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A DISCRETE RELATIVISTIC QUANTUM PHYSICS*

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ABSTRACT

By basing physics on a growing universe of bit strings generated by a simple recursive algorithm quantum mechanics, 3-momentum conservation, the Lorentz transformations for quantum events, the conserved quantum numbers of the first generation of the standard model for quarks and leptons and a flat space "big bang" cosmology can be constructed.

Quantum events are unique, discrete, irreversible, non-local and yet indivisible. Conventional quantum theory tries to embed them in a space-time continuum, — a step which in our view is the source of many conceptual difficulties such as the "collapse of the wave function", the EPR "paradox" and the infinities of second quantized field theory. We meet these problems here by basing our theory on a completely finite, discrete and recursive evolution operator called *program universe*[1]. We can then demonstrate systematically that our generating algorithm gives us a discrete metric, finite periodicities of "length" in "time", possible events only on integer recurrences of these lengths (and hence "interference"),"3-momentum conservation", quantized action, and the usual commutation relations for both linear and angular momentum. Further, the construction necessarily contains a limiting velocity, and entails the Lorentz transformations among the rational fraction velocities, etc. so constructed in both coordinate and momentum "space". Asymptotically, these spaces, and more significantly the common space in which their correlated evolution occurs, are proved to be limited to three "homogeneous and isotropic" dimensions.

The conceptual system which we use for this articulation of our theory has been developed by one of us (DMcG); a thorough discussion of the mathematical and philosophical structure of this system as it relates to physics will be presented elsewhere[2]. We start from the postulates of finiteness, discreteness, finite computability and absolute non-uniqueness. The rules of inference assumed in more conventional systems are replaced for us by explicitly constructed ordering operators which necessarily introduce sequential irreversibility. These act on *d-elements* (that as a consequence of our postulates can include *indistinguishables*) which occur in necessarily finite *ensembles* with ordinality less than or equal to their cardinality[3]. Here we invoke the fact that our postulates require that any specified attributes of a finite and discrete ensemble can be mapped onto an ordered sequence of 1's and 0's by asking whether they are present or absent in a reference collection. When such an ordered sequence combines with other sequences of the same bit length by XOR ("exclusive or", symmetric difference, addition (mod 2)= $+_2$, ...) it is called a "bit string"; because we treat the symbols "0", "1" as bits and/or as integers, we use the more general *discrimination* operation "$\oplus$" defined by $S^a \oplus S^b \equiv (..., b_i^a +_2 b_i^b, ...)_n = (..., (b_i^a - b_i^b)^2, ...)_n$; $b \in 0, 1$; $i \in 1, 2, ..., n$.

In order to generate a universe of such strings which grows, sequentially, in either number (SU) or length (N_U) we use program universe[1]. The main program starts with $PICK$, a routine that picks two different, arbitrary strings from the memory (whose content starts from the strings 0 and 1, in either order, and hence with $SU = 2, N_U = 1$), discriminates them and if this produces a novel string, adjoins it to the universe ($SU := SU + 1$) and then recurses to $PICK$. The only alternative recursion occurs whenever the program generates a string already in the universe; the operation which results, called $TICK$, increases each string independently by concatenating it with one arbitrary bit ($N_U := N_U + 1$) and recurses to $PICK$. It is easy to show[1] that the operation $TICK$ occurs only at a step where three strings connected with the generation process satisfy the constraint $S^a \oplus S^b \oplus S^c = (0, 0, ..., 0)_{N_U}$. When N_U is large these constraints will be satisfied by many combinations. These sequentially generated constraints are our model for the unique, nonlocal yet indivisible and *irreversible* events of quantum mechanics. Program universe also automatically allows us to define conserved quantum numbers associated with these events, which then can serve as the "Yukawa vertices" of a relativistic quantum mechanics. These organize themselves into the four levels of the *combinatorial hierarchy* of Amson, Bastin, Kilmister and Parker-Rhodes[4] which generates a sequence of increasing complexity ($3, 10, 137 \simeq \hbar c/e^2, 2^{127} + 136 \simeq 1.7 \times 10^{38} \simeq \hbar c/Gm_p^2 = (M_{Planck}/m_p)^2$) that *terminates* at the fourth level. We identify the first three levels with the quantum numbers of the first generation of the standard model of quarks and leptons, including a single candidate for a spin zero particle that differs from the "Higgs boson" in that it is pseudoscalar and not self-coupled[5].

