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HOW TO MEASURE ϵ'/ϵ WITH LATTICE QCD*

STEPHEN R. SHARPE

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

ABSTRACT

A pedagogical discussion is given of a lattice calculation of ϵ'. The method is outlined, and preliminary results are presented. They suggest that ϵ'/ϵ may be reduced from previous estimates by 60-70%.

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Practitioners of lattice QCD are nowadays using what may sound like a huge amount of computer time. The calculation discussed here required 500 Cray-2 hours, but this will soon be considered a small computation. What do we hope to discover? I will attempt to provide a partial answer in this talk. The work I discuss has been done in collaboration with Rajan Gupta, Gerry Guralnik, Greg Kilcup and Apoorva Patel. I will eschew technical details; they can be found in references 1 and 2. For a general introduction to lattice methods see the talk of Billiore.[3]

To illustrate what lattice calculations can and cannot do, I focus on the weak decay $K^- \to \pi^-\pi^0$, whose anatomy is exposed in Figure 1. The Standard Electroweak model tells us how the quarks couple to W bosons. The couplings needed for this particular decay are shown in the inner box. To connect this short distance interaction to the external particles we must dress the diagram with gluons and internal quark loops. We know how to do this using perturbation theory for internal momenta ranging down to $p \approx 2\text{GeV}$. This is represented in Figure 1 by region between the inner and outer boxes. The output of the perturbative calculation is the effective weak Hamiltonian ($\mathcal{H}_W$). It is for the remaining dressings, those outside the outer box, that we need lattice calculations. Here the momenta vary down to zero, the coupling constant becomes strong, perturbation theory is not useful, and so a truly non-perturbative method is required.

Lattice calculations are indeed non-perturbative, but they involve their own approximations. First, continuous space time is discretized – replaced with a mesh of finite spacing. When this spacing becomes very small it should be irrelevant. It is this mesh which cuts off the momenta at the high end. The second approximation is that the world is replaced by a box of finite size. Ours is 1.4 fm across, a typical size. This approximation means that it is not feasible to have more than one particle in the box. Indeed, the box is barely large enough to accommodate a single pion, and probably too small for a baryon.

These two approximations mean that, while the lattice can evaluate the effects of small internal momenta, it does so only for a discrete set. It should be emphasized, though, that this set can be systematically enlarged, the effects of the approximation being in this way reduced. I should also stress that numerical lattice calculations are brute force methods, which of themselves may lead to the correct answers without yielding any physical insight into the results. One should certainly not give up analytic approximations, such as the $1/N_c$ expansion discussed here by Bardeen.[4] A final comment is that, in the present calculation, we are using the quenched approximation. This means that we do not include internal quark loops in the lattice part of the calculation, though they are included in the perturbative calculations. This distinction is illustrated in Figure 1.

What do we want to calculate? The original motivation, and one which remains central, is to calculate the masses and decay constants of hadrons, and check that their ratios agree with those in the particle data book. This would provide a detailed non-perturbative check of the validity of QCD. This goal has by no means been attained. The properties of the light meson states come out reasonably (f_K/f_π, m_K/m_ρ, ...), but, even on the largest lattices available, the ratio m_{proton}/m_ρ comes out too high.[5-7] Nevertheless, encouraged by the successes for light mesons, we and others are attempting to extract further information about the structure of these lattice mesons. It is information about weak decays of kaons, such as illustrated in Figure 1, that I will concentrate on here.

As illustrated in Figure 1, what we need to calculate are the matrix elements (ME) of the effective weak Hamiltonian between hadronic states. The following table summarizes the ME that we, and our competition,[8,9] are trying to calculate. The labels "staggered" and "Wilson" refer to the two practical ways of putting fermions, in this case quarks, onto the lattice. In present calculations we use staggered fermions, while our competition uses Wilson fermions. I will not discuss the relative merits of the two methods, nor, indeed, why there is a choice to be made. Both methods should give the same answers for small enough lattice spacing.

