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FROM A NEW SMELL TO A NEW FLAVOUR—
 $B_d - \bar{B}_d$ MIXING, CP VIOLATION AND NEW PHYSICS*

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ABSTRACT

Using the preliminary data on $B_d - \bar{B}_d$ mixing from ARGUS, and the Standard Model with three families, we infer a lower bound on the top quark mass of 50-70 GeV; also, $B_s - \bar{B}_s$ mixing has to be close to maximal. We discuss how the prospects for observing CP violation in B^0 decays are enhanced and sketch alternative scenarios for New Physics.

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I. Introduction

Almost from the inception of B physics it was realized that dedicated research in this field had the potential to be as revealing (if not more so) as K physics. These expectations were based on rather general qualitative arguments; experimental information obtained in the meantime has actually strengthened these arguments: the “long” B lifetime $\tau(B) \sim 1 \text{ ps}$; $m(\text{top}) > 22 \text{ GeV}$; the intriguing evidence for $B^0 - \bar{B}^0$ mixing presented by UA1.¹

Most recently the ARGUS collaboration has found evidence for surprisingly strong $B_d - \bar{B}_d$ mixing. Their preliminary results on di-leptons observed in $e^+e^- \rightarrow \Upsilon(4s) \rightarrow B \bar{B}$ read²

$$y_p \equiv \frac{N(l^\pm l^\pm)}{N(l^+ l^-)} \Big|_{B^0 \bar{B}^0} = 0.234 \pm 0.067 \pm 0.031 \quad (1)$$

where it should be noted that for this reaction

$$y_p = r_d \equiv \frac{\Gamma(B^0 \rightarrow l^+ X)}{\Gamma(B^0 \rightarrow l^- X)} \quad (2)$$

holds³. The number in (1) is well above previous theoretical expectations which were based on the Standard Model with three families and $m_t \leq 45 \text{ GeV}$. This leads to two questions:

(A) To which degree do the ARGUS findings indicate the presence of New Physics and what kind of New Physics could it be?

(B) What are the consequences for even more ambitious studies, namely CP asymmetries in B decays?

II. Theoretical Predictions on $B^0 - \bar{B}^0$ Mixing in the Standard Model:

Since

$$r = \frac{x^2}{2 + x^2}, \quad x = \frac{\Delta m}{\Gamma} \quad (3)$$

one translates (1) into

$$x(B_d) \simeq 0.78 \pm 0.16 \quad (4)$$

In calculating Δm one has to deal with three critical input parameters: (i) the top mass m_t ; (ii) the KM parameter $V(td)$; (iii) the wavefunction of B^0 mesons; its relevant contribution is usually expressed in terms of Bf_B^2 when f_B is the meson decay constant and $B = 1$ corresponds to “vacuum saturation”. Thus

$$\Delta m \propto |V(td)|^2 \mathcal{D}(m_t^2) Bf_B^2 \quad (4)$$

$\mathcal{D}(m_t^2)$ is a known function of m_t that is obtained from computing the quark box diagram.⁵

ad (i): PETRA data give a direct lower limit on m_t ; from a comprehensive analysis of electro-weak processes one infers an upper bound in an indirect way.⁵

$$22 \text{ GeV} \leq m_t \lesssim 180 \text{ GeV} \quad (6)$$

ad (ii): With just three families one can invoke unitarity constraints to limit $V(td)$ severely. In the Wolfenstein representation one finds⁶: $V(cb) = A\lambda^2$, $V(ub) = A\lambda^3(\rho - i\eta)$, $V(ts) = A\lambda^2$, $V(td) = A\lambda^3(1 - \rho - i\eta)$; $\lambda = 0.22$. Using⁷

$$|V(cb)| = 0.045 \pm 0.008$$

$$\frac{|V(ub)|}{|V(cb)|} < 0.19 \quad (7)$$

one gets

$$A = 0.93 \pm 0.17 \quad \rho^2 + \eta^2 \lesssim 0.75 \quad (8)$$

η calibrates the strength of CP violation; as we will discuss in detail later on, one deduces $\eta \geq 0.2$ from ϵ_K . Accordingly we use

$$-0.8 \leq \rho \lesssim 0.8 \quad (9)$$

Therefore

$$\frac{|V(td)|^2}{|V(cb)|^2} \simeq \frac{|V(td)|^2}{|V(ts)|^2} = \lambda^2 ((1 - \rho)^2 + \eta^2) \lesssim 0.16 \quad (10)$$

with the limits being saturated by $\rho = -0.8$.

