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PHENOMENOLOGICAL IMPLICATIONS OF THE BLOWN-UP ORBIFOLDS*†

MIRJAM CVETIČ

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

ABSTRACT

We discuss the structure of the effective Lagrangian for the (2,2) Z_N orbifolds and the corresponding Calabi-Yau manifolds which are obtained by "blowing-up" the orbifold singularities. The method to "blow-up" such singularities is reviewed. Results are exact at the string tree-level. In particular the question of generating an intermediate scale M_I in such models is addressed. It is shown that for Z_N orbifolds (except one) *and* the corresponding blown-up orbifolds which are compactified on any six-torus T^6 which can be obtained by continuously deforming $T^4 \otimes T^2$, all the terms of the type $(\mathbf{27}\overline{\mathbf{27}})^K$ are absent from the effective superpotential, thus questioning the mechanism for generating a large intermediate scale for such compactifications.

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1. Introduction

Different compactifications of superstring theories whose four-dimensional effective field theories possess a realistic gauge group, $N = 1$ supergravity and quarks and leptons as elementary fields have been proposed recently.[1-12] They are believed to be consistent superstring vacua to all finite orders in string perturbation theory.

Here we will not discuss more general compactifications of the heterotic string, which require only (super)conformal invariance of the worldsheet action, with the contribution of the matter fields to the Virasoro and super-Virasoro central charges cancelling the ghost contribution, *i.e.* $\hat{c}_l = 26$ and $\hat{c}_r = 10$, plus modular invariance of scattering amplitudes.[8-12] We shall rather study phenomenological implications of originally proposed compactifications of the $E_8 \times E_8$ heterotic string[13] on Calabi-Yau manifolds[1] or left-right symmetric orbifolds,[4] in which the spin and gauge connections are identified. In these cases the theory possesses (2,2) worldsheet supersymmetry, *i.e.* there is both a left-moving (l) *and* a right-moving (r) $N = 2$ worldsheet superconformal algebra.[1,14,15]

Orbifolds are especially attractive because interactions on orbifolds can be calculated *exactly* at the string tree-level.[16,17] Thus all the parameters of the tree-level effective Lagrangian can be determined exactly, *i.e.* including contributions which are nonperturbative in the ratio $\sqrt{\alpha'}/R$, where α' is the string tension and R is the radius of the orbifold. For example, the effects of worldsheet instantons are automatically incorporated.

On the other hand the methods for explicitly studying string interactions on

Calabi-Yau manifolds is limited, partly due to the lack of an explicit metric. The field theory limit ($\sqrt{\alpha'}/R \rightarrow 0$)results [1,18] state that the numbers of particular types of massless modes are determined by the Hodge numbers, the topological invariants of the Calabi-Yau manifolds. Also, certain Yukawa couplings [19,20] are determined by similar topological considerations. Nonperturbative contributions to the effective Lagrangian for Calabi-Yau compactifications have been explored [21] by studying worldsheet instantons. One result of this analysis is that some parameters of the effective Lagrangian *can* be modified by worldsheet instanton contributions, which are proportional to $\exp(-R^2/\alpha')$. It has been shown [21] that Yukawa couplings as well as masses of the matter E_6 singlets receive nonzero corrections in general, while **27** and $\overline{\mathbf{27}}$ do not pair-up. However, the calculation is not entirely explicit, due to the unknown metric.

A complementary approach to studying the complete tree-level effective Lagrangians for Calabi-Yau models has been given [22] by choosing a Calabi-Yau manifold which is constructed by repairing ("blowing-up") the singularities of an orbifold. This approach makes use of the fact that each orbifold singularity is associated with massless scalar fields – blowing-up modes – whose potential is flat to all orders in the string loop expansion.[16,17] Thus any vacuum expectation value (VEV) of these modes corresponds to a vacuum solution to the string equations of motion, at least perturbatively in the VEV's. The case with all blowing-up modes having zero VEV corresponds to the orbifold limit, while nonzero VEV's for the mode located at a particular singularity corresponds to repairing that singularity. Scattering amplitudes in the repaired Calabi-Yau background – and hence also parameters of the effective Lagrangian – can be calculated by inserting successively larger numbers of *background* blowing-up modes into orbifold amplitudes. Although this method is perturbative in the blowing-up VEV's, it enables one to obtain *explicit* values for parameters of the blown-up orbifolds, thus giving *exact* results at the string tree-level.

In this paper we shall study the general structure of the effective Lagrangian of the Abelian Z_N orbifolds as well as their blown-up versions corresponding to the Calabi-Yau manifolds by using the above method.[22] We shall first summarize already obtained[22] results for parameters of dimension 4 or smaller for Z_3 and Z_4 orbifolds and their blown-up versions. Then we shall concentrate on higher dimensional operators, for general Z_N (blown-up) orbifolds, in particular $(\mathbf{27}\overline{\mathbf{27}})^K$ (with $K \geq 2$) terms of the superpotential. Such terms are relevant for generating an intermediate scale[23] M_I and therefore understanding the structure of such terms.

The rest of the paper is organized as follows. In sect. 2 we review general properties of Calabi-Yau and orbifold models, with the emphasis on the nature of interactions in the models. In sect. 3 we outline the calculation for the parameters of the effective Lagrangian for a general Abelian (blown-up) orbifold and summarize results for the parameters of dimension ≤ 4. In sect. 4 we revisit the intermediate scale mechanism and address the question of higher dimensional operators in the superpotential, which are of the form $(\mathbf{27}\overline{\mathbf{27}})^K$, $(K \geq 2)$. Phenomenological relevance of the obtained results is emphasized. Conclusions are given in sect. 5.

2. Features of Calabi-Yau and Orbifold Models

Calabi-Yau models give rise to $N = 1$ supergravity in four dimensions and gauge group:*

$$G = E_6 \times E_8. \tag{1}$$

The massless particle spectrum consists of the gauge and the gravity supermultiplets as well as zero modes (moduli) of the Ricci-flat (to $\mathcal{O}(\alpha')$) Calabi-Yau metric. In addition there are massless matter multiplets, **27**'s, $\overline{\mathbf{27}}$'s, and (perhaps) **1**'s (the so-called matter singlets) of E_6 which are all singlets of E_8.

