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THE NATURE OF BEAMSTRAHLUNG*)

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ABSTRACT

The physical nature of beamstrahlung during beam-beam interaction in linear colliders is reviewed. We first make the distinction between a *dense* beam and a *dilute* beam. We then review the characteristics of synchrotron radiation (SR) and bremsstrahlung, and argue that for a wide range of beam parameters beamstrahlung is SR in nature, even if the beam is dilute. Some issues concerning the specific conditions in beamstrahlung as SR are then discussed. Finally we suggest that in order to suppress beamstrahlung energy loss and to improve energy resolution, it is desirable to partition a bunch into a train of bunchlets, where the length of each bunchlet is shorter than the SR convergence length.

1. INTRODUCTION

For future e^+e^- linear colliders with center of mass energy at the TeV range, and luminosity around 10^{33} cm^{-2} sec^{-1}, it is inevitable that the e^+e^- bunches be focused down to miniscule dimensions. The high density of charged particles at the interaction point would provide strong electromagnetic fields viewed by the particles of the oncoming beam. The bending of particle trajectories under the influence of these EM fields is called disruption.[1)] During this bending particles would radiate, causing an energy loss of the beam; this is called beamstrahlung.[2)] Both effects of disruption and beamstrahlung are important to the design of linear colliders.[3)]

While disruption with negligible energy loss, which is a purely classical phenomena, is in principle understood (although in practice, the effect is convolutional and therefore needs computer simulations for detailed description of the phenomena), the nature of beamstrahlung still needs to be further clarified. In this paper we review the beamstrahlung in various beam parameter regimes. We then point out that it is desirable to partition each e^+e^- bunch into a train of bunchlets with longitudinal standard deviation σ_z^* shorter than the synchrotron radiation convergence length $\ell_{\rm SR}$.

2. DENSE BEAM *vs.* DILUTE BEAM

In the laboratory frame (also the center-of-mass frame) of a linear collider, an electron encountering a positron with an impact parameter b would have an effective interaction time $\Delta t_1 \sim b/\gamma c$, where c is the speed of light, due to the fact that the fields associated with relativistic particles span about an opening angle $\Delta\theta \sim 1/\gamma$. In turn, the corresponding effective distance of traverse through the fields of the oncoming particle is

$$\Delta\ell_1 = c\Delta t_1 \sim \frac{b}{\gamma} \quad . \tag{1}$$

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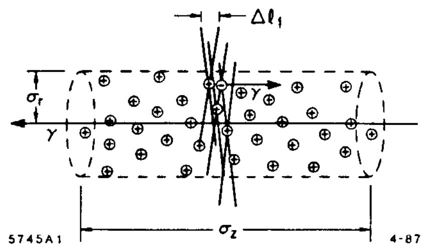

Fig. 1. Schematic diagram of a *dense* beam.

2.1 Dense beam

Consider an electron encountering the entire flux of the oncoming positron bunch. The flux is roughly

$$\frac{Nc}{\sigma_z} \equiv \frac{c}{\Delta\ell_2} \quad , \tag{2}$$

where $\Delta\ell_2 = \sigma_z/N$ is the mean longitudinal separation of target particles. The target beam is considered to be dense if $\Delta\ell_1 \gg \Delta\ell_2$. Taking a typical value of impact parameter to be one standard deviation in the transverse direction , *i.e.*, $b \sim \sigma_r$, the condition for a dense beam translates into

$$\frac{N\sigma_r}{\gamma\sigma_z} \gg 1 \quad . \tag{3}$$

In this case the background field provided by the particles in the oncoming bunch is continuous. (See Fig. 1.) For example, the Stanford Linear Collider (SLC) beam parameters are $\gamma = 10^5$, number of particles per bunch $N = 5 \times 10^{10}$, $\sigma_z = 1$ mm, and $\sigma_r = 1$ μm at the interaction point. Thus, $N\sigma_r/\gamma\sigma_z \cong 500 \gg 1$, and the beams are *dense.*

2.2 Dilute beam

A beam is said to be *dilute* if $\Delta\ell_2 \ll \Delta\ell_1$, or

$$\frac{N\sigma_r}{\gamma\sigma_z} \ll 1 \quad . \tag{4}$$

In this case the background field becomes discrete and the test particle would see the granularity of the target bunch. (See Fig. 2.) For example, in the conceptual accelerator of 5 TeV + 5 TeV discussed by Richter,[4] $\gamma = 10^7$, $N = 4.1 \times 10^8$, $\sigma_z \sim 10^{-3}$ mm and $\sigma_r \sim 10^{-3}$ μm, we have $N\sigma_r/\gamma\sigma_z \simeq 0.04 \ll 1$. The beams are therefore quite *dilute.*

