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Imploding Monopoles*

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ABSTRACT

We show that magnetic monopoles with mass greater than $M_{crit} \simeq M_{Pl}/\sqrt{\alpha}$, corresponding to a grand unification scale $M_W \gtrsim \sqrt{\alpha} M_{Pl}$, are unstable to gravitational collapse: they will form magnetic black holes. They evaporate via Hawking radiation until they reach $M_{mon} \simeq M_{crit}$ and thereafter remain as extreme Reissner-Nordstrom holes with zero Hawking temperature. We point out a striking analogy between the structure of monopoles and the mechanics of black holes, which becomes an identity for magnetic black holes.

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In grand unified theories of the strong, weak and electromagnetic interactions, including the currently fashionable superstring models[1], the symmetry breaking scale, characterized by the gauge vector boson mass M_W, is close to the Planck energy, $M_{Pl} = (\hbar c^5/G)^{1/2} = 1.3 \times 10^{19}$ GeV, the scale characteristic of gravitational interactions. Such theories generically contain stable magnetic monopoles[2]. Since they are heavier than the vector bosons, $M_{mon} \simeq M_W/\alpha$, it is natural to ask how gravity affects the structure of monopoles. In this letter, we show that classical monopole solutions of energy $M_{mon} \gtrsim M_{Pl}/\sqrt{\alpha}$ are unstable to gravitational collapse and ultimately become magnetically charged black holes[3].

These magnetic black holes are interesting in several respects. First, monopoles are one of the few hopes for a direct experimental probe of unification. Because they are stable, there may be a relic population of monopoles in the universe today, remnants of a symmetry breaking phase transition in the early universe, and experiments are underway to detect them. This work suggests that, in fact, black holes may be detected in the laboratory.

Second, it is well known that the properties of black holes suggest a deep connection between the dynamics of strong gravitational fields and the laws of thermodynamics[4]. In particle theories, magnetic monopoles serve as a similar arena for the study of strong *gauge* fields. The interplay between non-linear fields in general relativity and non-perturbative fields in gauge theories of elementary particles may help to indicate how black hole thermodynamics emerges from a 'supertheory' in which gravity and gauge interactions are eventually unified.

Third, these objects raise anew some old and partly unsettled issues of general relativity, such as the endstate of gravitational collapse, the cosmic censorship hypothesis (which states that spacetime singularities are always demurely hidden behind event horizons instead of being naked), and the ultimate fate of evaporating black holes.

In spontaneously broken gauge theories, monopoles arise as stable, spherically symmetric, static solutions of finite energy to the classical field equations[2]. Consider the simplest model with monopoles, an $SU(2)$ gauge theory with Higgs triplet ϕ^a, gauge fields W^a_μ, and gauge coupling e. The scalar field vacuum expectation value, $(\phi^a\phi^a)^{1/2} = v$, breaks the symmetry to $U(1)_{EM}$ and gives mass to two components of the vector field, $M_W^{\pm} = ev$. The magnetic charge $g = 1/e$ of the monopole is distributed over a core of radius R_c, giving rise to a long range magnetic field $\vec{B} = (g/r^2)\hat{r}$ on scales $r \gtrsim R_c$. Its magnetostatic energy is thus of order

$$E_{mag} \simeq \frac{1}{2}\int_{R_c}^{\infty} B^2 d^3r \simeq 2\pi g^2/R_c. \tag{1}$$

Unlike Dirac's point monopole ($R_c \to 0$), the 't Hooft-Polyakov monopole is an extended, *nonsingular* object.

The stability of the classical monopole solution, and thus the conservation of magnetic charge, are ensured by the topology of the gauge field vacuum. Far from the monopole core, ϕ^a takes on the hedgehog configuration, $\phi^a = vr^a/r$, which cannot be 'unwound' by a nonsingular gauge transformation. For this configuration to be nonsingular, at the origin the scalar field must be pinned at the symmetry restoring value $\phi^a = 0$, far from the vacuum state. The gradient of the scalar field is concentrated at $r \simeq R_c$, falling off exponentially at larger radii; the energy stored in the scalar field is thus

$$E_{scalar} \simeq \frac{1}{2}\int_0^{R_c} (\partial_i \phi)^2 d^3r \simeq \frac{1}{2}\int (\frac{\phi^2}{r^2}) d^3r \simeq 2\pi R_c v^2 \tag{2}$$

The balance between the terms in eqs.(1) and (2) makes the core stable.