Examining the structure of the events generated by program universe[1] in more detail, we see that each time we have a "1" in the same ordered position in two of the strings, we must have a "0" in the third. One "1" and two "0"s are not allowed, nor are three "1"s, but any number of ordered positions can contain "0"s in all three strings. Defining $k_i^x(n) = \Sigma_{i=1}^n b_i^x, x \in a, b, c$ it then follows immediately that $|k^a - k^b| \leq k^c \leq k^a + k^b$ (cyclic on a, b, c) for any event. Thus k, the number of "1" s in a string, can serve as a discrete metric. Note that the triangle resulting from our definition of events necessarily will make them non-local.

In order to locate the "origin" of our metric symmetrically in the finite and discrete interval allowed, we define $q_a \equiv [2k^a(n) - n]\lambda_a$ where λ_a has the dimensions of length. Then at each step of the generating operator q_a changes by $\pm\lambda_a$, the "step length", with the sign + or - determined by whether a "1" or a "0" is concatenated with the extant string. Calling the time $t = n\Delta t$, we see that we can define a velocity $v_a \equiv (2k^a/n - 1)V_x = \beta_a V_x$ where $V_x = \lambda_a/\Delta t$ is a *limiting velocity* achieved when all the steps are in the same direction; we also have an *event horizon* that grows with the number of steps the generating operator has taken. The dimensionless velocity $-1 \leq \beta_a \leq +1$ is the "information transfer velocity" of information theory.

We now note that if we consider a system that evolves with constant velocity $\beta_0 \equiv 2k_0/n_0 - 1$, strings which grow subject to this constraint, i.e. $n = n_T n_0, k = n_T k_0, 1 \leq n_T \leq n/n_0$ will have a periodicity $T \equiv n_T \Delta t = n_T \lambda / V_x$ specifying the events in which this condition can be met. Hence, in more complicated situations where there can be more than one "path" connecting strings with the same velocity to a single event, this event can occur only when the paths differ by an integral number of "d-wavelengths" λ. Thus our construction already contains the seeds of "interference" and an explanation of the "double slit experiment".

If we now associate a parameter m_a having the dimensions of mass with each string, and define $p_a \equiv m_a v_a = m_a \beta_a V_x = \beta_a m_a \lambda_a / \Delta t$ we see that $|p_a - p_b| \leq p_c \leq p_a + p_b$ provided only (as is required for consistency) $m_a \lambda_a / \Delta t$ is any finite constant independent of a. Thus the triangle closes in "momentum space" as well as "configuration space" and we find that our events can be interpreted as 3-momentum conserving 3-particle scattering events in the zero momentum frame with the "center of mass" at rest.

We have already seen that any system with "constant velocity" – at those "ticks" when events can occur – evolves by discrete steps $\pm\lambda$ in q *between* ticks. These steps occur in the *void* where space and time are meaningless. Since $\lambda/\Delta t = V_x$, each step occurs forward or backward with the limiting velocity. Thus we deduce a discrete *Zitterbewegung* from our theory. If we think of this as a "trajectory" in the pq phase space, each time step induces a step $\pm\lambda$ in q correlated with a step $\pm m V_x$ in p. Even in the case of a particle "at rest", this must be followed by two steps of the opposite sign to return the system to "rest".

Thus there is, minimally, a four-fold symmetry to the "trajectory" in phase space corresponding to the generation periodicity we discovered above.