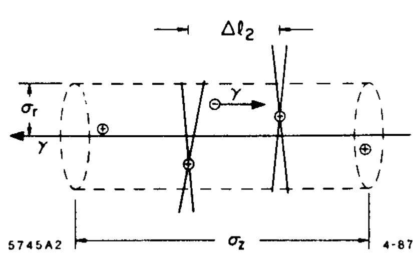

Fig. 2. Schematic diagram of a *dilute* beam.

In one version of the CLIC parameters,[5] where $\gamma = 2 \times 10^6$, $N = 5.4 \times 10^9$, $\sigma_z = 0.5$ mm and $\sigma_r = 65$ nm, we find $N\sigma_r/\gamma\sigma_z \simeq 0.35 \lesssim 1$. Therefore the beam is marginally dilute.

3. SYNCHROTRON RADIATION AND BREMSSTRAHLUNG

In terms of the physical nature of beamstrahlung, two well-known radiation mechanisms come into mind, *i.e.*, synchrotron radiation (SR) and bremsstrahlung (BR). Each mechanism has a different characteristic length.

3.1 Synchrotron radiation

By synchrotron radiation we mean the radiation of charged particles moving in circular orbits under a *uniform* background magnetic field infinite in extent. Quantum mechanically, the photons are emitted in a discrete manner. For each radiated photon, it takes a certain *convergence* length $\ell_{\rm SR}$ such that the radiation process can be completed. This length is found to be

$$\ell_{\rm SR}(\omega) = \frac{\rho}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3} \quad , \tag{5}$$

where ρ is the radius of the orbit and ω_c the critical frequency ($\omega_c \equiv 3c\gamma^3/2\rho$).

When ω_c is much less than the kinetic energy of the radiating particle $\mathcal{E} = \gamma mc^2$, the radiated photons are soft and in large quantity. This corresponds to the classical regime of SR. On the contrary, when $\omega_c \gg \mathcal{E}$, the photons would take away a substantial fraction of the particle's initial energy; therefore the conservation of energy-momentum before and after the radiation process and the noncommutativity between the photon field and the particle field have to be properly treated, and we are in the quantum mechanical regime.

A useful Lorentz invariant, dimensionless parameter that indicates the various regimes of SR is Υ, defined as

$$\Upsilon = \frac{2\omega_c}{3\mathcal{E}} = \gamma\,\frac{B}{B_c} \quad , \tag{6}$$

where $B_c = m^2c^3/e\hbar$. In the classical regime, $\Upsilon \ll 1$, whereas in the quantum regime, $\Upsilon \gg 1$.

The applicability of the SR picture to the problem of beamstrahlung can be qualified by the following inequality:

$$\Delta\ell_2 = \frac{\sigma_z}{N} \ll \ell_{\rm SR} \ll \sigma_z \quad . \tag{7}$$

When this is satisfied, the field provided by the opposite beam can be treated as homogeneous and infinite longitudinally. For the transverse dimensions similar arguments apply, *i.e.*, we require that

$$\Delta\theta\ell_{\rm SR} \sim \frac{\ell_{\rm SR}}{\gamma} \ll \sigma_r \quad . \tag{8}$$

3.2 Bremsstrahlung

Historically, bremsstrahlung refers to the radiation phenomena caused by the scattering of a test particle by target particles. In order for radiation to take place, it is necessary that some momentum q be transferred from the radiating particle to the target particles. The minimum of this momentum transfer, q_{min}, corresponds to the situation where the photon momentum $\vec{k}$ is parallel to the momentum of the radiating particle:

$$q_{min} = |\vec{p}_i| - |\vec{p}_f| - |\vec{k}| \approx \frac{mc}{2}\left(\frac{1}{\gamma_f} - \frac{1}{\gamma_i}\right) = \frac{\hbar\omega}{2c\gamma_i\gamma_f} \quad . \tag{9}$$

Let us define $q^2 = q_\parallel^2 + q_\perp^2$, where $q_\parallel$ and $q_\perp$ are the longitudinal and transverse components, respectively. We can then distinguish two characteristic regions of q values.[6] The first region is characterized by $q \approx q_\perp$. The momentum transfer is essentially in the direction transverse to the particle's instantaneous motion. In this region the value of $q_\perp$ is determined only by the action of the external field (*i.e.*, the scattering angle) and is not associated with the radiation process. In this region of *classical* momentum transfer, the radiation is essentially that in the Born approximation.

