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# BLOWN-UP ORBIFOLDS[⋆†]

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## ABSTRACT

A method to repair — "blow-up" — the singularities of the Abelian (2,2) orbifolds to obtain the corresponding (2,2) Calabi-Yau manifolds is presented. This approach makes use of the fact that with each orbifold singularity there are associated massless scalar fields — blowing-up modes — whose potential is flat to all orders in the string perturbation theory. The zero vacuum expectation values (VEV's) of the blowing-up modes correspond to the orbifold limit, while nonzero VEV's yield the corresponding Calabi-Yau manifold. One can then calculate explicitly, for such Calabi-Yau manifolds, the mass spectrum, Yukawa couplings, and all the other parameters of the effective Lagrangian by inserting successively all the background blowing-up modes with nonzero vacuum expectation value into the corresponding orbifold amplitudes. These results are *exact* at the string tree-level; however, they are perturbative in the blowing-up procedure. Mass spectra and Yukawa couplings for the blown-up $Z_3$ and $Z_4$ orbifolds are explicitly calculated. In particular all the $E_6$ singlets except the ones associated with the moduli-space of the blown-up orbifolds receive the mass; while the **27**'s and $\overline{\mathbf{27}}$'s do not pair up.

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A large variety of different compactifications of superstring theories have been proposed[1−12] in the last few years. However, the compactifications whose four-dimensional effective field theories possess a realistic gauge group, N=1 supergravity and quarks and leptons as elementary fields, can be divided into two classes of candidates, which are believed to be consistent superstring vacua to all finite orders in string perturbation theory:

1. Compactifications of the $E_8 \times E_8$ heterotic string[13] on a Calabi-Yau manifold or a left-right symmetric orbifold, in which the spin and gauge connections are identified. In these cases the theory possesses (2,2) worldsheet supersymmetry, *i.e.* there is both a left-moving (l) *and* a right-moving (r) $N = 2$ worldsheet superconformal algebra.[1,14,15]

2. More general compactifications of the heterotic string,[‡] which require only (super)conformal invariance of the worldsheet action, with the contribution of the matter fields to the Virasoro and super-Virasoro central charges cancelling the ghost contribution, *i.e.* $\widehat{c}_l = 26$ and $\widehat{c}_r = 10$, plus modular invariance of scattering amplitudes.[8−12][§] Space-time supersymmetric compactifications of this type — necessarily having at least (0,2) worldsheet supersymmetry — also appear to give rise to perturbatively stable vacua.[19−21] Some of these constructions seem to be isolated vacuum solutions, *i.e.* one cannot continuously deform such a vacuum solution into another. Many of them can be explicitly constructed[16] as asymmetric orbifolds.[11]

In this note we shall concentrate on the first class of models, from now on referred to as Calabi-Yau models and orbifold models.

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‡ In compactifications of the type II superstring theory, *i.e.* $\widehat{c}_l = \widehat{c}_r = 10$, massless excitations cannot be identified with the standard quarks, because massless triplets of $SU(3)$ and massless doublets of $SU(2)$ are never present in the same model.[16] Note also that compactifications of the bosonic string, with $\widehat{c}_l = \widehat{c}_r = 26$, cannot yield space-time fermions.

§ Note that for (0,2) Calabi-Yau backgrounds[2], *i.e.* configurations where the spin and gauge connections are not identified, conformal invariance is generically spoiled by worldsheet instantons.[17,18]

Orbifolds are especially attractive because interactions on orbifolds can be calculated *exactly* at string tree level.[22,23] Thus all the parameters of the tree-level effective Lagrangian can be determined exactly, *i.e.* including contributions which are nonperturbative in the ratio $\sqrt{\alpha'}/R$, where $\alpha'$ is the string tension and $R$ is the radius of the orbifold. For example, the effects of worldsheet instantons are automatically incorporated.

