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Fayet-Iliopoulos D terms in string theory*

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ABSTRACT

One loop scalar masses induced by Fayet-Ilipoulos D terms in string theory are calculated directly in the heterotic string theory for an arbitrary compactification which preserves space-time supersymmetry at the string tree level. The result is shown to be a total derivative in the moduli space of a torus with two punctures, and hence receives contribution only from the boundary of this moduli space.

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Investigation of non-renormalization theorems in string theories has been of great interest recently[1-10], since this is intimately connected to the question of vacuum stability. It was argued in ref.[11] that for a class of compactified heterotic string theories, consistency of anomaly cancellation with space-time supersymmetry requires some of the auxiliary D fields in the theory to develop a vacuum expectation value (vev) at one loop order. In this talk I shall show how to verify the appearance of such terms through explicit one loop string calculation. The results were derived in ref.[12] in collaboration with J. Atick and L. Dixon. Similar calculations have been carried out independently by Dine, Ichinose and Seiberg[13]. A string calculation of the anomaly coefficient, which, in turn, may be shown to be related to the coefficient of the Fayet-Iliopoulos D-term through low energy effective field theory considerations, has been carried out by Lerche, Nilsson and Schellekens[14]. Here I shall also give an alternative derivation of the results of refs.[12,13], where, unlike refs.[12,13], we may carry out the calculation keeping the external momenta strictly on-shell. In this formulation the effect of the D-term shows up as a total derivative in the moduli space of a torus with two punctures, and hence receives contribution only from the boundary of this space.

If there is a term in the effective action of the form $c^{(a)}D^{(a)}$, it induces a mass of the scalar field of the form,

$$gh\sum_{a} q^{(a)}c^{(a)} \tag{1}$$

Here the sum over a runs all the $U(1)$ gauge generators of the theory, $D^{(a)}$ is the auxiliary field associated with the $a'th$ $U(1)$ gauge group, g is the gauge coupling constant, $q^{(a)}$ is the charge carried by the scalar field under consideration under the $a'th$ $U(1)$ gauge group, and h is the chirality of the scalar field, defined to be the chirality of the fermionic superpartner of the scalar field. $c^{(a)}{}'s$ are the radiatively generated coefficients. In the absence of *a priori* knowledge of the D-term vertex operator, the most straightforward way of calculating the coefficients $c^{(a)}$ is to calculate the scalar masses to one loop, and then calculate the coefficients $c^{(a)}$ from eq.(1).

The first step in our calculation is to construct the vertex operators for the massless scalar fields. Let us denote by X^μ, ψ^μ ($1 \leq \mu \leq 4$) the free bosonic fields in two dimensions, and their superpartners, associated with the uncompactified dimensions. Let b, c denote the reparametrization ghosts, and β, γ the local supersymmetry ghosts. All other two dimensional fields will be denoted by φ^j, this includes the bosonic and the right-handed fermionic fields associated with the compactified dimensions, as well as the 32 left-handed fermions which are associated with the gauge group. We shall assume nothing about these internal fields φ^j, except that they give rise to a super-conformal field theory with the correct central charge, and has (2,0) world-sheet supersymmetry, a criteria which is intimately connected to the existence of unbroken $N = 1$ supersymmetry in the theory[15]. This gives rise to the conserved stress-tensors $T(z)$, $\bar{T}(\bar{z})$, two right-handed supersymmetry currents $T_F^{\pm}(z)$ and a right-handed $U(1)$ current $J(z)$.

We are now in a position to write down the vertex operator for a general scalar field. It was shown in ref.[12] that in the -1 picture[1], the zero momentum vertex operator for a general massless scalar field (except the dilaton and its associated axion field) has the form,

$$V_{-1}(k, z, \bar{z}) = e^{-\phi(z)} f(\varphi(z, \bar{z})) e^{ik \cdot X(z, \bar{z})} \tag{2}$$

where $f(\varphi)$ is an operator of conformal dimension $(\frac{1}{2}, 1)$, and ϕ is a bosonized ghost field, defined through the relation[1],

$$\gamma(z) = e^{\phi(z)} \eta(z), \quad \beta(z) = \partial_z \xi(z) e^{-\phi(z)}, \tag{3}$$

where η and ξ are fermionic fields of conformal dimensions (1,0) and (0,0) respectively. The $BRST$ current J_{BRST} in this theory may be written as a sum of three terms[1], given by,