In the second region where $q \approx q_\parallel \approx q_{min}$ the momentum transfer is not determined by the scattering angle of the particle, and the phenomena of quantum diffraction becomes important. From the uncertainty principle the virtual photon that carries the minimal momentum transfer can be absorbed anywhere within the coherence length ℓc,

$$\ell c = \frac{\hbar}{q_{min}} = \frac{2c\gamma_i\gamma_f}{\omega} \quad . \tag{10}$$

In a wide range of parameters that we study in beamstrahlung, for example, from SLC, CLIC, to the Richter scale, we always find that $q_\perp \gg q_\parallel$ due to the following observation:[7] For the sake of argument, let us consider bunches as uniform cylindrical slugs of charges. The total $q_\perp$ for a test charge with impact parameter σ_r is thus

$$q_\perp = 2e \int dz\, E_\perp = \frac{2Ne^2}{\sigma_r} \quad . \tag{11}$$

Thus we find

$$\frac{q_\perp}{mc} = \frac{2Nr_e}{\sigma_r} \simeq \begin{cases} 2.8 \times 10^2\ , & \qquad \text{SLC}\ , \\ 4.3 \times 10^2\ , & \gg 1, \quad \text{CLIC}\ , \\ 2.0 \times 10^3\ , & \qquad \text{Richter 5 TeV + 5 TeV} \end{cases} \quad . \tag{12}$$

Whereas typically

$$\frac{q_\parallel}{mc} \approx \frac{q_{min}}{mc} = \frac{1}{2}\left(\frac{1}{\gamma_f} - \frac{1}{\gamma_i}\right) \ll 1\ .$$

Therefore $q_\perp \gg q_\parallel$ and we have $q \approx q_\perp$. This means that the bremsstrahlung coherence length ℓc is irrelevant to our issue. More importantly, the applicability of bremsstrahlung to the problem of beamstrahlung lies only in the domain

$$\sigma_z \lesssim \ell_{\mathrm{SR}} \quad . \tag{13}$$

In this regime the spatial extent of the external field is too limited for synchrotron radiation to take place.

4. BEAMSTRAHLUNG AS SYNCHROTRON RADIATION

In the parameter regime where SR is applicable to beamstrahlung (*i.e.*, $\sigma_z/N \ll \ell_{SR} \ll \sigma_z$), one should take into account the specific nature of beam-beam interaction:

4.1 Uniformity of field strength

Typically the density distribution varies both longitudinally and transversely across a bunch. For *round* beams where $R = \sigma_x/\sigma_y = 1$, and define $\sigma_r = \sigma_x = \sigma_y$, the distribution function is proportional to $f_{R=1}$:

$$f_{R=1} = \frac{8}{\sqrt{3}} \frac{\exp\{-z^2/2\sigma_z^2\}}{\sqrt{2\pi}} \frac{1-\exp\{-r^2/2\sigma_r^2\}}{r/\sigma_r} \quad ,$$

and for flat beams ($R > 1$), (14)

$$f_{R>1} = \frac{2(1+R)}{\sqrt{3}} \frac{e^{-z^2/2\sigma_z^2}}{(R-1)^{1/2}} \cdot \left| w\left(\frac{x+iy}{\sqrt{2(R^2-1)}\sigma_y}\right) - e^{-[(x^2/2\sigma_x^2)+(y^2/2\sigma_y^2)]} \cdot w\left(\frac{x/R+iRy}{\sqrt{2(R^2-1)}\sigma_y}\right)\right| \quad ,$$

where $w(\varsigma)$ is the complex error function. In turn the field strength has the same functional variation. It is in principle possible to evaluate the radiation energy loss in this varying field by carrying out calculations based on first principles. It is, however, more desirable if there exist simple scaling relations where energy loss and other related physical quantities in beamstrahlung can be evaluated based on the knowledge of single particle radiation in a uniform field; namely, that of Sokolov and Ternov.[8]