On the other hand the methods for explicitly studying string interactions on Calabi-Yau manifolds is limited, partly due to the lack of an explicit metric. The field theory limit ($\sqrt{\alpha'}/R \to 0$) results[1,24] state that the numbers of particular types of massless modes are determined by the Hodge numbers, the topological invariants of the Calabi-Yau manifolds. Also, certain Yukawa couplings[25,26] are determined by similar topological considerations. Nonperturbative contributions to the effective Lagrangian for Calabi-Yau compactifications have been explored[18] by studying worldsheet instantons. One result of this analysis is that some parameters of the effective Lagrangian *can* be modified by worldsheet instanton contributions, which are proportional to $\exp(-R^2/\alpha')$. It has been shown[18] that Yukawa couplings as well as masses of the matter $E_6$ singlets receive nonzero corrections in general, while **27** and $\overline{\mathbf{27}}$ do not pair-up. However, the calculation is not entirely explicit, due to the unknown metric.

In this note we shall present a complementary approach to studying the complete tree-level effective Lagrangians for Calabi-Yau models by choosing a Calabi-Yau manifold which is constructed by repairing ('blowing-up') the singularities of an orbifold.[27] This approach makes use of the fact that each orbifold singularity is associated with massless scalar fields — blowing-up modes — whose potential is flat to all orders in the string loop expansion.[22,23] Thus any vacuum expectation value (VEV) of these modes corresponds to a vacuum solution to the string equations of motion, at least perturbatively in the VEV's. The case with all blowing-up modes having zero VEV corresponds to the orbifold limit, while nonzero VEV's for the mode located at a particular singularity corresponds to repairing that singularity. Scattering amplitudes in the repaired Calabi-Yau background — and hence also parameters of the effective Lagrangian — can be calculated by inserting successively larger numbers of *background* blowing-up modes into orbifold amplitudes. Although this method is perturbative in the blowing-up VEV's, it enables one to obtain *explicit* values for parameters of the blown-up orbifolds, giving *exact* results at the string tree-level.

In the following we shall review the general properties of the Calabi-Yau and the orbifold models, outline the calculation of the parameters of the effective Lagrangian for the blown-up orbifolds, and present explicit results for $Z_3$ and $Z_4$ blown-up orbifolds.

## Calabi-Yau and Orbifold Models

Calabi-Yau models give rise to $N = 1$ supergravity in four dimensions and gauge group*

$$G = E_6 \times E_8. \tag{1}$$

The massless particle spectrum consists of the gauge and the gravity supermultiplets as well as zero modes (moduli) of the Ricci-flat (to $\mathcal{O}(\alpha')$) Calabi-Yau metric. In addition there are massless matter multiplets, **27**'s, $\overline{\mathbf{27}}$'s and **1**'s of $E_6$ which are all singlets of $E_8$.

Due to the local right-moving superconformal invariance[1,14,15] one can use the picture-changing formalism, in which vertices for a given state appear with different ghost numbers for the bosonized right-moving superconformal ghost $\phi$; *i.e.* they appear in different "pictures".[15,29] Tree-level amplitudes involve collections of vertices such that the total ghost number equals $-2$.[15] The simplest form of the vertex operator for a space-time fermion is the $-1/2$ picture, while that for a space-time boson is the $-1$ picture. The picture-changing formalism

* Space-time supersymmetry implies that the Calabi-Yau spin connection has $SU(3)$ holonomy; the orbifold holonomy group is a discrete subgroup of $SU(3)$. In general the gauge group (1) could be broken further at the compactification scale by employing the Wilson-loop mechanism.[28] However, this will not affect the study of the general structure of the effective Lagrangian.

enables one to obtain vertices in other pictures. For example, the vertex for a space-time boson in the 0 picture is obtained in the following way:[15]

$$(V_B(z))_0 = \lim_{w \to z} \exp(\phi) T_F(w)(V_B(z))_{-1}. \tag{2}$$

Here $(V_B(z))_{-1}$ is the corresponding vertex operator in the $-1$ picture and

$$T_F = T_F^{\rm int}(X^i, \overline{X}^{\bar{i}}, \psi^i, \overline{\psi}^{\bar{i}}) + \partial X^\mu \phi^\mu \tag{3.a}$$

is the worldsheet supersymmetry generator[15] — the energy-momentum tensor. Here $X$ and $\psi$ are the string bosonic and fermionic coordinates, respectively; the indices $(i, \bar{i}) = (1,2,3)$ and $\mu = (1,2,3,4)$ denote the three complex internal and the four space-time dimensions, respectively. Partial derivatives are with respect to the right-moving worldsheet coordinate $z$. For an orbifold model, $T_F^{\rm int}$ takes the simple form

$$T_F^{\rm int} = \partial X^i \overline{\psi}^{\bar{i}} + \partial \overline{X}^{\bar{i}} \psi^i. \tag{3.b}$$

