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EXOTIC DAMPING RING LATTICES*

R. B. PALMER

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

AND

Brookhaven National Laboratory
Upton, Long Island, New York 11973

Abstract

We have looked at, and compared, three types of damping ring lattices:

a) conventional

b) wiggler lattice with finite α

c) wiggler lattice with $\alpha = 0$

and observed the attainable equilibrium emittances for the three cases assuming a constraint on the attainable longitudinal impedance of 0.2 ohms. The emittances obtained are roughly in the ratio 4:2:1 for a, b, and c.

Introduction

The equilibrium emittance in a conventional damping ring depends at high energies on quantum fluctuations and at low energies on intrabeam scattering. A minimum is obtained when the two contributions are matched. It is then found to depend critically on a parameter H that is dominated by the dispersion η and this in turn depends on the tune Q of the ring. We want a high Q for low emittance.

But a high Q implies a large ring, and a large ring at the required energy implies low bending fields. Low bending fields imply slow damping times and small momentum spread, both of which are clearly undesirable.

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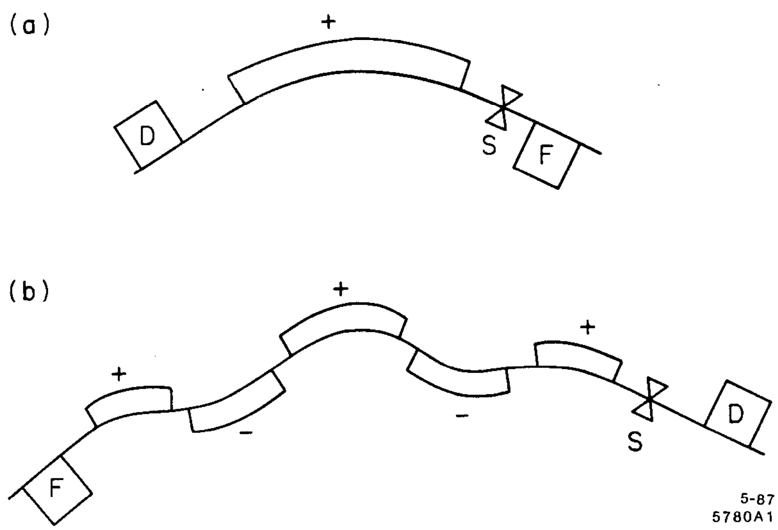

Fig. 1. One half cell of a) a conventional damping lattice and b) a wiggler lattice with $n = 2.5$ wiggles.

However, high Q, a large radius and high bending fields are possible if the bending is alternated (see Fig. 1). This is the idea behind a wiggler lattice.[1]

A problem, however, with a wiggler lattice is that the momentum compaction parameter α becomes very small and problems arise with longitudinal instabilities. α becomes small even in a normal ring if a high tune Q is chosen, but becomes even smaller in a wiggler ring and can even become negative. This gives us the possibility of looking at $\alpha = 0$ rings where, though longitudinally they may be unstable, the growth time of the instability is longer than the needed damping time.

In this note I examine what gains might be possible using these two ideas.

Equations

Assuming β_x and β_y are constants, I can take the values of the equilibrium normalized emittances from 1) quantum fluctuations[2] and 2) intrabeam scattering[3] to be

$$\epsilon_{qn} \approx 2.2 \times 10^{-10} \frac{1}{(J_x + \varsigma J_y)} \gamma^2 \langle HB \rangle \tag{1}$$

$$\epsilon_{cn} \approx \frac{1.2 \times 10^{-10}}{\varsigma^{1/2}} \left[\frac{N \langle H^{1/2} \rangle}{\epsilon_{zn} B^2 \gamma F_m \left(J_x + \varsigma J_y \right) \beta_y^{1/2}} \right]^{1/2} \quad . \tag{2}$$

I assume

$$J_x = J_y = 1 \quad ,$$
$$H \approx \frac{\eta^2}{\beta_x} + \beta_x (\eta')^2 \tag{3}$$

and B is the magnetic field in the bending magnets; ς is vertical/horizontal mixing parameter; $\epsilon_{zn} = \gamma \sigma_z dp/p$ is the invariant longitudinal emittance; F_w is the fraction of the circumferance filled with bending magnets; β_x and β_y are the focusing parameters in horizontal and vertical directions; and η is the transverse dispersion parameter.

$$\eta \approx \beta^2 / R \tag{4}$$

where R is the mean ring radius.

In a simple ring the contribution to H from η' is negligible, but in a wiggler ring, within the wiggler pole of length 2ℓ:

$$\eta' \approx \frac{z}{\rho} \quad (\text{for } z = -\ell \text{ to } +\ell) \tag{5}$$

where ρ is the bending radius within the wiggler, and z is measured from the center of each wiggle magnet pole.

