# EXTRA NEUTRAL GAUGE BOSONS IN ELECTRON-POSITRON COLLISIONS AT RESONANCE* 

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#### Abstract

A simple extension of the standard model to an $S U(2)_{L} \times U(1)_{Y} \times U(1)$ gauge symmetry is considered. The symmetry is broken by vacuum expectation values of one doublet and one singlet scalar field, resulting in a second massive neutral gauge boson. Electron - positron collisions at this $Z^{\prime}$ resonance are calculated. Aside from the possibility of $Z^{\prime}$ decays into exotic fermions which fill out the irreducible representations of grand unified theories, there generally are important decay modes into $W^{+} W^{-}$and $Z^{0} H^{0}$ coming from mixing between the $Z^{\prime}$ and the $Z$ of the standard model, even after imposing the constraints from experiment on the magnitude of the mixing angle.


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[^0]
## 1. Introduction

The $S U(2)_{L} \times U(1)_{Y}$ model of electroweak interactions ${ }^{[1]}$ is in good agreement with experiment within the range of energy that is available currently. Grand unified theories therefore need to be closely approximated by the standard model at low energies. In general, grand unified theories predict the existence of additional neutral gauge bosons as well as new fermions, with the prominent exception of $S U(5) .{ }^{[2]}$ Although most of the extra gauge bosons are superheavy, some theories allow the existence of lighter neutral gauge bosons with a mass near the scale of electroweak symmetry breaking. A recent example of some popularity was inspired by work on superstrings. ${ }^{[3]}$

Here we concentrate on an $S U(2)_{L} \times U(1)_{Y} \times U(1)$ effective theory, where the extra $U(1)$ is an abelian symmetry with its associated "hypercharge" $Y^{\prime}$. In particular, we will consider the $U(1)_{\chi}, U(1)_{\psi}$, and $U(1)_{\eta}$ that are possible $U(1)$ 's in broken $E_{6}$ grand unified theories. ${ }^{[4]}$ The symmetry will be spontaneously broken by a Higgs sector consisting of one doublet and one singlet. Our model will preserve the relation for the electric charge,

$$
\begin{equation*}
Q=I_{3}+\frac{Y}{2} \tag{1}
\end{equation*}
$$

As a consequence, the photon field will not mix with the extra neutral boson, $Z^{\prime}$.
Our paper is organized as follows: the model is presented in Chapter 2; the gauge boson mixing scheme is shown and the masses and mixing angles are calculated in terms of the parameters of the theory. All the vertices of the model are determined for a general extra "hypercharge" $Y^{\prime}$. In Chapter 3 we show our results for the different decay modes of the extra gauge boson, and $e^{+} e^{-}$cross sections at the $Z^{\prime}$ resonance. The processes considered are

$$
\begin{gather*}
e^{+} e^{-} \rightarrow Z^{\prime} \rightarrow f \bar{f}  \tag{2a}\\
e^{+} e^{-} \rightarrow Z^{\prime} \rightarrow W^{+} W^{-} \tag{2b}
\end{gather*}
$$

and

$$
\begin{equation*}
e^{+} e^{-} \rightarrow Z^{\prime} \rightarrow Z^{0}+\text { Higgs. } \tag{2c}
\end{equation*}
$$

The influence of a $Z^{\prime}$ on the cross sections and asymmetries at or near the $Z$ have been extensively studied. ${ }^{[5]}$ The cross section for $e^{+} e^{-} \rightarrow Z^{0} H^{0}$ at a $Z^{\prime}$ resonance has recently been treated by S . Nandi ${ }^{[6]}$ for the particular case corresponding to a $Z_{\chi}$. The $e^{+} e^{-} \rightarrow W^{+} W^{-}$reaction with the inclusion of a $Z^{\prime}$ has also been investigated recently in some particular cases. ${ }^{[7]}$ Our treatment differs in that we impose the experimental bounds on the mixing angle between the (bare) $Z$ and $Z^{\prime}$, and especially consider the energy region above that available at LEPII. In particular, we find that for large $Z^{\prime}$ masses decays into $W^{+} W^{-}$and into $Z^{0} H^{0}$ (if the Higgs boson exists at a sufficiently low mass) are dominant if the mixing angle is close to the maximum allowed value.

