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ON THE CONTRIBUTION OF ELECTROMAGNETIC PENGUINS TO $\epsilon'^{\star}$

STEPHEN R. SHARPE

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

ABSTRACT

I discuss the importance of the electromagnetic penguin (EMP) contribution to ϵ'. I confirm the corrections to earlier calculations found by Buras and Gerard (BG). Incorporating these corrections, I calculate the Wilson coefficients of the EMP operators using the full anomalous dimension matrices, and also using the large N (number of colors) approach of BG. I disagree with BG on the coefficient of the EMP operator dominant at large N: my result is of opposite sign and smaller in magnitude. This means that for large N the EMP contribution increases ϵ', but by less than 1 %. I also find that this Wilson coefficient is poorly estimated in the large N approach. I agree with the BG result for the coefficient of the EMP operator subdominant at large N. This coefficient is large, and is well estimated using the large N anomalous dimension matrices. I emphasize that the crucial factor determining the EMP contribution to ϵ' is the size of the hadronic matrix elements of the subdominant EMP operator. Though suppressed at large N, it increases ϵ' by 5-20% if one assumes vacuum insertion approximation, and by up to 100% if one uses the results from a recent lattice calculation.

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1. Introduction

It is of great importance to reduce the uncertainties in the theoretical calculation of ϵ'/ϵ. Together with experimental measurements, particularly that of the top quark mass, this would allow a stringent test of the standard model with three generations. This letter comments on part of the theoretical calculation of ϵ' – the contribution of the electromagnetic penguin (EMP) operators.

Some time ago Bijnens and Wise[1] pointed out that a variety of factors combine to enhance the contribution of EMP operators to ϵ'. More recently, Donoghue *et al*[2] made a detailed estimate of this contribution, and concluded that it reduces ϵ' by about 20%. However, a very recent paper of Buras and Gerard[3] finds errors in both the earlier papers. Ref. 1 had missed some terms in the anomalous dimension matrix, while Ref. 2 had used the wrong sign for the matrix elements of the EMP operators. The combined effect of these two errors, estimated in the large N (number of colors) approach of Ref. 4, was to decrease the size of the EMP contribution. Buras and Gerard estimated that it reduced ϵ' by 5-7%, which, as they stressed, is a very small contribution.

The purpose of this note is threefold. First, I want to point out a numerical error in the results of Buras and Gerard. Although I agree with their formulae, I disagree with their numerical value for the Wilson coefficient ($\tilde{c}_8$) of the EMP operator dominant at large N ($\mathcal{O}_8$). My result is of opposite sign to theirs, and is also smaller in magnitude. On the other hand, I agree with their result for the coefficient ($\tilde{c}_7$) of the subdominant EMP operator ($\mathcal{O}_7$).

The second purpose is to display the Wilson coefficients of the EMP operators calculated including the corrections found by Buras and Gerard, but with no approximations in the anomalous dimension matrices. This allows one to separate the effects of the corrections from those of the large N approximation. I display these coefficients as a function of the renormalization scale for a variety of values of m_t and Λ_{QCD}. I also show the Wilson coefficient of the strong interaction operator which provides the dominant contribution to ϵ'. Previous estimates have only provided the coefficients at the scale appropriate to the method of estimation of hadronic matrix elements being employed. Since these scales range from .1 GeV to 2.5 GeV, it is useful to combine all of them in a single curve.

The final, and most important, purpose is to point out how sensitively the EMP contribution to ϵ' depends on the value of certain hadronic matrix elements. To parameterize these matrix element I introduce three "B parameters". B_6 determines the strong interaction contribution, while B_7 and B_8 determine the contributions of the subdominant and dominant EMP operators, respectively. If all the B parameters are positive, as seems most likely, then, given the signs of the corrected Wilson coefficients, the EMP contribution always increases the size of ϵ'. However, the smallness of $\widetilde{c}_8$ implies that B_8 is essentially irrelevant. For large N, where $B_7 = 0$, the EMP contribution is proportional to B_8 and increases ϵ' by at most 1%. That the effect is small is in accord with the general conclusions of Buras and Gerard. My additional correction simply makes it smaller. In vacuum insertion approximation, in which all the B parameters are 1 by design, the part proportional to B_7 can push the contribution of the EMP up to 20%. And if one uses preliminary lattice results from Ref. 5, in which the EMP are enhanced, but the strong interaction operators suppressed, the EMP may contribute as much as the strong interaction operators to ϵ'.

2. Wilson Coefficients

I use the basis of operators $O_1 - O_8$ defined in Ref. 1. The imaginary parts of the Wilson coefficients of these operators are denoted $\widetilde{c}_1 - \widetilde{c}_8$, and are defined so that

$$\mathrm{Im}\, \mathcal{H}_W = \frac{G_F}{\sqrt{8}} \left(s_1 s_2 c_2 s_3 s_\delta\right) \sum_{i=1}^{8} \widetilde{c}_i O_i \ .$$

My coefficients are related to those of Ref. 3 by $\widetilde{c}_{1-6} = 2\,\widetilde{y}_{1-6}$ and $\widetilde{c}_{7,8} = 2\alpha_{em}\widetilde{y}_{7,8}$, and to those of Ref. 1 by $\widetilde{c}_{1-8} = 2\,\widetilde{C}_{1-8}$. O_1 - O_6 are the operators generated by mixing due to the strong interactions. O_5 and O_6 are the strong interaction penguin operators. $O_1 - O_6$ mix with the EMP operators, O_7 and O_8, only through diagrams involving photons. Thus both $\tilde{c}_7$ and $\tilde{c}_8$ are proportional to α_{em}.

In the imaginary part of $\mathcal{H}_W$, the dominant contributions to Kaon decays come from the operators O_6, O_7 and O_8. The issue in the following is the sizes of these contributions, and in particular the size of the electromagnetic penguins relative to the strong interaction penguins. Thus I shall mainly consider $\tilde{c}_6$, $\tilde{c}_7$ and $\tilde{c}_8$.

