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$B - \bar{B}$ Mixing and Relations Among Quark Masses, Angles and Phases*

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ABSTRACT

Using the recent data on $B_d^0 - \bar{B}_d^0$ mixing we establish constraints on the allowed values of the standard model parameters m_t, θ_{13} and δ. The allowed range is inconsistent with the Stech scheme for the quark mass matrices. For almost all allowed values of the parameters, the Fritzsch form of the mass matrices is also excluded. Only in the unlikely event that the experimental and theoretical quantities x_d, m_s/m_b, θ_{23}, B_K and f_B assume extreme values within their allowed ranges, a set of parameters exists which is consistent with all data and with the Fritzsch scheme. We then obtain the unique solution $m_t \sim 85\ GeV$, $\sin\theta_{13} \sim 0.0035$, $\delta \sim 100°$, yielding $\Gamma(b \to u\ell\bar{\nu}_\ell)/\Gamma(b \to c\ell\bar{\nu}_\ell) \sim 0.01$, maximal $B_s^0 - \bar{B}_s^0$ mixing, relatively small ϵ'/ϵ and $BR(K \to \pi\nu\bar{\nu}) \sim 1 \times 10^{-10}$.

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The quark sector of the three-generation standard model contains ten independent parameters: six quark masses, three mixing angles and one Kobayashi-Maskawa (KM) phase. Future theories may lead to a calculation of these parameters or at least to relations among them.

Seven of the ten parameters are reasonably well determined: the masses of the first five quarks and the mixing angles θ_{12} and θ_{23}. For the remaining three parameters m_t, θ_{13} and δ, we have fairly weak bounds.

Several theoretical and phenomenological models which go beyond the standard model have led to the derivation of relations among mixing angles, phases and quark mass ratios. The best known scheme is the one proposed by Fritzsch[1]. Another interesting scheme has been suggested by Stech[2] and variations on the two themes were proposed by Gronau, Johnson and Schechter[3] and by Shin[4].

The recent observation of $B_d^0 - \bar{B}_d^0$ mixing by the ARGUS collaboration[5] in DESY provides us with important new information on the three poorly determined parameters of the quark sector of the standard model. It also leads to severe constraints on the relations among masses, angles and phases. In this note we investigate the implications of the new data and show that the Stech scheme[2] (as well as its extension by Gronau, Johnson and Schechter[3]) is now clearly inconsistent with the data. We also show that the Fritzsch[1] matrix (and its extension suggested by Shin[4]) disagrees with the *central values* of the various experimental parameters and with *almost* all other allowed values of the parameters. The Fritzsch scheme will survive only if *all* the following conditions are met: $B_d^0 - \bar{B}_d^0$ mixing is at the lowest level allowed by the error quoted by the ARGUS experiment; the B-meson decay constant is approximately 0.2 GeV (at the top of its "reasonable" range); the hadronic factor B_K in the expression for the CP violating parameter ϵ obeys $B_K \sim 1$; the ratio m_s/m_b is around 0.022 (*e.g.* $m_s = 120\ MeV$, $m_b = 5.4\ GeV$); the value of the mixing angle θ_{23} is at the top of its allowed range. Only if all of these constraints are *simultaneously* fulfilled, the Fritzch matrix will remain consistent with the data. In such a case, we have $m_t \sim 85\ GeV$; $s_{13} \sim 0.0035$; $\delta \sim 100^\circ$ and the phases of the Fritzsch matrix must be close to the ones suggested by Shin.

We also comment on several consequences of these results for other experimental quantities.

In our analysis we use a choice of mixing angles based on the combined work of Maiani[6], Wolfenstein[7], Chau and Keung[8] and others. We follow the notation and the precise form of this matrix as given by Harari and Leurer[9]. For the purpose of the analysis presented here it is sufficient to neglect corrections of order θ_{ij}^2 and to assume $\cos\theta_{12} = \cos\theta_{23} = \cos\theta_{13} = 1$. The resulting form of the mixing matrix is:

$$V = \begin{pmatrix} 1 & s_{12} & s_{13}e^{-i\delta} \\ -s_{12} - s_{23}s_{13}e^{i\delta} & 1 & s_{23} \\ s_{12}s_{23} - s_{13}e^{i\delta} & -s_{23} & 1 \end{pmatrix} \tag{1}$$

Where $s_{ij} \equiv \sin\theta_{ij}$.

We have the following direct information on the three angles:

(*i*) $s_{12} = 0.221 \pm 0.002$.