## 2. The Model

We consider an effective low energy theory based on the gauge group $S U(2)_{L} \times$ $U(1)_{Y} \times U(1)_{Y^{\prime}}$, with generators $\vec{I}, Y$, and $Y^{\prime}$ respectively. The electric charge is given by Eq. (1). The gauge fields appear in the covariant derivative as

$$
\begin{equation*}
D_{\mu}=\partial_{\mu}-i g \vec{I} \cdot \vec{W}_{\mu}-i g_{Y} \frac{Y}{2} B_{\mu}-i g_{Y^{\prime}} \frac{Y^{\prime}}{2} B_{\mu}^{\prime} \tag{3}
\end{equation*}
$$

The symmetry of the vacuum is broken by one scalar doublet $\phi$, and one scalar singlet $\sigma$, that can be written in unitary gauge as

$$
\begin{align*}
\phi & =\frac{1}{\sqrt{2}}\binom{0}{v+\eta}  \tag{4a}\\
\sigma & =\frac{1}{\sqrt{2}}(w+\xi) \tag{4b}
\end{align*}
$$

where $v$ and $w$ are the vacuum expectation values, and $\eta$ and $\xi$ the neutral Higgs fields. Their hypercharges are determined through Eq. (1) to be $Y_{\phi}=-1$ and
$Y_{\sigma}=0$. The photon field will be a combination of $W_{3}$ and $B$, but not of $B^{\prime}$, due to Eq. (1):

$$
\begin{equation*}
A_{\mu}=W_{3 \mu} \sin \theta_{W}+B_{\mu} \cos \theta_{W} \tag{5}
\end{equation*}
$$

where $\tan \theta_{W}=g_{Y} / g$. However, the orthogonal combination

$$
\begin{equation*}
Z_{\mu}^{0}=W_{3 \mu} \cos \theta_{W}-B_{\mu} \sin \theta_{W} \tag{6}
\end{equation*}
$$

will no longer be a mass eigenstate. Using the standard model $Z_{0}$ and the $B^{\prime}$ as a basis, the mass matrix is given by

$$
M^{2}=\left[\begin{array}{cc}
M_{0}^{2} & -M_{0} a  \tag{7}\\
-M_{0} a & {M_{0}^{\prime 2}}^{2}+a^{2}
\end{array}\right],
$$

where we have defined

$$
\begin{align*}
M_{0} & =g_{Z} \frac{v}{2}  \tag{8a}\\
M_{0}^{\prime} & =g_{Y}, \frac{Y_{\sigma}^{\prime}}{2} w  \tag{8b}\\
a & =g_{Y}, \frac{Y_{\phi}^{\prime}}{2} v \tag{8c}
\end{align*}
$$

and $g_{Z} \equiv \sqrt{g^{2}+g_{Y}^{2}}=e / \cos \theta_{W} \sin \theta_{W}$. Whereas the mass of the charged gauge bosons is

$$
\begin{equation*}
M_{W}=g \frac{v}{2} \tag{9}
\end{equation*}
$$

as in the standard model, the mass matrix in Eq. (7) causes the mass eigenstates to generally be mixtures of the "bare" $Z_{0}$ and $B^{\prime}$. We call the net mass eigenstates
$Z$ and $Z^{\prime}$ :

$$
\begin{align*}
Z & =Z_{0} \cos \theta_{M I X}+B^{\prime} \sin \theta_{M I X}  \tag{10a}\\
Z^{\prime} & =-Z_{0} \sin \theta_{M I X}+B^{\prime} \cos \theta_{M I X} \tag{10b}
\end{align*}
$$

with the masses and the mixing angle given by:

$$
\begin{align*}
M_{Z, Z}^{2}= & \frac{1}{2}\left[M_{0}^{2}+{M_{0}^{\prime}}^{2}+a^{2} \pm \sqrt{\left(M_{0}^{2}+{M_{0}^{\prime}}^{2}+a^{2}\right)^{2}-4 M_{0}^{2}{M_{0}^{\prime}}^{2}}\right]  \tag{11a}\\
& \tan \theta_{M I X}=\frac{\Delta}{1+\sqrt{1+\Delta^{2}}}, \text { with } \Delta \equiv \frac{2 \mathrm{M}_{0} \mathrm{a}}{{\mathrm{M}_{0}^{\prime}}^{2}+\mathrm{a}^{2}-\mathrm{M}_{0}^{2}} \tag{11b}
\end{align*}
$$

Our convention in Eq. (11a) is that $M_{Z}$ is lower than $M_{Z^{\prime}}$. Assuming that $M_{0} \leq M_{0}^{\prime}$, this convention makes $M_{0}$ and $M_{0}^{\prime}$ the limiting values of $M_{Z}$ and $M_{Z^{\prime}}$ as $a$, and therefore the mixing angle $\theta_{M I X}$, goes to zero.