$$J_0(z) = c(z)T^{matter}(z) + c(z)(-\frac{3}{2}\beta(z)\partial_z\gamma(z) - \frac{1}{2}\partial_z\beta(z)\gamma(z)) - b(z)c(z)\partial_z c(z) \tag{4}$$

$$J_1(z) = \gamma(z)T_F^{matter}(z) \tag{5}$$

$$J_2(z) = \frac{1}{4}\gamma^2(z)b(z) \tag{6}$$

where $T^{matter}(z)$ and $T_F^{matter}(z)$ denote respectively the right-handed stress tensor and the supercurrent for the matter system, involving the fields X^μ, ψ^μ and φ^j. Using eqs.(2-6) we may construct the scalar field vertex operator in the zero picture[1],

$$\begin{aligned} V_0(k,z,\bar{z}) &= [Q_{BRST}, 2\xi(z)V_{-1}(z)] \\ &= [g(\varphi(z,\bar{z})) - ik_\mu\psi^\mu(z)f(\varphi(z,\bar{z}))]e^{ik\cdot X(z,\bar{z})} \end{aligned} \tag{7}$$

where,

$$g(\varphi) = 2\lim_{w\to z}(w-z)T_F^{matter}(w)f(\varphi(z)) \tag{8}$$

In deriving eq.(7) we have ignored total derivative terms, as well as terms which do not contribute to the relevant correlator due to ghost charge conservation.

We may similarly introduce vertex operators for the complex conjugate fields in the zero picture,

$$\tilde{V}_0(k,z,\bar{z}) = [\tilde{g}(\varphi(z,\bar{z})) - ik_\mu\psi^\mu(z)\tilde{f}(\varphi(z,\bar{z}))]e^{ik\cdot X(z,\bar{z})} \tag{9}$$

The scalar mass is then proportional to,*

$$\int d^2\tau \int d^2z_1 d^2z_2 \langle V_0(k,z_1)\tilde{V}_0(-k,z_2)\rangle_e \tag{10}$$

The calculation of the scalar mass proceeds in the following steps. (We shall only

* Here the subscript e denotes the sum over even spin structures only. Contribution from the odd spin structure, *i.e.* the periodic periodic sector, vanishes identically due to the zero modes of ψ^μ [12].

outline the main steps here, since the details have been given in ref.[12].)

i) First we show that $\langle g(z_1)\tilde{g}(z_2)e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle_e$ vanishes identically. This is shown by considering a correlator of the form

$$\langle P^+(z)P^-(w)g(z_1)\tilde{g}(z_2)e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle$$

where $P^+(z)$ is a particular component of the space-time supersymmetry generator in the $-\frac{1}{2}$ picture, and $P^-(w)$ is an operator of conformal dimension (0,0), constructed from the ghost fields and the space-time supersymmetry currents, such that,

$$P^+(z)P^-(w) \sim \frac{1}{z-w} \quad \text{as } z \to w \tag{11}$$

Thus $\langle g(z_1)\tilde{g}(z_2)e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle_e$ is given by the residue of the pole at $z = w$ of the correlator $\langle P^+(z)P^-(w)g(z_1)\tilde{g}(z_2)e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle$.† On the other hand, using space-time supersymmetry transformation properties of various fields one can show that $P^+(z)$ does not develop any singularity near $g(z_1)$ or $\tilde{g}(z_2)$. Finally we note that the correlator $\langle P^+(z)P^-(w)g(z_1)\tilde{g}(z_2)e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle$ is periodic as a function of z with periods 1 and τ. Using complex function theory one can then show that the correlator $\langle P^+(z)P^-(w)g(z_1)\tilde{g}(z_2)e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle$ does not have any pole as a function of z. In other words, the residue of $\langle P^+(z)P^-(w)g(z_1)\tilde{g}(z_2)e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle$ at $z = w$ must vanish. This shows that,

$$\langle g(z_1)\tilde{g}(z_2)e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle_e = 0 \tag{12}$$

† There is a slight complication due to the fact that in defining $\langle P^+(z)P^-(w)g(z_1)\tilde{g}(z_2)e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle$ we sum over all spin structures, whereas we are interested in calculating $\langle g(z_1)\tilde{g}(z_2)e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle_e$ where we sum over even spin structures only. However, the contribution to $\langle P^+(z)P^-(w)g(z_1)\tilde{g}(z_2)e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle$ from the odd spin structure may be explicitly shown to be singularity free in the $z \to w$ limit.

ii) We are now left with the correlator,

$$\int d^2\tau \int d^2z_1 \int d^2z_2 k_\mu k_\nu \langle f(z_1)\tilde{f}(z_2)\psi^\mu(z_1)\psi^\nu(z_2)e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle_e \qquad (13)$$

