benefits we can derive from it are two-fold: (a) we will improve our understanding of strong interactions at the interface between the perturbative and non-perturbative regime. (b) We can apply these lessons to other dynamical systems where the underlying forces are much less known. The most urgent need for these lessons arises in bottom decays where the charged current couplings $V(ub)$ and $V(cb)$ are not or are fairly poorly known; where $B^0 - \overline{B}^0$ mixing which apparently has been observed has to be interpreted properly; where Penguins could make a small, yet significant, impact and where—most importantly and most ambitiously—CP violation might be observed.

B. There is no guarantee that the Standard Model represents the complete cast of forces and states. On the contrary there is widespread suspicion in the community that it is incomplete. One can also draw from historical precedent, namely, K decays: the observation of the $\theta - \tau$ puzzle, of $K^0 - \overline{K}^0$ mixing and of $K_L \to \pi\pi$ lead to the realization that "New Physics" was present; namely, parity violation, the charm quark and CP violation. Something like this might happen again in charm decays: the observation of $D^0 - \overline{D}^0$ mixing, CP violation, etc., would—apart from their intrinsic fascination—represent almost zero-background signals for New Physics.

These lecture notes will be organized as follows: in Section 2 we discuss the qualitative picture that can be obtained from a simple approach based on quark diagrams; Section 3 contains a more quantitative description of exclusive charm decays based on a largely phenomenological approach; its success will prompt us to consider more formal approaches in Section 4; namely, a $1/N$ treatment and QCD sum rules. In Section 5 we analyze how further lessons can be obtained from F, $\Lambda_c, \ldots$ decays and from once and twice Cabibbo suppressed decays and how they can profitably be applied in B decays. Section 6 is devoted mainly to heresies, i.e. New Physics in D decays producing flavor changing neutral currents, $D^0 - \overline{D}^0$ mixing and CP violation. The whole discussion is summed up in Section 7 and an outlook on future developments is presented.

2. A Qualitative Picture from Quark Diagrams

The effective Lagrangian for Cabibbo allowed charm decays is obtained by dressing the bare current-current coupling that includes the intermediate W boson with gluon lines and then integrating out the W boson.The result reads[1]

$$\mathcal{L}(\Delta C = 1) \propto \frac{c_+ + c_-}{2}\, \bar{s}_L \gamma_\mu c_L \, \bar{u}_L \gamma_\mu d_L + \frac{c_+ - c_-}{2}\, \bar{u}_L \gamma_\mu c_L \bar{s}_L \gamma_\mu d_L$$

$$c_\pm = \left[\frac{\alpha_S(M_W^2)}{\alpha_S(m_c^2)} \right]^{\gamma_\pm}, \qquad \gamma_+ = \frac{2}{b} = -\frac{1}{2}\gamma_-, \qquad b = 11 - \frac{2}{3} n_F \; . \tag{1}$$

For D^0 decays there are two quark diagrams with different topologies, as shown in Fig. 1a,b, while for D^+ there is only one—Fig. 1a. The open circle in the decay vertex denotes both the couplings given in Eq. (1). The diagrams in Fig. 1a represent the spectator mechanism since the anti-quarks do not participate directly in the c quark decays. On dimensional grounds one obtains for the corresponding decay width[2]

$$\Gamma_{\text{Spect}}(D^+) = \Gamma_{\text{Spect}}(D^0) \propto G_F^2 m_c^5 \; . \tag{2}$$

The diagram in Fig. 1b represents the "W-exchange" or "weak annihilation" process (hereafter summarily referred to as WA). On general grounds one finds:

$$\Gamma_{\rm WA}(D^0) \propto G_F^2 |f_D|^2 m_s^2 m_D \, . \tag{3}$$

f_D—the decay constant of D mesons (the precise definition will be given later)—depends on $|\phi(0)|$, the D meson wavefunction at the origin. Its presence reflects the pointlike nature of weak forces at this scale; m_s—the strange quark mass—represents the helicity suppression for spin one currents, in complete analogy to $\pi \to \ell \nu_\ell$ decays. Since

$$f_D \sim 0.2 \; GeV, \qquad m_s \lesssim 0.5 \; GeV \ll m_c \sim 1.5 \; GeV \tag{4}$$

one predicts $\Gamma_{\rm Spect} \gg \Gamma_{\rm WA}$ and expects therefore a universal charm lifetime which is very simply related to the muon lifetime:

$$\tau_D \simeq \tau(\text{charm}) \simeq \Gamma^{-1}_{\rm Spect} \simeq \frac{1}{2+3} \, \tau_\mu \left(\frac{m_c}{m_\mu}\right)^5 \sim 7 \times 10^{-13} \, \text{sec} \tag{5}$$

where the last relation holds for $m_c \simeq 1.5 \; GeV$.

Comparing (5) with the experimental findings[3]

$$\tau(D^0) \simeq 4.3 \times 10^{-13} \, \text{sec} \qquad \tau(D^+) \simeq 10 \times 10^{-13} \, \text{sec} \tag{6}$$

i.e. $\langle \tau_D \rangle \simeq 7 \times 10^{-13}$ sec one should be duly impressed—after all, Eq. (5) represents an extrapolation over more than six orders of magnitude.

After having savored this moment of triumph it is however appropriate to note that the agreement between expectation and the data is certainly not perfect since experimentally

$$\frac{\tau(D^+)}{\tau(D^0)} \sim 2.5 \qquad (7)$$

holds—in contrast to the expectation of a universal lifetime. The situation is clearly not as dramatic as in K decays— where $\tau(K^+)/\tau(K_s) \sim 135$ holds—but there is a definite need for refining our theoretical picture.

One word of caution is in order here: drawing cute diagrams is one thing, deriving reliable predictions is another. After all the D mass is such that charm hadrons are placed into an environment of clear resonances. Therefore final state interactions a priori cannot be neglected and one has to allow for a non-trivial hadronization process.

Some examples can illustrate the meaning of this caveat:

(1) *Helicity suppression:*
As already mentioned the amplitude for $D^0 \to s\bar{d}$ vanishes like m_s as $m_s \to 0$. Alternatively one can look at a WA transition between hadrons, for example $D^0 \to P_1P_2$ where P_1, P_2 denote pseudoscalar mesons; its amplitude is proportional to $m^2_{P_1}, m^2_{P_2}$. Thus it vanishes as $m_s > m_{u,d} \to 0$ (for Goldstone bosons P_1, P_2)—as anticipated. For $D^0 \to PV$ decays on the other hand where V denotes a vector meson the situation is quite different: the amplitude does not go to zero anymore for vanishing quark masses. (Technically this comes about due to the presence of a new variable, the spin vector for V; dynamically it is effected by a nearby 0^- resonance with the quantum numbers of $\overline{K}^0$ which will contribute to $D^0 \to PV$, but not to $D^0 \to PP$.)

Thus we see that considerable care has to be employed when one appeals to the concept of duality for equating quark transitions with transitions between hadrons.

(2) $D^0 \to K^-\pi^+$ *vs.* $\overline{K}^0\pi^0$:

For $c_+ = 1 = c_-$ there is only one spectator diagram each for $D^0 \to K^-\pi^+$ and $D^0 \to \overline{K}^0\pi^0$, Fig. 2 , from where one reads off

$$\frac{\Gamma(D^0 \to \overline{K}^0\pi^0)}{\Gamma(D^0 \to K^-\pi^+)} \simeq \frac{1}{2}\,\frac{1}{N^2} = \frac{1}{18} \tag{8}$$

where N denotes the number of colors.

For $c_- > 1 > c_+$ there are two more contributions due to the induced charm changing neutral current and one finds

$$\frac{\Gamma(D^0 \to \overline{K}^0\pi^0)}{\Gamma(D^0 \to K^-\pi^+)} \simeq \frac{1}{2}\left(\frac{2c_+ - c_-}{2c_+ + c_-}\right)^2 \sim \frac{1}{50}\,. \tag{9}$$

The superficial lesson of Eq. (9) would be to expect a tiny branching ratio for $D^0 \to \overline{K}^0\pi^0$; the correct lesson is to realize that this prediction is highly unstable under a change in parameters. Therefore one cannot ignore the impact of final state interactions (hereafter referred to as FSI): looking at the isospin decomposition of $K\pi$

$$\begin{aligned} \left|K^-\pi^+\right\rangle &= \frac{1}{\sqrt{3}}\left(\frac{3}{2},\frac{1}{2}\right)_I + \sqrt{\frac{2}{3}}\left(\frac{1}{2},\frac{1}{2}\right)_I \\ \left|\overline{K}^0\pi^0\right\rangle &= \sqrt{\frac{2}{3}}\left(\frac{3}{2},\frac{1}{2}\right)_I - \frac{1}{\sqrt{3}}\left(\frac{1}{2},\frac{1}{2}\right)_I \end{aligned} \tag{10}$$

illustrates this point when one keeps in mind that the phase shifts are not expected to be even approximately the same for the $I = 3/2$ and $I = 1/2$ channels.

(3) *Color Coherence in D^+ Decays:*

Considering $D^+ = (c\bar{d}) \to (s\bar{d})_{\overline{K}^0}(u\bar{d})_{\pi^+}$ one notices that there are two $\bar{d}$ quarks in the final state[2] . Thus one has to address the issue of coherence already on the quark level. It is sometimes argued that the ensuing interference has to be negative since the identical states —$\bar{d}$—are fermions. This conclusion—that there is destructive interference—is correct, the reasoning is however misleading. A cleaner argument is based on V spin: V spin groups u and s quarks into a doublet, while d and c are singlets. D^+ is thus a V spin singlet whereas D^0 and F^+ form a doublet; secondly, Bose statistics requires $\overline{K}^0$ and π^+ which are V spin partners to form a $V = 1$ state; thirdly, in $\mathcal{L}_{\text{eff}}(\Delta C = 1)$ there is a $\Delta V = 0$ term with coefficient c_- and a $\Delta V = 1$ term with coefficient c_+. Combining these three observations one notes that $\Gamma(D^+ \to \overline{K}^0\pi^+)$ depends on c_+^2 whereas $\Gamma(D^0 \to K\pi)$ contains also c_-^2. Therefore $D^+ \to \overline{K}^0\pi^+$ is *suppressed* relative to $D^0 \to K\pi$ by $(c_+/c_-)^2$.

The conclusions I want to draw at this point are:

(*a*) the simplest picture—the pure spectator mechanism—already gives the correct order of magnitude for charm lifetimes.

(*b*) Even so there is a definite need for refining the theoretical description since the measurements yield[3] :

$$- \frac{\tau(D^\pm)}{\tau(D^0)} \sim 2.5 > 1 \qquad (11a)$$

$$- \frac{\Gamma(D^0 \to \overline{K}^0\pi^0)}{\Gamma(D^0 \to K^-\pi^+)} \sim 0.4 \gg \frac{1}{18} \qquad (11b)$$

$$-\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} \sim 3 - 4 > 1 \ . \tag{11c}$$

(*c*) To achieve this necessary improvement one has to go beyond the purely probabilistic treatment of a parton model approach, coherence effects have to be included as well.

First we will address the challenge posed by $\tau(D^+)/\tau(D^0) > 1$. One reply to this challenge consists of including WA diagrams which—on the Cabibbo allowed level—contribute to D^0, but not to D^+ decays. This diagram is added incoherently since WA generates only a $q_1\overline{q}_2$ final state in the first step while the spectator process leads directly to a $q_1\overline{q}_2q_3\overline{q}_4$ final state. (In passing it should be noted that WA can therefore proceed only if the quantum numbers of the final state are such that they can be carried by a $q\overline{q}$ pair; it is exactly those "non-exotic" channels that are affected by FSI.) Therefore one obtains quite naturally:

$$\frac{\tau(D^+)}{\tau(D^0)} \simeq 1 + \frac{\Gamma_{\mathrm{WA}}(D^0)}{\Gamma_{\mathrm{Spect}}(D)} > 1 \ . \tag{12}$$

To reproduce the observed lifetime ratio (see Eq. (11a)) from this mechanism alone one has to require

$$\frac{\Gamma_{\mathrm{WA}}(D^0)}{\Gamma_{\mathrm{Spect}}(D)} \sim 1 \ . \tag{13}$$

Thus one has to find a way around the arguments given above for $\Gamma_{\mathrm{WA}} \ll \Gamma_{\mathrm{Spect}}$. This could conceivably be done in three steps:

(*i*) On the quark level the spectator process $c \to s\, q_1\overline{q}_2$ leads to a three-body final state while WA $c\overline{u} \to s\overline{d}$ yields a two-body final state; phase space thus favors WA.

(*ii*) The helicity suppression can be circumvented in two ways. Close to the D meson there could be a resonance with the quantum numbers of the K which could overcome the helicity suppression[4]. It should be noted that such a resonance would produce only PV, but not PP final states where $P[V]$ stands for pseudoscalar [vector] meson. Alternatively, as a more intriguing possibility[5,6], the D meson wave function might contain a non-vanishing component of $c\bar{q}g$ where g denotes one (or more gluons). In that case $c\bar{q}$ can form quite naturally a spin-one configuration—hence no helicity suppression for WA!

(*iii*) The weak forces are still pointlike, WA therefore depends, as before, on the $c\bar{q}$ overlap wavefunction at zero distance. It should be noted however that this is not the same quantity which is measured in $D^+ \to \ell^+\nu_\ell$ and which is referred to as f_D. Either there are strong resonance effects which make these arguments somewhat academic or—as argued above—WA proceeds predominantly in the presence of gluons. In that case one probes a part of the hadronic wavefunction, namely, $c\bar{q}+$ glue, that is not accessible in purely leptonic D decays. We will refer to the $c\bar{q}$ overlap in the presence of gluons as $\tilde{f}_D$. There is actually a qualitative argument for

$$|\tilde{f}_D| > |f_D| \,. \tag{14}$$

The D meson is described by a $c\bar{q}$ pair connected via a color flux tube made up by gluons. Such a system can produce a purely leptonic final state – as in D^+, $F^+ \to \ell^+\nu_\ell$ – only if the $c\bar{q}$ pair reabsorbs the flux tube before it annihilates. For this to happen the flux tube has to occupy the straight line of approach of c and $\overline{q}$. This should however be a rather unlikely configuration; it appears much more likely—due to the independent motion of the flux tube—that substantial amounts of it and thus of hadronic energy survive the $c\overline{q}$ annihilation. WA therefore contributes almost exclusively to nonleptonic and semi-leptonic decays—the latter for $F^+[D^+]$ mesons on the Cabibbo allowed [disallowed] level. Continuing this argument leads furthermore to the expectation that WA will contribute mainly to genuine multi-body final states and not to two-body modes.

There are apparently two ways to interpret $\widetilde{f}_D$:

(A) It reflects a $(c\overline{q}g)$ or $(c\overline{q}gg)$, etc. component in the meson wavefunction; thus it evidently reflects non-perturbative dynamics.

(B) One invokes bremsstrahlung to procure the required gluon background. This appears to be a perturbative phenomenon which could be treated quantitatively. That impression is however misleading as can be seen in the following way: a non-relativistic model calculation yields for D^0 transitions[6]

$$\widetilde{f}_D^2 \propto f_D^2 \left(\frac{m_c}{m_u}\right)^2 . \tag{15}$$

From it one reads off that the integration implicit in this ansatz receives most of its contributions from momenta p around $m_u \sim$ 300 MeV, say $m_u \lesssim p \lesssim 2m_u$. This is however the realm of "soft" dynamics which cannot be treated perturbatively. Alternatively one can note that the gluon bremsstrahlung of D mesons which are color singlets has an intrinsic infrared cut-off which is provided by the internal structure of the meson; its scale is given by the inverse Compton wavelength of the light quark; i.e. its mass m_u. Thus one is still very sensitive to soft gluons and a perturbative treatment cannot be trusted. In heavy quarkonia $(Q\overline{Q})$, on the other hand, the situation is quite different since there is only one scale $\sim m_Q$.