The physical $Z$ mass, $M_{Z}$, will then generally be lower than its standard model value, $M_{0}$. The present agreement of these two values to about 3 GeV , ${ }^{[8]}$ places an upper bound on the magnitude of the mixing angle. Figure 1 shows this bound on $\theta_{M I X}$, as well as the much weaker bound that arises from the structure of the mass matrix in Eq. (7).

The Lagrangian for the scalar sector,

$$
\begin{equation*}
L_{s}=\left|D_{\mu} \phi\right|^{2}+\left|D_{\mu} \sigma\right|^{2}+V_{1}(\phi)+V_{2}(\sigma) \tag{12}
\end{equation*}
$$

gives rise not only to the masses of the gauge bosons, but also to the gauge boson-Higgs interactions. The interaction terms, in leading order in the physical Higgs fields, $\eta$ and $\xi$, are given by ${ }^{[6]}$

$$
\begin{equation*}
L_{Z-H}=-\left(2 \frac{M_{0}^{\prime}}{v} \cos \theta_{M I X} \sin \theta_{M I X}\right) Z_{\mu}^{\prime} Z^{\mu} \eta+\left(2 \frac{{M_{0}^{\prime}}^{2}}{w} \cos \theta_{M I X} \sin \theta_{M I X}\right) Z_{\mu}^{\prime} Z^{\mu} \xi \tag{13}
\end{equation*}
$$

Since $M_{0} \leq M_{0}^{\prime}$, we assume that $\xi$ (the Higgs associated with the $Z^{\prime}$ ) is heavier than $\eta$ (the Higgs associated with the $Z$ ), and that $Z^{\prime}$ decays involving the $\xi$
are kinematically forbidden. In this manner, only the first term in Eq. (13) is relevant and therefore, in order to ease the notation, we will refer to this light neutral Higgs simply as $H^{0}$ from now on.

The fermion-gauge boson interaction can be obtained from the fermion sector of the Lagrangian. The couplings of the neutral gauge bosons to the left- and right-handed fermions are

$$
\begin{align*}
& L_{f}=g_{Z}\left(I_{3}-Q_{f} \sin ^{2} \theta_{W}\right) \cos \theta_{M I X}+g_{Y^{\prime}} \frac{Y_{L, f}^{\prime}}{2} \sin \theta_{M I X}  \tag{14a}\\
& R_{f}=g_{Z}\left(-Q_{f} \sin ^{2} \theta_{W}\right) \cos \theta_{M I X}+g_{Y}, \frac{Y_{R, f}^{\prime}}{2} \sin \theta_{M I X}
\end{align*}
$$

for the physical $Z$, and

$$
\begin{align*}
& L_{f}^{\prime}=-g_{Z}\left(I_{3}-Q_{f} \sin ^{2} \theta_{W}\right) \sin \theta_{M I X}+g_{Y^{\prime}}, \frac{Y_{L, f}^{\prime}}{2} \cos \theta_{M I X} \\
& R_{f}^{\prime}=-g_{Z}\left(-Q_{f} \sin ^{2} \theta_{W}\right) \sin \theta_{M I X}+g_{Y^{\prime}} \frac{Y_{R, f}^{\prime}}{2} \cos \theta_{M I X} \tag{14b}
\end{align*}
$$

for the $Z^{\prime}$.
The values of $Y_{L}^{\prime}$ and $Y_{R}^{\prime}$ depend on the symmetry breaking pattern of the unified theory. For example, in the case of an $E_{6}$ unified theory, a possible symmetry breaking pattern is:

$$
\begin{align*}
E_{6} & \longrightarrow S O(10) \times U(1)_{\psi} \\
S O(10) & \longrightarrow S U(5) \times U(1)_{\chi} \tag{15}
\end{align*}
$$

In this case our $U(1)_{Y^{\prime}}$ could be either $U(1)_{\psi}, U(1)_{\chi}$, or a combination of them. A particular combination is $U(1)_{\eta}$, which was an early favorite in superstring phenomenology. ${ }^{[3]}$ The values of the hypercharges $Y_{L}^{\prime}$ and $Y_{R}^{\prime}$ for these three different models are given in Table 1. The normalization we use for the extra coupling constant is $g_{Y^{\prime}}=\sqrt{5 / 3} g_{Y}$. We assume no significant difference between these two $\mathrm{U}(1)$ couplings due to their possible running from different grand unified scales. ${ }^{[4]}$