(*ii*) $s_{23} = 0.043^{+0.007}_{-0.009}$. This result is based on the following assumptions: $\tau_b = (1.16 \pm 0.16) \times 10^{-12}\ sec$ (ref. 10); $BR(b \to c\ell\bar{\nu}_\ell) = 0.121 \pm 0.008$ (ref. 11); $m_b = 5.0 \pm 0.3\ GeV$; $m_c = 1.5 \pm 0.2\ GeV$. The values of m_b and m_c appear in the phase space factors relating the measured b lifetime and the relevant matrix element. It is not entirely clear which values should be used and we have allowed for fairly generous errors. In some cases we will need to use the product $\tau_b s_{23}^2$. Since the errors on s_{23} contain the errors on τ_b, the product has smaller errors than s_{23}^2 alone. We obtain: $\tau_b s_{23}^2 = (3.25 \pm 1.10) \times 10^9\ GeV^{-1}$.

(*iii*) The only direct limit on s_{13} is obtained from the experimental bound[11]:

$$\frac{\Gamma(b \to u\ell\bar{\nu}_\ell)}{\Gamma(b \to c\ell\bar{\nu}_\ell)} < 0.08$$

In order to minimize the ambiguities which are due to the b-quark mass, we use the ratio $q = \frac{s_{13}}{s_{23}}$ rather than using s_{13}. For the values of m_b and m_c listed above, we obtain $s_{13} \leq 0.011$ and:

$$q \equiv \frac{s_{13}}{s_{23}} \leq 0.22.$$

The direct lower experimental bound on m_t is still 23 GeV. A recent combined analysis of all measurements of $\sin^2\theta_W$ showed[12] that the standard model radiative corrections are consistent with the data only if $m_t \leq 180\ GeV$. We therefore consider only the range $23\ GeV \leq m_t \leq 180\ GeV$.

At present, there are only two additional experiments which can help us to further limit the above parameters and establish bounds on the KM-phase δ. These are the value of $\epsilon = 2.3 \times 10^{-3}$ in the $K^0 - \bar{K}^0$ system and the new measurement[5] of the $B_d^0 - \bar{B}_d^0$ mixing parameter $r_d = 0.21 \pm 0.08$. Each of these two parameters depends on m_t, θ_{13} and δ as well as on several theoretical parameters. In both cases we can use the data in order to exclude certain ranges of the parameters.

The results of the first part of our analysis are similar to those recently obtained by several groups of authors[13]. We therefore do not give here all the details of the analysis, but quote the assumptions and the main results.

We use the following expression[14] for ϵ:

$$|\epsilon| = C \cdot B_K \cdot s_{23}^2 q \sin\delta \left[(\eta_3 f_3(y_t) - \eta_1) y_c s_{12} + \eta_2 y_t f_2(y_t) s_{23}^2 (s_{12} - q\cos\delta)\right]$$

Where:

$$C = \frac{G_F^2 f_K^2 M_K M_W^2}{6\pi^2\sqrt{2}\Delta M_K} = 4 \times 10^4 \quad ; \quad y_i = \frac{m_i^2}{M_W^2} \quad (i = c, t)$$

$$f_2(y_t) = 1 - \frac{3}{4}\frac{y_t(1+y_t)}{(1-y_t)^2}\left(1 + \frac{2y_t}{1-y_t^2}\ln(y_t)\right)$$

$$f_3(y_t) = \ln\left(\frac{y_t}{y_c}\right) - \frac{3}{4}\frac{y_t}{1-y_t}\left(1 + \frac{y_t}{1-y_t}\ln(y_t)\right)$$

We use $G_F = 1.166\times10^{-5}\ GeV^{-2}$; $f_K^2 = (0.16\ GeV)^2$; $M_K = 0.498\ GeV$; $M_W = 82\ GeV$; $\Delta M_K = 3.52\times10^{-15}\ GeV$; $\eta_1 = 0.7$; $\eta_2 = 0.6$; $\eta_3 = 0.4$. The three parameters η_i are QCD corrections. We also use the values of m_c and s_{23} listed above. The parameter B_K is unknown and is usually believed to be somewhere in the range $\frac{1}{3} \leq B_K \leq 1$.