Minimizing the total energy with respect to R_c, one finds the core radius is $R_c \simeq M_W^{-1}$, and the monopole energy is

$$M_{mon} \simeq 4\pi M_W/e^2 \simeq 4\pi g v. \tag{3}$$

Since the magnetic energy density falls off as $1/r^4$ and the scalar energy density even faster, the bulk of the monopole mass is stored in the core. In this estimate, we have not included the potential energy, $U(\phi) = (\lambda/8)(\phi^2 - v^2)^2 \simeq \lambda v^4/8$, associated with the scalar field trapped in the core. As a result, eqn.(3) turns out to be a rigorous lower limit on the monopole mass, the Bogolmony-Prasad-Sommerfield (BPS) bound[5].

What happens to this picture if we include the effects of gravity on the monopole? Not surprisingly, the existence of nonsingular, time independent, 'topologically stable' solutions to the coupled gauge and gravitational field equations can be proven[6]; these solutions just describe the curvature of space in the vicinity of the monopole. But we must now inquire whether such configurations are *gravitationally* stable as well[3].

In general relativity, a necessary condition for the spacetime to be static is that the monopole core must be larger than its Schwarzschild radius[7], $R_c \geq 2GM_{mon}$. Applying this to eqn.(3), we find that monopoles more massive than the critical value[8]

$$M_{mon} > M_{crit} \simeq \left(4\pi/e^2\right)^{1/2} M_{Pl} \tag{4}$$

cannot exist in a time independent configuration. From eqn.(3), this implies instability[9] for a vector boson mass $M_W \gtrsim (e^2/4\pi)^{1/2} M_{Pl}$. For a typical gauge coupling, $\alpha \sim 10^{-2}$, the critical vector mass is $M_W \simeq 10^{18}$ GeV, corresponding to $M_{mon} \simeq 10^{20}$ GeV $\simeq 10^{-4}$ gm. (Note the critical density is $\rho \simeq M_{crit}/R_c^3 \simeq 10^{92}$ gm/cm^3!) Although we have used the flat space solution for the monopole mass and radius in deriving eqn.(4), the self-consistent solution to the coupled Einstein-Yang-Mills-Higgs equations gives a similar estimate for the critical mass[3].

By analogy with stellar collapse, we expect the monopole core will implode to a singularity, forming a black hole. Let us perform a gedanken experiment to follow the evolution of a supercritical monopole. Imagine starting with a marginally subcritical configuration, $M_{mon} = M_{crit} - \epsilon$. By an extension of Birkhoff's theorem[7], on scales $r \gg R_c$, the asymptotic solution for the metric has the Reissner-Nordstrom form appropriate to the spacetime outside a spherically symmetric charged body[10],

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{Gg^2}{r^2}\right) dt^2 + \left(1 - \frac{2GM}{r} + \frac{Gg^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2. \tag{5}$$

For $M \lesssim M_{crit}$, i.e., for $GM^2 < g^2$, the spacetime can be extended to a nonsingular interior.

Now we add a small increment of mass to the core, enough to push it above the stability limit. Again, according to Birkhoff's theorem, the exterior ($r > R_c$) solution continues to have the static Reissner-Nordstrom form of eqn.(5), even as the core collapses. Thus, if a black hole forms, the long range magnetic field is left intact: the black hole has magnetic hair. This was to be expected: since the magnetic field lines are anchored at infinity, magnetic charge is conserved in the evolution of the hole. To lose its charge during collapse, the system would have to radiate a magnetically charged particle, *i.e.*, a monopole. If, as eqn.(3) indicates, *all* monopoles are heavy, this clearly cannot occur. Furthermore, the BPS lower bound on the monopole mass implies that the collapsing monopole cannot shed enough energy through electromagnetic or gravitational radiation to go 'subcritical', even if the initial configuration is highly asymmetric.