In conclusion, WA can be enhanced to a significant level only by invoking some non-perturbative phenomena (though the pattern exhibited in Eq. (15) might still be true for some other reason). Thus at present one cannot predict the strength of WA.

Instead of giving up on WA one can adopt the following strategy: (a) postulate some strength for WA like $\Gamma_{\mathrm{WA}}(D^0) = \Gamma_{\mathrm{Spect}}$ (charm); (b) then predict some other rates. Such a program can indeed be pursued on the semi-quantitative or at least qualitative level. It leads to predictions on global rates like lifetimes, on Cabibbo suppressed decays and on some special decay modes:

(i) WA, unless it is driven by the accidental presence of an appropriate resonance, leads quite unambiguously to

$$\tau_{\Lambda_c} < \tau_{D^0} < \tau_{D^+} \tag{16}$$

since in Λ_c decays WA does not have to contend with helicity suppression and its amplitude is proportional to the enhanced coefficient c_- (the quarks form color antisymmetric combinations in baryons).

For F decays no such clear statement can be made and two scenarios are conceivable

$$\tau_F < \tau_{D^0} < \tau_{D^+} \tag{17a}$$

$$\tau_{D^0} < \tau_F < \tau_{D^+} \ . \tag{17b}$$

The first case, (17a), appears somewhat more probable, yet the second one, (17b), could be realized in a scenario as described by Eq. (15) which leads to

$$\tilde{f}^2_{D^0} \sim \left(\frac{m_s}{m_u}\right)^2 \tilde{f}^2_F \sim (2-3) \times \tilde{f}^2_F \ .$$

Isospin arguments yield

$$\Gamma(D^+ \to \ell\nu X) = \Gamma(D^0 \to \ell\nu X) + \mathcal{O}(\sin^2\theta_c) \tag{18}$$

and therefore

$$\frac{\tau(D^+)}{\tau(D^0)} \simeq \frac{b_{\mathrm{SL}}(D^+)}{b_{\mathrm{SL}}(D^0)} \tag{19}$$

up to corrections of order θ_c^2, θ_c = Cabibbo angle. $\Gamma(D^0 \to \ell\nu X)$ and $\Gamma(F^+ \to \ell\nu X)$ are related by V spin and not by I spin; since WA is very likely to break V spin invariance one arrives at the general expectation:

$$\frac{b_{\mathrm{SL}}(F^+)}{b_{\mathrm{SL}}(D^0)} > \frac{\tau(F^+)}{\tau(D^0)} \ . \tag{20}$$

(ii) WA contributes directly to those Cabibbo suppressed D^+ modes that evolve from a ($u\bar{d}$+ glue) system; those should exhibit an enhancement over their naively expected level:

$$\frac{\Gamma(D^+ \to \pi'\mathrm{s})}{\Gamma(D^+ \to K + \pi'\mathrm{s})} > tg^2\theta_c \ . \tag{21}$$

(iii) There are those special decay modes that—it seems—can be generated only via WA on an appreciable level. Some prominent

examples of this family:

$$BR(D^0 \to \overline{K}^0 \phi) \underset{\text{WA}}{\sim} 0.5 - 1\% \tag{22}$$

$$BR(F^+ \to \pi'\text{s}) \underset{\text{WA}}{\sim} 3 - 5\% \tag{23}$$

$$BR(\Lambda_c^+ \to \Delta^{++} K^-) \underset{\text{WA}}{\sim} \text{few}\% \ . \tag{24}$$

The valence quark structure of these final states cannot be produced via spectator decays in a direct way—a contention we return to in the next chapter.

The second answer to the challenge provided by $\tau(D^+)/\tau(D^0) \sim 2.5$ consists of generalizing color coherence as introduced above for $D^+ \to \overline{K}^0 \pi^+$ to all D^+ decays. From V spin arguments one concludes

$$\tau(D^0) \simeq \tau(F^+) \ , \qquad b_{\text{SL}}(D^0) \simeq b_{\text{SL}}(F^+) \tag{25}$$

(in Section 5 we will give some estimates on the size of V spin breaking in this scheme). For Λ_c decays no identical quanta appear in the final state and thus

$$\tau(\Lambda_c) \simeq \tau(D^0) \ , \qquad b_{\text{SL}}(\Lambda_c) \sim b_{\text{SL}}(D^0) \ . \tag{26}$$

Coherence effects in the Cabibbo suppressed $D^+ \to \pi$'s transition should play the same role as in $D^+ \to K + \pi$'s since on the quark level they read $(c\bar{d}) \to u\bar{d}d\bar{d}$ and $(c\bar{d}) \to u\bar{d}s\bar{d}$. For $D^+ \to K\overline{K} + \pi$'s no such coherence emerges: $(c\bar{d}) \rightarrow u\bar{s}s\bar{d}$ and therefore[8]

$$\frac{BR(D^+ \rightarrow K\overline{K} + \pi\text{'s})}{BR(D^+ \rightarrow K + \pi\text{'s})} > tg^2\theta_c \simeq 0.05 \ . \tag{27}$$

To summarize Chapter 2:

(*i*) $\tau(D^+)/\tau(D^0) > 1$ can be accommodated (though not really explained) by WA or Color Coherence.

(*ii*) Both schemes make new genuine predictions beyond $\tau(D^+)/\tau(D^0) > 1$: for example Cabibbo suppressed D^+ decays should be enhanced relative to the $tg^2\theta_c$ level. According to Color Coherence [WA] this enhancement should occur in the $D^+ \rightarrow K\overline{K} + \pi$'s [$D^+ \rightarrow \pi$'s] modes.

(*iii*) For pure Color Coherence one expects

$$\tau(\Lambda_c^+) \sim \tau(F) \simeq \tau(D^0) < \tau(D^+)$$

these relations should be reflected also in the corresponding relations between the semi-leptonic branching ratios.

In WA one obtains instead

$$\tau(\Lambda_c) < \tau(D^0) < \tau(D^+) \qquad \frac{b_{\rm SL}(F)}{b_{\rm SL}(D^0)} > \frac{\tau(F)}{\tau(D^0)} \ .$$

(*iv*) WA leads to some special decay modes.

Two important warnings should be added at this point:

(*a*) So far we have completely ignored non-trivial hadronization effects as expressed by final state interactions, resonances, etc.

(*b*) It is very hard on the level of reasoning pursued so far to make the statements listed above under (i)-(iv) more quantitative.

3. The Stech *et al.* Approach to Charm Decays or "Modesty Rewarded"

It is important to note that by now almost all theoretical treatments of charm decays employ the same effective Lagrangian with coefficients $c_{\pm}$ of more or less the same numerical size. These schemes—the Spectator Ansatz, WA, etc.—differ only in the way they deal with the hadronic matrix elements that enter. There is one class of decay modes where these matrix elements appear to be simpler and therefore more tractable: these are the two-body decays $D, F \rightarrow PP$, PV, VV. At first there arises the concern that one might embark on a somewhat academic exercise since these decay channels might make up only a minor part of all decays. Nevertheless one proceeds in the spirit of "modesty."

The approach by Stech and co-workers[9] (Fakirov-Stech, Bauer-Stech, Bauer-Stech-Wirbel) is based on five ingredients:

(i) use the appropriate $\mathcal{L}_{\text{eff}}$ with QCD coefficients $c_{\pm}$.

(ii) Ignore WA contributions.

(iii) Draw diagrams for all the different quark decay topologies; two typical examples are shown in Fig. 2a,b. The $u\bar{d}$ pair in Fig. 2a forms a color singlet and thus carries all the quantum numbers of a pion; the quark current is then simply replaced by the hadronic current with the quantum numbers of the pion. The situation is less straightforward in Fig. 2b: the $s\bar{d}$ pair does not necessarily form a color singlet. When one replaces the quark current by its hadronic counterpart one has to include a new parameter ξ of *a priori* unknown size. Naive counting of the color degrees of freedom would lead to $\xi \sim 1/N_c = 1/3$; yet ξ is used here as a free parameter:

$\xi = 0$ means color matching is necessary for forming a hadron, $\xi = 1$ it is not.

One then writes down for the transition *amplitude*

$$
\begin{aligned}
T(D \to f) \propto\ & a_1 \left\langle f \left| (\overline{s}_L \gamma_\mu c_L)_H (\overline{u}_L \gamma_\mu d_L)_H \right| D \right\rangle \\
& + a_2 \left\langle f \left| (\overline{u}_L \gamma_\mu c_L)_H (\overline{s}_L \gamma_\mu d_L)_H \right| D \right\rangle
\end{aligned} \tag{28}
$$

with

$$
\begin{aligned}
a_1 &= \frac{1}{2}\,(c_+ + c_-) + \frac{\xi}{2}\,(c_+ - c_-) \\
a_2 &= \frac{1}{2}\,(c_+ - c_-) + \frac{\xi}{2}\,(c_+ + c_-)
\end{aligned} \tag{29}
$$

where the subscript H in Eq. (28) refers to the use of *hadronic* currents carrying the quantum number of the quarks shown.

(iv) For the final states $f = PP$, PV, VV one employs—and this is a crucial assumption—a factorization ansatz

$$\langle f \,|\, J_\mu J_\mu \,|\, D \rangle \simeq \langle P, V \,|\, J_\mu \,|\, 0 \rangle \ \langle P, V \,|\, J_\mu \,|\, D \rangle \ . \tag{30}$$

Although the number of hadronic matrix elements is thus doubled, they become much more tractable:

$$\langle P \,|\, J_\mu \,|\, 0 \rangle = i f_P k_\mu \tag{31}$$

$$\langle V \,|\, J_\mu \,|\, 0 \rangle = i f_V m_V \epsilon_\mu^V \tag{32}$$

$$\langle P \mid J_\mu \mid D \rangle = \left(k^D + k^P - \frac{m_D^2 - m_P^2}{q^2}\, q \right)_\mu F_1(q^2) +$$

$$+ \frac{m_D^2 - m_P^2}{q^2}\, q_\mu F_0(q^2)$$

$$q = k^D - k^P, \quad F_1(0) = F_0(0) \tag{33}$$

$$\langle V \mid J_\mu \mid D \rangle = \frac{2}{m_D + m - V}\, \epsilon_{\mu\alpha\beta\gamma}\, \epsilon_\alpha^V k_\beta^D k_\gamma^V V(q^2)$$

$$+ i \Big\{ \epsilon_\mu^V (m_D + m_V) A_1(q^2)$$

$$- \frac{\epsilon^V \cdot q}{m_D + m_V}\, (k^D + k_V)_\mu A_2(q^2)$$

$$- 2\, \frac{\epsilon^V \cdot q}{q^2}\, m_V q_\mu A(q^2) \Big\} + 2i\, \frac{\epsilon^V \cdot q}{q^2}\, m_V q_\mu A_0(q^2)$$

$$A_3(q^2) = \frac{m_D + m_V}{2m_V}\, A_1(q^2) - \frac{m_D - m_V}{2m_V}\, A_2(q^2)$$

$$A_3(0) = A_0(0) \tag{34}$$

ϵ^V denotes the spin vector of V.

The q^2 dependence of the various formfactors $F_i, A_i(q^2)$ is assumed to be dominated by the nearest pole. E.g. for $D \rightarrow K$

transitions one uses

$$F_1(q^2) \simeq \frac{h_1}{1 - q^2/m_{F^*}^2} . \tag{35}$$

The residues of the pole term, e.g. h_1, are determined by the overlap integral of the appropriate hadronic wave functions as obtained in a relativistic harmonic oscillator model.

(v) Strong final state interactions (= FSI)—phase shifts, absorption, etc.—are included as best as possible.

A few comments might help in elucidating this lengthy recipe.

- Ingredients (ii)-(v) concern the treatment of matrix elements.
- In principle there are only two free parameters: a_1, a_2. In practice however there exists some considerable leeway since the final state interactions involved are not well known or determined.
- The parameters $c_\pm$ and ξ are *quite different in origin*: $c_\pm$ originates in the renormalization of the effective operator $\mathcal{L}_{\text{eff}}(\Delta C = 1)$ whereas ξ is connected with hadronic matrix elements. Off-setting changes in ξ by varying $c_\pm$ to maintain the same a_1, a_2 runs the risk of being little more than numerology.
- On the justification for ingredient (iv) the following can be said at this point: concerning factorization, try it! On f_π, f_K—take them from experiment, i.e. $f_\pi \simeq 133$ MeV, $f_K \sim 160$-170 MeV. On the residues h_i—they contain the biggest model uncertainties; at least make sure that your model reproduces f_π, f_K correctly.

Having prepared the machinery one proceeds to apply it: there are three types of transitions:

- class I transitions where the amplitude is proportional to a_1 only. these are the D^0 decays leading to two charged mesons: $D^0 \to M_1^{\pm} M_2^{\mp}$.
- class II transitions controlled by a_2: $D^0 \to M_1^0 M_2^0$.
- class III transitions: $D^+ \to M_1^+ M_2^0$. Here the amplitude depends on a linear combination: $a_1 + (1+SB)a_2$. The quantity SB represents the amount of $SU(3)_{\rm FL}$ breaking; in its absence one finds $a_1 + a_2 = (1+\xi)c_+$ - as inferred from the V spin considerations presented in Section 2.

Fitting the (slightly revised) MARK III data on D^0, $D^{\pm}$ branching ratios one finds

$$a_1^{\rm exp} \simeq 1.2 \pm 0.1 \qquad a_2^{\rm exp} \simeq -0.5 \pm 0.1 \; . \tag{36}$$

Comparing these numbers with the expressions obtained when inserting typical QCD values for $c_{\pm}$:

$$a_1^{\rm QCD} \simeq 1.3 - 0.6\,\xi \qquad a_2^{\rm QCD} \simeq -0.6 + 1.3\,\xi \tag{37}$$

one sees that the experimental numbers are well reproduced for $\xi \simeq 0$! In particular class II transitions depend very sensitively on the value of ξ. For example for $\xi = 1/3$ one gets $a_2^{\rm QCD} \simeq -0.2 \neq a_2^{\rm exp}$. Another useful relation is provided by

$$(a_1 + a_2)^2 (a_1 - a_2) = (1-\xi^2)(1+\xi)\, c_+^2 c_- \simeq (1-\xi^2)(1+\xi) \tag{38}$$

where we have used $c_+^2 c_- = 1$ which is an identity on the leading log level[1] and still holds within a few percent on the two-loop level.

Before commenting on the quality of the fit to the data, one apparently technical, though important comment, has to be made. Considerable care has to be applied when extracting the coefficient a_1, a_2 from the data. For naively one would conclude if $\xi = 0$

$$\frac{\Gamma(D^0 \to \overline{K}^0 \pi^0)}{\Gamma(D^0 \to K^- \pi^+)} \simeq \frac{1}{2} \left(\frac{c_+ - c_-}{c_+ + c_-}\right)^2 \sim 0.1$$

which is by a factor of ~ 4 too small compared to the data. This embarrassment is avoided mainly by the intervention of FSI. The three $D \to K\pi$ amplitudes can be expressed in terms of two isospin amplitudes with isospin 1/2 and 3/2 in the final state:

$$\begin{aligned} A(D^0 \to K^- \pi^+) &= \frac{1}{\sqrt{3}} \left(\sqrt{2}\, A_{1/2} + A_{3/2}\right) \\ A(D^0 \to \overline{K}^0 \pi^0) &= \frac{1}{\sqrt{3}} \left(-A_{1/2} + \sqrt{2}\, A_{3/2}\right) \qquad (39) \\ A(D^+ \to \overline{K}^0 \pi^+) &= \sqrt{3}\, A_{3/2} \ . \end{aligned}$$

MARK III measurements yield for the phase shift between the two isospin amplitudes

$$\delta_{1/2} - \delta_{3/2} \simeq 77^\circ \ .$$

This is a very reasonable value for FSI.[12] Including this effect (see prescription (v) above) allows one to increase $\Gamma(D^0 \to \overline{K}^0 \pi^0)/\Gamma(D^0 \to K^- \pi^+)$ from 0.1-0.2 to the experimental number while still using values for a_1, a_2 as given in Eq. (36).

