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ZERO RANGE SCATTERING THEORY in configuration space*

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The "zero range scattering theory" for three particles[1] as an operator dynamics in a free particle basis can be defined by the factorization of the Faddeev amplitudes

$$\mathbf{M}_{ab}(z) - \mathbf{t}_a(z)\delta_{ab} = \mathbf{t}_a(z)\mathbf{Z}_{ab}(z)\mathbf{t}_b(z)$$

Each projected amplitude $< \mathbf{Z}_{ab} >= Z_{ab}(\underset{\sim}{p}_a, \underset{\sim}{p}_b; z)$ where the momenta of the spectators are in the zero momentum frame $\underset{\sim}{p}_a + \underset{\sim}{p}_b + \underset{\sim}{p}_c = 0$. This reduces the basic dynamics to an off-diagonal equation of the Lippmann-Schwinger type

$$\mathbf{Z}_{ab}(z) = \bar{\delta}_{ab}\mathbf{V}_{ab}(z) + \Sigma\bar{\delta}_{ac}\mathbf{V}_{ac}(z)\mathbf{t}(z)\mathbf{Z}_{cb}(z)$$

The effective interaction is non-local and energy dependent

$$< V_{ab}(z) >^{-1} = p_a^2/2\mu_b + p_b^2/2\mu_a + \underset{\sim}{p}_a \cdot \underset{\sim}{p}_b/m_c - z$$

As Sandhas has often pointed out, any cluster description make the N-particle propagator ($R_0^{-1}(z) = E - z$) into the (N-1)-cluster interaction and the two particle interaction t_c into the (N-1) cluster propagator.

Although the further articulation of this theory is straightforward, the way we part company with Hamiltonian dynamics still seems to cause problems for some people. Recently Eric Schmid[2] made this clear by saying that "zero range" is equated for many people to "δ-function interaction" which L.H.Thomas, in what Delves once called the only significant contribution to three particle theory for three decades, showed implied an infinite three particle binding energy. So Schmid asked me for a configuration space description of the theory; hence this paper. Actually, the factorization which defines the theory confines the "zero range" aspect to the first or last scattering, and hence to the *convergent* asymptotic description of the scattering amplitude; any number of "non-interacting" particulate degrees of freedom can be added. The *dynamics* in our theory comes (for three particle systems) from the single particle exchange which connects (in a provably unitary way) each "zero range" two particle scattering to a *different* one.

Using $\underset{\sim}{r}_a$ as the variable conjugate to $\underset{\sim}{p}_a$ (which implies that $\underset{\sim}{r}_a + \underset{\sim}{r}_b + \underset{\sim}{r}_c = 0$), the configuration space wave function in each channel is

$$\psi_{\underset{\sim}{p}_a^0}(\underset{\sim}{r}_a) = \int d^3p_a e^{i\underset{\sim}{p}_a\cdot\underset{\sim}{r}_a} \frac{\Sigma_b Z_{ab}(\underset{\sim}{p}_a, \underset{\sim}{p}_b(\underset{\sim}{p}_a^0); \frac{1}{2}[(p_a^0)^2(1/m_a + 1/m_{bc})] - \epsilon_{bc})}{p_a^2 - (p_a^0)^2 - i0^+}$$

where $m_{bc} = m_b + m_c - \epsilon_{bc}/c^2$ is the mass of the bound state to which m_a is spectator. To this solutions of the free particle equation must be added in order to satisfy the configuration space (scattering) boundary conditions. The integro-differential equation satisfied by these wave functions is

$$[\Delta_a + (p_a^0)^2]\psi_a = \Sigma_c\bar{\delta}_{ac}\int d^3r'_c V_{ac}(\underset{\sim}{r}_a, \underset{\sim}{r}'_c)\psi_c(\underset{\sim}{r}'_c)$$

Here V_{ac} is a non-local, energy dependent effective interaction which becomes complex above breakup threshold and is a cleanly derived example of an "optical potential". Clearly its meaning is confined to the context, and it cannot be used as an interaction term in a "Hamiltonian" for other situations. It is a double Fourier transform, whose structure is determined (with $z = \frac{1}{2}[p_a^2(1/m_a + 1/m_{bc})] - \epsilon_{bc}$) by the denominator given above.

For equal mass particles this effective interaction is simply an "exchange Yukawa potential" with a range given by the bound state radius and a distance parameter $|\underset{\sim}{r}_a + \frac{1}{2}\underset{\sim}{r}_b|$. Hence, when the binding energy goes to zero, the range goes to infinity, which is yet another way of understanding the Efimov effect. This formalism also gives a clean justification of the phenomenology presented by Adhikari at the Tokyo Workshop[3] which provides such a beautifully simple explanation of the Phillips plot connecting doublet scattering length, binding energy and "anomalous" low energy behavior of $kctn\delta_2$ for *both* n-d and p-d scattering. We also remind you that if a_2 is accepted from experiment, Barton and Phillips[4] have shown that this single proton exchange term, unitarized by N/D predicts a triton binding energy of 6.48 Mev. In our poster we will present quantitative extensions of this lowest order result for the three nucleon system.

1) H.P. Noyes, *Phys. Rev.* **C26** (1982) 1858.
2) Eric Schmid, private communication to HPN at Schladming, 1987.
3) S.K. Adhikari et al., in *Few Body Approaches...*, S. Oryu and T. Sawada, eds., World Scientific, Singapore, 1987, pp. 52-61.
4) G. Barton and A.C. Phillips, *Nucl. Phys.* **A132** (1969) 97.

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