For a supercritical monopole, $M > M_{crit}$, the metric (5) appears to have singularities at $r_{\pm} = GM \pm (G^2M^2 - Gg^2)^{1/2}$. However, as in the Schwarzschild ($g = 0$) case, these singularities can be removed by a change of coordinates and analytical extension of the manifold[11] to $r < r_{\pm}$, shown in Fig.1. The surface $r = r_+$ is then reinterpreted as an event horizon: an observer who crosses inside this surface cannot return again to the same region. Inside the event horizon, in region (II) between $r = r_-$ and $r = r_+$, even light rays emitted radially outward (say, from point p in Fig.1) are 'dragged back' and converge toward smaller radii. More generally, in cases without spherical symmetry, each closed spacelike two-surface S for which both families of null geodesics orthogonal to S are converging is said to form a *closed trapped surface*[11].

Although the metric inside the collapsing monopole core (the shaded region in Fig.1) is a complicated time-dependent function, especially for non-spherically symmetric initial conditions, we can nevertheless draw general conclusions about the nature of the collapse without a detailed numerical solution. The classical stress energy tensor of the vector and scalar fields in the core can be shown to satisfy the weak energy condition[12], $T_{ab}V^aV^b \geq 0$ for any timelike or null vector

V. From Einstein's equations, $R_{ab} - g_{ab}R/2 + \Lambda g_{ab} = 8\pi G T_{ab}$, this implies that light rays (null geodesics) converge under the influence of gravity, $R_{ab}V^aV^b \geq 0$ for any null vector V. Penrose's theorem[13] essentially states that the assumption of null geodesic convergence and the existence of a closed trapped surface imply that either a spacetime singularity or a Cauchy horizon must form. Note that this conclusion holds even if the initial configuration is not spherically symmetric. Since the evolution beyond the Cauchy horizon (the surface $r = r_-$ in the spherically symmetric case) cannot be predicted from data on a spacelike surface through the initial configuration, in either case predictability breaks down.

Surprisingly, if it forms, the singularity does not necessarily occur *inside* the collapsing core. Instead, since the core radius follows a timelike path, it may bounce[14] before reaching infinite density and reexpand into the other asymptotically flat region of Fig. 1. As far as observers in the original region (I) from which the monopole imploded are concerned, however, the core disappears forever. Furthermore, the Cauchy horizon, through which the bouncing core must pass, very likely becomes a singular surface as well[15], which strongly suggests that the monopole core itself collapses to a singularity. The originally nonsingular 't Hooft-Polyakov monopole has metamorphosed into a Dirac monopole, with a (gauge invariant) singularity of the gauge and gravitational fields at the origin.

This raises an apparent paradox. Black holes are expected to radiate particles by the Hawking process, their temperature rising in inverse proportion to their decreasing mass. We have shown, however, that these magnetic black holes carry a conserved charge which they cannot radiate away. How then do we reconcile topological stability, *i.e.*, charge conservation, with Hawking radiation?

The answer is well known: for charged black holes, the relation between temperature and mass is changed[4],

$$T_H = (1/8\pi GM)(1 - G^2 g^4/r_+^4) \tag{6}$$

and after reaching a maximum temperature, $T_{max} \simeq M_{Pl}/g \lesssim M_W$, they *cool* as they radiate (see Fig. 2). As the Hawking temperature approaches $T_H = 0$, they reach a minimum mass given by $GM^2 = g^2$; this is just of order the critical mass for collapse, $M_{BH} \simeq M_{Pl}/\sqrt{\alpha}$. This altered mass-temperature relation is identical to that for electrically charged black holes, which are also described by the Reissner-Nordstrom metric. An important difference is that black holes with net electric charge rapidly discharge by emitting light particles of like charge. In the magnetic case, since all monopoles are heavy, the black hole cannot discharge. As a consequence, due to Hawking radiation, $M_{Pl}/\sqrt{\alpha}$ is the *maximum* mass for 't Hooft-Polyakov monopoles. For supercritical monopoles, this upper limit is *below* the BPS lower bound on (nonsingular) monopole masses[16].