The one loop renormalization group (RG) equations determine the $\tilde{c}_i$ to be:

$$\tilde{c}_i = z_i \; T\left\{ \exp \int_{\ell n\, m_w}^{\ell n\, m_t} dt \gamma^6(t) \; \exp \int_{\ell n\, m_t}^{\ell n\, m_b} dt \gamma^5(t) \; \exp \int_{\ell n\, m_b}^{\ell n\, \mu} dt \gamma^4(t) \right\}$$

$$z_i = (1,1,0,0,0,0,0,0) \tag{2.1}$$

where $t = \ell n\, q$, and the symbol T implies a momentum ordering of the anomalous dimension matrices γ^f. In this equation, $\tilde{c}_i$ and z_i are row vectors, and the factor in curly braces is a matrix. The superscript on γ indicates the number of light flavors. Equation (2.1) displays the formula for $m_c < \mu < m_b$; similar formulae apply for other momentum ranges. The t dependence comes in through the factors of $\alpha_s(t)$ in the γ^f. For α_s I use the form suggested byRef. 6:

$$\alpha_s(t) = \frac{2\pi}{b_f(t - \ell n\, \Lambda_f)} \,, \qquad b_f = 11 - \frac{2}{3} f \,. \tag{2.2}$$

The Λ_f are chosen so that α_s is continuous. I treat the thresholds as abrupt, using the standard one loop effective field theory matching. It is likely that, for the $\tilde{c}_i$, a more complete treatment of threshold effects is not necessary[7] .

The full 8×8 anomalous dimension matrices γ^f were first given in Ref. 1. Buras and Gerard[3] pointed out that diagrams involving a photon dressing the operators O_5 and O_6 had been overlooked. These diagrams give two additional contributions to the γ^f:

$$\begin{aligned}\gamma_{57} &\to \gamma_{57} + \frac{\alpha_{em}}{2\pi}\cdot\frac{4}{3}\\ \gamma_{68} &\to \gamma_{68} + \frac{\alpha_{em}}{2\pi}\cdot\frac{4}{3}\,.\end{aligned} \tag{2.3}$$

The subscripts refer to the elements of the anomalous dimension matrices.

Because of the structure of the anomalous dimension matrices, only $\widetilde{c}_7$ and $\widetilde{c}_8$ are altered by the additions (2.3). It turns out that the major effect is on $\widetilde{c}_8$. This is simply seen in the large N approximation used by Refs. 3 and 7. In this approach one only keeps the operators $O_\pm = O_2 \pm O_1, O_6, O_7$ and O_8. Since O_5 does not appear in this approximation, the change to γ_{57} is irrelevant, and so $\widetilde{c}_7$ is completely unaffected. In this restricted basis the approximate anomalous dimension matrix ($\widetilde{\gamma}^f$) is, for $f = 3,4,5$

$$\frac{1}{2\pi}\begin{bmatrix} \left(3-\frac{3}{N}\right)\alpha_s & 0 & \frac{1}{3}\alpha_s & \frac{8}{27}(1+N)\alpha_{em} & 0 \\ 0 & \left(-3-\frac{3}{N}\right)\alpha_s & \frac{1}{3}\alpha_s & \frac{8}{27}(1-N)\alpha_{em} & 0 \\ 0 & 0 & -3N\frac{b_f}{11}\alpha_s & 0 & \frac{4}{3}\alpha_{em} \\ 0 & 0 & 0 & 0 & -3\alpha_s \\ 0 & 0 & 0 & 0 & -3N\frac{b_f}{11}\alpha_s \end{bmatrix}$$

The relevant part of $\widetilde{\gamma}^6$ is equal to the top left 2×2 submatrix of $\widetilde{\gamma}^{3,4,5}$. I have kept N explicit, though it is set to 3 in calculations.

The new term due to Equation (2.3) appears as the $\frac{4}{3}\alpha_{em}$ in the last column, third row. It is the same in the large N approximation as in the exact anomalous dimension matrix. It drives $\widetilde{c}_8$ in the opposite direction to the original γ_{78} ($= -3\alpha_s$) term. Buras and Gerard claim that the new term wins this competition, so that $\widetilde{c}_8$ is driven positive. It is this claim that I take issue with.

The advantage of the approximate $\widetilde{\gamma}^f$ is that it is easy to calculate $\widetilde{c}_i(\mu)$ analytically. I focus on $\widetilde{c}_8$. This only becomes non-zero at scales below m_t. Just below m_t one finds

$$\widetilde{c}_8(\mu) = \frac{\Delta^2}{(2\pi)^2}\,\frac{2}{9}\,\alpha_{em}\alpha_s(m_t)\,\left[5x^2 - \tfrac{7}{x}\right] + 0(\Delta^3)$$

$$x = [\alpha_s(m_t)/\alpha_s(m_w)]^{6/23} \qquad \Delta = \ell n(\mu/m_t)\ .$$

I stress that this is only true for the approximate anomalous dimension matrix $\widetilde{\gamma}$. The sign of $\widetilde{c}_8$ hinges on whether x is greater or less than $x_c \equiv (7/5)^{1/3} = 1.119$: if $x > x_c$ then $\widetilde{c}_8$ is positive, if $x < x_c$ then $\widetilde{c}_8$ is negative. For the parameters used by Ref. 3 ($\Lambda_4 = 0.3$ GeV, $m_t = 40$ GeV), and in fact for all allowed values of these parameters, $x < x_c$, and thus $\widetilde{c}_8$ starts out negative. I find that it remains negative as μ decreases, for nearly all parameters. In particular, as shown in Fig. 2, it remains negative for $\Lambda_4 = 0.3$ GeV, $m_t = 40$ GeV, in disagreement with the results of Buras and Gerard. The results for $\widetilde{c}_7$, on the other hand, which are shown in Fig. 1, are in agreement with those of Buras and Gerard.