Label	*Matrix Element*	*Application*	*staggered*	*Wilson*
ME1	$\left\langle K^0 \middle\vert \mathrm{Im}\ \mathcal{H}_W \middle\vert \overline{K}^0 \right\rangle$	ϵ	soon	now [8,9]
ME2*	$\langle K^0 \vert \mathrm{Re}\ \mathcal{H}_W \vert \pi\pi(I=0)\rangle$	$\Delta I = \frac{1}{2}$ rule	soon	soon [9]
ME3*	$\langle K^0 \vert \mathrm{Re}\ \mathcal{H}_W \vert \pi\pi(I=2)\rangle$	$\Delta I = \frac{1}{2}$ rule	soon	now [8,9]
ME4*	$\langle K^0 \vert \mathrm{Im}\ \mathcal{H}_W \vert \pi\pi(I=0)\rangle$	ϵ' (strong)	now [2]	not yet
ME5*	$\langle K^0 \vert \mathrm{Im}\ \mathcal{H}_W \vert \pi\pi(I=2)\rangle$	ϵ' (electromagnetic)	now [2]	now [8,9]

For technical reasons, only with staggered fermions can one calculate the strong interaction contribution to ϵ' at present. In fact, this, and the electromagnetic contribution to ϵ', are all that we have results on now. We have data for the other ME, but the signal is swamped by statistical errors and/or "wrap-around" contributions.[10] The "soon" in the table means that we are engaged in a bigger and better calculation which will reduce the statistical errors, and also avoids the wrap-around contributions. We hope to have results in 6-12 months. Thus the only point where both we and the competition calculate the same quantity is ME5, and on this we agree. For a discussion of the Wilson fermion results for ME1 and ME3 see the talks of Soni[8] and Martinelli[9] .

If we can succeed in calculating the ME in the table, then we can really pin down the QCD part of the properties of Kaons. ME1 gives us the "B parameter" needed in the estimate of ϵ. If we know this, and we know m_t, and we measure the KM parameters s_2 and s_3 from B meson decays, then we can deduce the KM phase δ. ME2 and ME3 will provide a stringent test of QCD because they are directly related to K^0 and K^+ decays respectively. In particular, the ratio ME2/ME3 must come out to be 22, this large number being due to the $\Delta I = 1/2$ rule. Finally, if we can calculate ME4 and ME5, we have the ingredients for a first principles prediction of ϵ'/ϵ. It is these possibilities that motivate the various groups to carry out these laborious calculations.

I will spend the rest of the talk discussing our measurement of ME4 and ME5, and its implications for ϵ'/ϵ. The first issue is to explain the asterixes beside the last 4 matrix elements in the table. These indicate that we do not, in fact, measure $K \to \pi\pi$ amplitudes, for our lattices are too small to accommodate the $\pi\pi$ final state, and, furthermore, our techniques are too weak. Instead, we use lowest order current algebra to relate the $K \to \pi\pi$ amplitude to a $K \to \pi$ amplitude which we can calculate on the lattice.[11] This is an approximation which one hopes works to about 30%.

The final ingredient we need is the effective weak Hamiltonian. In fact, for ME4 and ME5 we only need its imaginary part. We use the standard renormalization group technology to sum up the leading perturbative contributions represented by the region between the inner and outer boxes of Figure 1. This gives us $\mathcal{H}_W$ at the scale at which the lattice gluons become active, roughly the inverse lattice spacing, in our case 1.7 GeV. At this scale, it turns out that Im $\mathcal{H}_W$ is dominated by three operators, $\mathcal{O}_6$, $\mathcal{O}_7$, and $\mathcal{O}_8$, defined, for example, in Ref. 2. $\mathcal{O}_6$ is the strong interaction penguin operator, while $\mathcal{O}_7$ and $\mathcal{O}_8$ are electromagnetic penguin operators. One has Im $\mathcal{H}_W \approx \frac{G_F}{\sqrt{8}} \left(s_1 s_2 c_2 s_3 s_\delta\right) \sum_{i=6}^{8} \widetilde{c}_i \mathcal{O}_i$. The $\widetilde{c}_i$ are the Wilson coefficients, which I find to be[12] -$\widetilde{c}_6 = .08 - .09\ (.12 - .15)$, $-\widetilde{c}_7/\alpha_{em} = .15 - .22\ (.11 - .18)$, and $-\widetilde{c}_8/\alpha_{em} = .01 - .02\ (.01 - .03)$. The various numbers are for $\Lambda_{QCD} = .1\ (.3)$ GeVand $m_t = 30 - 70$ GeV. The coefficients of the electromagnetic penguins, $\widetilde{c}_7$ and $\widetilde{c}_8$, are proportional to α_{em}.