A detailed representation of the data is given in D. Hitlin's lectures and does not have to be repeated here. Instead we will elaborate on the quality of the Bauer-Stech fit:

(i) Their description of some 20-odd D^0, D^+ decay modes with just two fit parameters a_1, a_2 reproduces the data rather well, though not perfectly. This is certainly impressive even when one keeps in mind that some "poetic license" is created by the way in which one deals with FSI.

(ii) There are some decay modes—in particular $D^0 \to \overline{K}^0 \phi$, $\overline{K}^0 \omega$, $\overline{K}^0 \eta$—where this approach seems to seriously underestimate the experimental values. One can point out that the experimental findings on $D^0 \to \overline{K}^0 \omega$, $\overline{K}^0 \eta$ are far from settled and contain large uncertainties. Yet more importantly one notes that these three "problematic" modes are all class II transitions proportional to $(a_2)^2$; i.e. smallish; $BR(D^0 \to \overline{K}^0 \phi) = 0$ is even expected for a one-step decay here since the spectator picture gives $(c\overline{u}) \to s\overline{d}\, u\overline{u} \neq s\overline{d}\, s\overline{s}$. Having realized that it is then plausible to consider two-step processes involving rescattering leading to the same final state. E.g.

$$D^0 \to \text{“}K^- \rho^+\text{”} \to \overline{K}^0 \phi \; . \qquad (40)$$

The important point is that the first step in the reaction chain in (40) is a class I transition, i.e. sizeable, which in this case actually commands a branching ratio of more than 10%. Therefore even a small rescattering probability can lead to an overall branching ratio of 1-2% in these cases. Thus these two-step processes might even be the dominant source for $D^0 \to \overline{K}^0 \phi$, $\overline{K}^0 \eta$, $\overline{K}^0 \omega$. In particular the mode $D^0 \to \overline{K}^0 \phi$ has attracted considerable attentions since WA can generate it in a one-step process[14]

$$D^0 = (c\overline{u}g) \to s\overline{d}g \to s\overline{s}\, s\overline{d} \; . \qquad (41)$$

It might appear academic at this point yet it should be noted nevertheless that the WA process in (41), shown by Fig. 3a, describes a very different dynamical scenario than the rescattering reaction of (40), shown diagrammatically in Fig. 3b which could be called strong annihilation. In WA the $c\overline{u}$ quarks annihilate due to the weak forces. The latter are however pointlike, the $c\overline{u}$ quarks are therefore required to overlap; a measure of the probability for this to occur is provided by f_D. The WA rate is thus—as stated before—suppressed by $(f_D/m_D)^2$. In strong annihilation on the other hand it is the strong forces that drive rescattering or produce the $u\overline{u}$ annihilation. Their range is of order one Fermi—precisely like the typical $c\overline{u}$ separation. Therefore no suppression due to the finite range of the forces ensues—in contrast to WA. The two processes are thus clearly different in principle although in practice it represents a difficult task to disentangle the two.

Semi-leptonic $D \to \ell\nu K, K^*$ decays are treated in the same way and actually with more ease, since only one hadronic matrix element enters:
$\langle K, K^* \,|\, (\bar{c}_L \gamma_\mu s_L)_H \,|\, D^0 \rangle$. One then finds:[10]

$$\Gamma(D \to \ell\nu\, K) \simeq 8 \times 10^{10}\, \mathrm{sec}^{-1} \tag{42}$$

$$\Gamma(D \to \ell\nu\, K^*) \simeq 9.5 \times 10^{10}\, \mathrm{sec}^{-1} \ . \tag{43}$$

Let us summarize the Stech *et al.* approach:

(i) The theoretical prediction for $D \to \ell\nu\, K, K^*$ nearly saturates the observed semi-leptonic width:[3]

$$\begin{aligned} \Gamma(D \to \ell\nu\, K, K^*)_{\text{theoret}} &\sim (17 - 18) \times 10^{10}\, \mathrm{sec}^{-1} \\ \Gamma(D \to \ell\nu\, X)_{\text{exp}} &\sim (18 - 19) \times 10^{10}\, \mathrm{sec}^{-1} \ . \end{aligned} \tag{44}$$

There are MARK III data[3] on $\Gamma(D \to \ell\nu\, K\pi)$ which case some doubt on the success of prediction (43); we will come back to this point in Section 5.

(ii) As explained in more detail in D. Hitlin's lecture the modes $D \to PP$, PV make up a large fraction of all non-leptonic D decays. Most of the remainder could be due to $D \to VV$. Thus the two-body modes seem to dominate non-leptonic D^0, D^+ decays—a surprise, yet a pleasant one: "modesty rewarded".

(iii)

$$\frac{\Gamma(D^0 \to \ell\nu\, K, \ell\nu\, K^*,\, PP,\, PV,\, VV)}{\Gamma(D^+ \to \ell\nu\, K, \ell\nu\, K^*,\, PP,\, PV,\, VV)} \sim 2-3 \tag{45}$$

i.e. the observed lifetime ratio is reproduced due to an interference in two-body non-leptonic D^+ decays that is destructive since $a_1 \cdot a_2 < 0$, see (36).

(iv) The semi-leptonic D^0 branching ratio is reproduced correctly, i.e.

$$\frac{\Gamma(D^0 \to \ell\nu\, K, K^*)}{\Gamma(D^0 \to \ell\nu\, K, \ell\nu\, K^*,\, PP,\, PV,\, VV)} \sim 8\% \,. \tag{46}$$

It should be noted that (iii) and (iv) are achieved without any WA! The "low" D^0 semi-leptonic branching ratio of around 7.5% is thus no unambiguous evidence for substantial WA. Hadronization effects, formfactors, etc.—as exemplified by (46)—lower the semi-leptonic branching ratio from its naive value of 12 [15]% obtained when using the quark-level expression

$$b_{\mathrm{SL(charm)}} \sim \frac{1}{2 + \frac{3}{2}\,(c_+^2 + c_-^2) + \frac{3}{2}\,\xi(c_+^2 - c_-^2)} \tag{47}$$

with $\xi = 0[1/3]$.

Therefore it is fair to state that so far no phenomenological need for WA contributions has been demonstrated in D^0, D^+ decays. FSI on the other hand do play a major role as seen most clearly when analyzing the isospin amplitudes in $D \to K\pi$, $K\rho$, $K^*\pi$: the MARK III data yield for the ratios of isospin 1/2 and 3/2 amplitudes and their phase shifts[3]

$$\begin{aligned} |A_{3/2}/A_{3/2}| &= 3.67 \pm 0.27, \quad 3.22 \pm 0.97; \quad 3.12 \pm 0.40 \\ \delta_{1/2} - \delta_{3/2} &= (77 \pm 11)^\circ, \quad (84 \pm 13)^0, \quad (0 \pm 26)^0 \end{aligned} \tag{48}$$

for the three modes $D \to K\pi$, $K^*\pi$, $K\rho$ respectively. Another observation can be made here: if WA played the major role in explaining the $D^+ - D^0$ lifetime difference due to the presence of a K' resonance one would not expect a nearly universal value for $|A_{1/2}/A_{3/2}|$ as indicated by the data in (48). For

$$D^0 \to \text{“}K'\text{”} \to PV$$

can proceed whereas

$$D^0 \to \text{“}K'\text{”} \to PP$$

cannot. Secondly, WA contributes to $A_{1/2}$ only. Yet keeping the error bars in mind one cannot draw really quantitative conclusions from this at present. More precise data would be quite desirable.

Since the data on D^+, D^0 decays do not exhibit any clear need for WA and since WA does not represent an intellectually lucid concept one might be tempted to forget about it all together. This would however be ill-advised.

(i) The fact that WA has not been established as a major source of D decays does not imply that it is insignificant there. If WA were the dominant generator for $\tau(D^+)/\tau(D^0) > 1$ it could conceivably—though personally I am rather skeptical about it—lead to $\tau(B^+)/\tau(B^0) \sim 1.5$ and could well open up new avenues for CP studies in B decays.

(ii) WA though not dominant could still be significant, say on the 20% level. If this were not due to the lucky presence of an appropriate resonance, it would reveal—as discussed in Section 2—the presence of gluons in the D meson wavefunction thus bringing them one step closer to a role in spectroscopy. Although this would not be a truly surprising result, it would be a nice one nevertheless.

(iii) Even if WA were rather unimportant for the overall D decay rates it could have a significant impact on some rare modes, like once and twice Cabibbo suppressed modes. This would affect our conclusion on the role of Penguins (see Section 5) and $D^0 - \overline{D}^0$ mixing (see Section 6).

(iv) WA could be much more significant in charm baryon than in charm meson decays.

In summary: the Bauer-Stech ansatz represents a considerable step forward in understanding charm decays. It is rather successful and its very success shows that there is a fairly simple dynamical pattern underlying most of charm decays. This realization should however not lead to complacency. The ansatz is based on *ad hoc* assumptions like factorization and valence quark description and on a rather *ad hoc* treatment of FSI. On a more technical level the ansatz is not very successful in Cabibbo suppressed D^0 decays:[2,3]

$$\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} \sim \begin{cases} 1.44 & \text{“theoret”} \\ 3-4 & \text{experim.} \end{cases} \tag{49}$$

A more detailed discussion of this point will be given in Section 5.

There are two ways to progress from here:

(A) One generalizes the Stech *et al.* approach by including WA and Penguin diagrams and determines their relative weight phenomenologically from a comprehensive data analysis.[11]

(B) One starts from a more theoretical treatment of hadron dynamics.

I will choose the second avenue in Section 4—partly because I believe that it will offer more theoretical insight—partly because I am quite skeptical at present that approach (A) allows a satisfactory treatment of FSI.

4. Creeping Towards a Truly Theoretical Description of Charm Decays—The $1/N$ Approach and QCD Sum Rules

That D^0, D^+ decays are rather well described if $\xi \simeq 0$ is used appears somewhat miraculous at this point: if one thinks of ξ as representing the impact of low energy strong interactions—say, soft gluons, etc.—one would hardly expect to find a universal value of ξ; after all the kinematics, etc., are different for the various two–body modes. And a priori ξ could have taken any value of order $\pm$ 1.

The same surprise emerges in $B \to \psi X$ decays which in the language of Bauer–Stech are Class II transitions and thus very sensitive to the value of ξ. Theoretically one predicts[15,16,9] :

$$\mathrm{BR}(B \to \psi X) \simeq 1 - 2\% \cdot (3a_2)^2 \tag{50}$$

$$\mathrm{BR}(B \to \psi K^*) \simeq 4 \cdot 10^{-3} \cdot (3a_2)^2 \tag{51}$$

Comparing Eq. (50, 51) with the data[17]

$$\mathrm{BR}(B \to \psi + X) \sim 1.2\% \tag{52}$$

$$\mathrm{BR}(B \to \psi K^*) \sim (4 \pm 2) \cdot 10^{-3} \tag{53}$$

leads to

$$|a_2^{\mathrm{exp}}| \sim \frac{1}{3} \tag{54}$$

Using short–distance coefficients $c_\pm$ appropriate for $\Delta B = 1$ transitions—$c_+ \sim 0.82, c_- \sim 1.5$—one obtains

$$a_2^{\mathrm{QCD}} \simeq -0.34 \pm 1.7\xi \tag{55}$$

and $\xi \simeq 0$ again emerges as a natural solution—this time for a much more massive hadronic system, the B meson:

$$\xi\ (\text{charm}) \simeq \xi\ (\text{bottom}) \tag{56}$$

An ocurrence like that begs for an explanation and it was Buras et coworkers who first gave one within the framework of the $1/N$ approach where they extended considerably the earlier work by other authors[18] .

A. The $1/N$ Approach

The usual effective Lagrangian as given in Eq. (1) is employed. The matrix element is expanded into a power series in $1/N$, N being the number of colors:

$$< M_1, M_2|\mathcal{L}_{\text{eff}}(\Delta C = 1)|D> = \sqrt{N}\left\{b_0 + \frac{b_1}{N} + O\left(\frac{1}{N^2}\right)\right\} \tag{57}$$

The leading term (in $1/N$) has coefficient b_0, the next–to–leading b_1. We do not concern ourselves with mathematical niceties like whether such a power series is well–defined. The main assumption implicit in Eq. (57) is then the hope that the first term or at worst the first two terms of this expansion already represent a decent approximation to the full (yet unknown) result.

Very simple and almost self–evident rules are established for computing the coefficient in the $1/N$ expansion:

1. Draw all quark level diagrams with two weak vertices that contribute to a decay process; mesons are represented by their valence quarks.

2. Assign a weight N to each closed quark loop, Fig. 4a.

3. Each meson wavefunction, denoted by a cross in Fig. 4b, introduces a normalization factor $1/\sqrt{N}$.

4. Gluon lines are replaced by $q\bar{q}$ lines, see Fig. 4c; the quark–gluon coupling carries a weight $1/\sqrt{N}$.

It is very easy to reproduce the following basic results:

1. There are two types of diagrams that contribute on the leading $1/N$ level, i.e., to b_0, namely one of the spectator- and one of the WA-type, see Fig. 5a,b.

2. There are three types of diagrams on the next–to–leading level contribution to b_1: a spectator diagram, Fig. 6a, a WA (or W–exchange) diagram, Fig. 6b, and diagrams involving FSI, Fig. 6c.

3. For later reference it should be noted that "hair–pin" diagrams[18] give suppressed contributions—not surprisingly, since they represent OZI-forbidden processes. For example the hair–pin diagram in Fig. 7 contributes only on the $1/(N\sqrt{N})$ level, i.e., is suppressed by N^2 in amplitude relative to the leading term.

Two pleasant surprises are immediately read off from these and the corresponding D^+ diagrams:

1. The leading $1/N$ diagrams factorize into the same kind of matrix elements as appear in the Stech et al. ansatz.

2. It is quite evident that the interference in $D^+ \to \bar{K}^0\pi^+$ decays has to be destructive since it is proportional to $(c_+ + c_-)(c_+ - c_-) = c_+^2 - c_-^2 < 0$!

A few typical predictions are listed below and compared with the data on one hand and with the (naive) Standard Approach (i.e., $\xi = \frac{1}{3}$) on the other hand (for a more complete listing see ref. 3,18):

Decay Mode	Standard Approach	Large N Approach	Data
	[Decay widths in units of $10^{10}\,\mathrm{sec}^{-1}$]		
$D^0 \to K^-\pi^+$	14.6	20.1	9.8±1.5
$D^0 \to \bar{K}^0\pi^0$	0.26	4.1	4.4±1.0
$D^0 \to K^-\rho^+$	26.1	36.0	25±3.5
$D^0 \to \bar{K}^0\rho^0$	0.20	3.3	2.3±0.7
$D^0 \to \bar{K}^0\eta$	0.13	2.1	3.5±1.6
$D^0 \to \bar{K}^0\omega$	0.20	3.2	7.4±3.4
$D^0 \to \bar{K}^0\phi$	$\simeq 0$	0.2-0.5	2±1.2
$D^0 \to K^+K^-$	1.4	1.9	1.2±0.3
$D^0 \to \pi^+\pi^-$	1.0	1.4	0.33±0.12

Such a comparison leads to the following conclusions:

1. A leading $1/N$ treatment is not only less disastrous then the naive Standard Approach, but it produces also a decent representation of the data.