I now present my results for the $\widetilde{c}_i$ without the large N approximation. I calculate them by diagonalizing the γ^f numerically. One check is that I reproduce the values quoted by Ref. 6 for $\widetilde{c}_1$ through $\widetilde{c}_6$, and those of Ref. 1 (using the uncorrected γ^f) for $\widetilde{c}_7$ and $\widetilde{c}_8$. I use the same numerical method to calculate the coefficients in the large N approximation (following the prescription of Ref. 7 for α_s), and a second check is to compare with the analytic results of Ref. 7. For all calculations, I use $m_c = 1.5$ GeV, $m_b = 4.5$ GeV and $m_w = 82$ GeV.

I display the results for $\widetilde{c}_6$, $\widetilde{c}_7/\alpha_{em}$ and $\widetilde{c}_8/\alpha_{em}$ in a series of plots against μ. Throughout I have kept $\alpha_{em} = 1/137$ independent of μ. These plots are cut-off at the lower end when $\alpha_s(\mu) = 1$. Figures 1 and 2 show the effects of using the corrected anomalous dimension matrices on $\widetilde{c}_7$ and $\widetilde{c}_8$ for $\Lambda_4 = 0.3$ GeV and $m_t =$ 40 GeV. Clearly $\widetilde{c}_7$ is little affected by the correction, while $\widetilde{c}_8$ is considerably reduced. However, the sign of $\widetilde{c}_8$ is unchanged, and is opposite to that found by Buras and Gerard.

The large N approximation (for the same parameter values) is also shown in these figures. For further comparison, the results for $\widetilde{c}_6$ in the full evaluation as well as in the large N evaluation are shown in Fig. 3. The large N approximation is reasonable for $\widetilde{c}_6$ and $\widetilde{c}_7$, but poor for $\widetilde{c}_8$. However, $\widetilde{c}_8/\alpha_{em}$ is small, and the large N approximation can only hope to do well for large quantities. The figure does indicate that care should be taken at small μ (where $\alpha(\mu) \approx 1$), since quite large differences between the real answer and its large N approximant can develop.*

The same general comments apply for other values of the parameters Λ_4 and m_t. Thus I only display the results for the full evaluation of $\widetilde{c}_6$, $\widetilde{c}_7$ and $\widetilde{c}_8/\alpha_{em}$ (the latter multiplied by 5), and the large N evaluation of 5 $\widetilde{c}_8/\alpha_{em}$. These graphs give some idea of the range of variation of the coefficients. The strong interaction penguin $\widetilde{c}_6$ is in agreement with the calculation of Ref. 6, and I show it only to display its rapid variation with μ, and to compare it to $\widetilde{c}_7$ and $\widetilde{c}_8$. In addition, it is useful for lattice calculations to have these coefficients for a variety of μ ($\approx$ 1/lattice spacing).

$\widetilde{c}_7/\alpha_{em}$ is quite large, but varies significantly with Λ_4 and m_t. It is almost independent of μ for $\mu < 3$ GeV, a result which is exact for $\mu < m_c$ in the large N approximation. $\widetilde{c}_8/\alpha_{em}$ also varies considerably, but, at least for $\mu > 1$ GeV, it is always small. For the ranges 26 GeV $< m_t < m_w$ and $\Lambda_4 < .3$ GeV, and for μ such that $\alpha_s(\mu) < 1$, $\widetilde{c}_8$ is always negative if the full anomalous dimension matrices are used. In large N approximation positive values of $\widetilde{c}_8$ are obtainable in the region of parameter space where m_t is small, Λ_4 is large, and μ is small. An example is given below. $\widetilde{c}_8$ grows rapidly as μ decreases, but only becomes comparable to $\widetilde{c}_6$ and $\widetilde{c}_7/\alpha_{em}$ at very small μ, at which point perturbation theory is untrustworthy in any case. Lattice calculations require $\mu \sim$ 1.5-2.5 GeV, and for this range $\widetilde{c}_8$ is always small and negative.

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* The fact that, in Figure 2, the uncorrected $\widetilde{c}_7$ and the large N $\widetilde{c}_7$ are very close is not significant. I would also like to stress that, were it not for the peculiar factors of $b_f/11$ in $\widetilde{\gamma}_{66}$ and $\widetilde{\gamma}_{88}$, the large N value of $\widetilde{c}_6$ would grow much faster at small μ than in the full calculation (see Figure 3).

3. Implications for ϵ'/ϵ

The master formulae used by Refs. 3, 2 and 5 are:*

$$\left[\frac{\epsilon'}{\epsilon}\right] = 3 \times 10^{-3} \left|\frac{s_1 s_2 c_2 s_3 s_\delta}{2\ 10^{-4}}\right| \left|\frac{\widetilde{c}_6}{.1}\right| \left(\frac{125\ \mathrm{MeV}}{m_s}\right)^2$$

$$\times\ B_6\ (1 - \Omega_{\eta+\eta'} + \Omega_{EMP})$$

$$\Omega_{EMP} = 0.23\ \left(\frac{\widetilde{c}_7 B_7 + 3\widetilde{c}_8}{3\alpha_{em}\widetilde{c}_6}\right)\ \frac{B_8}{B_6}$$

where the generalized B parameters are:

$$B_6 = \langle K^0|O_6|\pi\pi\rangle\ /\ \langle K^0|O_6|\pi\pi\rangle_{VIA}$$

$$B_8 = \langle K^0|O_8|\pi\pi\rangle\ /\ \langle K^0|O_8|\pi\pi\rangle_{VIA}$$

$$B_7 = 3\,\langle K^0|O_7|\pi\pi\rangle\ /\ \langle K^0|O_8|\pi\pi\rangle = B_8^{-1}\ \langle K^0|O_7|\pi\pi\rangle\ /\ \langle K^0|O_7|\pi\pi\rangle_{VIA}\ .$$

Reasonable values for the KM parameters s_i and c_i have been used. The matrix elements, coefficients, and m_s are all to be evaluated at a common scale. $\Omega_{\eta+\eta'}$ contains isospin breaking contributions involving $\pi - \eta$ and $\pi - \eta'$ mixing. Reference 2 finds $\Omega_{\eta+\eta'} = 0.35 - 0.45$, while Ref. 3 find 0.27, the difference coming from the different values of $(m_u - m_d)/m_s$ used. Whatever its precise value, this term suppresses ϵ' by a significant amount.