The $B_d^0 - \bar{B}_d^0$ mixing paramater r_d is related to $x_d = \frac{\Delta M}{\Gamma}$ by the following relation:

$$r_d = \frac{x_d^2}{2 + x_d^2}.$$

The expression for x_d is[13]:

$$x_d = \tau_b \frac{G_F^2}{6\pi^2}\eta M_B(B_B f_B^2)M_W^2 y_t f_2(y_t)|V_{td}^* V_{tb}|^2$$

In addition to the previous parameters we now also use $M_B = 5.28\ GeV$; $V_{tb} = 1$; $\eta = 0.85$. Here η is, again, a QCD correction. We also have:

$$|V_{td}|^2 = s_{23}^2\left[s_{12}^2 + q^2 - 2s_{12}q\cos\delta\right]$$

Note that the expression for x_d contains the combination $\tau_b s_{23}^2$ for which we use the value quoted earlier. We also have here two additional unknown parameters: The B decay constant f_B and the hadronic parameter B_B which is analogous to the usual B_K mentioned above. Based on previous analysis[13] we assume $B_B \sim 1$ and $f_B = 0.15 \pm 0.05\ GeV$. What we really use is $B_B f_B^2 = (0.15 \pm 0.05\ GeV)^2$.

We can write:

$$x_d = C' \cdot y_t f_2(y_t)[s_{12}^2 + q^2 - 2s_{12}q\cos\delta]$$

where:

$$C' = (\tau_b s_{23}^2)\frac{G_F^2}{6\pi^2}\eta M_B(B_B f_B^2)M_W^2|V_{tb}|^2 = 5.1^{+7.0}_{-3.6}$$

We now use the experimental values $\epsilon = 2.3 \times 10^{-3}$ and $x_d = 0.73 \pm 0.18$ (corresponding to $r_d = 0.21 \pm 0.08$) in order to limit further the allowed range of the three poorly determined parameters m_t, s_{13} and δ. We do it in the following way: In principle, we express the known quantities ϵ/C and x_d/C' as functions of m_t, s_{13}, δ. We select a value of m_t and plot the two allowed curves for ϵ/C and x_d/C' on the $q - \delta$ plane. If the two curves intersect below the upper limit $q \leq 0.22$, the intersection point is an allowed solution for (m_t, s_{13}, δ).

In practice, x_d has a large experimental error and C' has both a theoretical ambiguity and an experimental error. We therefore obtain a wide band rather than a curve for x_d/C' in the $q - \delta$ plane, for each possible value of m_t. The central curve and the two edges of the band correspond to $x_d/C' = 0.14^{+0.47}_{-0.09}$, as given by the most extreme values assumed above for x_d, $(B_B f_B^2)$, $(\tau_b s_{23}^2)$. Note that the edge of the band corresponds to the very unlikely situation that *all* these parameters simultaneously obtain their most extreme allowed values. For ϵ/C, we produce a band whose width is determined by the allowed range of s_{23} and m_c. It is narrower than the x_d/C' band. However, we must repeat the analysis for several different values of B_K in the usually accepted range.

The overlap regions of the two bands which also lie below $q = 0.22$ provide us with a conservative evaluation of the allowed range of m_t, s_{13}, δ.

Figure 1 shows the relevant bands for $B_K = 1$ and $m_t = 45,\ 65,\ 85,\ 180\ GeV$. In each case the allowed range of q and δ values is shown by the shaded area. There is no allowed region for any value of m_t below 43 GeV. The *central* values of the two bands meet in the allowed region only for $m_t \geq 85\ GeV$. The lowest allowed value of q is obtained for the largest m_t and is around 0.03 implying $s_{13} \geq 0.0015$. At that same m_t value, δ can be anywhere between 50° and 175°. On the other hand, for m_t values around $50 - 60\ GeV$, s_{13} must be very close to 0.01 and δ is between 100° and 165°. Values near the boundaries of the two bands are improbable in the sense that they require all parameters to assume their extreme values simultaneously.

For lower values of B_K, the ϵ band corresponds to higher values of s_{13} and the allowed region is substantially smaller. A sample of such results for $B_K = 0.4$ is shown in figure 2 for $m_t = 65,\ 85\ GeV$.

It is easy to see from the figures that, for low m_t values, the new information obtained from $B_d^0 - \bar{B}_d^0$ mixing excludes a substantial part of the region which remained allowed by all previous experiments. For larger m_t values, the new data do not provide us with much new information.

Additional details of the above analysis will be presented elsewhere. The results discussed so far are consistent with those of previous authors[13].