ad(iii)

$$\langle B^0 | J_\mu J_\mu | \bar{B}_0 \rangle \equiv \frac{8}{3} B f_B^2 m_B^2 \quad (11)$$

Different theoretical approaches have been employed to determine this hadronic matrix element:

$$B f_B^2 \sim \begin{cases} (60 - 130 \text{MeV})^2 & \text{MIT bag model}^8 \\ (100 - 150 \text{MeV})^2 & \text{potential models}^9 \\ (115 \pm 15 \text{MeV})^2, (190 \pm 30 \text{MeV})^2 & \text{QCD sum rules}^{10,11} \\ (120 \text{MeV})^2 / \alpha_S & B^* - B \text{ mass splitting}^{12} \end{cases} \quad (12)$$

The discrepancy between the results of ref.10 and 11 is due to a different choice for the on-shell b quark mass. Detailed studies of exclusive B decays represent very sensitive tests for our understanding of meson wave functions. Yet many more precise data than available at the moment and more theoretical work is needed before definite conclusions can be reached.

Putting everything together we conclude that a reasonable way to express theoretical expectations on Δm is in units of a calibration factor F ,

$$F = \frac{|V(td)|^2}{(0.01)^2} \frac{Bf_B^2}{(150 \text{ MeV})^2} \quad (13)$$

Our discussion leads to the following range for F

$$F \sim 0.5 - 7$$

The resulting values for $\Delta m/\Gamma$ as a function of m_t are shown in Figure 1 from which we draw the following conclusions:

(i) Even a large value $x = 0.75$ (corresponding to $r_d \simeq 0.22$) could be realized in the Standard Model: if $\rho \simeq -0.8$ then $r_d \simeq 0.22[0.11]$ is obtained with $m_t \gtrsim 60[50] \text{ GeV}$; $\rho \geq 0$ however leads to $m_t \gtrsim 120[100] \text{ GeV}$.

(ii) The process $Z \rightarrow t\bar{t}$ is therefore kinematically forbidden if $r_d \geq 0.11$; observation of this process on the other hand would then point to the presence of New Physics in $B_d - \bar{B}_d$ mixing.

(iii) If for example $\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c) \leq 0.02$ or if $Bf_B^2 \leq (150 \text{ MeV})^2$ were found to hold then $F \leq 3$ would follow; accordingly $r_d = 0.22[0.11]$ would yield $m_t \geq 100[75] \text{ GeV}$.

(iv) Since all theoretical calculations agree on $Bf_B^2(B_d) \leq Bf_B^2(B_s)$ one finds (see eq. (10))

$$x(B_s) \geq 6.4x(B_d) \quad (14)$$

and therefore $r_s \geq 0.84$ for $r_d \geq 0.10$!

(v) If $R = 1.5$ were to hold then—as said before³—one deduces $r_d \sim 0.47$ from the ARGUS data; this in turn implies $m_t \geq 130 \text{ GeV}$ for $F \leq 3$.

III. Examples of New Physics:

A discussion of some more specific ansatz for New Physics is appropriate for two reasons: (i) More experimental input and growing theoretical sophistication will decrease our uncertainties and can thus strengthen the case for New Physics. (ii) Another phenomenon that is even more subtle than $B^0 - \bar{B}^0$ mixing, namely CP violation in B^0 decays is greatly affected by the dynamics underlying $B^0 - \bar{B}^0$ mixing and the possible presence of New Physics. This topic will be treated in Section IV. Here we present two complementary models for New Physics.

(A) An ansatz with four families: Adding a fourth family increases the complexity of the (now) 4×4 KM matrix quite significantly: three more angles and two more phases enter—in addition to the masses of the new fermions. Therefore no firm predictions can be made; instead one designs possible and internally consistent cases. Here we employ the following scenario:¹³ generalizing the Wolfenstein parametrization from three to four families we obtain as a possible solution:

$$\begin{aligned} V(t'd) &= B\lambda^3[(\gamma - \alpha) + i(\delta - \beta)] \\ V(t's) &= B\lambda^2(\alpha + i\beta), V(t'b) = B\lambda \end{aligned} \tag{15}$$

with t' being the fourth up-type quark; $B, \alpha, \beta, \gamma, \delta$ are the new (real) KM parameters with $|B|, \alpha^2 + \beta^2, \delta^2 + \gamma^2 \leq 1$. Just one example to illustrate the point: keeping $m_t = 40$ GeV and $m_{t'} = 200$ GeV fixed one can obtain $x_d \simeq 0.65$ (corresponding to $r_d \simeq 0.17$) in such a scenario for $F = 3$. At the same time it is quite possible to find $r_s < 0.80$ in this scenario, i.e., $B_s - \bar{B}_s$ mixing which is substantially suppressed relative eq. (14).