If we now recall from classical mechanics[6] that for any momentum which is a constant of the motion we can transform to angle and action variables with $\oint p_J dq_J = J$ where J has the dimensions of action, $p_J = J/2\pi$ and q_J is cyclic, we have an immediate interpretation. In the classical case the "period" goes to infinity for a free particle; for us we have already seen that we have a *finite* period $T = \lambda/V_x$. Therefore we can immediately identify $m_a \lambda_a V_x = J = n_T h$; we have constructed Bohr-Sommerfeld quantization within our theory.

To go on to the commutation relations, we take the usual step in the geometrical description of periodic functions of taking the $q_J J$ plane to be the complex plane $(q, 2\pi i p)$. Then the steps around the cycle in the order $qpqp$ are proportional to $\pm 2\pi(1, i, -1, -i)$ where $\pm$ depends on whether the first step is in the positive or negative direction or equivalently whether the circulation is counterclockwise or clockwise. We have now shown that $qp - pq = \pm i\hbar$ for free particles; this results holds for any theory which uses a discrete free particle basis. When we go to three dimensions (see below), the commutation relations for angular momentum follow immediately.

So far we have succeeded in deriving the formal structure of quantum mechanics in terms of an invariant, quantized phase space volume and an arbitrary, finite limiting velocity for the attribute referred to as "the number of 1's". In general there will be a different limiting velocity for each attribute. But if we wish to model the events of which contemporary physics takes cognizance, we know that all physical attributes are directly or indirectly coupled to electromagnetism. Therefore the limiting velocity of physics, c, will be the *smallest* of these limiting attribute velocities simply because it refers to the attribute with the maximum cardinality. Any ensemble of attributes specified by a more limited description involves a "supraluminal" velocity without allowing supraluminal communication of information. Hence we can expect to find correlation between and synchronization of events in space-like separated regions; from our discrete point of view the existence of the effects demonstrated in Aspect's and other EPR-Bohm experiments is anticipated and in no way paradoxical. We guarantee Einstein locality for *causal* events, that is for those initiated by the transfer of *physical* information.

Consider a free particle of mass m which is created, or engages in, an event at $x(0) = 0 = x_0, ct = 0 = ct_0, n = n_0$ and engages in a second event at $x(t - t_0) = x - x_0 = \beta c(t - t_0)$ after $n - n_0$ generations of our ordering operator. Going to light cone coordinates[7], $x_{\pm}(\beta) = ct \pm x = (1 \pm \beta)ct = [n - n_0]\lambda(\beta)$, $[n > n_0]$, we have that $x_+/x_- = (1+\beta)/(1-\beta) = k/(n-k)$ and $x_+x_- = (1-\beta^2)\lambda^2$; note that $x_+(-\beta) = x_-(\beta)$. For consistency with our finite postulate we must require $0 < k < 1$; no massive event can lie on the event horizon.

If we now transform the description of this event to coordinates in which the "particle" is at rest ($x'_+ = x'_-$) by $x'_+ = f(-\beta)x_+$; then $x_+(-\beta) = x_-(\beta)$. gives us also that $x'_- = f(\beta)x_-$. From our postulate of homogeneity in the absence of specific cause (absolute non-uniqueness), if we transform back to the original description we must require that $f(\beta)f(-\beta) = 1$. Hence $x'_+/x'_- = f^2(-\beta)(1+\beta)/(1-\beta) = 1$ and $f^2(\beta) = (1+\beta)/(1-\beta) = k/(n-k)$. Letting $\gamma^2 = [\frac{1}{2}(f + 1/f)]^2 = [1-\beta^2]^{-1}$ and transforming back from light cone to space-time coordinates $x' = \gamma(x + \beta ct), t' = \gamma(ct + \beta x)$, QED. Calling the unit of length in the rest system λ_0, we have that $x'_+x'_- = n^2\lambda_0^2$. Since n is a global (invariant) ordering parameter, and the transformation must hold for any allowed value of β, $\lambda^2(\beta) = (1-\beta^2)\lambda_0^2$. Therefore our finite, discrete theory has the appropriate Lorentz invariance for all states whose velocities are specified by bit strings.