Having established the allowed range for the parameters m_t, s_{13} and δ, we now proceed to consider the proposed relations among masses, angles and phases. We first note that all quark masses mentioned in our phenomenological analysis up to now are essentially the "physical" masses. In QCD language, the quark masses are "running" parameters, defined only for a specific energy scale. The "physical" m_t values appearing in figures 1 and 2 and in the above analysis should be interpreted as values of $m_t(m_t)$. In fact, we should also include a first order QCD correction, obtaining[15]:

$$m_t^{phys} = m_t(m_t)\left[1 + \frac{4}{3\pi}\alpha_s(m_t)\right].$$

In order to relate m_t^{phys} to, say, $m_t(1\ GeV)$ we must use the usual equation for the running mass[15]:

$$m(\mu) = \bar{m} \cdot \left(1 - \frac{2\beta_1\gamma_0}{\beta_0^3}\frac{\ln L + 1}{L} + \frac{8\gamma_1}{\beta_0^2 L}\right)\left(\frac{L}{2}\right)^{-2\gamma_0/\beta_0}$$

where:

$$\begin{aligned}\beta_0 =& 11 - \frac{2}{3}N_f \ ; \ \gamma_0 = 2 \\ \beta_1 =& 102 - \frac{38}{3}N_f \ ; \ \gamma_1 = \frac{101}{12} - \frac{5}{18}N_f \\ L =& \ln(\mu^2/\Lambda^2) \ ; \ \Lambda = 0.1 \ GeV\end{aligned}$$

and $\bar{m}$ is the renormalization group invariant mass. A useful approximate relation for m_t^{phys} in the interesting range between 40 GeV and 180 GeV is $m_t^{phys} \sim 0.6 m_t(1\ GeV)$. However, in the following discussion we use the full expression for the running m_t.

We consider two main schemes which were proposed for the structure of quark mass matrices. The first is the Stech scheme[2] according to which the mass matrices M^u and M^d are both Hermitian and can be described by:

$$M^u = S \ ; \ M^d = \alpha S + A$$

where S, A are, respectively, a symmetric and an antisymmetric matrix and α is a free parameter. Without loss of generality we can always choose a basis in which M^u is a real diagonal matrix with matrix elements $m_u, -m_c, m_t$. In such a basis M^d has the form:

$$M^d = \begin{pmatrix} \alpha m_u & iA & iB \\ -iA & -\alpha m_c & iC \\ -iB & -iC & \alpha m_t \end{pmatrix}$$

The Stech matrices have seven real parameters and they lead to three relations among quark masses, angles and phases. We use as input the seven best-determined parameters (masses of the first five quarks and s_{12}, s_{23}). The masses should all be chosen at the same energy scale and we select 1 GeV and use the values advocated by Gasser and Leutwyler[15]:

$$m_d/m_s = 0.051 \pm 0.04 \ ; \ m_u/m_c = 0.0038 \pm 0.0012;$$

$$m_s/m_b = 0.033 \pm 0.011 \ ; \ m_c(1\ GeV) = 1.35 \pm 0.05\ GeV.$$

All relations involve only mass ratios, but we need the value of m_c in order to derive m_t from the ratio $\frac{m_c}{m_t}$. We use the s_{12}, s_{23} values quoted earlier.

One of the predictions of the Stech scheme is:

$$s_{23} \approx \sqrt{\frac{m_s}{m_b} - \frac{m_c}{m_t}}$$

If we scan the entire range of all the relevant parameters, this gives $m_t(1\ GeV) \leq 72\ GeV$, leading to $m_t^{phys} \leq 46\ GeV$. Our earlier analysis of the ϵ and x_d data showed that even when all uncertainties were pushed to their limits, we obtained $m_t^{phys} \geq 43\ GeV$. That means that the only remaining hope for the Stech scheme is to have $m_t \sim 45\ GeV$. We may now use another prediction of the Stech scheme which restricts s_{13} and δ. We obtain:

$$q \cos\delta \approx -\frac{m_s}{m_b} s_{12}$$

This correspond to a curve in the $q - \delta$ plane. The resulting curve for the case of $m_t \sim 45\ GeV$ is shown in figure 3a. While the Stech curve passes through the region allowed by all earlier data, it is clearly in contradiction with the tiny allowed region which remains after the new $B_d^0 - \bar{B}_d^0$ mixing data. The Stech scheme is therefore now excluded by the data.

Gronau, Johnson and Schechter[3] proposed a scheme in which the Stech assumptions are supplemented by the assumptions of the Fritzsch scheme. Having excluded the Stech scheme, we are automatically excluding any model which uses the Stech assumptions as part of its input. That applies, of course, to the model of reference 3.