Putting all this together one obtains the master formulae

$$\left[\frac{\epsilon'}{\epsilon}\right] = 3 \times 10^{-3}\ B_6 \left|\frac{s_1 s_2 c_2 s_3 s_\delta}{2\ 10^{-4}}\right| \left|\frac{\widetilde{c}_6}{.1}\right| \left(\frac{125\ \text{MeV}}{m_s}\right)^2 \left(1 - \Omega_{\eta+\eta'} + \Omega_{EMP}\right)$$

$$\Omega_{EMP} = 0.23 \left(\frac{\widetilde{c}_7 B_7 + 3\widetilde{c}_8}{3\alpha_{em}\widetilde{c}_6}\right) \frac{B_8}{B_6}$$

where the generalized B parameters are:

$$B_6 = \left\langle K^+|O_6^{subt}|\pi^+\right\rangle \ / \ \left\langle K^+|O_6^{subt}|\pi^+\right\rangle_{VIA}$$

$$B_7 = 3\left\langle K^+|O_7|\pi^+\right\rangle \ / \ \left\langle K^+|O_8|\pi^+\right\rangle$$

$$B_8 = \left\langle K^+|O_8|\pi^+\right\rangle \ / \ \left\langle K^+|O_8|\pi^+\right\rangle_{VIA} \ .$$

There are many uncertain factors in these formulae. Starting with the least uncertain, $\Omega_{\eta+\eta'}$ is known to be in the range .27[13] to $\approx$ .4[14] . The strange quark mass, evaluated at a scale of 1.7 GeV, could well be smaller than 125 MeV, which would decrease ϵ'/ϵ. The product of KM angles is more uncertain, and, as just discussed, the Wilson coefficients are very sensitive to Λ_{QCD} and m_t.

The final uncertainty is in the three B parameters. It is this uncertainty that the lattice calculations can reduce or remove. These parameters are defined in a similar way to the original B parameter appearing in the evaluation of ϵ. The superscript *subt* refers to a technical detail I will not discuss. In their definitions VIA stands for vacuum insertion approximation. This is an approximate way of evaluating matrix elements which consist of a product of two factors, say $A \times B$. Instead of the exact value $\langle A \times B\rangle$, the VIA uses $\langle A\rangle \times \langle B\rangle$, an approximation which ignores correlations. In Figure 1 VIA would mean not including any gluon lines which join the left-hand quark loop to the right-hand loop. VIA is simple enough that it can be carried out in the continuum to give an estimate of the ME, yielding, by construction, $B_6 = B_7 = B_8 = 1$. But VIA can also be made on the lattice, and so it can serve as a connection between lattice and continuum. Lattice measurements of B parameters are less subject to systematic errors than those of the matrix elements themselves.

Figure 2 shows our results for the B parameters.[2] The errors shown are statistical, and do not account for the systematic error of using the quenched approximation. The lattice we use is $12^3 \times 30$, a moderate size lattice by today's standards. We have data only for three values of $m_\pi m_K$, and the lightest two lie between the physical value (shown in the Figure) and the physical m_K^2. To avoid confusion I should stress that on the lattice one has the freedom to vary the quark masses, and thus to consider worlds in which the lattice pions and kaons have different masses from the physical ones. The data show that B_6 is considerably below 1 for small $m_\pi m_K$. Thus the correlations included in the calculation of ME4 are very important. B_8, on the other hand, is much closer to 1, while B_7 is very close to 1.