The term in question here is Ω_{EMP}. Notice that, if positive, Ω_{EMP} increases ϵ', now that the correct sign found in Ref. 3 is used. The value of Ω_{EMP} depends upon the coefficients $\widetilde{c}_{6-8}$, and on the three B parameters. In the definitions of these B parameters the subscript VIA refers to vacuum insertion approximation. Assuming VIA one can estimate all three matrix element in the continuum, and the B parameters are constructed to equal 1 in this approximation.

★ I have used the experimental values for ϵ and $K \to 2\pi$ decays. Ref. 3 calculate these within their approximation and thus have a slightly different formula. These differences do not affect Ω_{EMP}, however, and this is the main focus of discussion.

There are three approaches to calculating the B parameters. The first, used in Ref. 2, is to use the vacuum insertion approximation and set $B_6 = B_7 = B_8 = 1$. This presumably applies best at a scale in the range 0.5 GeV - 1.0 GeV. The crucial factor is $(\tilde{c}_7 + 3\tilde{c}_8)/3\alpha_{em}\tilde{c}_6$. At $\mu = 1$ GeV, and for $m_t = 30$ to 70 GeV and $\Lambda_4 = 0.1$ (0.3) GeV, this factor is 0.56 to 0.69 (0.22 to 0.32) corresponding to $\Omega_{EMP} = 0.13$ to 0.16 (0.05 to 0.07). At μ such that $\alpha_s(\mu) = 1$, the factor is 0.53 to 0.87 (0.26 to 0.50), and Ω_{EMP} is thus 0.12 to 0.20 (0.06 to 0.12). Thus the size of Ω_{EMP} is not very sensitive to μ, but strongly dependent on m_t and Λ_4. For $\mu = 1$ GeV most of the contribution comes from $\tilde{c}_7$; for smaller μ, $\tilde{c}_8$ becomes equally important at large m_t. Thus this rough independence of Ω_{emp} on μ only follows because $\tilde{c}_7$ and $\tilde{c}_8$ have the same sign.

The large N approach of Refs. 3 and 7 also sets $B_6 = B_8 = 1$, but has $B_7 = 0$. These values are supposed to apply for scales $\mu \approx 0.8 - 1.0$ GeV, at which scale the short and long distance physics are matched. The relevant ratio of coefficients is $\tilde{c}_8/\alpha_{em}\tilde{c}_6$. If one uses the large N anomalous dimension matrices, and the same ranges of parameters as above, one finds this ratio to be +0.007 to +0.021 (-0.003 to +0.013). Note that for $m_t = 30$ GeV and $\Lambda_4 = 0.3$ GeV $\tilde{c}_8$ is slightly positive at $\mu = 1$ GeV. The large Λ_4, small m_t region of parameter space is the only one for which this happens. As Fig. 2 shows, it does not happen for $m_t = 40$ GeV. Furthermore, if on uses the complete γ^f to evaluate $\tilde{c}_6$ and $\tilde{c}_8$, then $\tilde{c}_8$ is always negative and $\tilde{c}_8/\alpha_{em}\tilde{c}_6 = 0.013$ to 0.031 (0.004 to 0.025). However, for both of the methods of evaluation the EMP contribution is very small, making at most a 1% contribution ϵ'.

The difference between vacuum insertion and large N approximations is in the value of B_7. While it is true that there are many terms of non-leading order in N, among which B_7 is only one, the smallness of $\tilde{c}_8$ makes it essential to attempt a calculation of these terms. Also it is quite possible that there are large corrections to the relations $B_6 = B_8 = 1$. One way to address these issues is to attempt a calculation on the lattice of $B_{6,7,8}$. Such calculations are in their infancy, and the numerical results should not be taken too seriously. Reference

5 finds $B_6 \approx 0.5$, $B_7 \approx 1$ and $B_8 \approx 1.2$, for $\mu \approx 1.7$ GeV. This converts to $\Omega_{EMP} = 0.41$ to 0.57 (0.19 to 0.34). Thus the EMP contribution is larger than in the other two approximations, and may be as large as the strong interaction penguin contribution, $1 - \Omega_{\eta+\eta'}$. However, this is mainly due to a reduction in the size of the strong interaction penguin, so that, overall, ϵ'/ϵ is reduced, by 60-70%, compared to what it would be if $B_6 = B_7 = B_8 = 1$.

4. Conclusions

A full evaluation of the coefficients $\widetilde{c}_7$ and $\widetilde{c}_8$ shows that only $\widetilde{c}_8$ is much affected by the corrections to the anomalous dimension matrix found by Buras and Gerard.[3] $\widetilde{c}_8$ is considerably reduced in magnitude, but not changed in sign, contrary to the result of Ref. 3.

The consequences of this for ϵ'/ϵ depend upon the values of m_t, s_2, s_3 and s_δ, and upon the parameters B_6, B_7 and B_8. As far as the EMP contribution is concerned, the crucial issue is the size of the matrix element of O_7. If, as assumed in the vacuum insertion approximation, or as found in the lattice calculation, $B_7 \approx 1$, then the EMP contribution enhances ϵ', possibly by as much as a factor of two.