Naively, this correlator will also vanish, since $\langle\psi^\mu\psi^\nu\rangle \sim \delta^{\mu\nu}$, and $k_\mu k_\nu \delta^{\mu\nu}$ vanishes on-shell. It turns out, however, that the integration over z_1 produces a factor of $\frac{1}{k^2}$ from the region of integration $z_1 \sim z_2$, and hence the final answer is finite in the $k \to 0$ limit. In order to get a $\frac{1}{k^2}$ singularity we need the integrand to have the form,

$$\frac{1}{(z_1-z_2)^{1+k^2}}\frac{1}{(\bar{z}_1-\bar{z}_2)^{1+k^2}} \qquad (14)$$

We now write down the relevant operator product expansions,

$$\langle e^{ik\cdot X(z_1)}e^{-ik\cdot X(z_2)}\rangle \sim \left|\frac{1}{\vartheta_1(z_1-z_2)}\right|^{2k^2} \qquad (15)$$

$$\psi^\mu(z_1)\psi^\nu(z_2) \sim \frac{1}{z_1-z_2} + O(z_1-z_2) \qquad (16)$$

$$f(\varphi(z_1))\tilde{f}(\varphi(z_2)) \sim \frac{1}{(z_1-z_2)(\bar{z}_1-\bar{z}_2)^2} + \cdots + \frac{N(z_2,\bar{z}_2)}{(\bar{z}_1-\bar{z}_2)} \qquad (17)$$

where eq.(17) defines the operator N. It follows from dimensional analysis that N must have conformal dimension (1,1). Combining these results we see that in order to get a term of the form given in eq.(14), we must pick up the leading term from the right hand side of eq.(16), and the term proportional to N in eq.(17). After carrying out the z_1 integral we see that the scalar mass is proportional to,

$$\int d^2\tau \int d^2z_2 \langle N(z_2,\bar{z}_2)\rangle_e \qquad (18)$$

iii) Next we show that $N(z,\bar{z})$ may be written as,

$$N(z,\bar{z}) \sim \sum_a q^{(a)} J(z) U^{(a)}(\bar{z}) + \tilde{N}(z,\bar{z}) \qquad (19)$$

where $J(z)$ is the right-handed $U(1)$ current associated with the $(2,0)$ superconformal algebra, $U^{(a)}(\bar{z})$ is the left-handed $U(1)$ current associated with the $a'th$

$U(1)$ gauge group, and $\tilde{N}$ is an operator for which $\langle \tilde{N}(z,\bar{z})\rangle_e$ vanishes on the torus.‡ Using eqs. (1), (18) and (19) we get,

$$c^{(a)} \sim \int d^2\tau \langle J(z) U^{(a)}(\bar{z})\rangle_e \tag{20}$$

From this we see that $J(z)U^{(a)}(\bar{z})$ may be identified with the vertex operator of the auxiliary field $D^{(a)}$.

iv) Using some manipulations involving the space-time supersymmetry generators of the theory, it can be shown that,

$$\langle J(z)U^{(a)}(\bar{z})\rangle_e \sim (Im\tau)^{-3}\left(\overline{\eta(\tau)}\right)^{-2} e^{-\frac{3i\pi\tau}{4}} e^{\frac{11i\pi\bar{\tau}}{4}} \big\langle \langle U^{(a)}(\bar{z})\rangle \big\rangle_{PP} \equiv (Im\tau)^{-2} F(\tau,\bar{\tau}) \tag{21}$$

where,

$$\big\langle \langle U^{(a)}(\bar{z})\rangle \big\rangle_{PP} \equiv \int [D\varphi]_{PP} e^{-S(\varphi)} U^{(a)}(\bar{z}) \tag{22}$$

The subscript PP denotes the fact that we put periodic boundary condition on the right-handed fermions along both cycles of the torus, while summing over all spin structures for the left-handed fermions with appropriate weights.

iii) Finally one can calculate $\big\langle \langle U^{(a)}(\bar{z})\rangle \big\rangle_{PP}$ using the operator formalism,

$$\big\langle \langle U^{(a)}(\bar{z})\rangle \big\rangle_{PP} = Tr_P\{(-1)^{F^{int}} U^{(a)}(\bar{z}) \bar{P}_{GSO} e^{2\pi i L_0^{(int)}\tau} e^{-2\pi i \bar{L}_0^{(int)}\bar{\tau}}\} \tag{23}$$

where the trace is taken over all states with periodic boundary conditions on the internal fermions. $L_n^{(int)}$ and $\bar{L}_n^{(int)}$ are the Virasoro generators of the internal conformal field theory. $F^{(int)}$ counts the number of internal right-handed fermions. $\bar{P}_{GSO}$ denotes the appropriate GSO projection operator in the left-handed sector. If $G_0^{(int)}$ denotes the generator of (1,0) supersymmetry for the superconformal field theory involving the fields φ, then the contribution to the trace