We now consider the Fritzsch scheme[1]. Without loss of generality we can choose a basis[16] in which the Fritzsch matrices have the form:

$$M^u = \begin{pmatrix} 0 & a^u & 0 \\ a^u & 0 & b^u \\ 0 & b^u & c^u \end{pmatrix} \quad M^d = \begin{pmatrix} 0 & a^d e^{i\phi_1} & 0 \\ a^d e^{-i\phi_1} & 0 & b^d e^{i\phi_2} \\ 0 & b^d e^{-i\phi_2} & c^d \end{pmatrix}$$

Here we have eight real parameters and therefore two predictions for relations among masses, angles and phases.

The Fritzcsh form enables us to express the three angles and the KM-phase in terms of the six quark masses and the two arbitrary phases ϕ_1, ϕ_2 which appear in the matrices. The relations are[16]:

$$s_{12} \approx \left| \sqrt{\frac{m_d}{m_s}} - e^{-i\phi_1} \sqrt{\frac{m_u}{m_c}} \right|$$

$$s_{23} \approx \left| \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}} \right|$$

$$s_{13} \approx \left| \frac{m_s}{m_b} \sqrt{\frac{m_d}{m_b}} + e^{-i\phi_1} \sqrt{\frac{m_u}{m_c}} \left(\sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}} \right) \right|$$

$$\frac{\sin\delta}{\frac{s_{12}s_{23}}{s_{13}} - \cos\delta} \approx \frac{\sin\phi_1}{\cos\phi_1 - \sqrt{\frac{m_d m_c}{m_s m_u}}}$$

The second of these four equations gives the bound:

$$m_t \leq \frac{m_c}{\left(\sqrt{\frac{m_s}{m_b}} - s_{23} \right)^2}$$

Using the entire allowed range of the parameters, this leads to $m_t(1\ GeV) \leq 145\ GeV$ which yields $m_t^{phys} \leq 88\ GeV$. This bound for m_t is valid only for the lowest allowed value of $\frac{m_s}{m_b}$ and the highest allowed value of s_{23}, namely: $\frac{m_s}{m_b} = 0.022$; $s_{23} = 0.05$. For other values, stronger bounds for m_t are obtained. We can now restrict our attention only to m_t values below 88 GeV.

For each such value of m_t, we have already extracted (figure 1) the region in the $q-\delta$ plane which is allowed by all present data. Now we take the same m_t value and find, in the same $q-\delta$ plane, the region allowed by the Fritzsch conditions. We again cover all allowed values of the various parameters. We find that for m_t^{phys} values below 82 GeV there is no overlap between the allowed Fritzsch domain and the region of the $q-\delta$ plane allowed by the data. The only overlap is obtained for $m_t \sim 85$ GeV. The tiny overlap region is shown in figure 3b. It is clear from the figure that all values of all parameters are essentially determined by that solution. We obtain $m_t \sim 85$ GeV; $q \sim 0.07$ leading to $s_{13} \sim 0.0035$; $\delta \sim 100°$; $B_K \sim 1$; $x_d \sim 0.55$; $B_B f_B^2 \sim (0.2\ GeV)^2$; $s_{23} \sim 0.05$ and $\frac{m_s}{m_b} \sim 0.022$.

At this point one may take two different points of view: A pessimist would say that the probability that all the experimental and theoretical parameters will eventually settle on their most extreme allowed values is very small and that it is therefore unlikely that the Fritzsch form of the mass matrix will survive. An optimist would argue that the Fritzsch scheme now provides us with a complete determination of all the parameters of the three-generation standard model and allows us to predict all the results of all future experiments which depend on these parameters. We leave it for the reader to decide which point of view is more likely to prevail.

The (unlikely) unique solution which we have obtained here leads to some interesting consequences:

(i) Shin[4] proposed that the two arbitrary phases of the Fritzsch matrices, ϕ_1 and ϕ_2, might have the simple values $\phi_1 = 90°$; $\phi_2 = 0$. We find that, with some effort, our unique Fritzsch solution can be consistent with this hypothesis. If, in addition to pushing all the above parameters to their extreme values, we also take the highest allowed value of s_{12} (namely 0.223) and the lowest allowed values for $\frac{m_d}{m_s}$ and $\frac{m_u}{m_c}$, we find that for $m_t = 88$ GeV, $s_{13} = 0.003$, $\delta = 93°$ we can have a solution which is consistent with the Fritzsch form, the Shin hypothesis and all the data. This situation is even more unlikely but cannot be completely ruled out.