For this purpose it is essential to define an effective SR parameter $\overline{\Upsilon}$ for the entire target bunch.[9] In the case of bi-Gaussian density distribution of Eq. (14), it is found by integrating over the entire bunch that

$$\overline{\Upsilon} \simeq \frac{\sqrt{3}}{4} \frac{N r_e \lambda\!\!\!^{-}_c \gamma}{\sqrt{\sigma_x \sigma_y}\sigma_z} \left[\frac{2R^{1/2}}{1+R}\right] \quad , \tag{15}$$

where r_e is the classical electron radius and $\lambda\!\!\!^{-}_c$ the Compton wavelength. The local $\Upsilon(x,y,z)$ is then related to $\overline{\Upsilon}$ via

$$\Upsilon(x,y,z) = f_R(x,y,z)\,\overline{\Upsilon} \quad . \tag{16}$$

4.2 The effect of granularity

In the case of *dilute* beams, the fields are physically discrete as viewed by a test particle. Though on the average the test particle would bend towards the axis, locally the dilute scattering centers may deflect the particle inward or outward stochastically. This "wiggler" effect due to the granularity would therefore superimpose some ripples to the smooth trajectory associated with the global bending. The mean periodicity of the ripples is expected to be the mean separation between particles $\Delta\ell_2 = \sigma_z/N$, with the corresponding frequency

$$\omega_d \sim \frac{c\gamma^2}{\Delta \ell_2} = \frac{cN\gamma^2}{\sigma_z} \quad . \tag{17}$$

Since we are in the regime where $\Delta \ell_2 \ll \rho/\gamma$, we deduce that

$$\omega_d \gg \frac{c\gamma^3}{\rho} \sim \omega_c \quad . \tag{18}$$

In the case of the Richter scale, $\omega_c \gg \mathcal{E}$, thus both ω_c and ω_d are kinematically forbidden.[10] The same conclusion was reached by Blankenbecler and Drell[11] through explicit calculations.

As for CLIC, even though $\omega_c < \mathcal{E}$, one can easily verify that ω_d is still so large that $\omega_d \gg \mathcal{E}$, and would not be seen. We therefore conclude that in a wide range of beam parameters the effect of granularity in the case of a dilute beam would not be seen.

4.3 Finite length of the target

Given the total power of radiation $P(\overline{\Upsilon})$ from an electron and the photon emission rate $N(\bar{\Upsilon})$, one can deduce scaling laws of various physical quantities related to beamstrahlung if another parameter Γ is introduced. This is because dimensionally the fractional energy loss per electron is

$$\frac{\Delta \mathcal{E}}{\mathcal{E}} \propto P(\overline{\Upsilon}) \cdot \left(\frac{\Delta z}{c} \cdot \frac{1}{\mathcal{E}} \right) \quad . \tag{19}$$

It is thus useful to introduce Γ as available energy per unit length[9]:

$$\Gamma \equiv \frac{9}{16} \frac{\gamma \lambda\!\!\!^{-}_c}{\sigma_z} \quad . \tag{20}$$

From computer simulations, Noble[9] has deduced a set of remarkably simple scaling laws for beamstrahlung with negligible disruption based on the two parameters $\overline{\Upsilon}$ and Γ, which includes the following relations for average energy loss δ, average photon number $\langle N_\gamma \rangle$, and average photon energy $\langle \hbar\omega/\mathcal{E} \rangle$:

$$\begin{aligned} \delta \equiv \left\langle \frac{\Delta \mathcal{E}}{\mathcal{E}} \right\rangle &= \frac{2}{3} \frac{\alpha}{\Gamma}\, g\left(\overline{\Upsilon}\right) \quad , \\ \langle N_\gamma \rangle &= \frac{5}{2\sqrt{3}} \frac{\alpha}{\Gamma}\, h\left(\overline{\Upsilon}\right) \quad , \\ \left\langle \frac{\hbar\omega}{\mathcal{E}} \right\rangle &= \frac{4}{5\sqrt{3}} \frac{g\left(\overline{\Upsilon}\right)}{h\left(\overline{\Upsilon}\right)} \quad , \end{aligned} \tag{21}$$

where

$$g\left(\overline{\Upsilon}\right) \simeq \begin{cases} \overline{\Upsilon}^2 \, , & \overline{\Upsilon} \ll 1 \, , \\ 0.556\, \overline{\Upsilon}^{2/3} \, , & \overline{\Upsilon} \gg 1 \, , \end{cases}$$