(B) **An ansatz with flavour-changing scalar couplings:** Since no good reason has been found for having just one Higgs doublet, there might be many more; in particular, models with three Higgs doublets have received a good deal of attention¹⁴. These models exhibit quite naturally flavour-changing Yukawa couplings that contain CP violation unless definite countermeasures are taken. Couplings $\bar{s}d\Phi, \bar{b}d\Phi, \bar{c}u\Phi$ may not be suppressed by KM angles. For this reason, $K^0 - \bar{K}^0$ mixing tends to require the mass of such scalars to be relatively heavy $\sim 30TeV$.

If such scalars were responsible for some fraction of $K^0 - \bar{K}^0$ mixing (and CP violation), then their impact on $B^0 - \bar{B}^0$ mixing would be very significant and $x_d \simeq 0.6$ could be generated very naturally. In that case there is then no clear reason for $D^0 - \bar{D}^0$ mixing to be *absent* on the 0.1-1 % level.

IV. CP Asymmetries in B^0 Decays:

(A) **Semi-leptonic B^0 decays:** It is fairly easy to see that a CP asymmetry in semi-leptonic B^0 decays cannot be sizeable. For

$$a_{SL} = \frac{\sigma(B^0 \bar{B}^0 \rightarrow \ell^+ \ell^+ X) - \sigma(B^0 \bar{B}^0 \rightarrow \ell^- \ell^- X)}{\sigma(B^0 \bar{B}^0 \rightarrow \ell^+ \ell^+ X) + \sigma(B^0 \bar{B}^0 \rightarrow \ell^- \ell^- X)} \quad (16)$$

is given by

$$a_{SL} = \frac{Im \frac{\Gamma_{12}}{M_{12}}}{1 + \frac{1}{4} \left| \frac{\Gamma_{12}}{M_{12}} \right|^2} \quad (17)$$

$\left| \frac{\Gamma_{12}}{M_{12}} \right|$ is controlled by m_b^2/m_t^2 and therefore quite small. This argument is further strengthened if $r_d \geq 0.1$; in the Standard Model one has to make m_t rather heavy to produce the required M_{12} while Γ_{12} is hardly affected. Accordingly, one then

estimates

$$a_{SL}(B_d) \leq 10^{-3}, a_{SL}(B_s) \leq 10^{-4} \quad (18)$$

If there is New Physics in $B^0 - \bar{B}^0$ mixing as sketched above then

$$a_{SL}(B_d) < a_{SL}(B_s) \sim 0(1\%) \quad (19)$$

is a possible though not highly likely scenario.

(B) Non-leptonic B^0 decays. If f denotes a decay mode common to B^0 and \bar{B}^0 —a property then shared by its CP conjugate \bar{f} —one finds¹⁵

$$\Gamma(B^0[\bar{B}^0](t) \rightarrow f[\bar{f}]) \propto e^{-\Gamma t} \left\{ (1 + \cos\Delta mt) \times |\rho_f|^2 [1] + (1 - \cos\Delta mt) \times 1[|\rho_f|^2] - [+] 2\sin\Delta mt \operatorname{Im} \frac{p}{q}(\rho_f) \right\} \quad (20)$$

with $\rho_f = \frac{A(B^0 \rightarrow f)}{A(\bar{B}^0 \rightarrow f)}$, $\frac{p}{q} = \frac{1+\epsilon}{1-\epsilon}$; for simplicity we have set $\Delta\Gamma = 0$ $|p|^2 = |q|^2$ which should be excellent approximations. The size of the observable CP asymmetry thus depends both on Δm and $\operatorname{Im} \frac{p}{q} \rho_f$.

Δm has no intrinsic connection to CP violation and can thus be taken from data on like-sign di-leptons: $x_d \sim 0.75$ produces hardly a suppression of the observable asymmetry. This is true also if one integrates over all decay times which yields a factor $\frac{x}{1+x^2}$. For B_s mesons with $x_s \geq 6.4x_d$ on the other hand an excellent time resolution is essential.