(*ii*) The low value $s_{13} \sim 0.0035$ suggests that it will be difficult to observe non-charmed b-decays and that their actual rates are a full order of magnitude below the present limits. It is therefore clear that any direct observation of $b \rightarrow u\ell\bar{\nu}_\ell$ in the near future will eliminate our unlikely unique solution and will exclude the Fritzsch scheme (and the Shin hypothesis).

(*iii*) The low value of s_{13} also leads to relatively small values for the parameter $\frac{\epsilon'}{\epsilon}$. It is well known that the theoretical determination of this quantity is subject to major uncertainties. Our unique solution gives:

$$\frac{\epsilon'}{\epsilon} \sim 0.005 \left(\frac{Im\widetilde{C}_6}{-0.1}\right)\left(\frac{\langle\pi\pi|Q_6|K^0\rangle}{1.0\ GeV^3}\right)$$

where $\widetilde{C}_6, Q_6$ are not well determined and are defined in reference 17. For any value of the uncertain theoretical parameters, this corresponds to ϵ'-values which are approximately one third of the values obtained by the current upper limit on s_{13}. This is certainly consistent with the present data.

(*iv*) Using our unique solution we can predict that $B_s^0 - \bar{B}_s^0$ mixing is practically maximal. Assuming that M_B, B_B, f_B and τ_b are essentially the same for B_d^0 and B_s^0, we find:

$$\frac{x_s}{x_d} \sim \frac{|V_{ts}|^2}{|V_{td}|^2}$$

With our values, we must have $x_d \sim 0.55$ and $|V_{ts}| = 0.05$; $|V_{td}| = 0.011$. Consequently, we find $x_s \sim 11$ and $r_s = \frac{x_s^2}{2+x_s^2} = 0.98$. Note, however, that if, instead of choosing the Fritzsch solution, one uses the central values of x_d, the parameter r_s is even closer to one. The UA1 data[18] on $B - \bar{B}$ mixing is consistent with $r_s \sim 1$.

(v) Our complete set of parameters also predicts that, for three light neutrinos: $BR(K \to \pi\nu\bar{\nu}) \sim 1 \times 10^{-10}$.

We summarize: we have used the recent ARGUS data for $B_d^0 - \bar{B}_d^0$ mixing in order to restrict the allowed domain of values for m_t, s_{13} and δ. We then compared this domain with the Stech and Fritzsch forms of the quark mass matrices. The Stech form is excluded by the new data, even if all uncertainties are stretched to their limits. The Fritzsch form is almost excluded, with a tiny allowed region remaining for $m_t \sim 85\ GeV$, $s_{13} \sim 0.0035$, $\delta \sim 100°$. This last unique solution leads to many definite experimental predictions. It should not be too difficult to rule it out by further data.

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FIGURE CAPTIONS

Figure 1: Allowed range of $q = \frac{s_{13}}{s_{23}}$ and δ for $m_t = 45, 65, 85, 180$ GeV and $B_K = 1$. Heavy and light solid lines give, respectively, the central value and the edges of the band allowed by ϵ. Heavy and light dashed lines give the corresponding band for x_d. The upper edge of the x_d band is often outside the graph. The dotted line is the upper limit for q. The shaded area is the final allowed region.

Figure 2: Allowed range of q and δ for $m_t = 65,\ 85$ GeV and B_K=0.4. The meaning of the lines and shaded area are as in figure 1.

Figure 3: (a) Allowed range (shaded) of q and δ for $m_t = 45$ GeV, $B_K \leq 1$ and the curve (dot-dash) representing the Stech scheme. The two do not overlap. Higher m_t values are excluded by the Stech scheme. Lower m_t values are excluded by the data. Solid, dashed and dotted lines are defined as in figure 1. (b) Allowed range of q and δ for $m_t = 85$ GeV, $B_K \leq 1$. The small shaded area is the overlap of the region allowed by the data and the region allowed by the Fritzsch form of the mass matrices. Higher m_t values are inconsistent with the Fritzsch assumptions. Lower m_t values show no overlap between the two allowed regions. Solid, dashed and dotted lines are defined as in figure 1.