The left- (right-) moving $N = 2$ superalgebra of a (2,2) model incorporates a $U(1)_l$ $(U(1)_r)$ current algebra, generated by $J_l = -i\sqrt{3}\bar{\partial} H_l$ $(J_r = -i\sqrt{3}\partial H_r)$, where $H_l(\bar{z})$ $(H_r(z))$ is a free left- (right-) moving scalar field. Vertex operators can be classified according to their $H_{l(r)}$ charge. One can, for example, determine the $H_r$ charges for vertices for the massless chiral supermultiplets in various pictures. One finds that

$$\begin{aligned} H_r &= 1, & &-1 \text{ picture}, \\ H_r &= -\frac{1}{2}, & &-\frac{1}{2} \text{ picture}, \end{aligned} \tag{4}$$

for the four dimensional chiral superfield with positive chirality.

Another feature of these compactifications is that *every* such vacuum can be continuously deformed to a nearby vacuum of the same (2,2) type.[18,22,23,27] In field theoretical language this corresponds to a flat potential for massless scalars which correspond to the 'moduli' of the compactified space.[27] In the Calabi-Yau case the moduli are identified with the zero modes of the metric. Namely, giving vacuum expectation values (VEV's) to the moduli in one conformally invariant background generates a nearby background configuration which is also a vacuum solution, at least perturbatively in these VEV's. This procedure can be carried out explicitly for the case of deforming an orbifold into the corresponding Calabi-Yau manifold by giving VEV's to the 'blowing-up' modes[22,23] and will be examined in detail later.

Orbifolds as a special limit of particular Calabi-Yau manifolds possess the following *additional* features:

1. *Enlarged gauge group.* In addition to the gauge group (1) there is a gauge group $G_0 \subseteq SU(3)$ which commutes with the discrete holonomy group of the orbifolds, *e.g.* the $Z_N$ holonomy group for a $Z_N$ orbifold.

2. *Enlarged symmetry of the effective Lagrangian.* A $Z_N$ orbifold possesses a $Z_N$ symmetry which can be described as an additional selection rule on interactions. Blowing-up modes carry nonzero charge under these symmetries. Thus many nonzero parameters of the Calabi-Yau manifold become zero in the orbifold limit, including certain mass terms and Yukawa couplings of matter multiplets.

3. *Increased worldsheet symmetry.* In particular, the $U(1)_{(l,r)}$ worldsheet symmetry of the (l,r)-sector is enlarged to $[U(1) \times U(1) \times U(1)]_{(l,r)}$ for a $Z_N$ orbifold. Thus, instead of the two conserved charges $H_{(l,r)} \equiv \sum_{i=1}^{3} H_{i,(l,r)}$, there are now six conserved charges $H_{1,(l,r)}$, $H_{2,(l,r)}$, and $H_{3,(l,r)}$. This enlarged symmetry enables one to construct *exactly*[22,23] the vertex operators for the emission of massless states at the string tree-level.

## CALCULATION OF THE AMPLITUDES FOR THE BLOWN-UP ORBIFOLDS

The calculation of parameters of the effective Lagrangian in a particular theory reduces to the study of the corresponding amplitude of massless particles emitted from the string propagating in this particular background. For the blown-up orbifold this would correspond to calculating the corresponding amplitudes by including in the *orbifold* amplitudes a successive number of vertices corresponding to the blowing-up modes $b_M$, which now have nonzero VEV's.

We shall concentrate on the following Yukawa-type n-point function:

$$\langle V_{F_1} V_{F_2} V_{B_1} \ldots V_{B_{(n-2)}} \rangle \tag{5}$$

Here $V_{F_i}$ and $V_{B_j}$ denote the vertices for the emission of the massless fermionic and bosonic mode respectively.

This amplitude enables one to probe the parameters of the superpotential for the blown-up theory directly, unlike the amplitude for n-bosons*. Also the gaugino masses can be computed directly, thus determining the new gauge group in a direct way†.