The extension to momentum space is immediate, since once we have identified the limiting velocity of our quantum mechanical treatment V_x with c, λ_0 - the step length in a system at rest - is simply the Compton wavelength h/mc. Hence $E = \gamma m_0 c^2, p = \gamma\beta m_0 c$. For $p_\pm = E/c \pm p$ we have that $p_+ p_- = m_0^2 c^2, p_+/p_- = k/(n-k)$ and $\frac{1}{2}(p_+ x_- + p_- x_+) = Et - px$. Further, the internal periodicity we computed from the invariant phase space volume becomes h/mc^2, just as it did for deBroglie, and the interference phenomena we derived can be identified as a discrete version of deBroglie waves, when proper account is taken of the difference between phase and group velocity.

So far as we can see, any measurable world satisfying our postulates is restricted to three dimensions, since by spatial dimensions we must mean the cardinal number of independent generators of *ordered* sequences (which can be mapped onto bit strings as already noted) that can be synchronized homogeneously across all dimensions. Feller[8] has pointed out (in the context of independent Bernoulli trials) that the probability that the accumulated number of 1's $[k^a(n) = \Sigma_{i=1}^n b_i^a]$ is the same for all sequences after n symbols have been generated is drastically limited by the number of independent sequences (which in our application of his result are identified as dimensions). Clearly, if D is the number of dimensions, this probability is

$$u_n = \frac{1}{2^{nD}}[\binom{n}{0}^D + \binom{n}{1}^D + ... + \binom{n}{n}^D] \sim \frac{1}{\sqrt{r}}[\frac{2}{\pi n}]^{\frac{1}{2}(D-1)}$$

Thus for two or three dimensions, the probability that synchronization can be continued many times is proportional to $\Sigma_n n^{-1/2}$ or $\Sigma_n n^{-1}$ respectively, and is always finite no matter how long we continue it (up to our universal bound), but for four or more dimensions, the probability of synchronization across all dimensions is strictly bounded by zero for large numbers of recurrences. We prefer this simple derivation of 3+1 space-time to the "compactification" needed to reach the same conclusion in the conventional kind of "string theories".

Now that we have two ($\hbar$ and c) of the three dimensional constants needed to connect a fundamental theory to experiment in the 3-space in which physics operates, and which we have proved must be the asymptotic space of our theory, all that remains is to determine the unit of mass. But this has already been done for us by the combinatorial hierarchy result $2^{127} + 136 \simeq 1.7 \times 10^{38} \simeq \hbar c/Gm_p^2 = (M_{Planck}/m_p)^2$ which tells us that we can either identify the unit of mass in the theory as the proton mass, in which case we can calculate (to about 1 % in this first approximation) Newton's gravitational constant, or if we take the Planck mass as fundamental, calculate the proton mass. Connection to laboratory phenomena is then achieved by what we call the *counter paradigm*[1]:

any **elementary event**, *under circumstances which it is the task of the experimental physicist to investigate, can lead to the firing of a counter.*

In an earlier work[9] the counter paradigm, together with Stein's random walk model, allowed us to derive a propagator for relativistic quantum scattering theory. Now that we have constructed the commutation relations, we (or the reader) could provide a more conventional construction, once we have provided the interaction terms. These "driving terms" arise directly by identifying our 3-momentum conserving events as "Yukawa vertices" as follows. Take $A : (10) \parallel S^a(n,k)$, $B : (01) \parallel \bar{S}^a(n,k)$, $C : (11) \parallel S(n,n)$ where "$\parallel$" is string concatenation and the "bar operator" $\bar{S}$ $(= S(n,n) \oplus S = 1_n \oplus S)$ interchanges 0's and 1's. Note that the three strings define an event, as required, and that B has opposite velocity to A. If we allow the first two elements in the string $(b_1 b_2)$ to define a quantum number $h_z = b_1 - b_2$, which is conserved in the reaction $A + B \to C$, we can identify the "bar" operation with changing particle to antiparticle. This is the usual Feynman rule: change particle quantum numbers to their negatives and reverse the velocity. Further, if we only reverse the velocity, the quantum number does not reverse, showing that it is a *helicity* state properly connected to the direction of particle motion. Then this particular process is particle-antiparticle annihilation to a massless and spinless quantum "moving" with the velocity of light.