An important check to be made on our results is that they have the correct "chiral behavior", i.e. that they vary according to the dictates of current algebra for small m_π and m_K. ME4 is expected to be proportional to $m_\pi m_K$, with no constant term, nor any terms proportional to m_π^2 or m_K^2. For small enough m_π and m_K one can show that this must be true on the lattice, given the absence of "wrap-around" contributions.[15] This let-out means that the argument is stronger for the VIA data than for the real ME4, since in VIA there are no "wrap-around" contributions. Our results for ME4 $/m_\pi m_K$ are shown in Figure 3. Our three points are too few for a stringent test, but the real ME4 appears to have the expected chiral behavior, while the VIA data do not. My optimistic interpretation of these data, based on the theoretical arguments just mentioned, is as follows. For the real ME4, the lowest two mass points are showing the onset of chiral behavior, and this behavior will continue for lower masses. On the other hand, the two lowest mass VIA points are in a transition region between high mass, where VIA works well, and low mass, where the correct chiral behavior obtains. Chiral behavior should set in by the physical $m_\pi m_K$. This interpretation predicts that the VIA results for ME4 $/m_\pi m_K$ will continue roughly at the level of the lowest mass point for all lower masses. Clearly other interpretations are possible, and, more clearly still, lattice data at low masses are needed to resolve the issue.

If we take these results seriously we have $B_6 \approx .5$, $B_7 \approx 1.$ and $B_8 \approx 1.2$. Relative to VIA, this doubles Ω_{EMP}, but the overall effect on ϵ'/ϵ is a reduction of 60-70%. The reader can plug her or his favorite values for the uncertain factors into the master equations to obtain a value of ϵ'/ϵ. A reasonable range seems to be $(1-2) \times 10^{-3}$, below the present limit but accessible by experiments in progress.

It is worth breaking down the contributions to ϵ'. The smallness of B_6 means that the strong interaction penguin contributes less than in VIA. On the other hand, since $B_8 > 1$, the electromagnetic penguin contribution is slightly increased. Depending on the values of m_t, Λ_{QCD}, and $\Omega_{\eta+\eta'}$, these two changes mean that the electromagnetic contribution to ϵ' may be as large as that of the strong penguin, despite being proportional to α_{em}. This surprising result is due to a combination of factors. Unlike the strong penguins, electromagnetic penguins are not suppressed by the $\Delta I = 1/2$ rule, nor by the requirements of chiral symmetry, and, if our result is true, nor by the combined effects of non-perturbative gluons.

In conclusion, lattice evaluations of the ME of $\mathcal{H}_W$ are an essential ingredient in the theoretical calculations of Kaon weak decay properties. At present, the uncertainties in these calculations coming from uncertainties in the KM angles, in m_t, and in Λ_{QCD}, are comparable to that coming from the ME of $\mathcal{H}_W$. However, in the future, it is likely that the uncertainty coming from the ME will be dominant. Although the lattice calculations of these ME are preliminary, they do indicate that the results of a complete calculation may differ substantially from those of the approximation schemes used previously.

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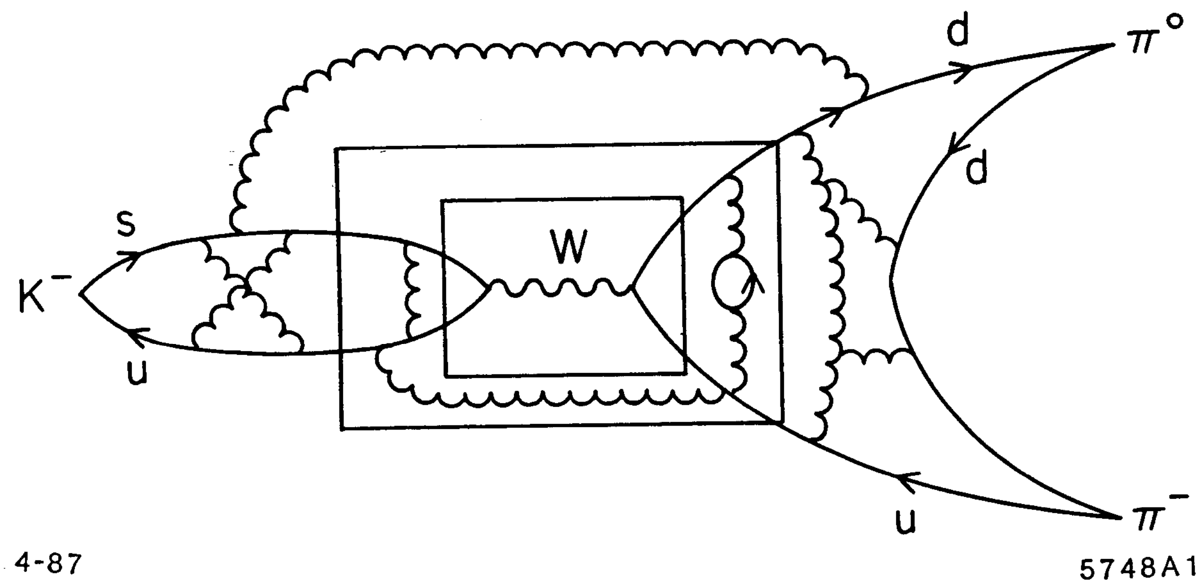