Substituting into Eq. (3):

$$\langle HB \rangle \approx B \beta_x \left(\frac{\beta_x^2}{R^2} + \frac{\ell^2}{2\rho^2} \right) \tag{6}$$

and

$$\langle H^{1/2} \rangle = \beta_x^{1/2} \Big\{ (1 - F_m) a$$

$$+ (F_m) \frac{1}{2} \left[\sqrt{a^2 + \ell^2} + \frac{a}{\ell} \log \left(\ell + \sqrt{a^2 + \ell^2} \right) - \frac{a}{\ell} \log a \right] \Big\} \tag{7}$$

where $a = \beta_x / R$.

In the finite α cases the impedance requirement is taken as

$$\frac{Z}{n} \leq \frac{(2\pi)^{3/2} \sigma_z E \alpha \sigma_p^2}{e^2 c N} \tag{8}$$

where σ_z is the rms bunch length; E is the electron energy in electron Volts; N is the number of electrons; e is the electron charge; and c is the velocity of light. The momentum speed is taken to be

$$\sigma_p = \frac{dp}{p} \approx \left(\frac{2}{J_z} \right) 1.1 \times 10^{-5} (\gamma B)^{1/2} \quad \text{(mks)} \tag{9}$$

assuming $J_z = 2$.

The longitudinal momentum compaction α for a simple ring is

$$\alpha \approx (\beta_x / R)^2 \tag{10}$$

but for the wiggler case we must include the effect of a finite η':

$$\alpha = \frac{\beta_x^2}{R^2} - \frac{F_m}{2} \frac{\ell^2}{\rho^2} \tag{11}$$

or for $F_m = 1$

$$\alpha \approx \left(\frac{\beta_x^2}{R^2} - \frac{\ell^2}{2\rho^2}\right) \quad . \tag{12}$$

Comparing this with Eq. (6) we see that as the η' term in H becomes significant in increasing H, it simultaneously becomes a significant reduction in α. For convenience I define a term F_α giving the relative contribution of the η' term:

$$F_\alpha = \frac{F_m \ell^2}{8\rho^2} \Big/ \frac{\beta_x^2}{R^2} \quad . \tag{13}$$

In the $\alpha = 0$ case, electrons of different momenta are synchronous, the beams are infinitely unstable with an infinite growth time. The machine is like a relativistic linac, and the effect of impedance is to produce "wakefields" that give a momentum spread between the front and back of a bunch. This momentum spread can, however, be corrected either in the ring, by operating at an appropriate rf phase or outside the ring in an rf section. The uncorrected energy spread:

$$\frac{\Delta E}{E} = IZ = \frac{NeRc}{\sqrt{2\pi}\sigma_z^2 E} \frac{Z}{n} \tag{14}$$

or if we set a bound on $\frac{\Delta E}{E}$ of 1.7% ($\cos\phi = 1-.025$; $\theta = 10°$), then:

$$Z/n \leq .017 \cdot \frac{\sqrt{2\pi}\sigma_z^2 E}{NecR} \quad . \tag{15}$$

Method

1. Using the above equations I select the operating energy (E) to set the equilibrium emittance from quantum fluctuations equal to that from intra-beam scattering. (Since intrabeam scattering falls with energy and quantum fluctuations rise, the combined emittance is at a minimum when they are approximately equal.)

2. I chose β_x depending on the operating energy scaling from the SLAC damping ring:

$$\beta_x = .77 \left(\frac{\gamma}{2.4 \times 10^3} \right)^{1/2} \quad .$$

I take $\beta_y = 4\beta_x$.

3. I assume the fraction of the circumference filled with magnets $F_m = .33$. Given R this then determines the bending field B for a conventional ring. For the wiggler rings I keep $B = 1.5$ Tesla.

4. I take

$$N = 2 \times 10^{10}$$

$$\varsigma = .01$$

$$\epsilon_{zn} = .024$$

these all being taken from the examples given in my introductory talk.

5. For the finite α wiggler case I chose the wiggler pole tip lengths to have

$$F_\alpha = .1$$

For the $\alpha = 0$ case

$$F_\alpha = 0 \quad .$$

I now vary R and plot the equilibrium emittance and Z/n requirement as a function of R, for the three cases

a) conventional

b) wiggler $F_\alpha = .1$

c) $\alpha = 0$

Figure 2 shows these plots.