The mass terms for the fermions $\psi_1$ and $\psi_2$ arising from the chiral multiplet is obtained by choosing the appropriate vertex operators $V_{F_1}$ and $V_{F_2}$ while all the bosonic vertices $V_{B_j}$ correspond to the vertices for the blowing-up modes. On the other hand the mass term for the mixing between the fermions, $\psi_i$, and the gauginos can be obtained by inserting in (5) vertices for blowing-up modes as well as their complex conjugates, because this arises from the D-term as well.

Yukawa couplings for two fermions and the boson of the chiral multiplets are obtained from (5) by taking all but one bosonic vertices $V_{B_j}$ to be the vertices for the blowing-up modes. Similarly, one can calculate any higher point function in the superpotential, thus obtaining all the terms in the effective Lagrangian. We shall mainly concentrate on the masses and Yukawa couplings, while higher dimensional terms are explored elsewhere.[27]

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⋆ Note, that the in this case one is probing the scalar potential, which is the mixture of the F- *and* D-terms.

† The new gauge group can in principle be determined also by calculating the gauge boson masses. However, this appears to be more complicated.

With this procedure the value of the parameters in the superpotential can be determined in principle to all orders in the blowing-up procedure,[‡] *i.e.* by calculating the particular amplitude (5) for all the possible insertions of the $b_M$ vertices.[§]

Since all the vertices are in the orbifold limit, they can be constructed exactly.[22,23] In the $-1$ picture of the r-sector $(V_B)_{-1}$ is in general of the following form:

$$(V_B)_{-1} = \exp(-\phi)\psi^i \exp(ik_\mu X^\mu) f(\partial_{\bar{z}} X, \partial_{\bar{z}} \bar{X}, \bar{\tilde{\psi}}), \qquad \text{untwisted sector} \quad (6.a)$$

$$(V_B)_{-1} = \exp(-\phi) \prod_i \sigma_i s_i \exp(ik_\mu X^\mu) g(\partial_{\bar{z}} X, \partial_{\bar{z}} \bar{X}, \tilde{s}), \qquad \text{twisted sector} \quad (6.b)$$

Here $\mu = 1, \ldots, 4$ and $(i, \bar{i}) = 1, \ldots, 3$ again refer to the four space-time and the six compactified dimensions, respectively. The bosonic twist fields $\sigma^i$ and the fermionic twist fields $s^i$ correspond to the emission of the massless state from the propagating string with the twisted boundary conditions in the r-sector for the bosonic $X^i$ and the fermionic $\psi^i$ coordinates, respectively.[23] Fermionic fields are presented in terms of the three bosonic $U(1)_r$ charges:

$$\psi^i = \exp[i(H_i)_r], \qquad \bar{\psi}^i = \exp[-i(H_i)_r] \qquad (7.a)$$

$$s^i = \exp[ik_i/N(H_i)_r], \qquad \bar{s}^i = \exp[-ik_i/N(H_i)_r] \qquad (7.b)$$

The three separate charges $(H_i)_r$ should satisfy constraint (4), namely $H_r = \sum_i (H_i)_r = \sum_i k_i/N = 1$. For example, for the $Z_3$ orbifold $k_i/N = \frac{1}{3}$, $i = 1, 2, 3$.

‡ In the supersymmetric theory as ours this is actually true to all orders in the string loop corrections, due to the non-renormalization theorem[19].

§ One could argue that this result is also *exact* in the blowing-up procedure if $b_M$'s, and not for example $1/b_M$'s, correspond to the representation of the fields in the Lagrangian. Then, since the superpotential should be an analytic function of the fields; *i.e.* the terms in the effective superpotential cannot be generated from interaction terms which have a nonpolynomial dependence on $b_M$ fields. Note, that this statement is not true for the gaugino masses and the chiral multiplet fermionic masses which mix with gauginos. In this case these masses do not arise from the F term, thus the argument of the analyticity does not apply.