2. It is more transparent and thus more "user–friendly" than the Stech et al. ansatz.

3. Yet its fit to the data suffers from some evident deficiencies:

- The ratio $\dfrac{\Gamma(D^0 \to \bar{K}^0\pi^0)}{\Gamma(D^0 \to K^-\pi^+)} \simeq 0.2$ is too small by a factor of roughly two relative to the data.

- $\mathrm{BR}(D^0 \to \bar{K}^0\phi) \sim 0.1 - 0.2\%$ appears too small by a factor of at least five relative to the measured numbers. WA contains a leading $1/N$ contribution and in $D \to$ PV transitions there is no automatic helicity suppression (in contrast to $D \to PP$ modes) yet it is a class II transition, i.e., proportional to $\frac{1}{4}(c_+ - c_-)^2$ and suffers from a hadronic formfactor suppression. (One should keep in mind that this last point does not represent an ironclad argument).
- $\dfrac{\mathrm{BR}(D^0 \to K^+K^-)}{\mathrm{BR}(D^0 \to \pi^+\pi^-)} \simeq 1.4$, i.e., apparently too small again by a factor of two or more.
- The predictions on class I transitions over-shoot the experimental numbers, in particular after the recent MARK III recalibration. Re-scattering processes which have not been included here (see comment 2. below) will decrease this excess; this problem therefore appears more as a technical shortcoming than a fundamental defect.

The $1/N$ approach, as practiced so far, is still not fully satisfactory as a theoretical description:

1. The terms non-leading in $1/N$ are dropped by fiat, not by theoretical reasoning. The $1/N$ approach developed so far represents a nice way to memorize our knowledge on charm decays, but by itself it does not advance our understanding of these phenomena.
2. Self-consistency requires us to ignore FSI; yet there is clear evidence for their significance in the data.
3. Non–leading terms, in particular non-factorizable contributions

like-FSI cannot be calculated in such a simple way.

Yet the relative phenomenological success of this easy and transparent ansatz clearly suggests that the various non–leading terms have - by and large - a strong tendency to cancel each other.

B. QCD Sum Rules

(a) General Procedure:

It has become customary to state that all problems of hadronization will be solved some day once and for all by Monte Carlo simulations on a sufficiently large lattice. I am not so sure however that this will happen before this millennium is over. People who share my relative lack of patience have to employ other, maybe less reliable vehicles to approach hadronization. At present the most promising theoretical technology, in my judgment, is based on the judicious use of sum rules, as pioneered by the ITEP group. We will give here only an outline of the general strategy since a more detailed description goes well beyond the scope of these lectures and can readily be found in the original papers[19] or in the thorough review by Reinders, et al.[20] .

One always starts from a correlation function between two or more (generalized) currents; they are chosen such that they carry the same quantum numbers as the hadrons one wants to study. A well–known example is provided by the reaction $e^+e^- \rightarrow$ charm (at energies well below the Z^0 mass). The appropriate current is a Lorentz vector $j_\mu = \bar{c}\gamma_\mu c$ and one considers the time–ordered two–point function

$$i \int d^4\mathrm{x} e^{iq\cdot x} < 0|T\{j_\mu(\mathrm{x}) j_\nu(0)\}10 > \equiv (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi^V(q^2) \quad (58)$$

Next one invokes analyticity to write a dispersion relation for the polarization function $\Pi(q^2)$. (In general it will contain subtraction terms, which can be removed by applying the operator $\frac{\partial}{\partial q^2}$ a sufficient number of times. Since we will do that anyway for different reasons, we will ignore this complication in the following.)

$$\Pi(q^2) = \frac{q^2}{\pi} \int \frac{\mathrm{Im}\, \Pi(s)}{s(s-q^2)} ds \tag{59}$$

For $e^+e^- \to$ charm there is a very simple representation for $\mathrm{Im}\, \Pi^V(q^2)$ due to the optical theorem:

$$\begin{aligned} \mathrm{Im}\, \Pi^V(q^2) &= \frac{9}{64\pi^2\alpha^2}\, \sigma(e^+e^- \to \text{had.}) \\ &\cong \frac{\pi}{e_q^2} \sum_R \frac{m_R^2}{g_R^2}\, \delta(q^2 - m_R^2) + \\ &+ \frac{1}{4\pi}\left(1 + \frac{\alpha_s}{\pi}\right)\, \theta(q^2 - s_{\text{thresh.}}) \end{aligned} \tag{60}$$

We have made two approximations in the last line where we have expressed the total cross section as a sum over a discrete spectrum of (say charmonium) resonances R with mass [width] m_R [g_R] and a continuum of open flavour production above the threshold (say $s_{\text{thresh.}} > (2m_D)^2$) : firstly we have used a narrow resonance width ansatz and secondly we have inserted an expression for the continuum cross section that goes to first order in α_s only.

The first big theoretical assumption consists in employing an operator product expansion (=OPE) for the time ordered current–

current product:

$$i \int d^4 x e^{q \cdot x} T\{j_\mu(x) j_\nu(0)\} = (g_{\mu\nu} q^2 - q_\mu q_\nu) \cdot$$

$$\cdot \left[c_0(q^2) I + \sum_k c_k(q^2) \mathcal{O}_k \right] \quad (61)$$

where the c_i are the Wilson coefficients, I denotes the identity operator and $\mathcal{O}_i$ the other local, gauge invariant operators. Analyses so far have included five such operators, namely quark and gluon "bilinears" $\mathcal{O}_m = m\bar{q}q$, $\mathcal{O}_G = G^a_{\mu\nu} G^a_{\mu\nu}$ of dimension four, and three manifestly non-bilinear operators of dimension six, namely

$$\mathcal{O}_\Gamma = \bar{q}\gamma_\mu q \bar{q}\gamma_\mu q,$$

$$\mathcal{O}_\sigma = \tilde{m}\bar{q}\sigma_{\mu\nu} \frac{\lambda_a}{2} q G^a_{\mu\nu},$$

$$\mathcal{O}_f = f_{abc} G^a_{\mu\nu} G^b_{\nu\gamma} G^c_{\gamma\mu}.$$

The next step is equally important and unique to this approach: a perturbative treatment gives for all VEV's $< 0|\mathcal{O}_k|0 >= 0$ (apart from $< 0|I|0 >$ of course). Allowing for

$$< 0|\mathcal{O}_k|0 > = f_k \neq 0 \quad (62)$$

therefore incorporates some - and hopefully the most significant - non–perturbative aspects. Thus new unknowns f_k are introduced, like the quark and gluon condensates $< 0|\bar{q}q|0 >$, $< 0|G \cdot G|0 >$;

yet they are process-independent quantities that can - at least in principle - be fitted in one reaction and then used in many other processes.

Putting everything together one arrives at

$$\text{LHS} \simeq \text{M}_n(q_0^2) \equiv \frac{1}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2)\Bigg|_{q=q_0^2} = \frac{1}{\pi}\int ds \frac{\text{Im}\,\Pi(s)}{(s+q_0^2)^{n+1}} =$$

$$\simeq \text{RHS} \tag{63}$$

$$\text{RHS} \simeq \frac{1}{e_q^2}\;\frac{m_R^2}{g_R^2}\;\frac{1}{(m_R^2+q_0^2)^{n+1}}\;[1+\Delta_n(q_0^2)] \tag{64}$$

$$\text{LHS} \simeq \frac{1}{n!}\left(\frac{d}{dq^2}\right)^n \left\{c_0(q^2) + \sum_k c_k(q^2) f_k\right\} \tag{65}$$

where the following comments have to be made:

(a) Eq. (63) represents the duality concept: one equates (or at least approximates) the right-hand side RHS which is expressed in terms of hadronic parameters m_R,g_R, etc., at a somewhat unphysical momentum scale q_0 with the left-hand side LHS calculated on the quark–gluon level where non–perturbative effects are included via the VEV's f_k.

(b) In Eq. (64) we have factored out the lowest lying resonance R whereas the higher lying resonances and the continuum are lumped together into $\Delta_n(q_0^2)$; its leading term is given by

$$\Delta_n(q_0^2) \sim \left(\frac{m_R^2+q_0^2}{m_{R'}^2+q_0^2}\right)^{n+1}, \quad m_{R'} > m_R \tag{66}$$

(c) $< 0|I|0 >= 1$ was used in Eq. (65).

Applying Eq. (63-65) truly resembles a voyage between Scylla and Charybdis: the higher n is chosen, the more sensitive RHS becomes for the resonance parameters M_R, g_R since $\Delta_n(q_0^2) \rightarrow 0$ as $n \rightarrow \infty$; however at the same time the use of OPE in LHS with a *finite* number of operators $\mathcal{O}_k$ becomes more problematic. No clearcut criterion can be given for dealing with this road hazard. One assumes that it is treated in a satisfactory way numerically if one can find a "stability region" in (n, q_0^2) for LHS $\simeq$ RHS; i.e., if LHS and RHS do not vary too dramatically under changes of n and q_0^2. Therefore one has to apply these sum rules *judiciously.* An improvement is often, though not always achieved if a Borel transformation is applied to Eq. (63).

$$\begin{aligned} L_M \Pi(q^2) &\equiv \lim \frac{1}{(n-1)!}(q^2)^n \left(\frac{d}{dq^2}\right)^n \Pi(q^2) \\ &= \frac{1}{\pi M^2} \int ds e^{-s/M^2} Im \ \Pi(s) \end{aligned} \tag{63a}$$

as $n, q^2 \rightarrow \infty$ with $\frac{q^2}{n} = \mathrm{M}^2$ kept fixed.

The general procedure can thus be summarized briefly:

1. Pick the appropriate current for your problem; for pseudoscalar Goldstone bosons one even has a choice, namely to use either $\bar{q}\gamma_5 q$ a là Reinders et al. or $\bar{q}\gamma_\mu\gamma_5 q$ a là Shifman et al.
2. Write down the relevant n-point function in these currents.
3. Compute the Wilson coefficients (as many as possible/necessary) in the resulting OPE.
4. Obtain the significant VEV's from previous fits to other measured quantities.

5. Find the "stability regime" in LHS $\simeq$ RHS and extract numbers.

(b) Applications, part I: Some Couplings and Masses

Two point functions of currents carrying open charm (like $\bar{c}\gamma_5 q$, $\bar{c}\gamma_\mu\gamma_5 q$ or $\bar{c}\gamma_\mu q$) allow us to obtain an estimate of f_D once the experimental m_D is used. The authors of Ref. 21 find

$$f_D(\text{QCD SR}) \sim 150 - 180\ MeV \tag{67}$$

in rough agreement with other theoretical estimates[22,23]

$$f_D \sim \begin{cases} 150 - 230\ MeV & \text{potential models} \\ 100 - 170\ MeV & \text{MIT bag models} \end{cases} . \tag{68}$$

Similarly one finds for the mass splittings between s-wave, D, and p-wave, D^{**}, states[24]

$$\langle m_{D^{**}} - m_D \rangle \sim 400 - 500\ MeV \tag{69}$$

as in potential models.

(c) Applications, part II: Two-body Decays of Charm mesons

Very recently Blok and Shifman[24,25] have developed a treatment of

$$D \to PP,\ PV \tag{70}$$

decays that is based on QCD sum rules. They analyzed a four-point correlation function between three currents—one carrying the quantum number of D, the other two those of the final state mesons generically referred to as A and B—and the weak Lagrangian $\mathcal{L}_W$:

$$\begin{aligned}\Pi_{\mu\nu}(Q_1, Q_2, q) = \int d^4x \int d^4y \int d^4z\, e^{iQ_2\cdot x + iq\cdot y}\cdot \\ \cdot \langle 0 | \{ j_A(y) j_B(x) j_D(0) \mathcal{L}_W(z) \} | 0 \rangle \ .\end{aligned} \tag{71}$$

This relation can be visualized as follows: one starts from three types of skeleton (or bare) diagrams as shown in fig. 8; fig. 8a represents WA, fig. 8b,c, Spectator processes.

These bare diagrams are then dressed by gluon exchanges, both soft and hard. If these gluon lines stay inside the triangle or the external loop, then the factorization property of the amplitude is not changed. If on the other hand the gluons communicate between the triangle and the external loop the diagram becomes definitely non-factorizable. In addition light quark (u, d, s) lines and soft gluon lines are cut to include the non-perturbative VEV's in the OPE treatment of LHS.

In the OPE Blok and Shifman use the five operators $\mathcal{O}_k$ (in addition to the identity I) that were introduced above after eq. (61). The Wilson coefficients are calculated at $Q_1^2 \sim Q_2^2 \sim q^2 \sim -(1.5\ GeV)^2$, i.e. in the Euclidean region. When extrapolating to the (physical)

Minkowski region to make contact with RHS one has to concern oneself with three types of singularities:

(*i*) a pole at the D mass—the objective of the analysis.

(*ii*) Cuts at higher masses corresponding to excited D mesons and to the $D+\pi\pi$, etc., continuum. Their importance is suppressed by performing a Borel summations a la eq. (63a).

(*iii*) Physical cuts, etc., corresponding to resonances, FSI that affect the final state AB. It is argued—and this might be the gravest theoretical uncertainty—that this duality ansatz when initiated in the Euclidean region possesses no sensitivity to such singularities.

I am not convinced that this rather complex analysis has already reached maturity; it is very likely indeed—judging from past experience—that at least some numbers will change. Nevertheless I want to present some of their numbers, be it only to strongly encourage and stimulate further work in this direction:

Decay Mode	Theoret. BR
$D^0 \to K^-\pi^+$	6.4%
$\overline{K}^0\pi^0$	1.5%
$K^-\rho^+$	15%
$\overline{K}^0\rho^0$	0.8%
$K^{-*}\pi^+$	9 %
$\overline{K}^{0*}\pi^0$	3 %
$\overline{K}^0\omega$	1.5%
$\overline{K}^0\phi$	1.3%

The following points should be especially noted:

(i) The agreement with the "old" MARK III branching ratios is quite good! These "predictions" cannot be confronted immediately with the recalibrated branching ratios: for a few of the experimental data act as input to determine the relative weight of the three diagrams in fig. 8—as is typical for the duality ansatz underlying QCD sum rules.

(ii) Most of the numbers are actually very similar to those obtained in the $1/N$ ansatz. Thus it lends theoretical justification for dropping non-leading $1/N$ terms in most cases.

(iii) This treatment represents a theoretical improvement also in the sense that it does include non-leading terms. For example

$$BR(D^0 \rightarrow \overline{K}^0 \phi) \sim 1.3\% \tag{72}$$

is generated purely from WA—without help from FSI! This implies furthermore that $\tilde{f}_D > f_D$ (see the discussion in Section 2) since $f_D \leq 0.2\ GeV$ in this treatment, Eq. (67).

(iv) WA emerges as a significant though not dominant process contributing roughly 20% to $\Gamma(D^0)$.

After listing all these successes some of the short-comings have to be mentioned as well:

(a) The present treatment does not allow for taking FSI properly into account.

(b) It is then not surprising that the ratio

$$\frac{BR(D^0 \rightarrow \overline{K}^0 \pi^0)}{BR(D^0 \rightarrow K^0 \pi^+)} \sim 0.2$$

turns out to be rather small.

(c) $\Gamma(D^0 \to K^+K^-) \simeq \Gamma(D^0 \to \pi^+\pi^-)$ is found in clear conflict to the data. Yet this equality is largely built into the ansatz from the beginning since $SU(3)_{\rm FL}$ breaking effects are ignored (like $(\langle \bar{s}s \rangle \neq \langle \bar{d}d \rangle)$).

Interlude: An Assessment of Our Theoretical Understanding of Charm Decays

The Stech *et al.* treatment revealed that—by and large—a fairly simple dynamical scenario underlies D^0, D^+ decays.

The $1/N$ approach allows us to express this overall simplicity in a concise manner. Yet—like Feynman diagrams and personal computers before—it does not necessarily deepen our understanding although it certainly facilitates most computations. Still it wets the appetite for a deeper understanding.