‡ This is shown by using a trick very similar to the one used for showing the vanishing of $\langle g(z_1)\tilde{g}(z_2) e^{ik\cdot X(z_1)} e^{-ik\cdot X(z_2)}\rangle_e$.

from any state $\mid n>$ and $G_0^{(int)} \mid n>$ cancel each other, since $G_0^{(int)}$ commutes with $L_0^{(int)}$, $\bar{L}_0^{(int)}$, $\bar{P}_{GSO}$ and $U^{(a)}(\bar{z})$, but anti-commutes with $(-1)^{F^{(int)}}$. Thus only the states satisfying $G_0^{(int)} = 0$ ($\rightarrow L_0^{(int)} = (G_0^{(int)})^2 + \frac{3}{8} = \frac{3}{8}$) contribute to the correlator. This completely determines the τ dependence of $\left\langle\langle U^{(a)}(\bar{z})\rangle\right\rangle_{PP}$, and from eq.(21) we see that $F(\tau, \bar{\tau})$ must be independent of τ.

Since the final answer must be modular invariant, $F(\bar{\tau})$ must be a modular function of weight zero. Using eqs.(21) and (23), and using the fact that the state with lowest $\bar{L}_0^{(int)}$ eigenvalue (=0) does not carry any $U^{(a)}(1)$ charge, one can show that $F(\bar{\tau})$ is bounded by a constant as $Im\ \tau \rightarrow \infty$. This, in turn, implies that $F(\bar{\tau})$ must be a constant. Hence we may evaluate it by calculating its value as $Im\ \tau \rightarrow \infty$. In this limit it receives contribution only from the massless states, and may be written down explicitly in terms of the massless spectrum of the theory. The final result for the coefficient of the D-term is,

$$c^{(a)} = \frac{g}{192\pi^2} \sum_i n_i q_i^{(a)} h_i \tag{24}$$

where n_i is the number of massless fermionic (or bosonic) states carrying $U^{(a)}(1)$ charge $q_i^{(a)}$ and chirality h_i.

This concludes our discussion of the calculation of the D tadpole generated at one loop order in the string theory. The implication of these results on vacuum stability and supersymmetry breaking has been discussed in ref.[12]. In calculating this tadpole we ran into a dimension (1,1) operator which could be interpreted as the vertex operator of an auxiliary D field. As was shown in ref.[12], these vertex operators do have the correct space-time supersymmetry transformation properties in order to be interpreted as the vertex operators for the auxiliary D fields. In ref.[12] we also constructed the vertex operators of the auxiliary F fields for a general scalar super-multiplet.

The existence of a one loop D-tadpole is expected to induce a two loop dilaton tadpole proportional to $\sum_a c^{(a)}c^{(a)}$. Recently it has been shown that such a contribution is indeed present, and arises as a boundary term in the moduli space of genus two Riemann surface[16].

In the calculation of the scalar masses that I have described so far, the external momenta were kept slightly off-shell, and were set to zero only at the end of the calculation. I shall now give an alternative derivation of eq.(18), by setting the external momenta to be on-shell from the very beginning. A considerable simplification occurs when we set the external momenta to zero, however, now we have to be careful not to throw away any total derivative terms from the vertex operators. The reason is the following. Typically, if we are calculating a two point correlation function of the form $\langle V_1(k,z_1)V_2(-k,z_2)\rangle$, then the correlator behaves as $\sum_{m,n}(z_1-z_2)^{-m-k^2}(\bar{z}_1-\bar{z}_2)^{-n-k^2}C_{m,n}$, where the sum over m and n runs over a fixed set of numbers, reflecting the conformal dimensions of operators present in the theory. If a correlator can be written as a total derivative of another correlator, then, after integration over z_i, we may express the contribution as a boundary term at $z_1 = z_2$. (A suitable regularization scheme is to integrate over the region $\mid z_1 - z_2 \mid \geq \epsilon$, and take $\epsilon \to 0$ at the end of the calculation). If we allow k^2 to be non-zero, then, by suitable analytic continuation we may go to a region in the k^2 space where the boundary terms are zero. However, if we set $k^2 = 0$ from the very beginning, then we loose this freedom, and must include the contribution from the boundary terms. (Of course, we still have to regularize the amplitude by subtracting off terms which diverge as $\epsilon \to 0$, but should keep the finite terms). As we shall see, this procedure gives the same answer obtained above by continuing k^2 away from zero.