On the other hand, functions $f$ and $g$, which carry the information of the l-sector, should be constructed explicitly, due to the lack of the local superconformal invariance in the l-sector. Then the $U(1)_{li}$ charges corresponding to the fermionic fields, $\tilde{s}^{\bar{i}}$, are determined by the lattice vector and the bosonic derivatives $\partial_{\bar{z}}X$ are determined by the type of bosonic creation operators in the l-sector. For example, a state of the twisted sector represented as $\bar{\tilde{\alpha}}^{\bar{i}}_{-k_i/N}\,|k_1/N, k_2/N, k_3/N\rangle_l$ would have $g = \partial_{\bar{z}}\bar{X}^{\bar{i}}\tilde{s}$ with $\tilde{s} = \exp\left[ik_1/N(H_1)_l + ik_2/N(H_2)_l + ik_3/N(H_3)_l\right]$.¶

From $(V_B)_{-1}$, $(V_B)_0$ in the 0-picture is obtained by using eq. (2). From the form (3) for $T_F$ and eq. (7) one sees that in this case $(V_B)_0$ consists in general of terms with $H_r = 0, 2, \text{and} -1$, respectively.

For the fermionic vertices $V_F$ one can also analogously use the picture changing formalism in the r-sector,[23] while the structure of the l-sector remains the same as in (6). For example[23] the fermionic vertex for the untwisted and the singly twisted sector in the $-1/2$ picture can be written in the following way:

$$(V_F)_{-1/2} = \exp\left(-\phi/2\right)u\prod_j \exp\left(-H_j/2\right)\psi^i \exp\left(ik_\mu X^\mu\right)f(\partial_{\bar{z}}X, \partial_{\bar{z}}\bar{X}, \bar{\tilde{\psi}}), \tag{8.a}$$

untwisted sector

$$(V_F)_{-1/2} = \exp\left(-\phi/2\right)u\prod_i \sigma_i \exp\left(-H_i/2\right)s_i \exp\left(ik_\mu X^\mu\right)g(\partial_{\bar{z}}X, \partial_{\bar{z}}\bar{X}, \tilde{s}),$$

twisted sector.

(8.b)

Here $u$ refers to the spinor of the four uncompactified dimensions. Analogously

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¶ The function $g$ for the case of the blowing-up modes $b_M$ can be obtained in the following way. It turns out that the vertex operator of $b_M$ for the l-sector is the vertex operator in the 0-picture; *i.e.* this vertex is obtained by using (2). However, now all the notation applies to the l-sector. For example in the $Z_3$ orbifold one would get $g = \lim_{z\to w}\sum_{\bar{i}}\partial_{\bar{z}}\bar{\tilde{X}}^{\bar{i}}\bar{\tilde{\psi}}^{\bar{i}}\prod_j \exp\left[i\tilde{k}_j/N(H_j)_r\right]$ with $\tilde{k}_j/N = \frac{1}{3}$. The state $b_M$ is in turn described as

$$|0\rangle \times \left[\bar{\tilde{\alpha}}^1_{-1/3}\left|-\frac{2}{3},\frac{1}{3},\frac{1}{3}\right\rangle_l + \bar{\tilde{\alpha}}^2_{-1/3}\left|\frac{1}{3},-\frac{2}{3},\frac{1}{3}\right\rangle_l + \bar{\tilde{\alpha}}^3_{-1/3}\left|\frac{1}{3},\frac{1}{3},-\frac{2}{3}\right\rangle_l\right].$$

one can obtain the vertex operators for the other twisted sectors. Again the constraint (4) for $H_r$ is satisfied, *i.e.* $H_r = \sum_i k_i/N - \frac{3}{2} = -\frac{1}{2}$.

With the explicit form of the vertices (6,3,8), one can now evaluate the amplitudes in the background of the blown-up orbifolds, *i.e.* $\langle b_M \rangle \neq 0$. These amplitudes should obey the following selection rules.[23]

1. The total $\phi$ charge equals $-2$.
2. $(H_i)_r$ charges should be separately conserved.
3. $(H_i)_l$ charges should be separately conserved.
4. The amplitude should be twist invariant. By this one means that in the amplitude the twist numbers $J_i$ associated with the bosonic twist fields $\sigma^{J_i}$ of the $g^{J_i}$ twisted sectors should sum up to $(0 \mathrm{mod} N)$.
5. The amplitude should be invariant under the automorphisms of the lattice. In general the amplitudes depend on the bosonic coordinates $X^i$. Transformations on these coordinates which are in the group automorphism of the lattice should certainly leave the amplitudes invariant.
6. The location of the twist fields should satisfy the space group selection rules described in detail in ref. 23. They essentially determine the location of string states, *i.e.* the location of the fixed points at which the particular states are located.