It is certainly not clear whether treatments that invoke QCD sum rules can take us to this goal; but I believe that eventually they will take us a good distance along this road.

While the final verdict on the theoretical treatments is therefore still not in, it is nevertheless fair to say that the state of the art in theoretical reasoning has improved quite considerably in the last two years. Looking at simple quark-level diagrams is still useful (and fun)—but only as a starting point for a dynamical analysis, not as the end point!

After having made these statements which might be perceived as a clear symptom of overconfidence, I will attempt to redeem myself and some of my colleagues by adding a few words of caution:

(*i*) The success of our theoretical description is still somewhat unstable. More data and more precise data can jeopardize our contentment.

(*ii*) Success is actually quite elusive so far in some rare D decay modes.

These issues will be addressed in the next section.

5. Present and Future Cross Checks and Lessons

A. F Decays

(i) The first question obviously refers to the F total lifetime: do $\tau(D^0)$ and $\tau(F^+)$ agree to within, say, 10% as suggested by V spin symmetry with moderate breaking? WA on the other hand allows for very substantial V spin breaking. The most recent data give an affirmative answer[3] :

$$\frac{\tau(F^+)}{\tau(D^0)} = 1.0 \pm 0.1 \ . \tag{73}$$

(ii) Semi-leptonic branching ratios are the next most interesting global quantities. V spin symmetry gives

$$\frac{b_{\rm SL}(F^+)}{b_{\rm SL}(D^0)} \simeq \frac{\tau(F^+)}{\tau(D^0)} \simeq 1 \tag{74}$$

whereas WA quite naturally leads to

$$\frac{b_{\rm SL}(F^+)}{b_{\rm SL}(D^0)} \gtrsim \frac{\tau(F^+)}{\tau(F^0)} \ . \tag{75}$$

(iii) There are special decay modes—$F \to \pi$'s—that at first sight appear to be produced by WA only, like $D^0 \to \overline{K}^0 \phi$. Yet the same caveat applies in both cases: FSI can in principle generate such final states via strong annihilation and thus fake WA. Nevertheless the following argument can be made: (a) If WA is assumed to dominate $D^0 \to \overline{K}^0 \phi$, and (b) $\tau(F^+) \simeq \tau(D^0) < \tau(D^+)$ is largely attributed to WA, then one concludes

$$BR(F^+ \to \pi\rho) \, , \quad BR(F^+ \to \pi\omega) \sim 3 - 5\% \tag{76}$$

has to hold. Such branching ratios are then quite similar or even larger than $BR(F^+ \to \pi\phi)$. Very recent E691 and ARGUS data however reveal[3]

$$\frac{BR(F^+ \to \pi^+\rho^0)}{BR(F^+ \to \pi^+\phi)} < 0.08 \qquad (90\%\ C.L.) \tag{77}$$

thus ruling WA out as the dominant motor behind $D^0 \to \overline{K}^0\phi$ or F decays, or both.

There are two loop-holes in this argument. FSI could conceivably (though not very likely) conspire with WA in such a way that the transition $F^+ \to \pi^+\rho^0$ is interfered away. There is another more intriguing and more interesting scenario[13] : if hairpin diagrams were important one would expect to see the mode $F^+ \to \pi^+\omega$ as well as $F^+ \to \pi^+\phi$ (and $F^+ \to \pi^+G$ if a glueball state G were accessible), but not $F^+ \to \pi^+\rho^0$ since the ρ^0 has no isoscalar component! Yet two notes of caution can be added here:

(α) In the $1/N$ approach $F^+ \to \pi^+\omega$ is clearly suppressed relative to $F^+ \to \pi^+\phi$ as explained in Section 4. Furthermore fairly little FSI is expected to intervene in either the $\pi\omega$ or $\pi\phi$ final state. Finding comparable rates, i.e.

$$BR(F \to \pi\omega) \sim BR(F \to \pi\phi) \tag{78}$$

would therefore amount to a very hefty blow to $1/N$ practitioners. Incidentally the same is true for the QCD sum rule approach.

(β) WA could be important in F decays due to the accidental presence of a nearby π-like resonance which then decays strongly, i.e.

$$F^+ \xrightarrow[\text{WA}]{} \text{“}\pi'\text{''} \to f\ . \tag{79}$$

In that case *G parity* would forbid (or at least suppress) decays into an even number of pions while allowing decays into an odd number of pions:

$$F^+ \to \text{``}\pi' \not\to 4\pi \tag{80}$$

$$F^+ \to \text{``}\pi' \to 3\pi \tag{81}$$

Since these theoretical remarks might reflect just theoretical biases, it is important, if not even mandatory to search for $F \to \pi\omega$ as best as possible.

(iv) "What E691 taketh away with one hand, it giveth back with the other!" E691 has presented highly intriguing evidence for $F^+ \to (\pi^+\pi^-\pi^+)_{\mathrm{non-res}}$ [3]

$$\frac{BR(F^+ \to (\pi^+\pi^-\pi^+)_{\mathrm{non-res}})}{BR(F^+ \to \phi\pi^+)} = 0.29 \pm 0.07 \pm 0.05 \; . \tag{82}$$

As emphasized before, it is dangerous to draw firm conclusions from one or two decay modes. Nevertheless it is tempting to invoke (82) as evidence for WA playing a significant, though somewhat suppressed role (on the $\sim 20\%$ level) in F decays. For if (82) were produced via FSI from the Spectator process (or by resonance enhanced WA as discussed above Eq. (79)) why then is there no signal for $F^+ \to \pi^+\rho^0$? Keep in mind that WA is expected—as outlined in Section 2—to contribute more to three- than to two-body decays.

(v) No reliable conclusion on F decays can be drawn before a sizeable number of exclusive decay modes has been measured. More generally one would like to know whether two-body modes are equally important, i.e. dominant in non-leptonic F decays as they are in D^0, D^+ decays. Stech *et al.*[9] predict for $\tau(F) = \tau(D^0)$:

$$BR(D^0 \to PP,\ PV,\ VV) \sim 0.67 \pm 0.10 \tag{83a}$$

$$BR(D^+ \to PP,\ PV,\ VV) \sim 0.56 \pm 0.15 \tag{83b}$$

$$BR(F^+ \to PP,\ PV,\ VV) \sim 0.56 \pm 0.08 \tag{83c}$$

These numbers should be compared to the total non-leptonic decay widths. With $b_{SL}(D^0) \sim 0.075$,$b_{SL}(D^+) \sim 0.17$ and *assuming* $b_{SL}(D^0) = b_{SL}(F^+)$ one concludes from the data

$$BR(D^0 \to non - lept.) \simeq 0.85$$

$$BR(D^+ \to non - lept.) \simeq 0.65$$

$$BR(F^+ \to non - lept.) \simeq 0.85$$

Two-body final states are thus still expected to dominate non-leptonic F decays - yet by a somewhat smaller margin than in D^0/D^+ decays. Ignoring the VV final states one obtains in the Stech ansatz

$$BR(D^0 \to PP,\ PV) \sim 0.44 \tag{84}$$

$$BR(F^+ \to PP,\ PV) \sim 0.31 \tag{85}$$

whereas the Blok-Shifman treatment leads to[25]:

$$BR(D^0 \to PP,\, PV) \sim 0.45 \tag{86}$$

$$BR(F^+ \to PP,\, PV) \sim 0.20\ . \tag{87}$$

Detailed data on F decays and more theoretical work is clearly needed to see whether the apparent difference between (84) and (85) or between (86) and (87) or between (85) and (87) is really significant or just reflects the theoretical uncertainties.

(vi) One can search semi-leptonic F decays for glueball candidates — in particular if (75) were found to hold— and one can look for $F^+ \to p\bar{n}$. These are admittedly "odd-ball" searches, but should be undertaken nevertheless.

B. Charmed Baryon Decays

Since the important points here have been nicely covered in Professor Huang's lecture[26] , I can be extremely brief and make mainly a comment on what *cannot* be learned in charmed baryon decays.

Recently the unfortunate claim has been made that a comparison between $\Gamma(\Lambda_c \to e\nu\Lambda)_{\text{theor.}}$ and $\Gamma(\Lambda_c \to e\nu\Lambda X)_{\text{exp}}$. yields

$$|V(cs)| \sim 1/3 \tag{88}$$

in clear conflict with

$$|V(cs)|_{3\ \text{fam.}} \simeq \cos\theta_c \simeq 0.97\ . \tag{89}$$

If true it would present strikingly clear, though indirect evidence for the existence of a fourth family. Such an analysis rests on three ingredients: (a) experimental data; (b) baryonic form factors; (c) the weak coupling $|V(cs)|$. It is obvious that Eq. (88) is completely inconsistent with all the data on D meson decays since it would lengthen decay times by a factor of 10! Therefore the baryonic form factors employed were probably grossly inadequate, the data used conceivably wrong and quite possibly both happened. The important point is that charm baryon decays allow interesting studies of hadronization effects—but not of weak couplings! Those are much better studied in the decays of charm mesons since they are much easier to produce and find and they also represent a simpler bound state.

Recently there has been a slight increase in the experimental value for $\tau(\Lambda_c)$, yet it is still considerably shorter than $\tau(D^0)$ [3] :

$$\left\langle \frac{\tau(\Lambda_c)}{\tau(D^0)} \right\rangle = 0.44 \begin{array}{l} +0.07 \\ -0.05 \end{array} . \tag{90}$$

This points to an important contribution of WA to Λ_c decays—which, as emphasized before—is not that surprising. The preliminary findings that

$$\Lambda_c \to p\pi K \tag{91}$$

is *not* dominated by K^* or Δ resonances is quite consistent with this picture.

C. Cabibbo Suppressed Decays

As already discussed in Section 2 (above Eq. (27)) color coherence leads to an enhancement of the Cabibbo suppressed modes $D^+ \to K\overline{K} + \pi$'s. One can employ the Stech *et al.* ansatz to translate this general expectation into a specific prediction and compare it to the MARK III data[9,3] :

$$\frac{BR(D^+ \to \overline{K}^0 K^+)}{BR(D^+ \to \overline{K}^0 \pi^+)} \simeq \begin{cases} 0.33 & \text{theoret.} \\ 0.317 \pm 0.086 \pm 0.048 & \text{MARK III} \end{cases} \tag{92}$$

$$\frac{BR(D^+ \to \pi^0 \pi^+)}{BR(D^+ \to \overline{K}^0 \pi^+)} \simeq \begin{cases} 0.04 & \text{theoret.} \\ < 0.15 (90\% \; C.L.) & \text{MARK III .} \end{cases} \tag{93}$$

Equation (92) represents a nice success, both qualitatively and quantitatively whereas more sensitive data are needed for $D^+ \to \pi^0 \pi^+$—a point to which we will return later.

A rather similar pattern seems to hold for $D^+ \to \overline{K}^{0*} K^+$ vs. $D^+ \to \overline{K}^{0*} \pi^+$—yet at present the data are far from conclusive.

The situation is less clear for Cabibbo suppressed D^0 decays. Theoretically there is no obvious, clear reason why such decays should be significantly enhanced like it is in the case of D^+ decays. If resonance enhanced WA were a major source of (Cabibbo allowed) D^0 decays one could entertain the idea that Cabibbo suppressed transition rates are even further decreased due to the absence of an appropriate resonance.

It is actually in these modes that charm decays revealed their first puzzling feature:

$$\frac{BR(D^0 \to \pi^+ \pi^-)}{BR(D^0 \to K^- \pi^+)} = 0.033 \pm 0.010 \pm 0.006 \lesssim tg^2 \theta_c \simeq 0.05 \tag{94}$$

$$\frac{BR(D^0 \to K^- K^+)}{BR(D^0 \to K^- \pi^+)} = 0.122 \pm 0.018 \pm 0.012 > tg^2 \theta_c \; . \tag{95}$$

Defining

$$R \equiv \frac{BR(D^0 \to K^- K^+)}{BR(D^0 \to \pi^- \pi^+)} \tag{96}$$

one might guess in a very naive spectator ansatz

$$R \sim \frac{|V(cs)V(us)|^2}{|V(cd)V(ud)|^2} \times \text{Phase space} \sim \text{Phase space} \lesssim 1$$

where we have used the tight constraints on the KM parameters that hold in a three family ansatz. Yet $SU(3)_{\text{FL}}$ breaking, as exemplified by $f_K > f_\pi$, has to be included and one expects $R > 1$ on quite general grounds. More specifically one finds

$$R \sim \begin{cases} 1.44 & \text{“Stech”} \\ 1.4 & \text{“1/N”} \end{cases} . \tag{97}$$

(As already state, $SU(3)_{\text{FL}}$ breaking has not been incorporated into the QCD sum rule approach yet.) Thus there is a very apparent conflict between theoretical expectations (97) and experimental findings (94,95). On the theoretical side this discrepancy could be fed from three different sources.

(*i*) an underestimate of $SU(3)_{\text{FL}}$ breaking;

(*ii*) large FSI, rescattering, etc.

(*iii*) Penguin operators, which were ignored in "Stech" and in "$1/N$" since they are non-leading.

Source (iii) is clearly the most intriguing one. In addition to the usual Penguin operators there are the so-called "sideways" or "horizontal" Penguins. I will ignore those for now since in a $1/N$ expansion they are even more suppressed ($\sim 1/N\sqrt{N}$) than the ordinary Penguins ($\sim 1/\sqrt{N}$).

Penguin transitions are certainly not a major force in charm decays: they are always Cabibbo suppressed, and, as usual, the Penguin operator commands only a small coefficient in $\mathcal{L}_{\text{eff}}(\Delta C = 1)$. In addition there is the technical problem that Penguin operators are, strictly speaking, not local anymore since the mass of the virtual strange quark is considerably lighter than that of the external charm quark. In summary: no reliable prediction on the strength of Penguin transitions can be made; it is only clear that they are small. Yet it is important to keep two things in mind[27] :

(a) Penguin and Spectator transitions contribute coherently to $D \to K\overline{K}$, $\pi\pi$.

(b) The two processes contribute with the same sign to $D \to K\overline{K}$, but with the opposite sign to $D \to \pi\pi$ because of $|V(cd)| \simeq -\sin\theta_c$!

Including Penguin transitions thus further enhances R and, because of the coherence, their impact is magnified. For example a mere 20% Penguin amplitude raises R from its values in (97) to

$$R(0.2 \text{ Penguin}) \sim 3.2 \ . \tag{98}$$

If that were the resolution to the puzzle posed by (94,95) we would have learned a very important lesson with far-reaching consequences:

- it would be the first positive evidence for Penguins;
- it would have repercussions for estimates on ϵ'/ϵ_K;
- it would strengthen the belief in the relevance of Penguin operators in B decays, with important consequences for rare decay modes and CP asymmetries in them.

Yet before one jumps to such conclusions one has to address more mundane explanations for $R \sim 3-4$.

ad (i):

There are two modes that are not affected by FSI and/or WA: $D^+ \to \overline{K}^0\pi^+$, $\pi^0\pi^+$. Both final states are exotic in the isospin context, i.e. $I(\overline{K}^0\pi^+) = 3/2$, $I(\pi^0\pi^+) = 2$. Thus WA does not contribute at all and FSI are not expected to play a significant role. Therefore

$$\frac{\Gamma(D^+ \to \pi^0\pi^+)}{\Gamma(D^+ \to \overline{K}^0\pi^+)} = \frac{1}{2}\, tg^2\theta_c \times SB \tag{99}$$

where $SB \neq 1$ represents $SU(3)_{\rm FL}$ breaking.

ad (ii)

The decays

$$D^0 \to \overline{K}^0 K^0,\ \pi^0\pi^0 \tag{100}$$

can be produced from the Spectator process via rescattering. WA can generate them, too - in particular also, contrary to claims in the literature, $D^0 \to \overline{K}^0 K^0$ unless $SU(3)_{\rm FL}$ symmetry is obeyed in the hadronization process. Different authors [28] using a different ansatz obtain widely different predictions for $D^0 \to \overline{K}^0 K^0$, $\pi^0\pi^0$. Data are therefore required to settle this rather relevant issue.