Figure 1. The anatomy of the weak decay $K^- \rightarrow \pi^- \pi^0$. The curly lines are gluons.

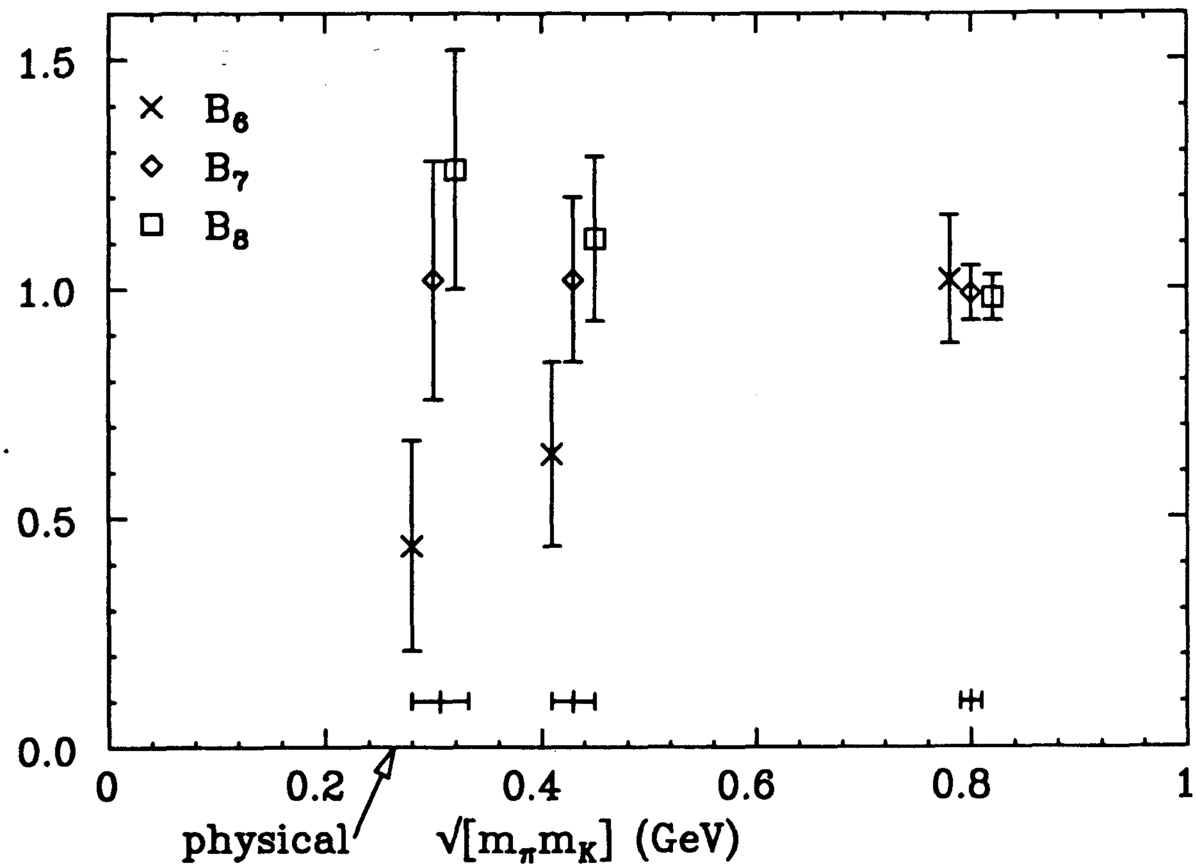

Figure 2. Lattice results for B parameters. The horizontal bars show the statistical error in the lattice values of $\sqrt{m_\pi m_K}$. The data are separated horizontally for clarity.