Selection rules (1-4) can be in general trivially satisfied. The worldsheet fermionic degrees of freedom are taken care of by applying the selection rules (1-3). Note also that the $(H_i)_l$ conservation essentially implies that the amplitude should be gauge invariant. Also some amplitudes could be determined to be zero by simply applying the selection rule 5.

When calculating the amplitudes (5) which probe the terms of the superpotential, one sees that only the terms of $(V_B)_0$ with $H_r = 0$ contribute. Namely, using (4) (*i.e.* for $V_{(-1/2,-1)}$, $H_r = -\frac{1}{2}, 1$, respectively), one sees that only the terms of $(V_B)_0$ proportional to $\partial X^i \bar{\psi}^{\bar{i}}$ survive in such amplitudes in order to conserve the total $H_r$ charge. Thus, the terms in $(V_B)_0$ coming from $\partial X^\mu \psi^\mu$, *i.e.* terms proportional to the four-dimensional external momenta $k^\mu$, do not contribute. Then such amplitudes assume the following form in general:

$$\langle V_{-1/2} V_{-1/2} V_{-1} V_0 \ldots V_0 \rangle \propto \left\langle \partial_z X^i, \ldots, \partial_{\bar{z}} X^j, \ldots, \partial_{\bar{z}} \bar{X}^{\bar{k}} \right\rangle_{\sigma^{J_1} \ldots \sigma^{J_n}} . \qquad (9)$$

This in turn implies that the effective superpotential calculated in this way cannot be mimicked by a massless exchange of gauge or gravitational particles because the amplitudes of such exchanges would be proportional to $k^2$ which are absent in our case. This is a plausible result, since it only confirms that the interactions arising from the D-term cannot mimic the F-terms. Thus if one obtains a zero amplitude for a certain term, this is a genuine zero value of the corresponding term in the superpotential. On the other hand the first nonzero value for a certain amplitude in the blowing-up procedure would also *directly* determine the value of the corresponding term in the superpotential.

EXPLICIT RESULTS- MASS SPECTRUM

1. *Four-dimensional N=1 supergravity multiplets.* We have explicitly verified that these multiplets remain massless, thus confirming that preserving the supersymmetry of the blown-up orbifolds remains intact.

2. *Gauge multiplets.* To the leading order in blowing-up procedure we checked explicitly that the additional gauge group $G_0$ is completely broken, thus leaving the gauge symmetry of the blown-up orbifold to be $E_6 \times E_8$. The mass-term for the mixing between the gauginos and the $E_6$ singlets $b_I$ of the twisted sector which have bosonic excitations in the l-sector is in the leading order of the blowing-up procedure of the following form:[⋆,†]

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⋆ The mixing between the gauginos and the singlets without bosonic excitations (in the l-sector) $1_i$, which do exist in the $Z_4$ orbifold turns out to be zero because the selection rule 5 is not satisfied.

† One also notices that the blowing-up modes $b_M$ which are a particular combination of $b_I$ modes *does not* couple to gauginos, in agreement with the observation that these modes remain massless also after the blowing-up procedure.

$$\left\langle V^{\lambda}_{-1/2} V^{b_I}_{-1/2} (V^{b_M}_{-1})^{\dagger} \right\rangle_{\sigma^J \sigma^{J^{-1}}} \propto \langle b_M \rangle^* \, \mathcal{O}(1) \tag{10}$$

3. **$27\overline{27}$** *pairing.* We studied the question whether **$27\overline{27}$** pair-up for the $Z_4$ orbifold.[‡] For the most general cubic $Z_4$ lattice we found that there is *no pairing-up* of **27** and **$\overline{27}$** to all orders in the blowing-up procedure. This result is due to the selection rule 5. Namely, the amplitude for this mass term is proportional schematically to:

$$m_{\mathbf{27},\overline{\mathbf{27}}} \propto \left\langle \partial X_3{}^{2n+1} \right\rangle \tag{11}$$

   Here the partial derivative is with respect to $z$ and $\bar{z}$. This equation is true for any number of the blowing-up mode insertions. One then notices that for the $Z_4$ orbifold one can independently rotate the third coordinate by 180°. This in turn ensures the zero value of amplitude (11) to all orders in the blowing-up procedure.[§]

4. $E_6$ *singlets.*

   (a) *Modes corresponding to the moduli space.* These are the modes corresponding to the moduli space of the six-torus and the blowing-up modes associated with the orbifold singularities. We checked explicitly that the modes have no mass terms[¶] in the background of the blown-up orbifold.