To estimate the size of $\operatorname{Im} \frac{p}{q} \rho_f$ one has to invoke a model. When f is a CP eigenstate—like $B_d \rightarrow \psi K_s$ or $B_d \rightarrow (K_S + \pi' s)_{D^0} + \pi' s$ —then $\frac{p}{q} \rho_f$ is, to an excellent approximation, a unit vector in the complex plane¹⁵ and is given by a

ratio of KM parameters alone; in other cases this is not true any more, but one can—with few exceptions—express reasonable order of magnitude estimates^{15–17} again by ratios of KM parameters. In this way one finds for $(b\bar{d}) \rightarrow c\bar{c}s\bar{d}, c\bar{u}d\bar{d}$ transitions

$$\text{Im} \frac{p}{q} \rho_f |_{B_d} \simeq -\frac{2\eta(1-\rho)}{(1-\rho)^2 + \eta^2} \quad (21)$$

As we have stressed before, large $B_d - \bar{B}_d$ mixing favors $\rho \simeq -0.8$ to improve the Standard Model's chances to reproduce it. The magnitude of η is inferred from ϵ_K :

$$B_K \eta \simeq \frac{|\epsilon_K|}{4.33A^2} \{f_3 S(x_c, x_t) - f_1 S(x_c) + 2.3 \times 10^{-3} A^2 (1-\rho) f_2 S(x_t)\}^{-1} \quad (22)$$

f_i denote the QCD radiative corrections, $S(x_i)$, and $S(x_c, x_t)$ the various quark box contributions with $x_i = \frac{m_i^2}{M_W^2}$; B_K enters in analogy to eq. (11).

For $30 \text{ GeV} \leq m_t \leq 180 \text{ GeV}$ one can employ a much simpler approximate expression.

$$B_k \eta \simeq \frac{0.53}{A^2} \{0.94 x_t^{0.1105} - 0.3 + A^2 (1-\rho) x_t^{0.8363}\} \quad (23)$$

where we have used $m_c \simeq 1.5 \text{ GeV}$; η thus drops fairly quickly with increasing m_t . For $B_k \sim \frac{2}{3}$ one finds

$$\eta \sim 0.5[0.2] \text{ if } m_t \simeq 60[130] \text{ GeV}. \quad (24)$$

Accordingly

$$\text{Im} \frac{p}{q} \rho_f |_{B_d} \sim -0.52[-0.22] \quad (25)$$

i.e., pleasantly large numbers. For $m_t \geq 90 \text{ GeV}$ one actually finds that $\Delta m \times \eta$ shows very little dependence on m_t since then $\eta \propto S^{-1}(x_t)$ and $\Delta m \propto S(x_t)$.

Therefore, quite generally one expects CP asymmetries of order 10 to 50 % in non-leptonic B_d decays!

A priori New Physics could either increase or decrease these asymmetries in B_d decays; in B_s decays it will most likely increase the corresponding asymmetries and could well lift them to the numerical level of eq. (25)¹³.

(C) **CP asymmetries in $\Upsilon(4s) \rightarrow B_d \bar{B}_d$:** There is another way to search for CP violation which has an improved chance to succeed if indeed $r_d \geq 0.10$: let f_1 and f_2 be two CP eigenstates of the *same* CP parity. Then, in principle, by observing *just one event* of the type

$$\Upsilon(4s) \rightarrow B^0 \bar{B}^0 \rightarrow f_1 f_2 \quad (26)$$

one has established CP violation.¹⁸ For the initial state has even CP parity whereas the final state, due to its P wave configuration, has odd CP parity. One finds for the rate

$$\text{rate } ([B^0(t)\bar{B}^0(t)]_{\Upsilon(4s)} \rightarrow f_1 f_2) \propto$$

$$e^{-\Gamma(t+\bar{t})} |\bar{A}_1|^2 |\bar{A}_2|^2 (1 - \cos \Delta m(t - \bar{t})) \left| 1 - \left(\frac{p}{q}\right) \rho_1 \rho_2 \right|^2 \quad (27)$$

where $\bar{A}_i = A(\bar{B}^0 \rightarrow f_i)$.