Due to the local right-moving superconformal invariance[1,14,15] one can use the picture-changing formalism, in which vertices for a given state appear with different ghost numbers for the bosonized right-moving superconformal ghost ϕ; *i.e.* they appear in different "pictures".[15,25] Tree-level amplitudes involve collections of vertices such that the total ghost number equals -2.[15] This simplest form of the vertex operator for a space-time fermion is the $-1/2$ picture, while that for a space-time boson is the -1 picture. The picture-changing formalism enables one to obtain vertices in other pictures. For example, the vertex for a space-time boson in the 0 picture is obtained in the following way:[15]

$$(V_B(z))_0 = \lim_{w \to z} \exp(\phi) T_F(w) (V_B(z))_{-1}. \tag{2}$$

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* Space-time supersymmetry implies that the Calabi-Yau spin connection has $SU(3)$ holonomy; the orbifold holonomy group is a discrete subgroup of $SU(3)$. In general the gauge group (1) could be broken further at the compactification scale by employing the Wilson-loop mechanism.[24] However, this will not affect the study of the general structure of the effective Lagrangian.

Here $(V_B(z))_{-1}$ is the corresponding vertex operator in the -1 picture and

$$T_F = T_F^{\rm int}(X^i, \bar{X}^{\bar{i}}, \psi^i, \bar{\psi}^{\bar{i}}) + \partial X^\mu \phi^\mu. \tag{3.a}$$

is the worldsheet supersymmetry generator[15] – the energy-momentum tensor. Here X and ψ are the string bosonic and fermionic coordinates, respectively; the indices $(i, \bar{i}) = (1,2,3)$ and $\mu = (1,2,3,4)$ denote the three complex internal and the four space-time dimensions, respectively. Partial derivatives are with respect to the right-moving worldsheet coordinate z. For an orbifold model, $T_F^{\rm int}$ takes the simple form:

$$T_F^{\rm int} = \partial X^i \bar{\psi}^{\bar{i}} + \partial \bar{X}^{\bar{i}} \psi^i. \tag{3.b}$$

The left- (right-) moving $N = 2$ superalgebra of a (2,2) model incorporates a $U(1)_l$ ($U(1)_r$) current algebra, generated by $J_l = -i\sqrt{3}\bar{\partial} H_l$ ($J_r = -i\sqrt{3}\partial H_r$), where $H_l(\bar{z})$ ($H_r(z)$) is a free left- (right-) moving scalar field. Vertex operators can be classified according to their $H_{l(r)}$ charge. One can, for example, determine the H_r charges for vertices for the massless chiral supermultiplets in various pictures. One finds that

$$\begin{aligned} H_r &= 1 \qquad && -1 \text{ picture} \\ H_r &= -1/2 \qquad && -1/2 \text{ picture} \end{aligned} \tag{4}$$

for the four dimensional chiral superfield with positive chirality.

Another feature of these compactifications is that *every* such vacuum can be continuously deformed to a nearby vacuum of the same (2,2) type.[16,17,21,22,26] In field theoretical language this corresponds to a flat potential for massless scalars which correspond to the "moduli" of the compactified space.† In the Calabi-Yau case the moduli are identified with the zero modes of the metric. Namely, giving vacuum expectation values (VEV's) to the moduli in one conformally invariant background generates a nearby background configuration which is also a vacuum solution, at least perturbatively in these VEV's. This procedure can be carried out explicitly for the case of deforming an orbifold into the corresponding Calabi-Yau manifold by giving VEV's to the "blowing-up" modes [16,17] as was examined in detail in Ref. 22.

Orbifolds are a special limit of particular Calabi-Yau manifolds. This is a six-torus T^6 with points being identified under a group P of discrete rotations θ:

$$\Omega = T^6/P. \tag{5}$$

This identification leaves some points or even two-tori, T_2 fixed. We shall confine our analysis to the Z_N orbifolds where the group of rotations $Z_N = (\theta^J, J = 1, \ldots, N-1)$ is also the discrete holonomy group which should be a subgroup of $SU(3)$ in order to end up with a four-dimensional supersymmetric theory. Also each discrete space rotation θ is accompanied by the corresponding discrete gauge connection γ. For (2,2) orbifolds one chooses $\theta = \gamma$, thus identifying spin

† It turns out [22,26] that the l-sector of the vertex operators for moduli is the same as the vertex operator in the 0 picture. In the theory with a local conformal invariance the vertex operators in the -1 and 0 pictures correspond to the lower and upper components of the worldsheet superfields-primary fields, [15,25] which in the case of "moduli" have conformal dimension $h = \frac{1}{2}$, *i.e.* they satisfy the constraint $2h = 1$ and $H = +1$ and $H = -1$ for worldsheet superfields-primary fields with positive and negative chirality, respectively. — Note also that the primary fields with $H = +1$ correspond to "moduli" transforming as (1,1) forms while those with $H = -1$ correspond to "moduli" transforming as (1,2) forms of the compactified space.

and gauge connection. The states of the four-dimensional theory should also be invariant under the diagonal transformation $g = (\theta, \gamma)$.

The massless spectrum[4] falls into the untwisted (g^0) and the twisted ($g^J, J = 1, \ldots, N-1$) sectors depending on whether the massless states arise as excitations of the string with periodic or twisted boundary conditions, respectively. Note that states arising from strings with twisted boundary conditions are located at a particular fixed point.

Orbifolds possess the following *additional* features:

1. *Enlarged gauge group.* In addition to the gauge group (1) there is a gauge group $G_0 \subseteq SU(3)$ which commutes with the discrete holonomy group of the orbifolds, *e.g.* the Z_N holonomy group for a Z_N orbifold. For Z_N orbifolds, G_0 is either $SU(3)$, $SU(2) \times U(1)$, or $U(1) \times U(1)$.