and

$$h\left(\overline{\Upsilon}\right) = \begin{cases} \overline{\Upsilon} \, , & \overline{\Upsilon} \ll 1 \, , \\ 1.012\, \overline{\Upsilon}^{2/3} \, , & \overline{\Upsilon} \gg 1 \, , \end{cases}$$

are the well-known functions for radiation power and emission rate in SR. For intermediate values of $\overline{\Upsilon}$, a numerical table for $g(\overline{\Upsilon})$ and $h(\overline{\Upsilon})$ is necessary, which can be found in the literature.

4.4 The effect of disruption

In reality both e^+ and e^- beams pinch each other into smaller sizes during the collision, forcing the value of $\overline{\Upsilon}$ to change. For disruption parameters $D_x, D_y \ll 1$, the equations of motion in the transverse dimensions are

$$\frac{dx}{dz} = -\frac{D_x}{\sigma_z} z \quad , \quad \frac{dy}{dz} = -\frac{D_y}{\sigma_z} z \quad . \tag{22}$$

It can be shown that for a given aspect ratio $R, D_x = D_y/R \equiv D/R$. Therefore, the bunch size after penetrating a distance σ_z is related to its initial size by

$$\sigma_x' = \sigma_x e^{-D/R} \quad , \quad \sigma_y' = \sigma_y e^{-D} \quad , \tag{23}$$

and the aspect ratio is changed to

$$R' = \frac{\sigma_x'}{\sigma_y'} = R e^{D(R-1)/R} \quad . \tag{24}$$

The beamstrahlung parameter $\overline{\Upsilon}$ in Eq. (15) is therefore modified into

$$\overline{\overline{\Upsilon}} = \left[\frac{(1+R)e^D}{1+Re^{D(R-1)/R}}\right]\overline{\Upsilon} \quad , \quad \text{for } D \ll 1 \quad . \tag{25}$$

To generalize this expression to arbitrary value of D, we replace the factor e^D by a general function $H_D^{1/2}$ whose functional behavior can only be obtained numerically. So for arbitrary D and R we have

$$\overline{\overline{\Upsilon}} = \left[\frac{(1+R)H_D^{1/2}}{1+RH_D^{(R-1)/2R}}\right]\overline{\Upsilon} \equiv H_B(D,R)\overline{\Upsilon} \quad . \tag{26}$$

In the limit for round beams, $R = 1$, and

$$\overline{\overline{\Upsilon}} = H_D^{1/2}\overline{\Upsilon} \quad . \tag{27}$$

This equation for the round beam limit agrees with the corresponding expressions in the literature.[12]

Taking again the example of CLIC, where $D = 0.91$ and $H_D = 3.5$, we find $\overline{\Upsilon} = 0.16$ and $\overline{\overline{\Upsilon}} = \sqrt{H_D}\,\overline{\Upsilon} = 0.29$. Plugging $\overline{\overline{\Upsilon}}$ into Eq. (20), we find that $\delta = 0.10$, $\langle N_\gamma \rangle = 2.17$, and $\langle \hbar\omega/\mathcal{E} \rangle = 0.048$. Other quantities obtained from a computer simulation[13)] show that the mean center of mass energy squared $\langle S/S_0 \rangle = 0.85$, and the *rms* $S/S_0 = 0.18$.

5. BUNCHLETS AND BEAMSTRAHLUNG SUPPRESSION

From the discussions above, we see that there would be substantial energy loss and energy spread through beamstrahlung in future linear colliders. The available center of mass energy for the colliding beams would therefore be less, and the energy resolution degraded. One way to suppress the beamstrahlung is to partition a typical bunch into a train of bunchlets such that the nature of beamstrahlung departs from synchrotron radiation.

Notice that in the quantum regime of SR the beamstrahlung energy loss $\delta \simeq 0.37\alpha\overline{\Upsilon}^{2/3}/\Gamma$. Since both $\overline{\Upsilon} \propto \sigma_z^{-1}$ and $\Gamma \propto \sigma_z^{-1}$, it is clear that there will be less energy loss for smaller σ_z if all other parameters are fixed. Therefore even in the range where $\sigma_z \gg \ell_{SR}$, the situation is in favor of short bunches when $\overline{\Upsilon} > 1$. What we are suggesting here is to go beyond this point and to work in the parameter range where $\sigma_z < \ell_{SR}$. In this limit the external field becomes so short that the edge effects of the field play an essential role in the radiation process. The entire target bunch acts more like a nucleus, and the radiation is turning more bremsstrahlung-like.