Note added: Buras and Gerard now find values for $\widetilde{c}_8$ of the same sign and similar magnitude to those I find. I thank them for clarifying correspondence and for comments on the manuscript.

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FIGURE CAPTIONS

1) Results for $\widetilde{c}_7(\mu)/\alpha_{em}$ for $\Lambda_4 = 0.3$ GeV, $m_t = 40$ GeV. The full calculation is shown by the solid line, the calculation with the older (uncorrected) anomalous dimension matrices of Ref. 1 is given by the dotted line, and the large N result is the dashed line.

2) Results for $\widetilde{c}_8(\mu)/\alpha_{em}$. Format as in Fig. 1.

3) Results for $\widetilde{c}_6(\mu)$ in large N (dashed) and with full calculation (solid). Parameters as in Fig. 1.

4) Results for $\widetilde{c}_6(\mu)$ (solid), $\widetilde{c}_7(\mu)/\alpha_{em}$ (dashed), $5 \times \widetilde{c}_8(\mu)/\alpha_{em}$ (dot-dashed) calculated with the full anomalous dimension matrices, and $5 \times \widetilde{c}_8(\mu)/\alpha_{em}$ calculated in large N approximation (dotted). Parameters are $\Lambda_4 = 0.3$ GeV, $m_t = 70$ GeV.

5) Same as Fig. 4 but for $\Lambda_4 = 0.1$ *GeV*, $m_t = 40$ GeV.

6) Same as Fig. 4 but for $\Lambda_4 = 0.1$ GeV, $m_t = 70$ GeV.

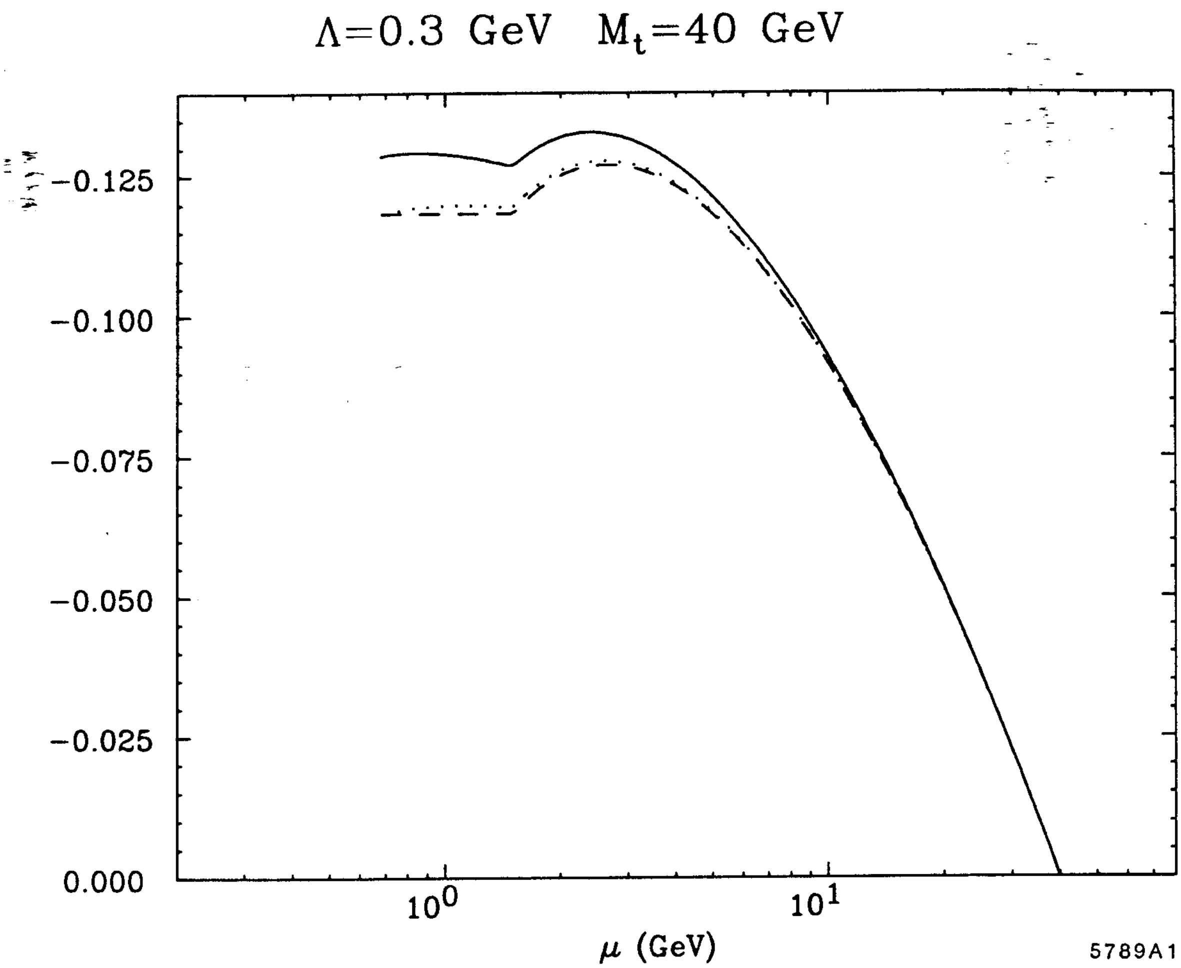

Λ=0.3 GeV M_t=40 GeV
-0.125
-0.100
-0.075
-0.050
-0.025
0.000
10^0
10^1
μ (GeV)
5789A1