2. *Enlarged symmetry of the effective Lagrangian.* A Z_N orbifold possesses a Z_N symmetry which can be described as an additional selection rule on interactions. Blowing-up modes carry nonzero charge under these symmetries. Thus many nonzero parameters of the Calabi-Yau manifold become zero in the orbifold limit, including certain mass terms and Yukawa couplings of matter multiplets.

3. *Increased worldsheet symmetry.* In particular, the $U(1)_{(l,r)}$ worldsheet symmetry of the (l,r)-sector is enlarged to $[U(1) \times U(1) \times U(1)]_{(l,r)}$ for a Z_N orbifold. Thus, instead of the two conserved charges $H_{(l,r)} \equiv \sum_{i=1}^{3} (H_i)_{l,r}$, there are now six conserved charges $H_{1\,l,r}$, $H_{2\,l,r}$, and $H_{3\,l,r}$.

H_i charges are classified[4] for *all* the Z_N orbifolds. The $(H_i)_r$ charges are related to the matrix of the discrete rotation θ acting on the three compactified coordinates. For example for Z_3 orbifolds $\theta = (\omega, \omega, \omega)$ in its diagonal form. Here $\omega = \exp(2\pi i/3)$. Such a θ in following determines $(H_i)_r$ charges of the singly-twisted sector (g^1) which are $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ in the -1 picture. In table I we give[4] $(H_i)_r$ charges of the singly-twisted sector for all the Z_N orbifolds possessing $N = 1$ supersymmetry.

N	$(H_1)_r$	$(H_2)_r$	$(H_3)_r$
3	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
4	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
6	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
$6'$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
7	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{4}{7}$
8	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{5}{8}$
$8'$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
12	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{1}{2}$
$12'$	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{7}{12}$

TABLE I. $(H_i)_r$ charges of the singly-twisted sector in the -1 picture for Z_N orbifolds possessing $N = 1$ supersymmetry.

$(H_i)_r$ charges in turn uniquely determine the r-sector of the vertex operator for emission of massless states at the string tree-level. For example in the -1 picture (emission of a massless boson) and the $-\frac{1}{2}$ picture (emission of a massless fermion) the r-sector of the vertex operators are the following:

$$(V_{B_r})_{-1} = \exp(-\phi)\psi^j \exp(ik_\mu X^\mu) \qquad \text{untwisted sector} \qquad (6.a)$$

$$(V_{B_r})_{-1} = \exp(-\phi) \prod_i \sigma_i s_i \exp(ik_\mu X^\mu) \qquad \text{twisted sector} \tag{6.b}$$

$$(V_{F_r})_{-1/2} = \exp(-\phi/2) u \prod_i \exp(-(H_i)_r/2) \psi^j \exp(ik_\mu X^\mu) \qquad \text{untwisted sector} \tag{7.a}$$

$$(V_{F_r})_{-1/2} = \exp(-\phi/2) u \prod_i \sigma_i \exp(-(H_i)_r/2) s_i \exp(ik_\mu X^\mu) \qquad \text{twisted sector.} \tag{7.b}$$

Here $\mu = 1, \dots, 4$ and $(i, \bar{i}) = 1, \dots, 3$ again refer to the four space-time and the six compactified dimensions, respectively and u refers to the spinor of the four uncompactified dimensions. The bosonic twist fields σ^i and the fermionic twist fields s^i correspond to the emission of the massless state from the propagating string with the twisted boundary conditions for the bosonic X^i and the fermionic ψ^i coordinates, respectively.[17] Fermionic fields are presented in terms of the three bosonic $U(1)_r$ charges:

$$\psi^j = \exp\left[i(H_j)_r\right], \quad \bar{\psi}^j = \exp\left[-i(H_j)_r\right] \tag{8.a}$$

$$s^j = \exp\left[ik_j/N(H_j)_r\right], \quad \bar{s}^j = \exp\left[-ik_j/N(H_j)_r\right]. \tag{8.b}$$

The three separate charges $(H_j)_r$ should satisfy the constraint that $H_r = \sum_j (H_j)_r = \sum_j k_j/N = 1$. For example, for the singly-twisted sector of the Z_3 orbifold $k_j/N = \frac{1}{3}, i = 1, 2, 3$.

On the other hand, the part of the vertices which carry the information of the l-sector, should be constructed explicitly, due to the lack of the local superconformal invariance in the l-sector; *i.e.* the picture changing formalism does not apply. However, for each state one can again explicitly determine the $(H_i)_l$ charges corresponding to the fermionic fields $\widetilde{s}^{\bar{i}}$. They are determined by the first three entries of the lattice vectors of the $\Gamma_8 \times \Gamma_8$ lattice. These three entries correspond to the additional gauge symmetry G_0. On the other hand the bosonic derivatives $\partial_{\bar{z}} \bar{X} (\partial_{\bar{z}} X)$ appear in the vertex operator whenever the state possesses a corresponding creation operator $\bar{\tilde{\alpha}}$ $(\tilde{\alpha})$.* For example a state of the twisted sector represented as $\bar{\tilde{\alpha}}^{\bar{i}}_{-k_i/N}\,|k_1/N, k_2/N, k_3/N\rangle_l$ would have the vertices (6.b,7.b) multiplied by the expression $g = \partial_{\bar{z}}\bar{X}^{\bar{i}}\tilde{s}$ with $\tilde{s} = \exp\left[ik_1/N(H_1)_l + ik_2/N(H_2)_l + ik_3/N(H_3)_l\right]$. In general the l-sector of the vertex operators should be constructed separately for each massless state. However, there is a general prescription for the vertex operators of the "moduli", *i.e.* the "blowing-up" modes corresponding to the "blowing-up" of the fixed points *and* the massless modes corresponding to the deformation of the six-torus T^6. It turns out that the l-sector of these vertex operators is the vertex operator in the 0 picture, obtained by using eq. (2) with $(V_B)_{-1}$ being of the form (6). However, now all the notation applies to the l-sector, *e.g.* $z \to \bar{z}$, $w \to \bar{w}$, $(H_i)_r \to (H_i)_l$, etc. For example the l-sector of the "blowing-up" modes of the Z_3 orbifolds is obtained in the following way. In this case there is one "blowing-up" mode b_M located at each of the 27 fixed points of the singly-twisted sector (g^1). This b_M transforms as (1,1) and therefore the corresponding vertex operator for the l-sector at zero external momentum can be obtained by using the first term of T_F (see eq. (3.b)) which annihilates the state with $H_l = 1$, *i.e.* the vertex operator in the -1 picture corresponds to the upper component of the worldsheet superfield with positive chirality. Then using (2) one obtains the l-sector of the vertex operator for such a b_M as $g = \lim_{\bar{z}\to\bar{w}}\sum_{\bar{i}}\partial_{\bar{z}}\bar{X}^{\bar{i}}\bar{\tilde{\psi}}^{\bar{i}}\prod_j \exp\left[i\tilde{k}_j/N(H_j)_l\right]$ with $\tilde{k}_j/N = \frac{1}{3}$. Such a b_M is then in turn described as $|0\rangle_r \times \left[\bar{\tilde{\alpha}}^{1}_{-1/3}\,|-\frac{2}{3},\frac{1}{3},\frac{1}{3}\rangle_l + \bar{\tilde{\alpha}}^{2}_{-1/3}\,|\frac{1}{3},-\frac{2}{3},\frac{1}{3}\rangle_l + \bar{\tilde{\alpha}}^{3}_{-1/3}\,|\frac{1}{3},\frac{1}{3},-\frac{2}{3}\rangle_l\right]$. On the other hand, for "moduli" which transform as (1,2) forms the corresponding vertex operator of the l-sector is obtained by acting with the second term of T_F on the vertex operator in the -1 picture corresponding to the upper component of the worldsheet superfield with negative chirality and thus $H_l = -1$. Such "moduli" for example appear in the case of Z_4 orbifolds (see Table I) in the