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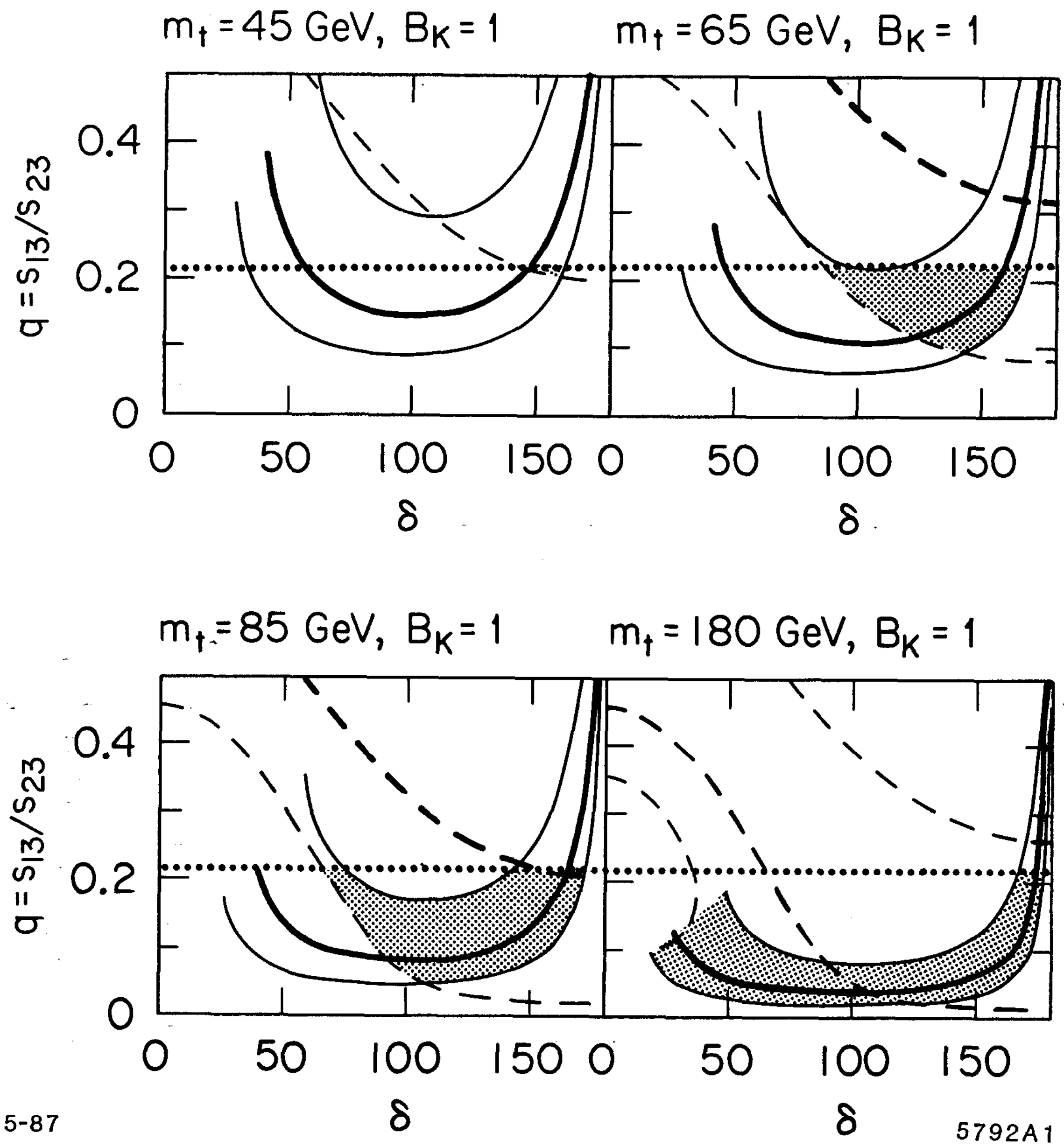
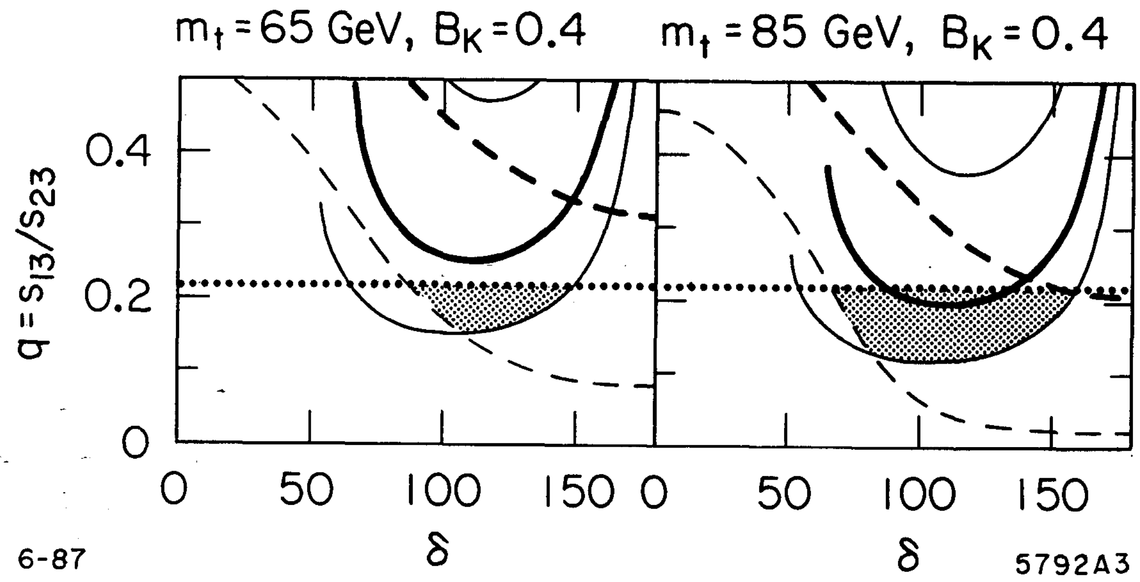