FIGURE 1

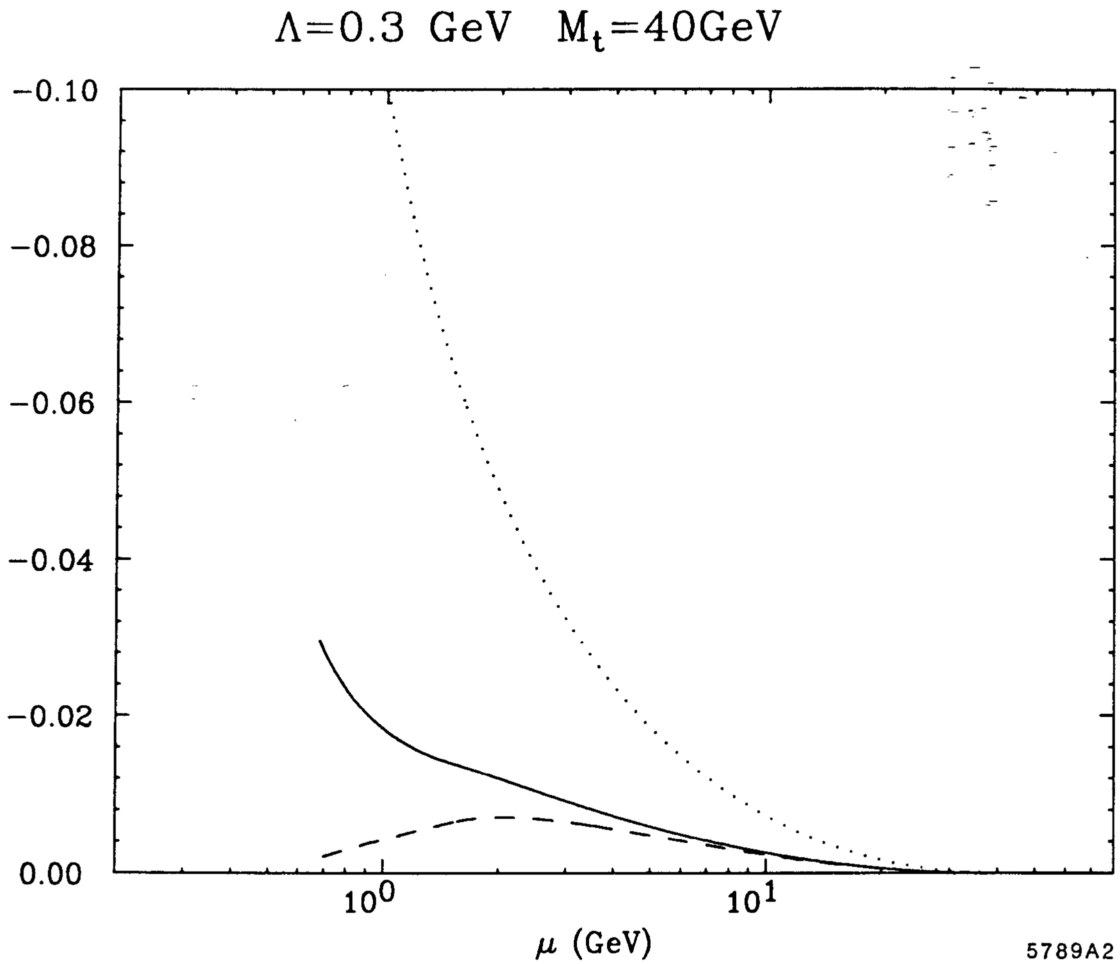
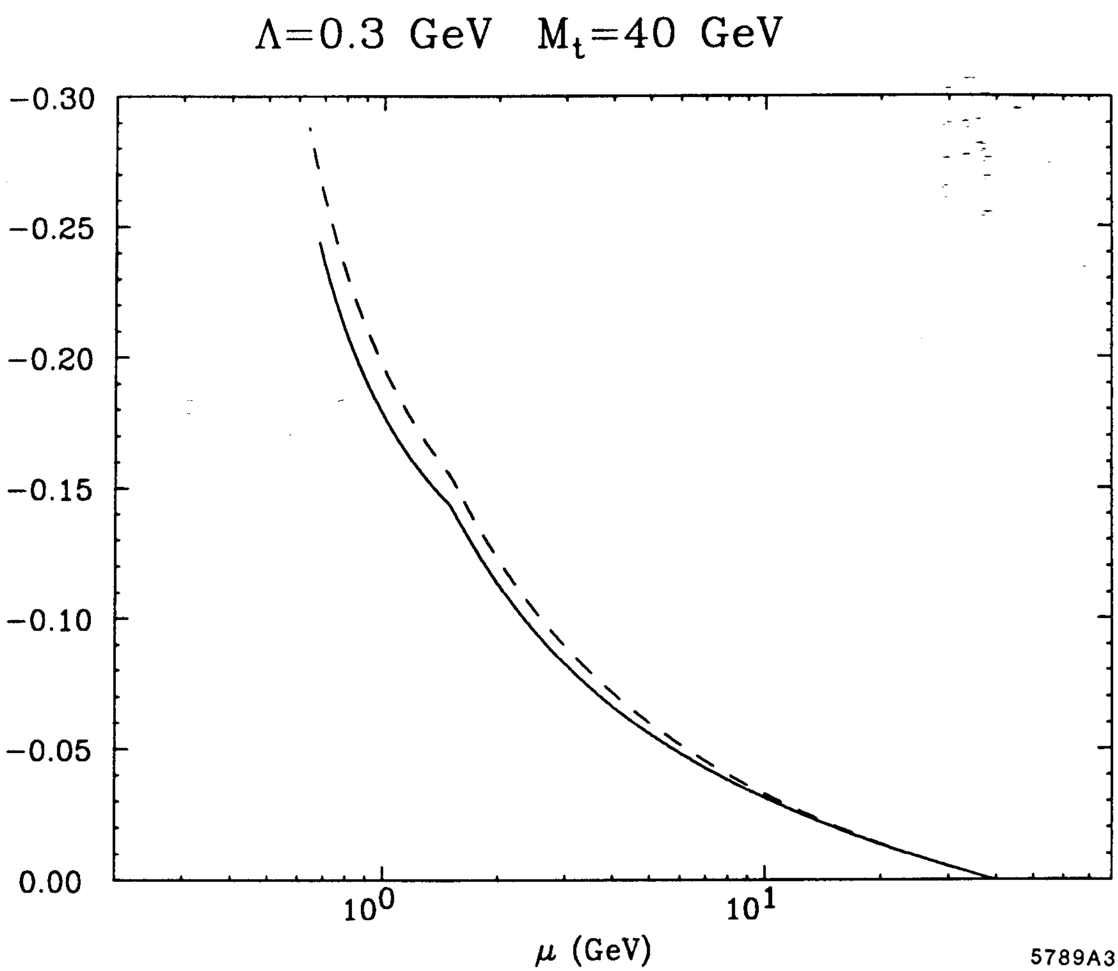