* This result of course emerges as a consequence of the first quantization of the bosonic string coordinates. Note that the only surviving term of $\partial_{\bar{z}}X$ $(\partial_{\bar{z}}\bar{X})$ coordinates arises from the first term in the "oscillator" expansion of the bosonic coordinates.

untwisted (g^0) and doubly-twisted (g^2) sectors.

One can also make general statements about the structure of the l-sector for **27** and $\overline{\mathbf{27}}$ of E_6. For the parts of **27** ($\overline{\mathbf{27}}$) transforming as **16** ($\overline{\mathbf{16}}$) of $SO(10)$ the vertex operator corresponding to the l-sector can be represented as vertex operators in the $-1/2$ picture with $H_l = -\frac{1}{2}$ ($H_l = +\frac{1}{2}$) (see eqs. (7)), respectively.* For example the first three entries of the lattice vectors (corresponding to the gauge symmetry G_0) for **16**'s for the singly-twisted sector of the Z_3 orbifold is $\left|-\frac{1}{6},-\frac{1}{6},-\frac{1}{6}\right\rangle_l$.

The above results also imply that in the orbifold limit the number of **27**'s ($\overline{\mathbf{27}}$'s) is always the same as the number of "moduli" which transform as (1,1) ((1,2)) forms of the compactified space.† General analysis for the $(H_i)_{l,r}$ charges for **27**'s, $\overline{\mathbf{27}}$'s, and the moduli will be given in sect. 4.

Along with these "moduli" there are also additional massless **1**'s of E_6, the so-called matter singlets. Some are states with bosonic excitations in the l-sector, while some appear without them. One can convince oneself by explicit construction that the number of the latter ones is at least equal to the number of $\overline{\mathbf{27}}$'s.‡ Those matter singlets which correspond to $\overline{\mathbf{27}}$'s can be obtained from the part of $\overline{\mathbf{27}}$'s transforming as a singlet under $SO(10)$ by changing one $(H_i)_l$ charge by two units. For example for the Z_4 orbifold (see Table I) in the doubly-twisted sector the singlet part of $\overline{\mathbf{27}}$'s is denoted as $\left|-\frac{1}{2},-\frac{1}{2},-1\right\rangle_l$ and the corresponding matter singlet is then denoted as $\left|-\frac{1}{2},-\frac{1}{2},1\right\rangle_l$.

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* It also turns out that for the parts of **27** ($\overline{\mathbf{27}}$) transforming as singlets of $SO(10)$ one can represent the l-sector of the vertex operator with the vertex in the -1 picture corresponding to the upper component of the worldsheet superfields with conformal dimension $h = 1$ and $H_l = +2$ ($H_l = -2$). (See Ref. 26 for details.)

† Note that in the "orbifold terminology" the definition for **27**'s ($\overline{\mathbf{27}}$'s) is just the opposite from the one in "Calabi-Yau terminology".

‡ It has been shown for Calabi-Yau models that in the *field limit*, *i.e.* $\alpha'/R^2 \to 0$, the number of massless matter singlets is bound from below by the Hodge number $h_{(2,1)}$, *i.e.* the number of (2,1) forms of the Calabi-Yau space.

3. Parameters of the Effective Potential – Results for the Operators of Dimension $\leq$ 4

The calculation of parameters of the effective Lagrangian in a particular theory reduces to the study of the corresponding amplitude of the massless states emitted from the string propagating in this particular background. In the orbifold limit the explicit form of the vertices for emission of massless states allows for a direct calculation of the parameters of the effective Lagrangian because all the background blowing-up modes have zero VEV's.

On the other hand for the blown-up orbifold this would correspond to calculating the corresponding amplitudes by including in the *orbifold* amplitudes a successive number of vertices corresponding to the blowing-up modes b_M, which now have nonzero VEV's.

It is most convenient to calculate[22] the following Yukawa-type n-point function:

$$\left\langle V_{F_1} V_{F_2} V_{B_1} \dots V_{B_{(n-2)}} \right\rangle \tag{9}$$

Here V_{F_i} and V_{B_j} denote the vertices for the emission of the massless fermionic and bosonic mode, respectively.