D. Semi-leptonic Decays

In the Bauer-Stech-Wirbel description one finds [10]

$$\Gamma(D \to \ell\nu K) \sim (7.6-8.3) \times 10^{10}\,\mathrm{sec}^{-1} \tag{101}$$

$$\Gamma(D \to \ell\nu K^*) \sim (7.7-9.5) \times 10^{10}\,\mathrm{sec}^{-1} \tag{102}$$

Adding them up results in a value that basically saturates the observed semi-leptonic width

$$\Gamma(D \to \ell\nu X) \sim (18.6 \pm 2.1)\ 10^{10}\,\mathrm{sec}^{-1}\ . \tag{103}$$

There is unfortunately one piece of data from MARK III[3] that casts a shadow over this picture: while their measurement of

$$\frac{BR(D \to K\pi e\nu)}{BR(D \to K\pi e\nu) + BR(D \to Ke\nu)} = 0.44\ {}^{+0.08}_{-0.09} \tag{104}$$

is quite consistent with (101,102),

$$\frac{BR(D \to K^* e\nu)}{BR(D \to K\pi e\nu)} = 0.55 \pm 0.13 \tag{105}$$

is not. A confirmation of (105), with smaller error bars, would raise some unpleasant theoretical concerns:

(i) From (104,105) one infers using central values only:

$$\frac{BR(D \to K^* e\nu)}{BR(D \to K\pi)} \sim 0.43 \tag{106}$$

i.e. quite different from (101,102). In the Grinstein *et al.* ansatz[29] it is even harder to accommodate. This does not mean that no model could be constructed that reproduces (106). But it implies that some of the overlap wavefunctions used by Stech *et al.* are inaccurate, in particular for the $D \to V$ matrix element. This would suggest that their predictions for $D \to VV$ modes are not quite reliable.

(ii) Finding a $\sim 20\%$ non-resonant contribution in semi-leptonic D decays is not really surprising. However, it suggests that the corresponding non-resonant contribution to semi-leptonic bottom decays will be even more important—which might or might not be a welcome result.

While $|V(cd)|$ is rather tightly constrained theoretically in a three family ansatz by $\sin\theta_c$, the experimental bounds on it as obtained from deep inelastic scattering are not overly impressive. Semileptonic D decays offer an independent handle on it when one has learned to deal with hadronization[9] :

$$\frac{BR(D \to \ell\nu\pi)}{BR(D \to \ell\nu K)} \sim (0.08 - 0.09)\,\frac{|V(cd)|^2}{0.05} \tag{107}$$

$$\frac{BR(D \to \ell\nu\rho)}{BR(D \to \ell\nu K^*)} \sim (0.07)\,\frac{|V(cd)|^2}{0.05}\,. \tag{108}$$

Additional dynamical information is gained from analyzing the shape of the energy spectrum of the charged leptons. This leads to the following challenge: is it possible to recover $|V(cd)|$ from the endpoint spectrum in $D \to \ell^+\nu X$ or does one have to rely on *exclusive* modes like $D \to \ell\nu\pi$, ρ. The answer to this challenge is of course of high relevance in B decays.

E. Doubly Cabibbo Suppressed D Decays (=DCSD)

Charm decays unlike K decays allow to observe a phenomenon that is quite typical for heavy flavor states—decays that are suppressed by more than one small KM angle:

$$\Gamma(D \to K^+\pi\text{'s}) \propto |V(cd)V^*(us)|^2 \simeq tg^4\theta_c\,. \tag{109}$$

As discussed in some detail in Section 6, the question of the real strength of DCSD is a crucial topic in searches for $D^0 - \overline{D}^0$ mixing. Here I want to raise another issue: looking for DCSD can be viewed as a search for New Physics. For Standard Physics, i.e. W exchange is highly suppressed in amplitude:

$$A(c \underset{\mathrm{W}}{\longrightarrow} d\bar{s}u) \propto tg^2\theta_c \ .$$

If there were New Physics mediated by charged Higgs fields for example one would guesstimate

$$A(c \underset{\mathrm{H}}{\longrightarrow} s\bar{d}u) \propto m_c m_d \qquad A(c \underset{\mathrm{H}}{\longrightarrow} d\bar{s}u) \propto m_c m_s \ .$$

Thus the "signal", i.e. New Physics, to "noise", i.e. Old Physics, ratio might quite possibly be greatly enhanced in DCSD, say by a factor $(m_s/m_d)^2/tg^4\theta_c \sim 4 \times 10^4$!

F. Applications in B Decays

(i) The value of $\tau(B^\pm)/\tau(B^0)$ or $b_{\rm SL}(B^\pm)/b_{\rm SL}(B^0)$ is both of intrinsic interest and serves as a crucial input parameter for many studies like $B^0 - \overline{B}^0$ mixing, etc. From our present understanding of $\tau(D^+)/\tau(D^0)$ I infer, quite conservatively I believe:

$$\frac{\tau(B^+)}{\tau(B^0)} \lesssim 1.3 \quad \textit{scaling from } \tau(D) \tag{110a}$$

to be compared with the present CLEO findings[17]

$$\frac{\tau(B^+)}{\tau(B^0)} \lesssim 2.0 \tag{110b}$$

An improvement in the experimental bound is still highly desirable.

(ii) Considerable emphasis is sometimes (mis-) placed on comparing the experimental numbers for $b_{\rm SL}(B)$, averaged over $B^\pm, B^0$, which are around 11-12%, with quite naive theoretical guesstimates around 14%. Applying the lessons gained in D decays to B decays one finds that hadronization effects, questions on the proper value of ξ, etc. can quite naturally change $b_{\rm SL}(B)$ from 14% down to 12% or so.

(iii) Two-body modes do not dominate B decays as they did D decays. Nevertheless they are not insignificant and actually quite crucial when searching for rare decays and CP asymmetries[30] .

(iv) On general theoretical grounds one expects[24]

$$f_F > f_D \simeq \frac{3}{2}\, f_B \, . \qquad (111)$$

Any bound on or number for f_F, f_D thus reflects on f_B which is a crucial parameter in describing $B^0 - \overline{B}^0$ mixing. It is amusing to note that the present MARK III upper limit

$$f_D \leq 340\ MeV$$

leads to

$$f_B \lesssim 220\ MeV$$

when invoking (111) which agrees with theoretical upper bounds obtained directly for f_B.

(v) Not too much is known on $|V(ub)|$ for certain beyond the statement that it is considerably smaller than $|V(cb)|$. As discussed in part **D.** detailed studies of semi-leptonic D decays will provide us with valuable insights into the relative merits of inclusive—$B \to \ell\nu X$—versus exclusive—$B \to \ell\nu\pi, \rho$—studies.

(vi) If Penguin transitions were identified in $D^0 \to KK$, $\pi\pi$ decays one would be on firmer ground to predict $BR(B \to K\pi,\, K\rho)$ and CP asymmetries in these modes.

6. Heresies: Exotic Decays, $D^0 - \overline{D}^0$ Mixing and CP Violation

A. Exotic D Decays

MARK III has searched for the decay $D^0 \to \mu e$ which is forbidden in the Standard Model and established an upper limit[3] :

$$BR(D^0 \to \mu e) < 1.5 \times 10^{-4} \qquad (90\% \; C.L.) \; . \tag{112}$$

Searches of this type might appear as a complete waste of time when one remembers that much smaller branching ratios (and even tinier decay widths) can be reached in K decays. Such a statement is overly pessimistic in general: firstly it ignores the possibility that exotic flavor changing currents could be produced by the exchanges of Higgs fields which couple much more strongly to a heavy flavor like charm than to strangeness; furthermore it is conceivable though not guaranteed that exotic flavor changing neutral currents possess much larger mixing angles to up-type quarks like $c\overline{u}$ than to down-type quarks like $s\overline{d}$.[31]

After this general pronouncement it has to be added however that $D^0 \to e\mu$ does not provide the most promising field to search for New Physics. This becomes obvious by comparing $D^0 \to e\mu$ to $D^+ \to \mu^+\nu_\mu$:

$$BR(D^0 \to e\mu) < 1.5 \times 10^{-4} \tag{113}$$

$$\Gamma(D^0 \to e\mu) \lesssim 3.4 \times 10^{8} \, \mathrm{sec}^{-1} \tag{114}$$

$$BR(D^+ \to \mu^+\nu) < 8.4 \times 10^{-4} \tag{115}$$

$$\Gamma(D^+ \to \mu^+\nu) \lesssim 8.4 \times 10^8 \, \mathrm{sec}^{-1} \ . \tag{116}$$

$D^+ \to \mu^+\nu$ is a Standard Model process due to real WA; it is tiny due to the combined effect of helicity suppression $(m_\mu/m_D)^2$ and wavefunction overlap suppression $(f_D/m_D)^2$. For $f_D = 200$ MeV one actually predicts $BR(D^+ \to \mu^+\nu) \simeq 2.9 \times 10^{-4}$.

Unfortunately quite analogous suppression factors enter in $D^0 \to e\mu$: the new interaction has to be local, therefore (f_D/m_D) again enters in the amplitude; if it were due to a spin-one exchange, the same helicity factor (m_μ/m_D) emerges; since a spin-zero coupling would violate chiral invariance one invokes general arguments to conclude that the coupling had then to be proportional to the fermion mass, i.e. m_μ!

Much more promising decay modes are therefore those that do not suffer from this double suppression, namely

$$D \to \pi\nu\overline{\nu}, \ \rho\nu\overline{\nu}, \ \pi\mu e \quad \text{etc.} \tag{117}$$

One might add that quite a few grand unified models that have gained popularity in the last years contain lepto-quarks, i.e. bosons that connect leptons and quarks; they can quite naturally generate the transitions in (117).

A. $D^0 - \overline{D}^0$ Mixing

(a) Phenomenology:

Mixing means that the flavor eigenstate D^0 is not a mass eigenstate; thus its time evolution is not described by a single exponential:

$$\left|D^0(t)\right\rangle = g_+(t)\left|D^0\right\rangle + \frac{q}{p}\, g_-(t)\left|\overline{D}^0\right\rangle$$

$$\frac{p}{q} = \frac{1+\epsilon}{1-\epsilon}; \quad g_\pm(t) = \tfrac{1}{2}\, e^{-\frac{1}{2}\Gamma_1 t}\, e^{im_1 t}\left(1 \pm e^{-\frac{1}{2}\Delta\Gamma} e^{i\Delta m t}\right) \tag{118}$$

with $\Delta\Gamma = \Gamma_2 - \Gamma_1$, $\Delta m = m_2 - m_1$. Γ_i and m_i denote the width and the mass for the two mass eigenstates D_i. The rate for semi-leptonic D^0 decays leading to a "wrong-sign" lepton is then easily found to be

$$\Gamma(D^0(t) \to \ell^- X) \propto \left|\left\langle \ell^- X | D^0(t)\right\rangle\right|^2 \propto \left|\frac{q}{p}\right|^2 e^{-\Gamma t}(1 - \cos\Delta m t) \tag{119}$$

where in the last step we have assumed for simplicity $\Delta\Gamma = 0$—a point we will return to later.

It is this deviation from an exponential time dependence that represents the defining property for mixing.

A fixed target experiment like E691 can measure such a time evolution; in addition, as an extra bonus, D^* decays allow to flavor-tag the produced neutral D meson:

$$D^{+*} \to D^0 \pi^+ \qquad D^{-*} \to \overline{D}^0 \pi^- \, . \tag{120}$$

The most general expression describing $D^0 \to K^+\pi^-$ for small mixing reads[32]

$$\begin{aligned}\Gamma(D^0(t) \to K^+\pi^-) \propto e^{-\Gamma t}\Big\{ &(\Gamma t)^2 (x^2 + y^2) + 4tg^2\theta_c \left|\hat{\rho}_f\right|^2 \\ &+ 4y(\Gamma t)\, tg^2\theta_c\, \mathrm{Re}\,\frac{p}{q}\,\hat{\rho}_f - 4x(\Gamma t)\, tg^2\theta_c\, \mathrm{Im}\,\frac{p}{q}\,\hat{\rho}_f \Big\}\end{aligned} \tag{121}$$

with the definitions

$$x = \frac{\Delta m}{\Gamma} \; ; \quad y = \frac{\Delta \Gamma}{2\Gamma} \; ; \quad \frac{A(D^0 \to K^+\pi^-)}{A(D^0 \to K^-\pi^+)} = tg^2\theta_c \widehat{\rho}_f \; . \qquad (122)$$

The four terms in (122) are easily interpreted: the first one containing t^2 (in addition to the time exponential) is the pure mixing term. The second one with no additional time dependence represents DCSD. The third one linear in t is due to the interference between the amplitude for DCSD and $\Delta\Gamma$ mixing. The fourth one finally which is also linear in t describes interference between DCSD and Δm mixing; it represents a CP asymmetry which is evident from the appearance of $Im \frac{p}{q}\, \widehat{\rho}_f$. This fact can be understood immediately by recalling that $x\Gamma t = \Delta m t$ is just the first term in the expansion of $\sin \Delta m t$—a function that changes sign under $t \to -t$.

E691 has presented a preliminary analysis using *only the first two terms in* (122). They find[3]

$$r_D \simeq \tfrac{1}{2}\left(x^2 + y^2\right) \lesssim 0.5\% \qquad (123)$$

$$\left|\widehat{\rho}_f\right|^2 \simeq \frac{1}{tg^4\theta_c}\,(0.025 \pm 0.014) \simeq 9 \pm 5 \qquad (124)$$

A few comments are in order here:

(i) If CP invariance holds the fourth term in (122) has to vanish; however, the third term is still there in general. If it contributes with a positive sign, it will *lower* the upper bound in (123); if it interferes destructively it will *raise* it. I find the first possibility more likely though not guaranteed: if CP is not violated $\mathrm{Re} \frac{p}{q}\, \widehat{\rho}_f = -|\widehat{\rho}_f|$ holds due to $\epsilon = 0$, $V(cd) \simeq -\sin\theta_c$. Furthermore $\Delta\Gamma = \Gamma(D_-) - \Gamma(D_+)$ follows where $D_+[D_-]$ denotes the even [odd] CP eigenstate. Just counting the number of PP, PV channels available for D_+ versus D_- decays—the argument is not more sophisticated than that—suggests $\Gamma(D_+) > \Gamma(D_-)$ and therefore $y \,\mathrm{Re}\, \frac{p}{q}\, \widehat{\rho}_f > 0$.

(ii) The value for the strength of DCSD in $D \to K\pi$, Eq. (124) appears much too high since a computation based on factorization yields[33]

$$\left|\widehat{\rho}_f\right|^2 \simeq 2 \tag{125}$$

for $f = K\pi$.

Important cross checks can be obtained by studying $D^+(t) \to K^+\pi^-\pi^+$—where there can be no mixing—and $D^0(t) \to K^+\rho^-$; the strength of mixing—x and y—characterizes the decaying D meson and therefore should be equally present or absent in all appropriate decay modes. For DCSD the situation is quite different: for example one estimates[33]

$$\left|\widehat{\rho}_f\right|^2 \sim 0.5 \tag{126}$$

for $f = K\rho$ to be contrasted with (125)!

When one lacks sufficient time resolution—as it happens in $e^+e^- \to \psi'' \to D\overline{D}$ or $e^+e^- \to \Upsilon(4s) \to B\overline{B}$—one can study only time-integrated quantities. Decays of D^0 mesons into wrong-sign leptons are then searched for as a sign for mixing:

$$r_D = \frac{BR(D^0 \to \ell^- X)}{BR(D^0 \to \ell^+ X)} \simeq \left|\frac{q}{p}\right|^2 \frac{x^2+y^2}{2} \tag{127}$$

for $x^2, y^2 \ll 1$ as is appropriate for $D^0 - \overline{D}^0$ mixing.