For small mixing this rate is proportional to $(\Delta m/\Gamma)^2$ and thus highly suppressed—yet such a suppression disappears for $r_d \geq 0.1$. When considering $b \rightarrow c\bar{c}s, c\bar{u}d$ transitions one finds for the last factor

$$\left| 1 - \left(\frac{p}{q}\right)^2 \rho_1 \rho_2 \right|^2 \simeq \frac{(4\eta)^2 (1 - \rho)^2}{((1 - \rho)^2 + \eta^2)^2} \sim 1[0.2] \quad (28)$$

for $m_t \simeq 60[130]$ GeV. Thus also this factor which is intrinsically connected to CP violation does not produce a large suppression.

Therefore the main challenge consists of finding such final states. No reliable estimate for the combined branching ratios $BR(B^{0(-)} \rightarrow f_1)BR(B^{0(-)} \rightarrow f_2)$ can be given at present considering the scant experimental information. However future information will allow us to present quantitative scenarios.

V. Conclusions:

The ARGUS findings are full of promise for the future. They contain some possible hints for New Physics in $B_d - \bar{B}_d$ mixing—yet nothing definite can be said at the moment. Future theoretical and experimental work which is stimulated by this development will allow us to make a more convincing case for or against New Physics.

The second promise concerns CP violation in B^0 decays. Those asymmetries that require the presence of $B^0 - \bar{B}^0$ mixing have a much better chance to reach the level of observability than it was previously thought. Even so the task will not be easy.

After completion of this work we received a preprint by J.Ellis, J.Hagelin and S.Rudaz, CERN-TH.4679/87 containing a re-analysis of $B^0 - \bar{B}^0$ mixing quite similar to ours. See also L.L.Chau and W.-Y. Keung, UCD-87-02.

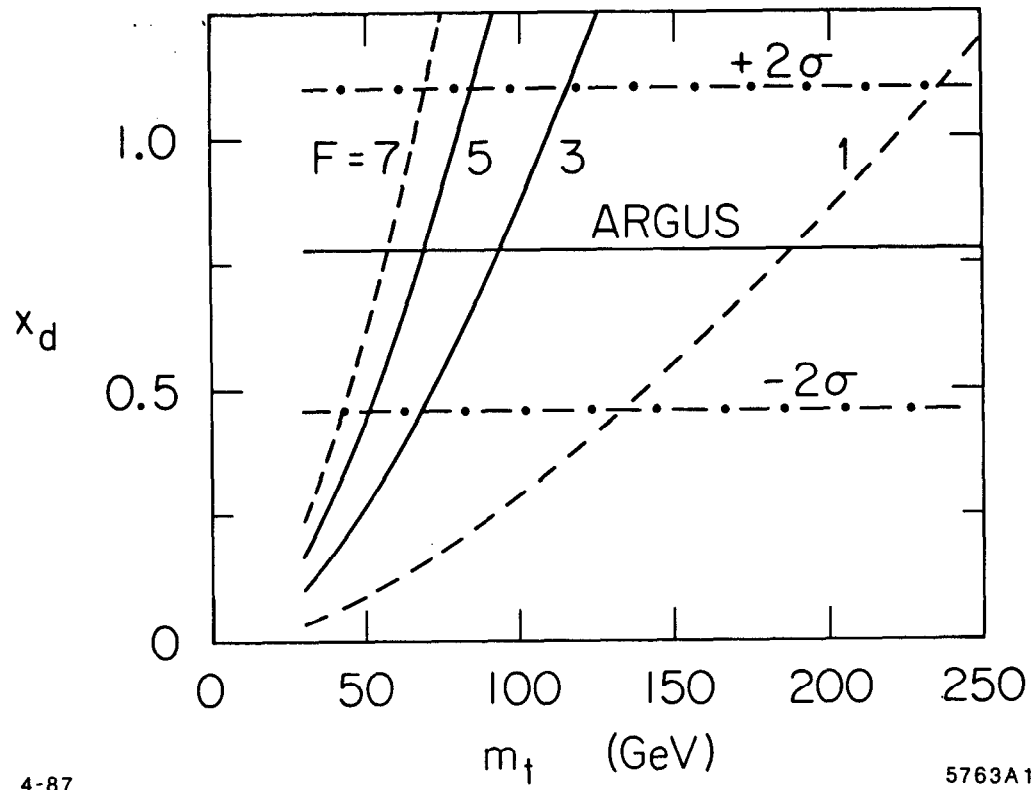
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Figure Caption

Fig. 1: Standard Model prediction for x_d as a function of m_t ; the theoretical uncertainties are represented by the different values of F. Also shown are the ARGUS results (for R=1).



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Fig. 1