This amplitude enables one to probe the parameters of the superpotential for the blown-up theory directly, unlike the amplitude for n-bosons.§ Also the gaugino masses can be computed directly, thus determining the new gauge group in a direct way.¶

The mass terms for the fermions ψ_1 and ψ_2 arising from the chiral multiplet is obtained by choosing the appropriate vertex operators V_{F_1} and V_{F_2} while all the bosonic vertices V_{B_j} correspond to the vertices for the blowing-up modes. On

§ Note that in this case one is probing the scalar potential, which is the mixture of the F- *and* D-terms.

¶ The new gauge group can in principle be determined also by calculating the gauge boson masses. However, this appears to be more complicated.

the other hand the mass term for the mixing between the fermions, ψ_i, and the gauginos can be obtained by inserting in (9) vertices for the blowing-up modes as well as their complex conjugates, because this arises from the D-term as well.

Yukawa couplings for two fermions and the boson of the chiral multiplets are obtained from (9) by taking all but one bosonic vertices V_{B_j} to be the vertices for the blowing-up modes. Similarly, one can calculate any higher point function in the superpotential, which shall be explored in sect. 4.

With the explicit form of the vertices (7,8,2), one can then evaluate the amplitudes in the background of the blown-up orbifolds, *i.e.* $\langle b_M \rangle \neq 0$. These amplitudes should obey the following selection rules.[17]

1. The total ϕ charge equals -2.
2. $(H_i)_r$ charges should be separately conserved.
3. $(H_i)_l$ charges should be separately conserved.
4. The amplitude should be twist invariant. By this one means that in the amplitude the twist numbers J_i associated with the bosonic twist fields σ^{J_i} of the g^{J_i} twisted sectors should sum up to $\mathcal{O}(\mathrm{mod}N)$.
5. The amplitude should be invariant under the automorphisms of the lattice, *i.e.* under the group of discrete rotations P. In general the amplitudes depend on the bosonic coordinates X^i. Transformations on these coordinates which are in the group of automorphisms of the lattice should certainly leave the amplitudes invariant.
6. The location of the twist fields should satisfy the space group selection rules described in Ref. 17. They essentially determine the location of string states, *i.e.* the location of the fixed points at which the particular states are located.

Selection rules (1-4) can be in general trivially satisfied. The worldsheet fermionic degrees of freedom are taken care of by applying the selection rules (1-3). Note also that the $(H_i)_l$ conservation essentially implies that the amplitude should be gauge invariant. Also some amplitudes could be determined to be zero by simply applying the selection rule 5.

It has been shown[22] that in the amplitudes (9) which probe the terms of the superpotential, only the terms of $(V_B)_0$ with $H_r = 0$ contribute, *i.e.* only terms ($V_{-1/2}$, V_{-1}, $H_r = -\frac{1}{2}, 1$, respectively) proportional to $\partial X^i \bar{\psi}^{\bar{i}}$ survive in such amplitudes in order to conserve the total H_r charge. Thus the terms in $(V_B)_0$ coming from $\partial X^\mu \psi^\mu$, *i.e.* terms proportional to the four-dimensional external momenta k^μ, do not contribute. Then such amplitudes assume the following form in general:[22]

$$\left\langle V_{-1/2}V_{-1/2}V_{-1}V_0 \ldots V_0 \right\rangle \propto \left\langle \partial_z X^i, \ldots, \partial_{\bar{z}} X^j, \ldots, \partial_{\bar{z}} \bar{X}^{\bar{k}} \right\rangle_{\sigma^{J_1} \ldots \sigma^{J_n}} . \tag{10}$$

This in turn implies that the effective superpotential calculated in this way cannot be mimicked by a massless exchange of gauge or gravitational particles; the amplitudes of such exchanges would be proportional to k^2 which are absent in our case.

Explicit calculations for the mass spectrum and Yukawa couplings have been done for the Z_3 and Z_4 blown-up orbifolds. It agrees with the general results of the worldsheet instanton calculations.[21] In particular, *all* the matter singlets acquire masses which are proportional to $\exp(-R^2/\alpha')$ while $\mathbf{27}$and $\overline{\mathbf{27}}$ do not pair-up.* Also, all the "moduli" remain massless† as expected. On the other hand, Yukawa couplings of the form $h_{ija}\mathbf{27}_i\overline{\mathbf{27}}_j\mathbf{1}_a$ for any pair (i,j) are nonvanishing for some a as well as Yukawa couplings of the type $h_{ijk}\mathbf{27}_i\mathbf{27}_j\mathbf{27}_k$ are nonzero in general. Some of these Yukawa couplings are nonzero already in the field theory limit, *i.e.* $\alpha'/R^2 \to 0$, while some become nonzero due to nonperturbative effects.

⋆ This result is general[26] for any orbifold or Calabi-Yau manifold barring nonperturbative effects of the modes corresponding to the moduli space of the VEV's.

† This is a general feature;[26] namely modes corresponding to the moduli space have *flat* potential for any orbifold or Calabi-Yau manifold, again barring nonperturbative effects in the VEV's of the moduli.

4. Intermediate Scale – $(\mathbf{27}\overline{\mathbf{27}})^K$ Terms of the Superpotential

We would now like to address the question of higher-dimensional operators, in particular $(\mathbf{27}\overline{\mathbf{27}})^K/M_K^{(2K-3)}, (K \geq 2)$ terms in the superpotential for a general Z_N (blown-up) orbifold. These terms can be relevant for generating the intermediate scale M_I which breaks the gauge symmetry G appearing at the compactification scale.[23] The need for M_I is inevitable for a large class of models in order to ensure a proper evolution of the gauge coupling constants down to the lowest energies, to prevent fast proton decay, $N-\overline{N}$ oscillations, and/or satisfy other constraints from the low-energy experiments. It is generally believed that M_I should be 10^{16} GeV or larger.[27,28]

The originally proposed intermediate scale mechanism[28] did not take into account the existence of the matter singlets. The M_I was generated by nonzero VEV of a particular S_i and $\overline{S}_j$, which are the singlets of the standard gauge group $SU(3)\times SU(2)\times U(1)$, arising from $\mathbf{27}_i$ and $\overline{\mathbf{27}}_j$, respectively. By choosing the flat direction of the D-term $\langle S_i\rangle \simeq \langle \overline{S}_j\rangle$ and assuming that the soft supersymmetry breaking terms would generate negative mass squared for these fields of order $M_W = 10^2$ GeV, the M_I is bound to be

$$S \equiv \langle S_i\rangle \simeq \langle \overline{S}_j\rangle \equiv M_I = \left[M_W M_K^{(2K-3)}\right]^{1/(2K-2)} \geq \sqrt{M_W} M_{pl} = \mathcal{O}(10^{11}\text{GeV}). \tag{11}$$

This result is obtained by using the fact that in the superpotential the term which would contribute to the part of the potential with the S fields only is the first nonzero term $(\mathbf{27}\overline{\mathbf{27}})^K/M_K^{(2K-3)}$. Since the value of $1/M_K$ is at most of order $1/M_{pl}$, M_I has the lower bound (11).