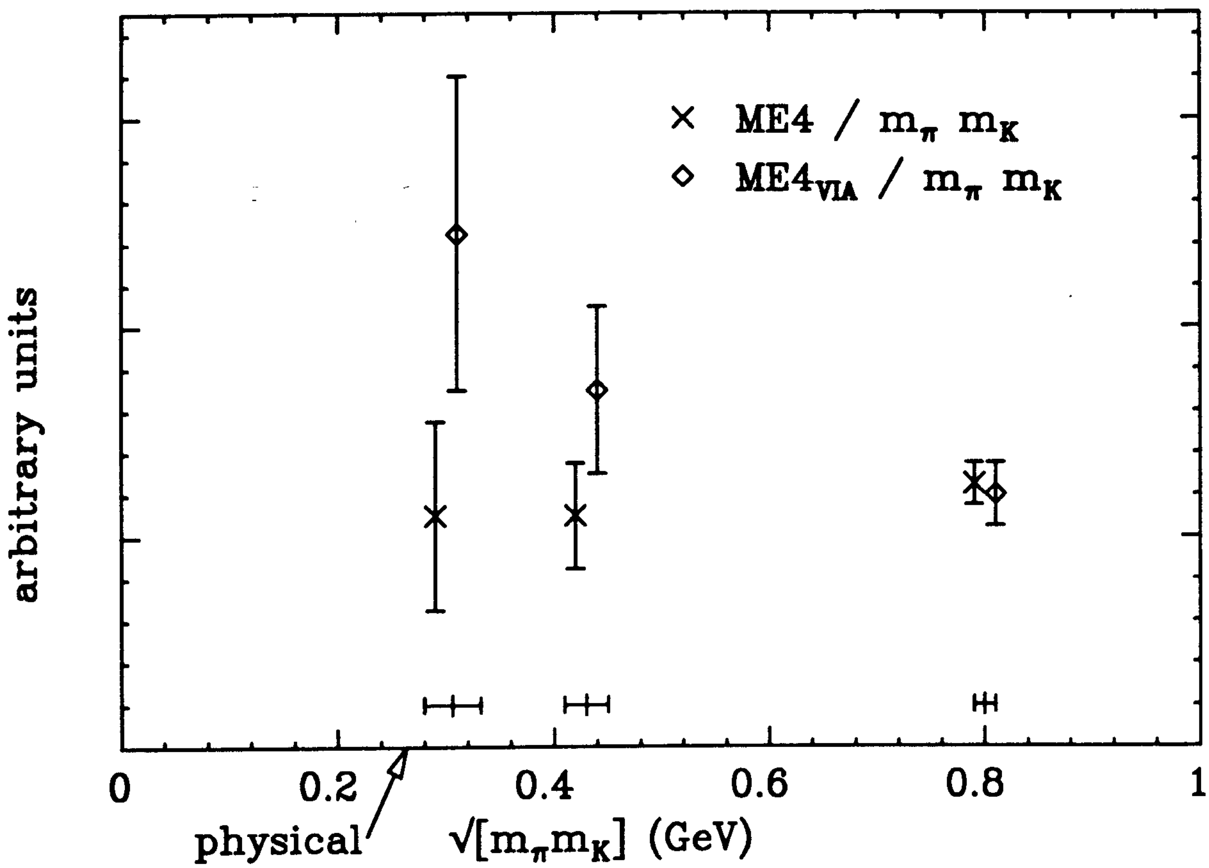

Figure 3. Testing the chiral behavior of the real ME4 and its values in VIA.