FIGURE 2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10^0
10^1
μ (GeV)
5789A3

FIGURE 3

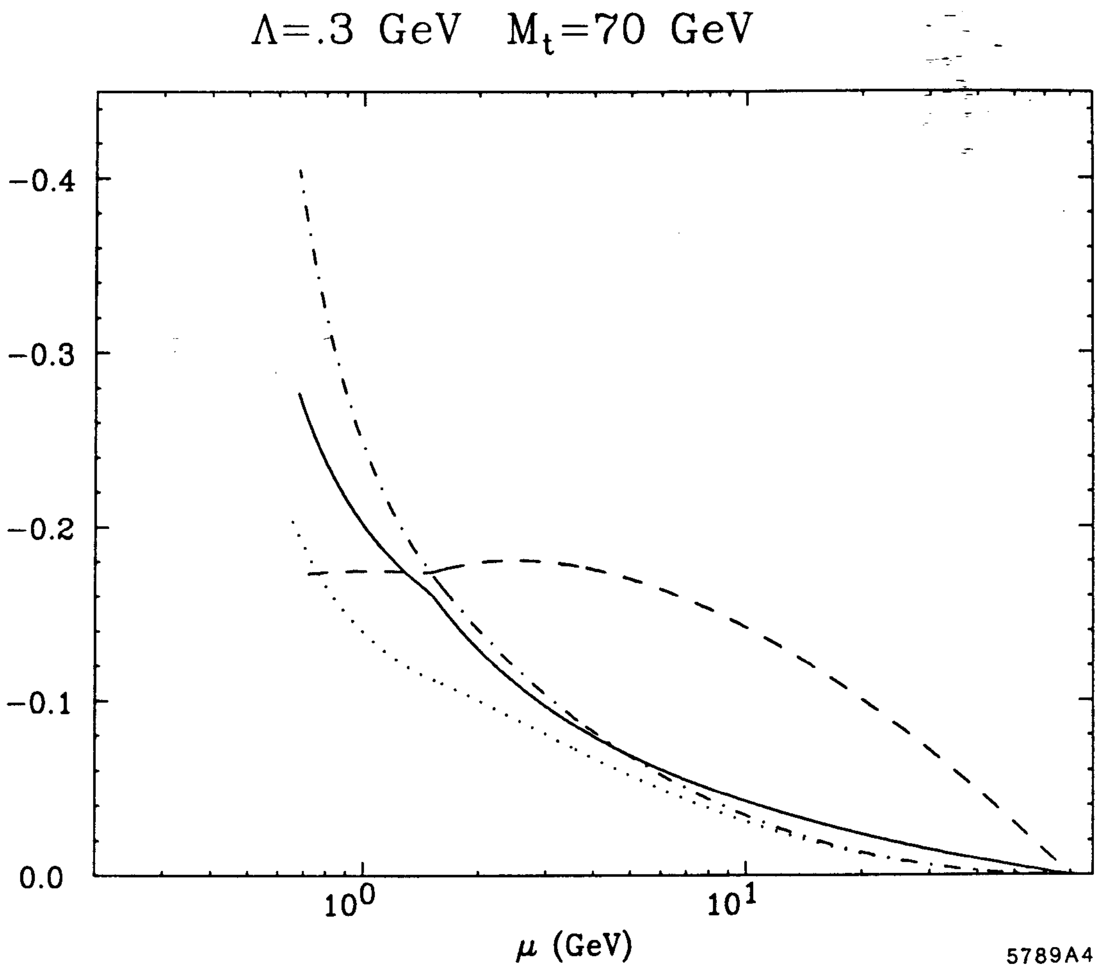