Two complications arise here that should be kept in mind:

(α) Strictly speaking, $r_D \neq 0$ by itself does not establish mixing, it does so only within the framework on the Standard Model! Remember the old debate on the $\Delta S = \Delta Q$ rule in K physics. When we discuss D^0 decays into "wrong-sign" kaons we will recognize this remark as not purely academic.

(β) In e^+e^- annihilation or hadronic collisions there are always charm-anticharm pairs produced.

To deal with complication (β) one employs correlations between the $D\overline{D}$, etc., decays; as we will see this takes care also of complication (α), at least in principle. The simplest correlation is that between the charge of leptons originating in semi-leptonic charm decays:

$$y = \frac{N(\ell^\pm \ell^\pm X)}{N(\ell^+ \ell^- X)} = \begin{cases} r_D & p \text{ wave} \\ \frac{2r_D}{1+r_D^2} & s \,\&\, p \text{ wave} \end{cases} \tag{128}$$

where $N(\ell\ell X)$ denotes the number of direct leptons of a given charge in $D^0\overline{D}^0 \to \ell\ell X$ with the $D^0\overline{D}^0$ pair in a relative p wave—like in $\psi'' \to D\overline{D}$—or in a configuration where s and p waves contribute equally—like a GeV or more above threshold.

It is intuitively clear why the ratio is suppressed in a p wave: Bose statistics tells us that one cannot have a $D^0(t_1)D^0(t_2)$ state in a p wave for $t_1 = t_2$.

The whole complexity of this program comes into play in a theoretical analysis of the MARK III study on[3]

$$e^+e^- \to \psi'' \to D^0\overline{D}^0 \to K^\pm K^\pm + \pi\text{'s} \,. \tag{129}$$

Most importantly one has to note that even within the Standard Model there is no strict $\Delta C = \Delta S$ rule; DCSD produce $\Delta C = -\Delta S$

transitions. Comparing the rates for $D^0\overline{D}^0$ being in a p wave or in an s wave configuration will allow us to disentangle the two effects[33,34]

$$\frac{N(K^\pm\pi^\mp, K^\pm\pi^\mp)}{N(K^\pm\pi^\mp, K^\mp\pi^\pm)} \simeq \begin{cases} \frac{1}{2}\left(x^2+y^2\right) & (p), \\ \frac{3}{2}\left(x^2+y^2\right) + 4tg^2\theta_c\left|\widehat{\rho}\right|^2 + 8ytg^2\theta_c\widehat{\rho}_f & (s). \end{cases} \tag{130}$$

It is important to note that without mixing the reaction

$$e^+e^- \to \psi'' \to D^0\overline{D}^0 \to (K^\pm\pi^\mp)(K^\pm\pi^\mp)$$

is forbidden by Bose statistics! The same can be shown to hold for $\psi'' \to D^0\overline{D}^0 \to (K^\pm\rho^\mp)(K^\pm\rho^\mp)$, $(K^{*\pm}\pi^\mp)(K^{*\pm}\pi^\mp)$, but not for $(K^*\rho)(K^*\rho)$ or $(K\pi\pi)_{\text{non-res}}(K\pi\pi)_{\text{non-res}}$. The large width of the ρ meson does not invalidate these results in principle, but of course dilutes their power considerably in practice.

Equation (130) also does not apply when $D^0\overline{D}^0$ decay into two different final states like $D\overline{D} \to (K\pi)(K\rho)$. There one finds when mixing is ignored, i.e. $x = y = 0$[33] :

$$\frac{N(K^\pm\pi^\mp,\, K^\pm\rho^\mp)}{N(K^\pm\pi^\mp,\, K^\mp\rho^\pm)} = \left|\widehat{\rho}_{K\pi} - \widehat{\rho}_{K\rho}\right|^2 tg^4\theta_c\ . \tag{131}$$

It does not vanish unless all DCSD had the same universal suppression factor $\widehat{\rho}tg^2\theta_c$. This is however very unlikely to hold. A computation involving factorizable contributions actually yields:

$$\widehat{\rho}_{K\pi} \simeq -1.4 \qquad \widehat{\rho}_{K\rho} \simeq -0.7\ . \tag{132}$$

This is another example of how dangerous it is to argue purely on the quark level and ignore modifications imposed by hadronization.

A last remark on future findings: comparing E691 and MARK III data and improving them might lead to the conclusion that there is no sign for $D^0 - \overline{D}^0$ mixing, say $r_D < 0.5\%$. At the same time we might find, say

$$\widehat{\rho}_{K\pi} \sim 3 \qquad \widehat{\rho}_{K\rho} \sim 2$$

i.e. considerably larger values than expected theoretically. As discussed in Section 5.E this might still signal the presence of New Physics.

(b) Standard Model Estimates:

On very general grounds one expects $D^0 - \overline{D}^0$ mixing to be small:

- $D^0 \to \overline{D}^0$ transitions are suppressed by $\sin^2\theta_c$; that is true also for $K^0 \to \overline{K}^0$ and $B^0 \to \overline{B}^0$ - however regular D decays, in contrast to K and B decays, are not suppressed by a small KM angle.
- Due to the GIM mechanism $D^0 \to \overline{D}^0$ transitions can proceed only due to $SU(3)_{\rm FL}$ breaking. $K^0 \to \overline{K}^0$ $[B^0 \to \overline{B}^0]$ is driven by the much larger $SU(4)_{\rm FL}$ $[(SU(6)_{\rm FL}]$ breaking.

The remaining question is only "how small is small"? Like in the case of $K^0 - \overline{K}^0$ mixing there are two types of contributions to Δm_D, one generated by the simple quark-box diagram and one due to long-distance physics:

$$\Delta m_D = \Delta m_{\rm D,\,box} + \Delta m_{\rm D,\,L.D.}\,. \qquad (133)$$

It has been recognized for a long time that $\Delta m_{\rm D,\,box}$ is very tiny, a major reason being that $SU(3)_{\rm FL}$ breaking enters via the ratio $(m_s^2 - m_d^2)/M_W^2 \ll 1$.

The situation is much less clear-cut for $\Delta m_{D,L.D.}$. One finds that there are at least 12 diagrams contributing with different signs, that soft gluons enter, etc. It is clearly hopeless to attempt a purely theoretical calculation.

Instead of giving up one can employ more phenomenological prescriptions to arrive at an estimate at least[33,35] . Let us consider

$$D^0 \to \text{``}PP \;\to \overline{D}^0$$

i.e. transitions mediated by a pair of two in general virtual pseudoscalar mesons $P = K, \pi$. This amplitude contains four parts representing the different intermediate states:

$$A(D^0 \to \overline{D}^0) = \sin^2\theta_c \left\{[K^+K^-] + [\pi^+\pi^-] - [\pi^+K^-] - [K^+\pi^-]\right\} \tag{134}$$

the K^+K^- $[\pi^+\pi^-]$ pair couples to both D^0 and $\overline{D}^0$ with strength $\sin\theta_c[-\sin\theta_c]$; $K^+\pi^-[\pi^+K^-]$ on the other hand couples to D^0 with $-\sin^2\theta_c[\cos^2\theta_c]$ and to $\overline{D}^0$ with $\cos^2\theta_c[-\sin^2\theta_c]$.

In the limit of $SU(3)_{\mathrm{FL}}$ symmetry the amplitude in (134) vanishes, yet remember

$$\frac{BR(D^0 \to K^+K^-)}{BR(D^0 \to \pi^+\pi^-)} \sim 3 - 4$$

as evidence for large symmetry breaking in exactly these modes!

To obtain a rough guesstimate one proceeds as follows: starting from the experimental numbers $BR(D^0 \to K^+K^-) \sim 0.6\%$, $BR(D^0 \to \pi^+\pi^-) \sim 0.2\%$ to which one adds 50% to include the

$K^0\overline{K}^0$ and $\pi^0\pi^0$ modes one arrives at

$$BR(D_+ \to K\overline{K} + \pi\pi^-) \simeq 2BR(D^0 \to K\overline{K} + \pi\overline{\pi}) \sim 0.025$$

(with $|D_+\rangle = CP\,|D_+\rangle$) where we have used the coherent nature of mixing. Thus very roughly

$$\Delta m_D \sim 0.03\,\Gamma_D \qquad \Delta\Gamma_D \sim 0.03\,\Gamma_D \qquad r_D \sim \mathcal{O}(10^{-3})\ . \qquad (135)$$

Needless to say the real value of r_D due to long distance dynamics could be considerably smaller. The point of this exercise was to show that values like those given in (135) are not clearly ruled out in the Standard Model.

There is a well-known strategy for improving the quality of our estimate:

- derive a dispersion relation;
- evaluate it using measured $\pi\pi$, $K\overline{K}$, $K\pi$ phase shifts.

However the actual execution of this program is rather non-trivial and is therefore unlikely to be undertaken unless $D^0 - \overline{D}^0$ mixing is found on the $r_D \sim 10^{-3}$ level.

To sum up: the Standard Model might allow for

$$x_D \sim 0.03\,, \quad y_D \sim 0.03\,, \quad r_D \sim 10^{-3}$$

but *not* for $r_D \sim 10^{-2}$.

C. New Physics Scenarios

It is very hard to see how new quarks, etc., that are then included in the box contribution can push r_D appreciably beyond the 10^{-3}

level. The situation changes if there are genuine flavor changing neutral currents on the tree level as one can encounter them in non-minimal Higgs models. It is quite possible to construct models where these exotic currents are mediated by the exchange of Higgs scalars with a mass of around a few TeV in such a way that they are not clearly identifiable in Δm_K, but produce

$$x_D \sim \mathcal{O}(0.1)\,, \quad y_D \lesssim 0.03\,. \tag{136}$$

Such models contain quite naturally CP violation in their Higgs sector that appears superweak in K decays, but is much stronger in heavy flavor decays.

D. CP Violation in Charm Decays

Observing CP violation in charm decays would be fascinating per se and would at the same time establish the presence of New Physics. The good news is that there are theoretical scenarios where D^0 decays could exhibit CP asymmetries of up to 10%, i.e. much larger than in K^0 decays. The bad news is that even under such favorable circumstances the experimental searches will be extremely challenging.

(a) D^0 decays:

the most promising decay modes, as we will see, are

$$D^0 \to K_S K^+ K^-,\; K^+ K^- \tag{137}$$

each commanding a branching ratio of roughly 0.5%. They are special in this context since these final states represent CP eigenstates. The time evolution of their transition rates takes a very interesting form

$$\Gamma(D^0(t) \to f) \simeq e^{-\Gamma t}\left(1 - \sin \Delta m t \,\mathrm{Im}\,\frac{p}{q}\rho_f\right) \qquad (138)$$

$$\Gamma(\overline{D}^0(t) \to f) \simeq e^{-\Gamma t}\left(1 + \sin \Delta m t \,\mathrm{Im}\,\frac{p}{q}\rho_f\right) \qquad (139)$$

with

$$\rho_f = \frac{A(\overline{D}^0 \to f)}{A(D^0 \to f)}, \quad f = K^+K^-, \quad K_sK^+K^- .$$

We have assumed $\Delta\Gamma = 0$ for simplicity. It is easy to show that either relation (138) or (139) establishes CP violation. Considering just (138) for small mixing, i.e. $\Delta m \ll \Gamma$ one gets

$$\Gamma(D^0(t) \to f) \simeq e^{-\Gamma t}\left(1 - (\Gamma t)\ x\mathrm{Im}\,\frac{p}{q}\rho_f\right) . \qquad (140)$$

$D^0 - \overline{D}^0$ mixing somewhat below the 1% level, say $r_D \simeq 5 \times 10^{-3}$ implies $x \simeq 0.1$. Such a mixing strength requires the presence of New Physics as discussed above which quite naturally leads to CP violation as well; thus $Im\,\frac{p}{q}\ \rho_f \sim 0.5$ could hold and

$$\Gamma(D^0(t) \to f) \simeq e^{-\Gamma t}(1 - 0.05(\Gamma t)) \qquad (141)$$

is not a ludicrous scenario. For $t \sim 2\tau_{D^0}$ this represents a 10% CP asymmetry. Furthermore, the time dependence is rather peculiar and not easily faked by backgrounds. Being able to resolve the time evolution is therefore highly desirable, though not essential in general.

An E691 and ARGUS type experiment is well suited for such an analysis:

(α) It has sufficient time resolution.

(β) It can employ the D^* trick to distinguish between D^0 and $\overline{D}^0$ decays. The final states obviously do not allow this distinction and summing over D^0 and $\overline{D}^0$ decays, see Eqs. (138) and (139), washes the effect out.

The situation is less favorable in this respect when one measures D decays just above production threshold.

(α) Since one cannot resolve the time evolution one has to rely on correlations like

$$D^0\overline{D}^0 \to (\ell^+/K^+ + X^-) + f \quad \text{vs.} \quad D^0\overline{D}^0 \to (\ell^-/K^- + X^+) + f \tag{142}$$

(β) Unfortunately these correlations cannot show a CP asymmetry in $\psi'' \to D^0\overline{D}^0$, i.e. when $D^0\overline{D}^0$ are produced in a p wave[30]. A CP asymmetry can become observable if $D^0\overline{D}^0$ form an s wave state as in, e.g.

$$e^+e^- \to D\overline{D}^* + \text{h.c.} \to D^0\overline{D}^0\gamma \; . \tag{143}$$

(b) $\psi'' \to D^0\overline{D}^0$:

If both neutral D mesons were seen to decay into a CP eigenstate of the same CP parity, e.g.

$$\psi'' \to D^0\overline{D}^0 \to (K^+K^-)_D(K^+K^-)_D \tag{144}$$

then CP violation would have been established. For the initial state is CP even whereas the final state (p wave!) is CP odd. Note that we are talking about a rate here and *not* a difference between two CP conjugate rates.

There is of course, as always, a drawback to this method as apparent from a different representation of the reaction chain in (144):

$$\psi'' \to D_+ D_- \underset{\text{Mix.}}{\longrightarrow} D_+ D_+ \to (K^+K^-)_D (K^+K^-)_D \ .$$

The rate (in almost all cases) requires $D^0 - \overline{D}^0$ mixing to have occurred and therefore very roughly

$$\text{relative rate} \sim x^2 BR(D \to f_1)\ BR(D \to f_2) \qquad (145)$$

where f_1, f_2 are CP eigenstates of the same CP parity. If one can employ only one such decay mode with a branching ratio of 1% one estimates a relative rate of at best 10^{-6} for $r_D \simeq 0.5\%$. Yet if one could sum over many appropriate channels like $D^0 \to K_S\omega$, $K_S\eta$, etc., one might obtain—very optimistically—up to 10% for each of the two branching ratios in (145) leading to a relative rate of $\sim 10^{-4}$!

(c) D^+, F^+, Λ_c^+ decays:

For CP asymmetries to become observable one needs the intervention of non-trivial FSI to generate different phase shifts for different isospin amplitudes. That (remember the discussion in Section 3, etc.) does not pose any problem. Secondly, one needs sizeable violations of weak universality; I consider that rather unlikely, but not impossible to happen. Therefore one should make an effort to compare as many CP conjugate rates as possible. For more detailled discussions, see the papers by L.-L. Chau and co-workers.

7. Summary and Outlook

Weak decays of kaons have been studied experimentally as well as theoretically for more than 30 years now. A pessimist might point out that despite all the efforts spent we still have not come up with an explanation for a basic and striking phenomenon, the $\Delta I = \frac{1}{2}$ rule which produced huge decay rate differences: $\tau(K^+)/\tau(K_s) \sim 135$!