First let us compare the following two situations: A magnet with length L, and a similar magnet but cut into two halves. Let the two shorter magnets be separated longitudinally such that no interference between the radiation from the separate magnets would occur. In the classical regime where $\Upsilon \ll 1$ the total power of the radiation is the same for the two cases, except that the shorter magnets tend to suppress the lower frequency spectrum in favor of higher frequencies. In the extreme limit where the original magnet is cut into a large number of short magnets with length $L^* < \ell_{SR}$, the radiation power spectrum would become independent of ω up to a maximum frequency $\omega^* \sim \omega_c\,(\ell_{SR}/L^*)$ (Fig. 3). Radiation is therefore suppressed for $\omega^* > \mathcal{E} = \gamma mc^2$ or equivalently for

$$\Upsilon > \frac{L^*}{\ell_{SR}} \quad . \tag{28}$$

Under this condition, the high-frequency spectrum beyond the kinetic energy of the radiating particle is energetically forbidden, and the total radiation power is reduced.

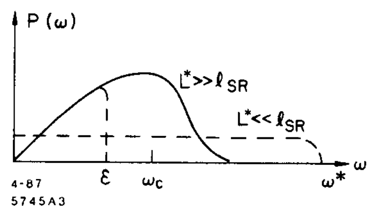

Fig. 3. The radiation power spectrum of bunchlets in the two asymptotic limits. The cut-off frequency ω^* is related to ω_c by $\omega^* = \omega_c(\ell_{SR}/L^*)$.

To invoke this radiation suppression mechanism in beamstrahlung, let us recall that in terms of Υ,

$$\frac{\rho}{\gamma} = \frac{\gamma \lambda\!\!\!^{-}_c}{\Upsilon} \quad . \tag{29}$$

For $\Upsilon \geq 1$, and for radiated photons at the kinematic limit $\mathcal{E}$, the convergence length is therefore

$$\ell_{SR}(\omega = \mathcal{E}) = \frac{\rho}{\gamma} \cdot \left(\frac{\omega_c}{\mathcal{E}}\right)^{1/3} = \left(\frac{3}{2}\right)^{1/3} \gamma \lambda\!\!\!^{-}_c \Upsilon^{-2/3} \quad .$$

Assuming that a bunch with length σ_z is now partitioned into n bunchlets, each with length σ_z^*, the requirement for beamstrahlung suppression is then

$$\sigma_z^* < \left(\frac{3}{2}\right)^{1/3} \gamma\, \lambda\!\!\!^{-}_c\, \Upsilon^{-2/3} \quad . \tag{30}$$

Next we insist that there is no constructive interference between the radiation from the separate bunchlets. For this purpose we require a photon radiated at the end of a bunchlet to travel long enough through the free space such that before both the radiating particle and the photon reach the next bunchlet, there is $\pi/2$ relative phase difference between the two particles.

Taking into account the Doppler shift, this translates into the following relation for interbunchlet spacing:

$$\Delta\ell^* \gtrsim \frac{\pi}{2}\gamma\lambda\!\!\!^{-}_c \quad . \tag{31}$$

For a 1 + 1 TeV collider, $\Delta\ell^* \simeq 1.2\ \mu$m. In the particular case where $\Upsilon \sim 1$, the other condition [*i.e.*, Eq. (30)] requires that $\sigma_z^* \sim .88\ \mu$m. We see that both the bunchlet length and the spacing are of the order of 1 μm. To retain the same luminosity in this arrangement, we would then have to stretch the total length of the bunch by about a factor of two.

Finally, since the power spectrum of this bunchlet arrangement is approaching a constant (*i.e.*, independent of ω) in the asymptotic limit, the photon emission rate is thus $\propto 1/\omega$, and we expect that the *rms* center of mass energy spread (*i.e.*, the energy resolution) should also be improved because the emission probability for hard photons is largely suppressed. Technically, it may be feasible to bunch such bunchlet trains by some kind of laser or FEL at $\sim 1\ \mu$m wavelength while the beam is still at reasonably low energy. More studies are necessary before one can be certain that this is a promising scheme.

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