FIGURE 4

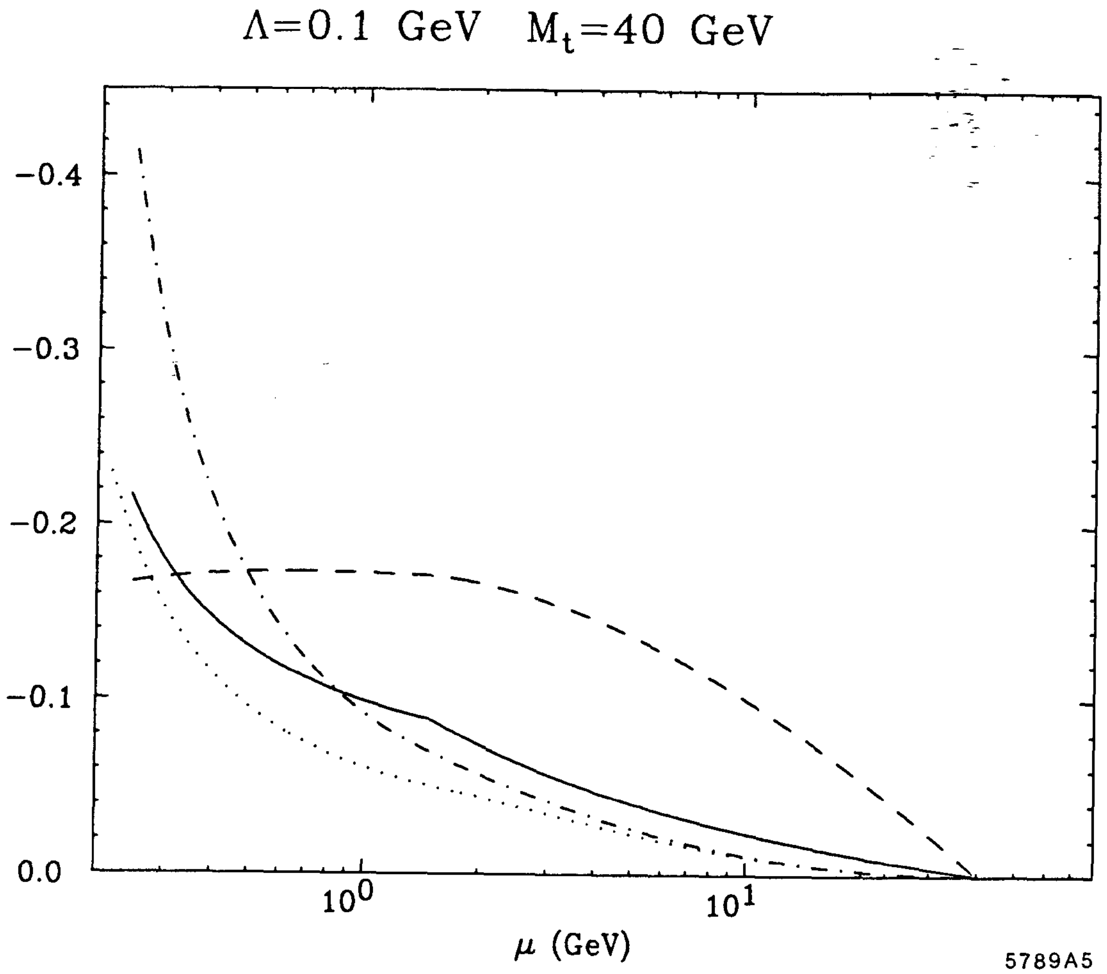

FIGURE 5

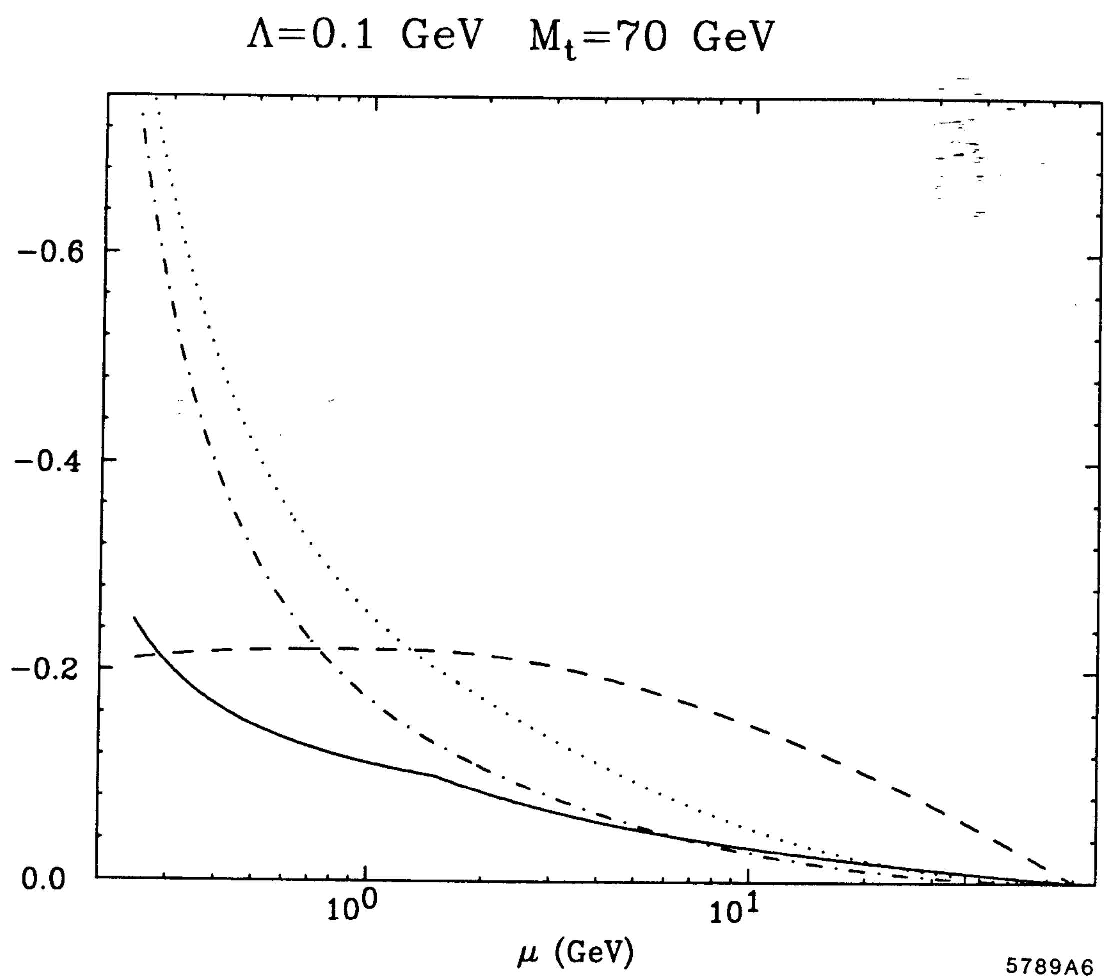

FIGURE 6