This originally proposed mechanism should be revisited because by now a better understanding of the structure of the effective Lagrangian has been gained, in particular the role of the matter singlets is well understood. As explained in the previous section, in general each $\mathbf{27}$ and each $\overline{\mathbf{27}}$ couple to a particular matter singlet with nonzero Yukawa couplings and the matter singlets become nonzero due to the nonperturbative effects.[21,22] What impact does this have for the low-energy phenomenology? Yukawa couplings $h_{ija}\mathbf{27}_i\overline{\mathbf{27}}_j\mathbf{1}_a$ in principle spoil the intermediate scale mechanism. However, since the matter singlets do acquire mass $m_1 \propto M_{pl}\exp(-R^2/\alpha') \gg M_I$ ‡ after integrating the heavy singlets out the effective contribution of the above Yukawa couplings is damped, *i.e.* $h^2_{ija} \rightarrow (h_{ija}M_I/m_1)^2$ due to the decoupling theorem.[30] Minimization of the potential yields in general an upper bound on M_I:

$$M_I = (M_W m_1/h_{ija})^{1/2} \leq \sqrt{M_W}M_{pl} = \mathcal{O}(10^{11}\mathrm{GeV}) \tag{12}$$

where the equality sign applies only if h_{ija} is damped exponentially as well. The above constraint for M_I can be evaded in a particular case[27] *only* if the relevant Yukawa coupling h_{ija} is absent due to a specially symmetric choice of the Calabi-Yau manifold.§ In this case the originally proposed intermediate scale mechanism[23] would remain intact, *i.e.* eq. (11) would be valid, provided $1/M_K \neq 0$.

It has been argued[21] that $1/M_K \neq 0$ for a general Calabi-Yau manifold, due to the worldsheet instanton contribution. This would in turn suggest that the intermediate scale mechanism is in general viable. However, here we would like to point out that this is *not* the case for the following particular Z_N orbifolds *and* their blown-up versions.

We will show that

$$1/M_K \equiv 0 \tag{13}$$

for all the Z_N (except the $Z_{6'}$, see Table I) orbifolds and their blown-up versions as long as they are compactified on a six-torus T^6 which can be obtained by

‡ Note that for $m_1 \leq M_I$ the effective Yukawa coupling would not have been damped, thus $M_I \ll 10^{11}$ GeV (see eq. (12)). Also, we have assumed that $\langle \mathbf{1} \rangle = 0$ which may not be the case in general.[29]

§ One should again point out that the explicit calculation shows[22] that this is not the case for the blown-up Z_4 orbifold. Even for the most symmetric cubic lattice of the torus T^6 *all* the matter singlet masses and the above Yukawa couplings are nonzero.

continuously deforming $T_0^6 = T^4 \otimes T^2$. By T_0^6 we mean the six-torus, which is a direct product of an arbitrary four-torus and a two-torus which are orthogonal with respect to each other. Therefore for T_0^6 the four basis vectors have entries corresponding to the T^4 coordinates only, *i.e.* $(a, b, 0)$, and the two basis vectors have entries corresponding to the T^2 coordinates only, *i.e.* $(0, 0, c)$.Here we have to choose the T^2 coordinate $i_0 = 3$.

For proving (13) we would need to analyze only the properties of the corresponding amplitude with respect to the $(H_{i_0})_{l,r}$ charge and the X_{i_0} worldsheet coordinate only. For this purpose we shall need the value of the $(H_{i_0})_{l,r}$ charges carried by $\mathbf{27}$'s, $\overline{\mathbf{27}}$'s, and the "moduli" b's (transforming as (1,1) forms) and $\bar{b}$'s (transforming as (1,2) forms) whose nonzero VEV's correspond to the deformation of T_0^6 ("moduli" of the untwisted sector) as well as to the blowing-up of the fixed points ("moduli" of the twisted sectors).

First one observes that in the untwisted sector at most one $\overline{\mathbf{27}}$ and the corresponding $\bar{b}_M$ can appear, and this for orbifolds which are generated by the discrete rotation θ which rotates one complex coordinate X_{i_0}, let's say the third one ($i_0 = 3$) by 180°.¶ These are all the orbifolds which have one $(H_{i_0})_r$ of the -1 picture equal to $\frac{1}{2}$. This in turn implies that $(H_{i_0})_r$ change for $\overline{\mathbf{27}}$ and the corresponding $\bar{b}_M$ from the untwisted sector (g^0)

$$\begin{aligned}
(H_{i_0})_r =&1 \qquad && -1 \text{ picture} && \\
(H_{i_0})_r =&1/2 && -1/2 \text{ picture} && \text{untwisted sector } (un_0). \qquad (14.a)\\
(H_{i_0})_r =&0 && 0 \text{ picture} &&
\end{aligned}$$

On the other hand the $(H_{i_0})_l$ for the part of this $\overline{\mathbf{27}}$ which transforms as $\overline{\mathbf{16}}$