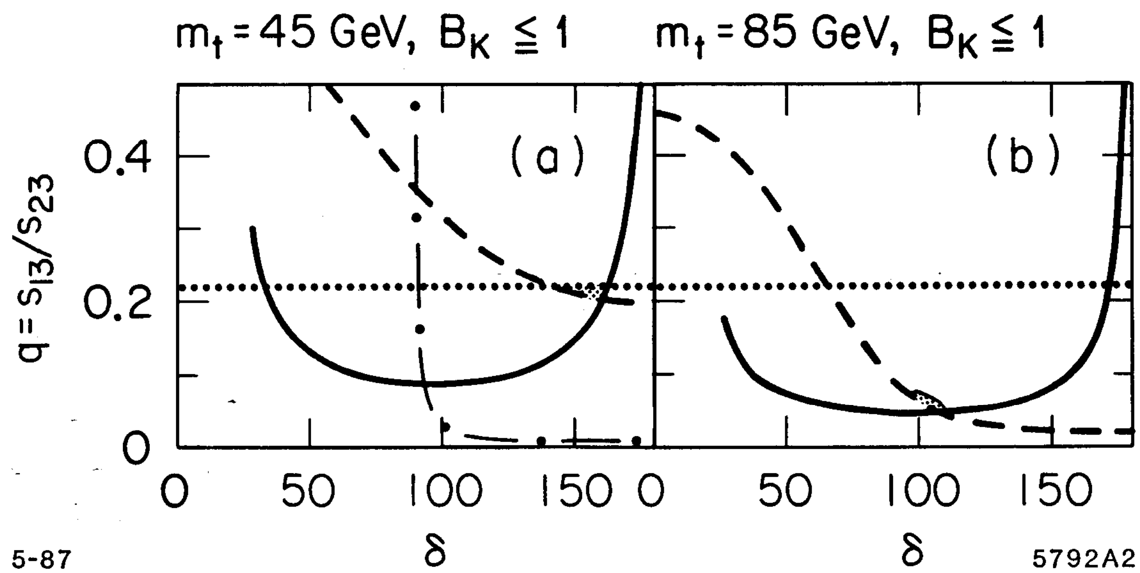
m_t = 45 GeV, B_K = 1
m_t = 65 GeV, B_K = 1
m_t = 85 GeV, B_K = 1
m_t = 180 GeV, B_K = 1
q = s13/s23
0.4
0.2
0
0
50
100
150
δ
5-87
5792A1

Fig. 1



m_t = 65 GeV, B_K = 0.4
m_t = 85 GeV, B_K = 0.4
q = s_13/s_23
0.4
0.2
0
0
50
100
150
0
50
100
150
δ
δ
6-87
5792A3

Fig. 2



m_t = 45 GeV, B_K ≦ 1
m_t = 85 GeV, B_K ≦ 1
(a)
(b)
q = s13/s23
0.4
0.2
0
0
50
100
150
0
50
100
150
δ
δ
5-87
5792A2

Fig. 3