These cooling magnetic black holes survive in this quiescent configuration unless they annihilate with a black hole of opposite charge in a burst of Hawking radiation. Amusingly, in the $T_H \to 0$ limit, one can construct static multi-monopole solutions with arbitrary spatial separations: black holes of like charge do not interact with each other, because the magnetic repulsion is precisely cancelled by the gravitational attraction[17].

These results suggest an analogy between charged black holes in general and flat space 't Hooft-Polyakov monopoles. Recall the BPS bound on the monopole mass is $M_{mon} = 2\pi R_c v^2 + 2\pi g^2/R_c \geq 4\pi g v$, while for charged black holes, the third law of thermodynamics, $T_H > 0$, may be written as $M_{BH} = r_+/G + g^2/4r_+ \geq g M_{Pl}$. Thus, BPS monopoles and charged black holes have identical 'mass-radius' relations, and the BPS inequality for monopoles has a similar form to the third law of black hole thermodynamics. In addition, as the monopole approaches the critical mass from below, the BPS bound smoothly evolves into the third law for the black hole it will become.

The laws of black hole mechanics can be used to derive other interesting features of collapsed monopoles. For example, in many grand unified theories, higher charge ($n = eg > 1$) flat space monopoles are unstable to decay into a bunch of $n = 1$ monopoles. On the other hand, supercritical monopoles obey the second law of black hole mechanics, $dA_{BH} \geq 0$. Since $A_{BH} \propto g^2$, higher charge magnetic black holes are classically stable and cannot bifurcate. Also, 't Hooft Polyakov monopoles are classical lowest energy configurations; quantum mechanically, they are pure, coherent states of roughly $N \simeq 1/\alpha$ vector and scalar fields. When a supercritical monopole collapses and settles to a charged $T_H = 0$ black hole, it ends up with a non-zero Bekenstein-Hawking entropy[4], given by $S = GM_{crit}^2/8 \simeq 1/\alpha$, just the entropy one would expect if the coherence of the original configuration were lost in the collapse.

Throughout this discussion, we have blithely ignored higher order corrections due to quantum gravity. This should be a reasonable approximation if the critical vector mass M_W and maximum Hawking temperature are well below the Planck scale, *i.e.*, for small coupling, $e^2 \ll 1$. We note that the critical monopole mass, core radius and event horizon are then significantly *above* the Planck scale. But this is precisely the criterion for one to be able to treat Hawking radiation in the usual semi-classical approximation. We conclude that in the small coupling regime, our treatment is self-consistent[18].

In many grand unified theories, monopoles can catalyze the decay of baryons at a strong interaction rate[19], a cross section much larger than naively expected, because low energy, s-wave fermions can penetrate to the monopole core and interact with the baryon-violating heavy vector and scalar bosons. In a subsequent paper, we will address the issue of baryon decay catalysis by magnetic black holes and astrophysical constraints on their abundance[3].

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outside the core and $\phi = 0$ inside the core, the effective value of Newton's constant would be different inside and outside the monopole.

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12. See refs.3, 9. Note that the strong energy condition is violated, so the stronger singularity theorems which rely on timelike convergence (see ref. 11) cannot be invoked. On dimensional grounds, quantum corrections to the stress energy, which may violate even the weak energy condition, are expected to be small on scales larger than the Planck length.

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FIGURE CAPTIONS

1) Penrose diagram for a collapsing monopole (shaded region). Light cones are at 45°, timelike lines at $< 45°$ from the vertical. The surfaces $(r = \infty)$ correspond to null infinity, $r = r_+$ to event horizons, $r = r_-$ to Cauchy horizons and $r = 0$ to a timelike singularity. The monopole core is pictured as bouncing and reemerging into 'another universe' although in fact this will not occur.