We do not have space here to develop these rules further, but refer the reader to Ref. 1, where we make a tentative identification of the first three levels of the hierarchy with (1) chiral electron-type neutrinos, (2) electrons, positrons and photons, (3) up and down quarks in a color octet. Level four will, we believe, provide weak-electromagnetic unification with weak coupling to the first three levels. The only unidentified state of the 137 provided by the first three levels of the hierarchy, once the γ (levels 1,2,3) and the W and Z_0 (level 4) have been shown to provide the lowest order weak-electromagnetic unification, is a neutral, pseudoscalar particle which couples to electron- positron pairs and has a weak coupling constant comparable to that of the zero helicity component of the Z_0. We have no place as yet for any Higgs bosons, so our scheme differs from the current wisdom in anticipating the discovery of a single spinless particle whose mass is within a factor of three of the mass of the W and Z_0, but of odd rather even parity. That our overall mass scheme should come out right is suggested by the success of the Parker-Rhodes calculation (Ref.3): $m_p/m_e = 137\pi/[(3/14)[1+2/7+(2/7)^2](4/5)] = 1836.151497....$ We have indications[1] that our cosmology will have a charged lepton and a baryon number consistent with current observation, and hence with an approximately flat space.

We dedicate this paper to our deceased colleague, Fredrick Parker-Rhodes, whose discovery of the combinatorial hierarchy and subsequent work on the problem of indistinguishability has contributed so much to this research. We sorely miss the criticisms of this paper he would have supplied.

REFERENCES

1. *Proc. of ANPA 7* (1986), Dr. F.Abdullah, Room E517, The City University, Northampton Square, London EC1V 0HB; see also SLAC-PUB-4008 and references and appendices therein.
2. D.O.McGoveran, et.al. "Foundations of a Discrete Physics", (in preparation).
3. In this respect our d-elements and ensembles (*d-sorts*) resemble the *indistinguishables* and *sorts* introduced by A.F.Parker-Rhodes in his *Theory of Indistinguishables* (*Synthese Library* **150**, Reidel, Dordrecht, Holland, 1981), although our constructive, process approach rapidly parts company with much of his subsequent development.
4. T.Bastin, *Studia Philosophica Gandensia* **4**, 77 (1966); —, H.P.Noyes, J.Amson and C.W.Kilmister, *Int'l J. Theor. Phys.*, **18**, 455-488 (1979).
5. H.Pierre Noyes, "A Combinatorial Hierarchy Prediction of a New Particle" (in preparation).
6. H.Goldstein, *Classical Mechanics*, 2nd Ed., Addison-Wesley, 1980, pp. 445-471.
7. Our discrete theory has the same velocity states as the discrete light cone quantization (DLCQ): H-C.Pauli and S.J.Brodsky, *Phys.Rev.* **D 32**, 2001 (1985); T.Eller, H-C. Pauli and S.J.Brodsky *Phys.Rev.* **D 45**, 1493 (1987).
8. W. Feller, *An Introduction to Probability Theory and its Applications*, Vol. I, 3^{rd} Edition (1968), p.316; for more detail see the first edition, p. 247 et. seq.
9. H.P.Noyes, C.Gefwert and M.J.Manthey, "A Research Program with No 'Measurement Problem' ", *Annals of the N.Y.Acad.Sci.* **480**, 553-563 (1986).