¶ This result is obtained by noticing that the $\overline{\mathbf{27}}$ transform as $(3 \times 3)_{\text{antisymmetric}} = \mathbf{6}^*$ under a continuous $SU(3)$ holonomy group. Noticing that the eigenvalues of the discrete rotation θ are $(\omega_1, \omega_2, \bar{\omega}_1\bar{\omega}_2)$ transform as 3 of the continuous $SU(3)$ holonomy group. One can then derive that the eigenvalues of states in the untwisted sector which transform as $\mathbf{6}^*$ are $(\bar{\omega}_1, \bar{\omega}_2, \omega_1\omega_2, \omega_1^2, \omega_2^2, \bar{\omega}_1\bar{\omega}_2^2)$. Here the $\bar{\omega}$ denotes a complex conjugate value of ω. —Thus, for $N = 1$ supersymmetric theory, *i.e.* all of the eigenvalues of θ should be different from 1, the number of physical states which transform as $\mathbf{6}^*$ of $SU(3)$ is at most 1. The latter is the case when one and only one of the eigenvalues of θ is 1, *e.g.* $\bar{\omega}_1\bar{\omega}_2 = -1$.

under $SO(10)$ and the corresponding $\bar{b}$ are, respectively:

$$\begin{array}{lll} (H_{i_0})_l = -1/2 & 16 & \\ & & \text{untwisted sector } (un_0). \\ (H_{i_0})_l = 0 & \bar{b} & \end{array} \qquad (14.b)$$

On the other hand the $\overline{\mathbf{27}}$'s and corresponding $\bar{b}$'s from the twisted sector come only from the n-twisted sectors (g^n) which leave at least one complex direction unaffected, *e.g.* $\theta^n = (\omega, \bar{\omega}, 1)$ in its diagonal form for $i_0' = 3$. It turns out that for all Z_N orbifolds, except the $Z_{6'}$ orbifold (see Table I),* the direction which remains unaffected is the *same* as the one which corresponds to rotation $\theta_{i_0} = 1$, *i.e.* $i_0' = i_0$. On the other hand for $\mathbf{27}$'s the three entries of the lattice vector corresponding to the gauge group $G_0 \subseteq SU(3)$ are $(k_1/N - 1/2, k_2/N - 1/2, k_3/N - 1/2)$ with $k_i/N \geq 0$ and $\sum_i k_i/N = 1$. (Note that $(H_i)_l$ charges correspond to the vertex operator in the $-1/2$ picture.)

This in turn implies that the $(H_{i_0})_r$ for $\overline{\mathbf{27}}$'s and $\bar{b}$'s coming from the twisted sector is fixed to be:

$$\begin{array}{lll} (H_{i_0})_r = 0 & -1 \text{ picture} & \\ & & \text{twisted sector } (tw_0). \\ (H_{i_0})_r = -1/2 & -1/2 \text{ picture} & \end{array} \qquad (15.a)$$

On the other hand $(H_{i_0})_l$ charges for the parts of $\overline{\mathbf{27}}$'s transforming as $\overline{\mathbf{16}}$'s of $SO(10)$ and $\bar{b}$'s are respectively:♣

$$\begin{array}{lll} (H_{i_0})_l = +1/2 & 16 & \\ & & \text{twisted sector } (tw_0). \\ (H_{i_0})_l = 0 & \bar{b} & \end{array} \qquad (15.b)$$

The constraints (14-15) for the $(H_{i_0})_{l,r}$ charges can be obtained by using the pic-

* This is the reason that in general one cannot prove (13) for $Z_{6'}$ orbifold.

♣ This can be shown in the following way. The part of $\mathbf{27}$ ($\overline{\mathbf{27}}$) transforming as $\mathbf{16}$ ($\overline{\mathbf{16}}$) under $SO(10)$ must have a representation in terms of $\Gamma_8 \times \Gamma_8$ lattice vector whose entries to be all $\pm\frac{1}{2}$ with eight entries corresponding to the E_6 gauge symmetry have an odd (even) number of plus signs. In order to have $\mathbf{27}$'s and $\overline{\mathbf{27}}$'s in a particular twisted sector, $\overline{\mathbf{27}}$'s can only be obtained by adding to the lattice vector of $\mathbf{27}$ a lattice vector Λ of $\Gamma_8 \times \Gamma_8$ lattice which has *one* ± 1 entry in one of the eight entries corresponding to E_6 and one $+1$ entry in one of the three entries corresponding to G_0, *i.e.* one must have for one i_0, $(H_{i_0})_l = k_{i_0}/N + 1/2$. The requirement that $\mathbf{27}$ and $\overline{\mathbf{27}}$ both lie at the same (zero) energy level further requires that $k_{i_0}/N = 0$. Q.E.D.

ture changing formalism. Note, that in the 0-picture $(H_{i_0})_{l,r}$ remains unaffected, because the energy-momentum tensor T_F acting on the vertex in the -1 picture cannot change $(H_{i_0})_{l,r} \equiv 0$. On the other hand the $(H_{i_0})_{l,r}$ charge of the **27**'s and $\bar{b}$'s (transforming as (1,1) form) is not restricted in general.

In order to calculate the parameter $1/M_K$ in the general background of the above Z_N (blown-up) orbifolds one has to evaluate the following amplitude:

$$1/M_K^{(2K-3)} \propto A = \langle \overline{\mathbf{27}}^{\bar{m}}_{un_0} \overline{\mathbf{27}}^{K-\bar{m}}_{tw_0} \mathbf{27}^{l}_{tw'} \mathbf{27}^{m_1}_{un'} \mathbf{27}^{m_2}_{un_0} \mathbf{27}^{K-m_1-m_2-l}_{tw_0} \times b^{\sigma_1}_{un'} b^{\sigma_2}_{un_0} \bar{b}^{\bar{\sigma}}_{un_0} b^{p}_{tw_0} \bar{b}^{\bar{p}}_{tw_0} b^{r}_{tw'} \rangle. \tag{16}$$

Here the fields symbolically denote the corresponding vertex operators. Here un_0 (tw_0) refer to the states of the untwisted (twisted) sector where **27**'s and $\bar{b}$'s (and also corresponding **27**'s and b's) appear. On the other hand un' (tw') refer to the states of the untwisted (twisted) sector with H charges such that *only* **27**'s and b's appear. Note also that states $\mathbf{27}_{un'}$ $(b_{un'})$ have the same $(H_{i_0})_{l,r}$ charges as $\mathbf{27}_{tw_0}$ (b_{tw_0}). Also recall that $(H_{i_0})_l$ of $\overline{\mathbf{27}}$'s (*i.e.* $\overline{\mathbf{27}}_{un_0,tw_0}$) is $-(H_{i_0})_l$ of the corresponding **27**'s (*i.e.* $\mathbf{27}_{un_0,tw_0}$). We also choose two $\overline{\mathbf{27}}$'s to be in the $-1/2$ picture (for the r-sector) and one **27** in the -1 picture, while the rest of the vertices are in the 0 picture. Note again that insertion of moduli from the untwisted sector in amplitude (16) corresponds to deformation of T_0^6 while moduli from the twisted sector correspond to the blowing up of the orbifold singularities.