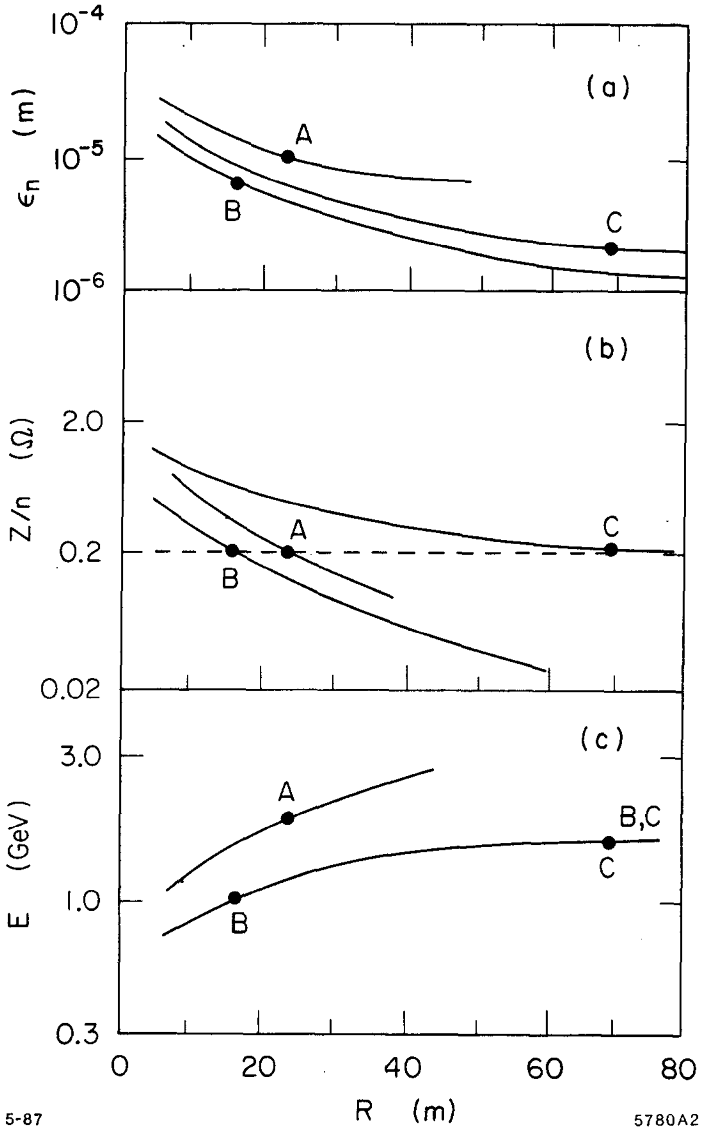

Fig. 2. a) Equilibrium emittance, b) impedance requirement and c) energy for equal quantum and intrabeam contributions, plotted as a function of radius R, for A) a conventional ring, B) a wiggler ring and C) an $\alpha = 0$ ring.

Conclusions

From Eq. (1a) we see:

1. That at any fixed radius the equilibrium emittances are lower for the wiggle lattices than for conventional, but that the $\alpha = 0$ case is not quite as good at the finite α case.
2. The impedance requirement is more severe for the wiggler than the conventional, but that if a radius is chosen to satisfy any given impedance requirement the wiggler still gives a lower equilibrium emittance.
3. The impedance requirement for the $\alpha = 0$ case is much easier.

If I apply a bound on Z/n of 0.2 ohms I then obtain best solutions for each of the cases (see Table 1). We see that the use of a wiggler lowers the achievable emittance by a factor of 2. The $\alpha = 0$ case lowers the emittance by at least another factor of 2 (the use of a larger phase advance would show a greater gain). We also note that the lower emittances of B and C come with faster damping times.

Table 1

			Conventional A	Wiggler B	$\alpha = 0$ C
Radius	R	m	20	20	70
Energy	E	GeV	1.9	1.1	1.5
Emittance	ϵ_w	m	10^{-5}	5×10^{-6}	2.4×10^{-6}
Damping Time	τ	msec	13	3.7	2.7
Focusing	β_x	m	1.14	.87	1.0
Momentum Spread	σ_p	10^{-3}	.55	.6	.73
Bunch length	σ_z	mm	8.5	18	11
Volts lost/turn	V	MV	.1	.2	1.3
Wiggles	n		1	2.5	2.5

References

1. K. Steffen, *The Wiggler Storage Ring,* Internal Report, DESY PET 79/05 (1979) and R. Palmer, *Cooling Rings for TeV Colliders,* SLAC–PUB–3883 and *Proc. Seminar on New Techniques for Future Accelerators,* Erice, Italy (1986).

2. M. Sands, *The Physics of Storage Rings,* SLAC–PUB–121 (1979), p. 110.

3. The approximation used here was taken from J. Bisognano et al., *Feasibility Study of Storage Ring for a High Power XUV Free Electron Laser,* LBL–19771 (1985). For a more basic reference, see J. LeDuff, Orsay Report LAL–134 (1965).