   (b) *Other $E_6$ singlets.* In general these modes acquire nonzero mass. For the $Z_3$ orbifold, for example, there are nine such modes located at 27

‡ Note, in the case of $Z_3$ orbifold there are no **$\overline{27}$**'s. Also the pure fact that the blown-up orbifolds are supersymmetric ensures that there is no mass term for **27**'s.

§ In ref. 27 we show that this result is general for any orbifold or Calabi-Yau manifold barring nonperturbative effects of the modes corresponding to the moduli space of the VEV's.

¶ In ref. 27 we show that this is a general feature; namely modes corresponding to the moduli space have *flat* potential for any orbifold or Calabi-Yau manifold, again barring nonperturbative effects in the VEV's of the moduli.

fixed points, which we denote as $b_{(ii'f_\alpha)}$ with $(i,i') = 1,\ldots,3$ and $f_\alpha$ denoting a particular fixed point.* In the leading order of the blowing-up procedure the mass terms among these modes assume the following form:

$$m_{[(ii'f_\alpha),(jj'f_\beta)]} = \sum_{\tilde{\nu},k} \epsilon_{i'j'k}\nu_i\nu_j\nu_k \exp\left[-\pi|\tilde{\nu}|^2/(2\sqrt{3})\right]\left\langle b_{(kkf_\gamma)}\right\rangle \tag{12}$$

with the coset vectors being determined as[23]

$$\tilde{\nu} \in (1-\theta^2)(f_\beta - f_\gamma + \Lambda). \tag{13}$$

Here the rotation $\theta = \exp(i2\pi/3)$ and $\Lambda$ is the lattice vector. Note that the blowing-up mode at the fixed point $f_\alpha$ corresponds to:

$$b_M = \sum_i b_{(iif_\alpha)}. \tag{14}$$

From the explicit form (12) one obviously concludes that these mass terms are in general nonzero. Also they are exponentially damped, thus indicating the nonperturbative instanton-type contribution. Note also, that the $b_M$'s do *not* possess any mixing term with any $b_{(ii'f_\alpha)}$.

In the case of the $Z_4$ orbifold there are also singlets without bosonic excitations in the l-sector, whose number is at least the number of $\overline{\mathbf{27}}$.** It turns out that there is no mass term among such singlets, because the corresponding amplitude is again proportional to the odd-powers of $X_3$ derivatives, thus not being invariant under the automorphisms of the lattice. However, the analysis of the *total* singlet mass matrix reveals that *all* those singlets become massive due to the mixing mass terms between the singlets with the bosonic excitations and the ones without them which are in general exponentially damped.

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* Note *e.g.*, $b_{(i1f_\alpha)} = |0\rangle_r \times \bar{\tilde{\alpha}}^i_{-1/3}\left|-\frac{2}{3},\frac{1}{3},\frac{1}{3}\right\rangle_l$; $(i = 1,2,3)$, located at $f_\alpha$. Thus, $b_M = |0\rangle_r \times \left\{\bar{\tilde{\alpha}}^1_{-1/3}\left|-\frac{2}{3},\frac{1}{3},\frac{1}{3}\right\rangle + \bar{\tilde{\alpha}}^2_{-1/3}\left|\frac{1}{3},-\frac{2}{3},\frac{1}{3}\right\rangle + \bar{\tilde{\alpha}}^3_{-1/3}\left|\frac{1}{3},\frac{1}{3},-\frac{2}{3}\right\rangle\right\}$.