The first step in our calculation is to construct the vertex operators for the massless scalar fields at zero momentum, but now being careful about not to throw away the total derivative terms. Using eqs.(4-7) we get,

$$\begin{aligned} V_0(z) &= [Q_{BRST}, 2\xi(z)V_{-1}(z)] \\ &= 2\partial_z(c(z)\xi(z)e^{-\phi(z)}f(\varphi(z)) + g(\varphi(z)) \end{aligned} \tag{25}$$

We may similarly introduce vertex operators for the complex conjugate fields in the 0 picture, with f and g replaced by $\tilde{f}$ and $\tilde{g}$ in eq.(25).

Naively one would think that the scalar mass may be calculated from the correlator $\langle V_0(z)\tilde{V}_0(w)\rangle_e$ on the torus.* However, on the torus, each of the fields b and c has a zero mode, and hence we must soak up these zero modes in a $BRST$ invariant way. Soaking up the b zero mode may be done in the standard way, by introducing a factor of $\int \eta_0(z)b(z)d^2z \int \bar{\eta}_0(w)\bar{b}(\bar{w})d^2w$ in the correlator, where $\eta_0(z)$ denotes the Beltrami differential dual to $d\tau$, τ being the Teichmuller parameter. It was proposed in ref.[1] that a $BRST$ invariant way to remove the c zero mode is to remove the z integration over the location of one of the vertices, and multiply the vertex by the product $c(z)\bar{c}(\bar{z})$.† This gives a $BRST$ invariant vertex, provided the original vertex $\hat{V}_0$ satisfies,

$$[Q_{BRST}, \hat{V}_0(z)] = \partial_z(c(z)\hat{V}_0(z)) \tag{26}$$

and,

$$[Q_{BRST}, c(z)] = c(z)\partial_z c(z) \tag{27}$$

Here, however, the vertices V_0, $\tilde{V}_0$ that we have introduced satisfy,

$$[Q_{BRST}, V_0(z)] = [Q_{BRST}, \tilde{V}_0(z)] = 0 \tag{28}$$

and so $c(z)V_0(z)$ or $c(z)\tilde{V}_0(z)$ do not give $BRST$ invariant vertex operators. (Multiplication by $\bar{c}(z)$ does not suffer from this problem).

We propose the following alternative scheme. Instead of multiplying $\tilde{V}_0(z)$ by $c(z)$, we multiply $\tilde{V}_{-1}(z)$ by $c(z)$, and then picture change to get a new vertex

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⋆ Here the subscript e denotes the sum over even spin structures only. Contribution from the odd spin structure, *i.e.* the periodic periodic sector, vanishes identically due to the zero modes of ψ.

† In the previous calculation we integrated over the location of both the vertices, but divided by an explicit factor of Im τ in writing down the correlator given in eq.(21).

$\hat{\tilde{V}}_0$. Thus,

$$\begin{aligned}\hat{\tilde{V}}_0(z) &= \bar{c}(\bar{z})[Q_{BRST}, 2\xi(z)c(z)\tilde{V}_{-1}(z)] \\ &= \bar{c}(\bar{z})c(z)\tilde{g}(\varphi(z)) - \frac{1}{2}\bar{c}(\bar{z})\gamma(z)\tilde{f}(\varphi(z))\end{aligned} \tag{29}$$

Finally, we must soak up the zero mode of ξ by inserting a factor of $\int \frac{d^2z}{Im\tau}\xi(z)$ in the correlator. The scalar mass term is then proportional to,