Using (14) one sees that $(H_{i_0})_{l,r} \equiv 0$ from $b_{un'}$'s, b_{un_0}, and $\bar{b}_{un_0}$ because in the 0 picture of the untwisted sector $H_i \equiv 0 (i = 1, 2, 3)$. However, from operation (2) the above modes give the following contribution to the amplitude (16) with respect to the i_oth bosonic coordinate:

$$\partial_z X_{i_0}^{\bar{\sigma}+\sigma_2} \partial_{\bar{z}} X_{i_0}^{\bar{\sigma}} \partial_{\bar{z}} \bar{X}_{i_0}^{\sigma_2}. \tag{17}$$

Note that this part of the amplitude is always invariant under the automorphisms of the lattice.♡ Analogously one can study the contribution from the rest of the vertices. Using (2,6,7,14,15) one arrives at the following contribution to the amplitude (16) from the i_0th coordinate:♠

$$\partial_z X^{\alpha}_{i_0} \partial_{\bar z} \bar X^{\beta}_{i_0}. \tag{18}$$

Conservation of $(H_{i_0})_{l,r}$ charge yields

$$\alpha = 2\bar m - 1 + \beta \tag{19.a}$$

with

$$\beta = m_2 - \bar m + N_l + N_r \tag{19.b}$$

where N_l and N_r refer to the sum of H_{i_0} charges in the -1 picture for $\mathbf{27}_{tw'}$'s and $b_{tw'}$'s. Obviously (18) is not invariant under the automorphism of the lattice θ_{i_0},◇ neither is then the total amplitude (16). Thus eq. (13) is proven for the above models.♯•

The absence of terms $(\mathbf{27}\overline{\mathbf{27}})^K/M_K^{(2K-3)}$ in the superpotential, for the Z_N (blown-up) orbifolds except $Z_{6'}$ again implies that either there cannot be any M_I or M_I should satisfy the bound (12).

♡ When $\theta_{i_0} = -1$ implies that $\bar\sigma \neq 0$ in general, *i.e.* there is $\bar b_{un_0}$. In this case (17) is invariant under θ_{i_0}. On the other hand when $\theta_{i_0} \neq -1$, there is no $\bar b_{un_0}$ and $\bar\sigma \equiv 0$. Then also in this case (17) is invariant under θ_{i_0}.

♠ Note that in eq. (18) there is no contribution of the type $\partial_{\bar z} X^{\gamma}_{i_0}$ which could in principle come from $\bar b_{tw_0}$'s. However, in such sectors where $\bar b_{tw_0}$'s appear, $H_{i_0} \equiv 0$ in the -1 picture and there could be no contribution of the type $\partial_{\bar z} X_{i_0}$ when one applies the picture changing (see eq. (2)) from the -1 to the 0 picture.

◇ For $\theta_{i_0} = -1$ and thus also $(\bar\sigma, \bar m) \neq 0$, A changes sign under the action of θ_{i_0}. On the other hand for $\theta_{i_0} \neq -1$ one has $\bar\sigma = \bar m \equiv 0$, *i.e.* there are no $\overline{\mathbf{27}}_{un_0}$'s and/or $\bar b_{un_0}$'s. In this case A picks up the phase θ_{i_0} under the action of this rotation.

♯ Here we would like to emphasize that for *any* Z_N (blown-up) orbifold (independent of the structure of the six-torus) the amplitude (16) is *always at most exponentially damped, i.e.* $\propto \exp(-R^2/\alpha')$. See Ref. 31 for details.

• Note that the proof for $Z_{6'}$ (blown-up) orbifold does not go through because one *cannot* choose the same i_0 direction for the classification of $(H_{i_0})_{l,r}$ charges of the untwisted and twisted sectors. Actually we obtained the form of the amplitude A which seems to be nonzero, in general. However, for the case of the $Z_{6'}$ orbifold limit, the first value of K for which $1/M_K$ is possibly nonzero is $K = 7$!

5. Conclusions

We studied the structure of the effective Lagrangian for the Z_N orbifolds and their blown-up versions. We discussed in detail the *general* structure of the vertices for the emission of the massless states for such modes, in particular for those of "moduli" as well as for **27**'s and $\overline{\mathbf{27}}$'s. These vertices are exact at the string tree-level. The calculation of the amplitudes in the general background, *i.e.* with "moduli" acquiring arbitrary VEV's, is outlined. This in turn allows one to extract the value of the corresponding parameters of the effective Lagrangian, which are *exact* at the string tree-level. Special care is given to the higher dimensional terms of the type $(\mathbf{27}\overline{\mathbf{27}})^K$ (with $K \geq 2$) in the superpotential and the importance of such terms to generate an intermediate mass scale, M_I in such models, is emphasized. We showed that for all the Z_N orbifolds and their blown-up versions in general background, such terms, if they are nonzero, are at most exponentially damped. However, interestingly we showed that *all* such terms are *absent* for all the Z_N orbifolds and their corresponding blown-up versions when compactified on a six-torus T^6 which could be continuously obtained by deforming $T^6_0 = T^4 \otimes T^2$, with T^4 and T^2 being orthogonal to each other. This certainly imposes strong phenomenological constraints on such models.

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