** Note that the number of such $E_6$ singlets is bound from below by the Hodge number $b_{(2,1)}$.[24]

EXPLICIT RESULTS- YUKAWA COUPLINGS

The phenomenologically interesting Yukawa couplings are the couplings of three **27**-plets, since these may determine the mass spectrum of the family generations, thus possibly shedding light on the fermion mass hierarchy problem. Another interesting Yukawa coupling to be determined is the one between the $\mathbf{27}\overline{\mathbf{27}}$ and the $E_6$ singlets.

Also it is of general interest to calculate effective terms of the superpotential of dimension 4 or higher, since these terms may be relevant for the mechanism to generate the intermediate scale.[30] These calculations will be presented elsewhere.[31]

1. *Yukawa couplings of the three* **27***'s.* Some of these Yukawa couplings are nonzero already in the orbifold limit, in particular those couplings whose total twist number is zero. For example for the $Z_3$ orbifold, the following Yukawa couplings are nonzero:[23]

$$h_{[\mathbf{27}_{(ii')},\mathbf{27}_{(jj')},\mathbf{27}_{(kk')}]} = \epsilon_{ijk}\epsilon_{i'j'k'} \tag{15.a}$$

$$h_{[\mathbf{27}_{f_\alpha},\mathbf{27}_{f_\beta},\mathbf{27}_{f_\gamma}]} = \sum_{\tilde{\nu}} \exp\left[-\pi|\tilde{\nu}|^2/(2\sqrt{3})\right] \tag{15.b}$$

with $\tilde{\nu}$ being defined in eq. (13). However, other Yukawa couplings become nonzero after the blowing-up procedure. For the $Z_3$ orbifolds one obtains the following explicit results in the leading order of the blowing-up procedure:

$$h_{[\mathbf{27}_{(ii')},\mathbf{27}_{f_\alpha},\mathbf{27}_{f_\beta}]} = \sum_{(\tilde{\nu},k)} \delta_{ik}\delta_{i'k}|\tilde{\nu}_k|^2 \exp\left[-\pi|\tilde{\nu}|^2/(2\sqrt{3})\right] \left\langle b_{(kkf_\gamma)}\right\rangle \tag{15.c}$$

$$\begin{aligned} h_{[\mathbf{27}_{(ii')},\mathbf{27}_{(jj')},\mathbf{27}_{f_\alpha}]} = &\sum_{(\tilde{\nu},k,l)} (1-\delta_{kl})\delta_{ik}\delta_{i'k}\delta_{jl}\delta_{j'l}|\tilde{\nu}_k|^2|\tilde{\nu}_l|^2 \\ &\times \exp\left[-\pi|\tilde{\nu}|^2/(2\sqrt{3})\right] \left\langle b_{(kkf_\beta)}\right\rangle \left\langle b_{(llf_\gamma)}\right\rangle \end{aligned} \tag{15.d}$$

with $\tilde{\nu}$ again being defined in eq. (13). $\mathbf{27}_{(ii')}$ with $(i,i') = 1,2,3$ refers to the nine different **27**-plets in the untwisted sector, while $\mathbf{27}_{f_\beta}$ refers to the **27**-plet arising from the string state located at the fixed point $f_\beta$. One sees from this explicit calculation that the blowing-up procedure can allow for an additional hierarchy in the Yukawa couplings. Namely, besides the exponentially damped terms $\propto \exp(-R^2/\alpha')$, there is an additional hierarchy $\propto R^2/\alpha' \times \langle b_M \rangle$. This may be relevant for the understanding of the fermion mass hierarchy. The explicit calculation within other possibly phenomenologically acceptable orbifolds is needed in order to further elaborate on this idea.

2. *Yukawa couplings of* $\mathbf{27}\overline{\mathbf{27}}$ *and singlets.* We checked that all the **27**-plets couple with *all* the $\overline{\mathbf{27}}$-plets and a particular singlet without the bosonic excitations in the leading order of the blowing-up procedure for the $Z_4$ orbifold. Some of these Yukawa couplings are again nonzero already in the orbifold limit, while the rest of these Yukawa couplings become nonzero after the blowing-up procedure. The value of these terms is again damped exponentially.

The explicit calculation of the mass terms and Yukawa couplings for the $Z_3$ and $Z_4$ blown-up orbifolds confirmed the general statement [18] about the structure of the effective Lagrangian for the Calabi-Yau manifold. Using the above method, the values of parameters are (can) be obtained explicitly.

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