$$\begin{aligned}I = &\int d^2\tau \int d^2z_1 \langle V_0(z_1)\hat{\tilde{V}}_0(z_2) \int d^2w_1\eta_0(w_1)b(w_1) \\ &\int d^2w_2\bar{\eta}_0(\bar{w}_2)\bar{b}(\bar{w}_2) \int \frac{d^2w_3}{Im\tau}\xi(w_3)\rangle_e \\ = &\int d^2\tau \int d^2z_1 \langle g(\varphi(z_1))\tilde{g}(\varphi(z_2))\bar{c}(z_2)c(z_2) \int d^2w_1\eta_0(w_1)b(w_1) \\ &\int d^2w_2\bar{\eta}_0(\bar{w}_2)\bar{b}(\bar{w}_2) \int \frac{d^2w_3}{Im\tau}\xi(w_3)\rangle_e \\ - &\int d^2\tau \int d^2z_1 \partial_{z_1} \langle c(z_1)\xi(z_1)e^{-\phi(z_1)}f(\varphi(z_1))\bar{c}(\bar{z}_2)e^{\phi(z_2)}\eta(z_2)\tilde{f}(\varphi(z_2) \\ &\int d^2w_1\eta_0(w_1)b(w_1) \int d^2w_2\bar{\eta}_0(\bar{w}_2)\bar{b}(\bar{w}_2) \int \frac{d^2w_3}{Im\tau}\xi(w_3)\rangle_e\end{aligned} \tag{30}$$

As shown before, the first term on the right hand side of eq.(30) vanishes identically. The second term is a total derivative in z_1, and it receives contribution only from the boundary at $z_1 = z_2$. In order to evaluate the second term, we first do the z_1 integral, restricted to the region $\mid z_1 - z_2 \mid \geq \epsilon$. This gives,

$$I = -\int d^2\tau \oint_C d\bar{z}_1 \langle \xi(z_1)e^{-\phi(z_1)}f(\varphi(z_1))e^{\phi(z_2)}\eta(z_2)\tilde{f}(\varphi(z_2)) \int \frac{d^2w_3}{Im\tau}\xi(w_3)\rangle_e' \tag{31}$$

where $\langle\ \rangle'$ denotes that we have explicitly performed the integration over the c and the b zero modes, thus removing the ghost insertions from the correlator. C

denotes a contour of radius ϵ-around the point z_2. We now take the $\epsilon \to 0$ limit, and throw away all terms which diverge as $\epsilon^{-p}(p > 0)$ in this limit, keeping only the finite terms. In other words, inside the integral, we should take the $z_1 \to z_2$ limit, and keep only those terms which diverge as $(\bar{z}_1 - \bar{z}_2)^{-1}$ in this limit. In order to do this, first let us note that the correlator involving the ξ, η and ϕ fields gives[7],

$$\frac{1}{Im\tau}\int d^2w_3 \langle \xi(z_1)e^{-\phi(z_1)}e^{\phi(z_2)}\eta(z_2)\xi(w_3)\rangle_\delta$$
$$= \frac{\eta(\tau)}{Im\tau}\int d^2w_3 \frac{\vartheta[\delta](w_3 - z_2)}{\vartheta[\delta](0)\vartheta[\delta](w_3 - z_1)} \frac{\vartheta\left[\begin{smallmatrix}\frac{1}{2}\\ \frac{1}{2}\end{smallmatrix}\right](w_3 - z_1)}{\vartheta\left[\begin{smallmatrix}\frac{1}{2}\\ \frac{1}{2}\end{smallmatrix}\right](w_3 - z_2)} \tag{32}$$

in the sector with spin structure δ.

This integral becomes $\frac{\eta(\tau)}{\vartheta[\delta](0)}$ in the limit $z_1 \to z_2$. Furthermore, the integrand is periodic as a function of w_3 with periods 1 and τ, and remains invariant under the transformation $z_1 \leftrightarrow z_2$, $w_3 \to -w_3 + z_1 + z_2$. As a result, the integral must be symmetric under the interchange $z_1 \leftrightarrow z_2$. This gives,

$$\int \frac{d^3w_3}{Im\tau}\langle \xi(z_1)e^{-\phi(z_1)}e^{\phi(z_2)}\eta(z_2)\xi(w_3)\rangle_\delta$$
$$= [1 + O((z_1 - z_2)^2)]\frac{\eta(\tau)}{\vartheta[\delta](0)} \tag{33}$$

Finally, we use the operator product expansion given in eq.(17). Combining these results we see that in order to get a term of order $(\bar{z}_1 - \bar{z}_2)^{-1}$ in (31), we must pick up the leading term from the right hand side of eq.(33), and the term proportional to N in eq.(17). This shows that the scalar mass is proportional to,

$$\int d^2\tau \langle N(z_2, \bar{z}_2)\rangle_e \tag{34}$$

which is precisely eq.(18).

This particular derivation, combined with the results of ref.[16], indicates that the effect of the vacuum expectation values of the auxiliary fields on physical scattering amplitudes is reflected as an obstruction to integration by parts in the moduli space, and gives rise to boundary terms.

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