Charm decays have been studied for a considerably shorter time span; namely, 10 to at most 15 years. It would be unjustified to claim that we understand charm decays fully; nevertheless I clearly believe that we have developed a decent overall understanding of charm decays which is actually considerably better than that in strange decays (including hyperon decays). The reason for that does not lie in a sudden jump in the intellectual power of theorists working in this field—after all, the $\Delta I = \frac{1}{2}$ rule has not been explained yet —but in the concurrence of three factors:

(*i*) Nature was apparently kind enough to provide us with a relatively simple dynamical system. See for example the two-body dominance in D decays.

(*ii*) Experimentalists worked hard enough to provide us with good, *comprehensive* data.

(*iii*) There was and still is a lively feed-back between theorists and experimentalists.

Nevertheless there cannot be any space for complacency. The basis for the statement that we have obtained a very decent understanding of charm decays is still unstable: a two sigma change in the data here and a two sigma change there and soon we would be in real trouble.

As far as experimental capabilities are concerned, charm physics is a mature field. As far as its theory is concerned, we are still in late adolescence. It is therefore both possible and very important

(a) to secure and widen the basis on which we place our claim of success

- by obtaining more precise data on D decays like $D \to VV$ modes or $D^0 \to \overline{K}^0 \omega, \eta, \eta'$
- by obtaining more data on F and charm baryon decays (semi-leptonic and non-leptonic branching ratios).

(b) to probe deeper by more detailed data on

- Cabibbo suppressed modes like $D^0 \to \overline{K}^0 K^0, \pi^0\pi^0, D^+ \to \pi^0\pi^+, D \to \ell\nu\pi, \rho$
- doubly Cabibbo suppressed decay like $D^+ \to K^+\pi^-\pi^+$.

The important lesson to remember from the MARK III experience is that just a few even well measured branching ratios are not sufficient to subject theoretical treatments to sensitive tests, that a comprehensive analysis involving many different channels is crucial.

Finally we should not let orthodoxy blind us for the potential for still making major new discoveries in charm decays like $D^0 - \overline{D}^0$ mixing, exotic rare decays or even, most ambitiously, CP violation. There are two historical lessons from K decays of importance here. Firstly

(*i*) Parity violation first exhibited itself in K decays;

(*ii*) $K^0 - \overline{K}^0$ mixing was discovered leading to postulating charm;

(*iii*) CP violation which showed, among many other things, the need for top and bottom quarks.

All of this represented New Physics at that time.

Secondly, the time scale over which real intellectual progress develops in this kind of physics is not one or two years—it is one or two decades!

The future of charm physics is therefore still full of promise - yet it can be realized only after very hard and patient work.

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It is a pleasure to thank Profs. Ye and Huang for inviting me to participate in such a stimulating and well organized meeting. I have very much enjoyed the many discussions and contacts I had with my Chinese colleagues. One note of apology might be in order: due to the very tight deadline for submitting the manuscript I was unable to include as extensive a list of references as I would have liked or regarded as appropriate.

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Figure Captions

Fig.1a: Spectator diagram for D^0,D^+ decays.

Fig.1b: WA diagram for D^0 decays.

Fig.2a: Spectator diagram for $D^0 \to K^-\pi^+$.

Fig.2b: Spectator diagram for $D^0 \to K^0\pi^0$.

Fig.3a: WA for $D^0 \to \bar{K}^0\phi$.

Fig.3b: Strong annihilation for $D^0 \to \bar{K}^{0\prime} \to \bar{K}^0\phi$

Fig.4a,b,c: 1/N counting rules.

Fig. 5a,b: Leading 1/N diagrams for $D^0 \to K\pi$; the broken line represents a weak current, either a charged current or a QCD induced neutral current.

Fig.6: Next-to-leading diagrams.

Fig.7: A hairpin diagram.

Fig.8a,b,c: The three skeleton diagrams for $D \to AB$ decays.

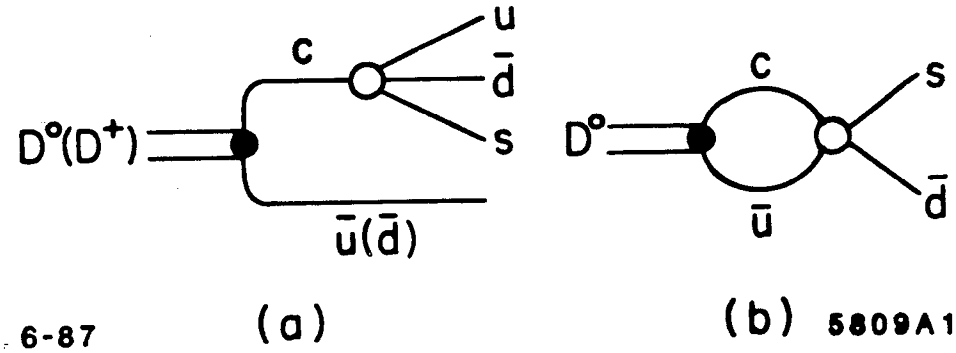


Fig. 1

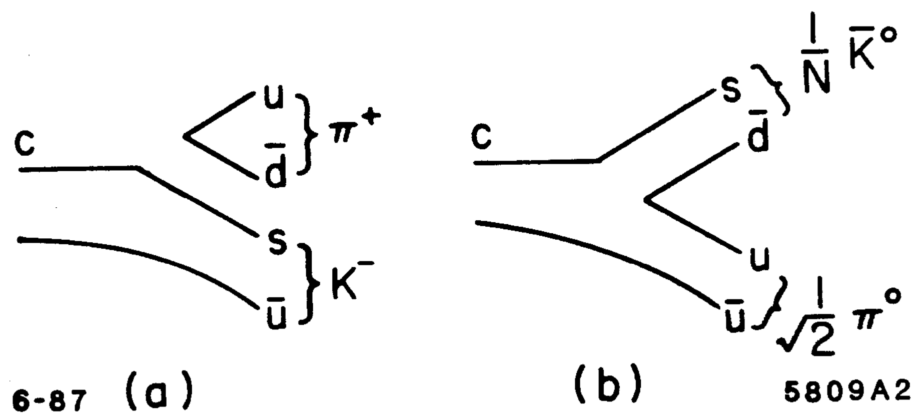
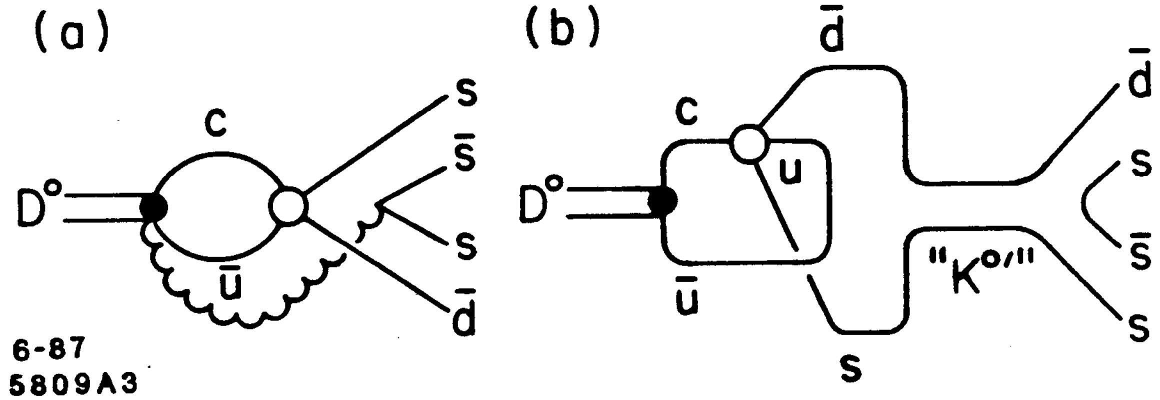
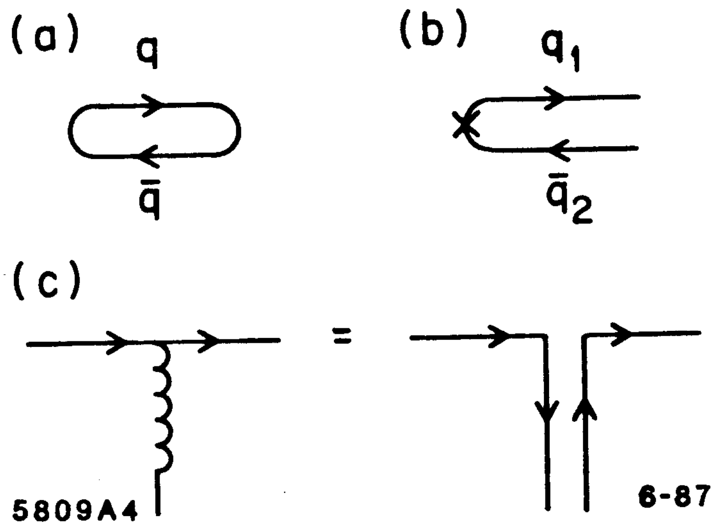


Fig. 2



(a)
(b)
c
D°
s
$\bar{s}$
s
$\bar{d}$
$\bar{u}$
6-87
5809A3
$\bar{d}$
c
u
D°
$\bar{u}$
s
$\bar{d}$
s
$\bar{s}$
s
"K°"

Fig. 3



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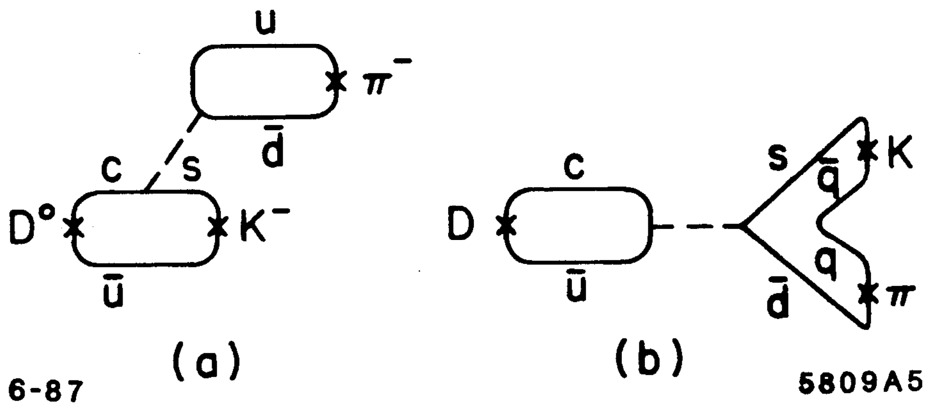
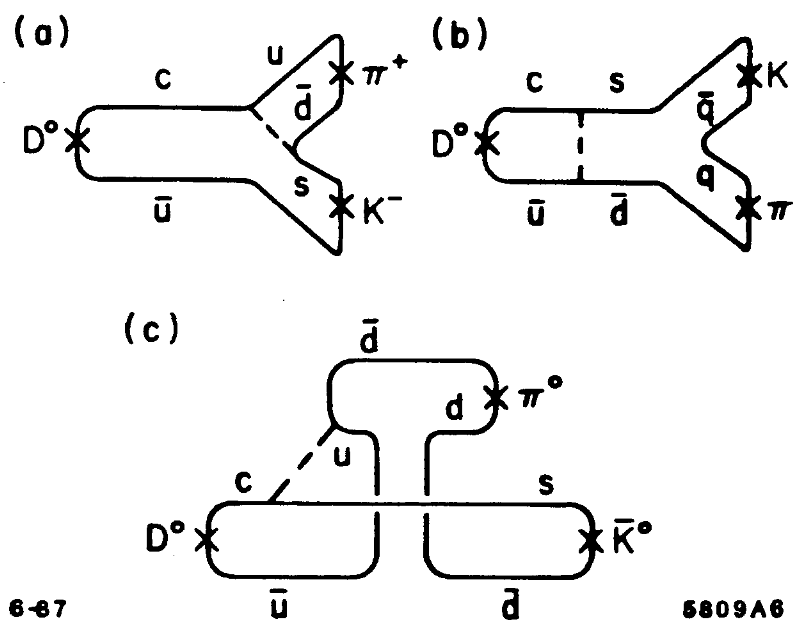


Fig. 5


(a)
c
u
π^+
$\bar{d}$
D°
s
K⁻
$\bar{u}$
(b)
c
s
K
$\bar{q}$
D°
q
$\bar{u}$
$\bar{d}$
π
(c)
$\bar{d}$
d
π^0
u
c
s
D°
$\bar{K}^0$
6-87
$\bar{u}$
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5809A6

Fig. 6

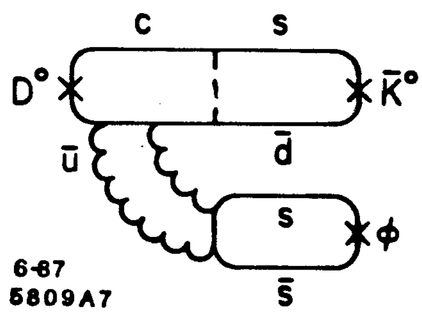

Fig. 7

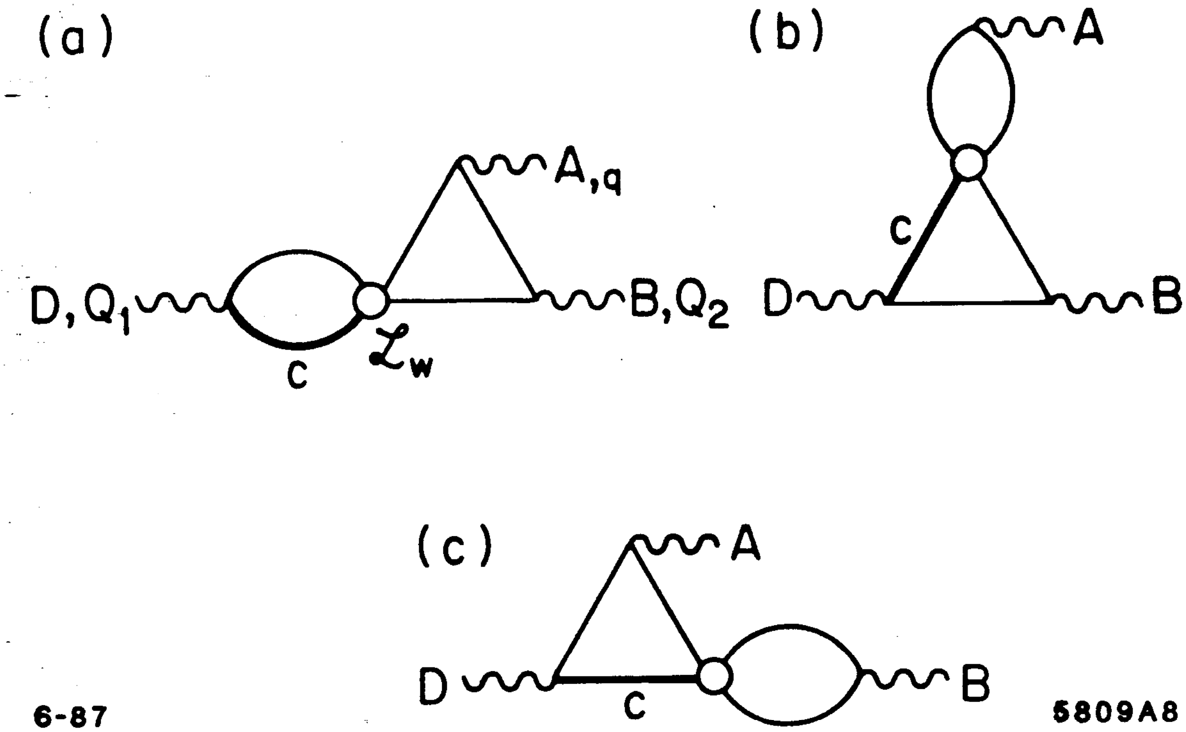

Fig. 8