2) Temperature-mass relation for a Schwarzschild black hole and a magnetic Reissner-Nordstrom hole. For $M \gg gM_{Pl}$, the two cases are indistinguishable.

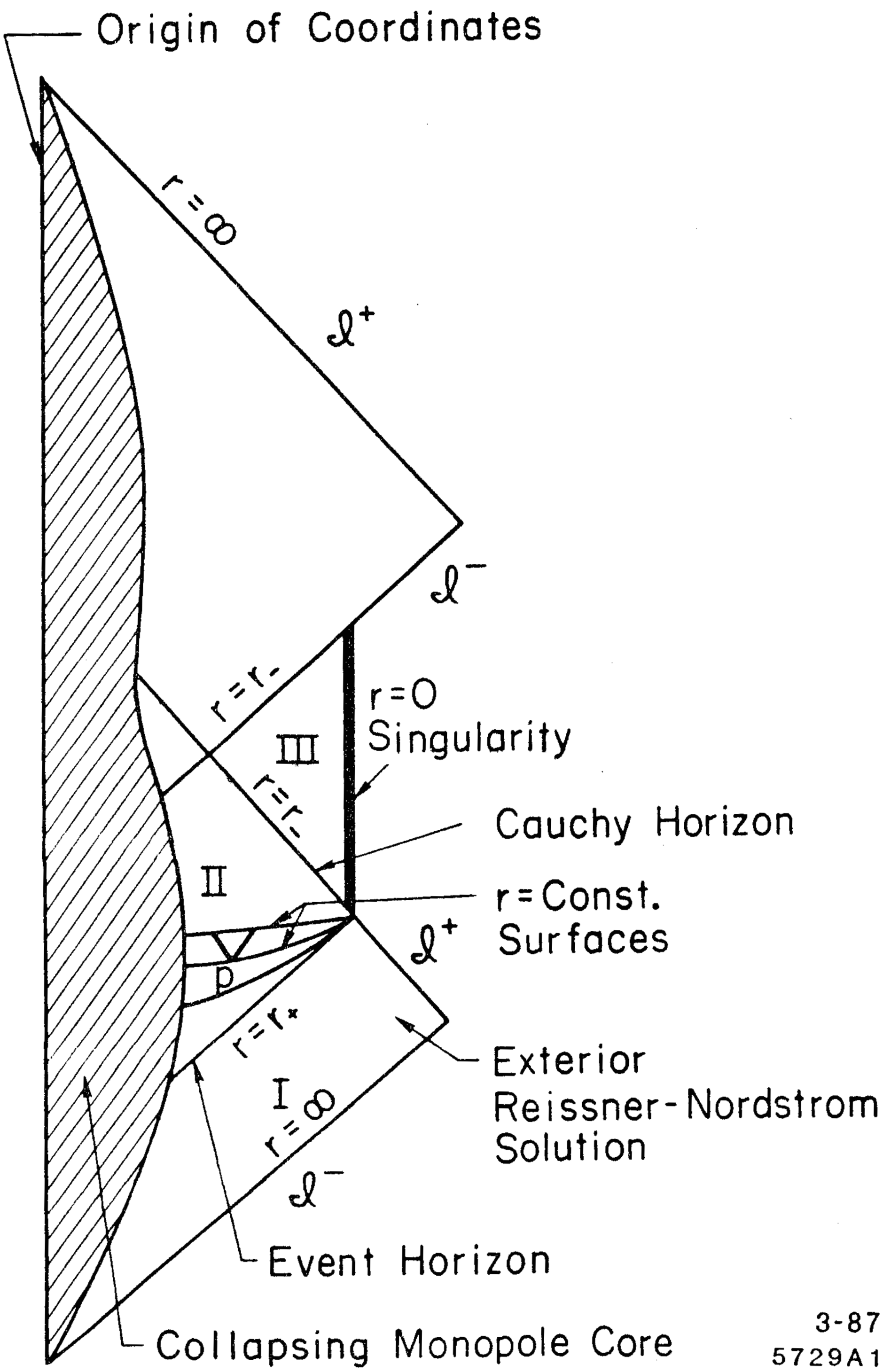
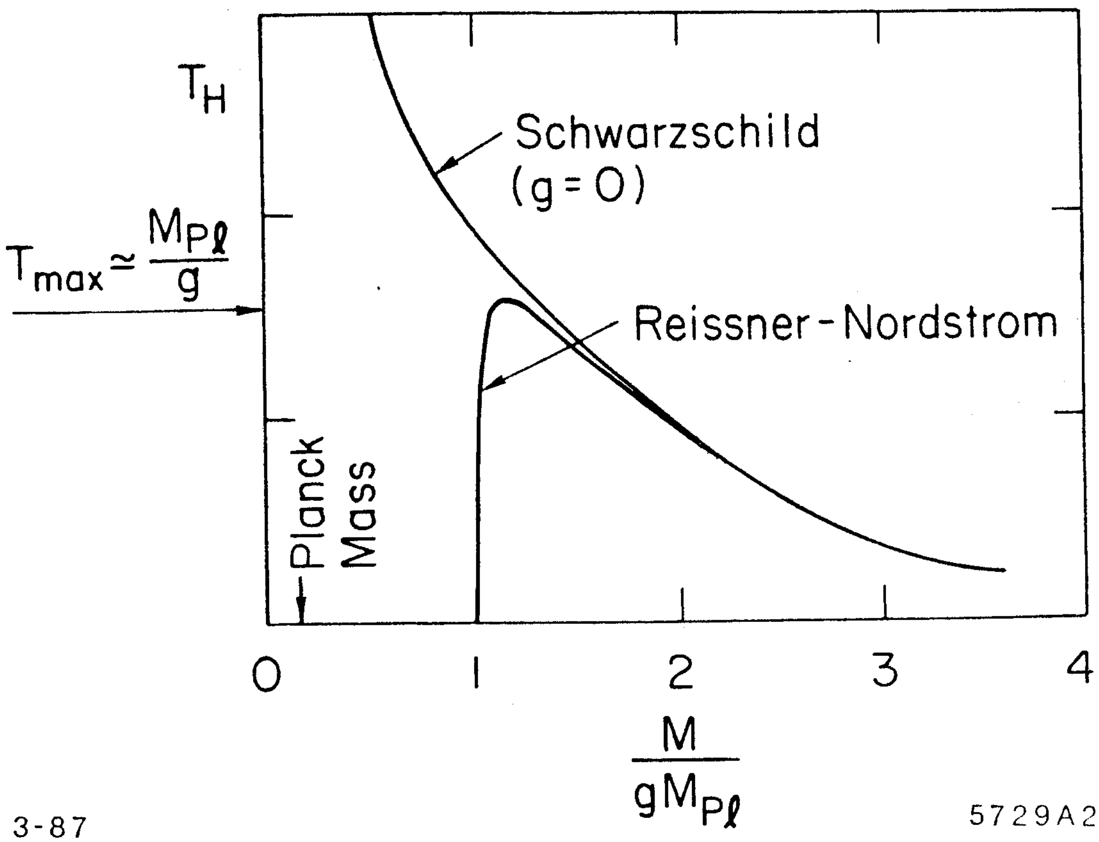

Origin of Coordinates
r = ∞
ℐ+
ℐ−
r = r−
r = 0
Singularity
III
r = r−
Cauchy Horizon
II
r = Const.
Surfaces
ℐ+
p
r = r+
Exterior
Reissner-Nordstrom
Solution
I
r = ∞
ℐ−
Event Horizon
Collapsing Monopole Core
3-87
5729A1

Fig. 1



T_H
Schwarzschild
(g = 0)
$T_{max} \simeq \frac{M_{P\ell}}{g}$
Reissner-Nordstrom
Planck
Mass
0
1
2
3
4
$\frac{M}{gM_{P\ell}}$
3-87
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Fig. 2