The three-gauge boson vertex can be obtained directly from the standard model, with the simple replacement of the $Z_{0}$ by its mass eigenstate decomposition $Z_{0}=Z \cos \theta_{M I X}-Z^{\prime} \sin \theta_{M I X}$. In this manner, we obtain the triple-gauge-boson vertices,

$$
\begin{align*}
L_{A W W} & =e V^{\mu \nu \sigma} W_{\mu}^{+} W_{\nu}^{-} A_{\sigma} \\
L_{Z W W} & =\cos \theta_{M I X} e \cot \theta_{W} V^{\mu \nu \sigma} W_{\mu}^{+} W_{\nu}^{-} Z_{\sigma}  \tag{16}\\
L_{Z W W} & =-\sin \theta_{M I X} e \cot \theta_{W} V^{\mu \nu \sigma} W_{\mu}^{+} W_{\nu}^{-} Z_{\sigma}^{\prime}
\end{align*}
$$

where $V^{\mu \nu \sigma}$ is the usual triple-gauge-boson vertex tensor.

## 3. Results

In this section we present some predictions of the model we are considering for electron-positron scattering at or near the peak of the $Z^{\prime}$ resonance. The decay modes of the $Z^{\prime}$ into fermion-antifermion pairs, Higgs plus $Z$, and pairs of charged gauge bosons are determined at tree level from the interaction Lagrangians given in the previous Section. For definiteness in the numerical work to follow, we concentrate on the case in which the decays into the heavier Higgs, $\xi$, and into exotic fermions are kinematically forbidden. Decays into $Z H^{0}, W^{+} W^{-}$and the standard three families of leptons and quarks, including the $t$ quark, are allowed.

The decay rates are given by the following expressions:

$$
\begin{equation*}
\Gamma\left(Z^{\prime} \rightarrow Z H^{0}\right)=\frac{1}{24 \pi}{f^{\prime}}_{Z H}^{2} p_{Z}\left[\left(\frac{E_{Z}}{M_{Z}}\right)^{2}+2\right] \tag{17}
\end{equation*}
$$

where $p_{Z}$ and $E_{Z}$ are the momentum and energy of the $Z$ in the $Z^{\prime}$ rest frame and $f_{Z H}^{\prime} M_{Z}$ is the $Z^{\prime} Z H^{0}$ coupling coming from Eq.(13), with $f_{Z H}^{\prime}$ given by

$$
\begin{equation*}
f_{Z H}^{\prime}=-g_{Z} \frac{\left(\frac{M_{Z \prime}}{M_{Z}}\right) \cos \theta_{M I X} \sin \theta_{M I X}}{\left(\cos ^{2} \theta_{M I X}+\left(\frac{M_{Z}}{M_{Z}}\right)^{2} \sin ^{2} \theta_{M I X}\right)^{3 / 2}} \tag{18}
\end{equation*}
$$

$$
\begin{gather*}
\Gamma\left(Z^{\prime} \rightarrow f \bar{f}\right)=\frac{1}{12 \pi} M_{Z^{\prime}}\left[1-\frac{4 m_{f}^{2}}{M_{Z^{\prime}}{ }^{2}}\right]^{\frac{1}{2}}\left[\frac{\left(L^{\prime 2}+{R^{\prime}}^{2}\right)}{2}\left(1-\frac{m_{f}^{2}}{M_{Z^{\prime}}^{2}}\right)+3 L^{\prime} R^{\prime} \frac{m_{f}^{2}}{M_{Z^{\prime}}^{2}}\right]  \tag{19}\\
\Gamma\left(Z^{\prime} \rightarrow W^{+} W^{-}\right)=\frac{1}{48 \pi} e^{2} \cot ^{2} \theta_{W} \sin ^{2} \theta_{M I X} M_{Z^{\prime}}\left[1-\frac{4 M_{W}^{2}}{M_{Z^{\prime}}^{2}}\right]^{\frac{1}{2}}  \tag{20}\\
\times\left[\frac{1}{4} \frac{M_{Z^{\prime}}^{4}}{M_{W}^{4}}+4 \frac{M_{Z^{\prime}}^{2}}{M_{W}^{2}}-17-12 \frac{M_{W}^{2}}{M_{Z^{\prime}}^{2}}\right]
\end{gather*}
$$

If $M_{Z^{\prime}}$ is much larger than any other mass considered, the above expressions reduce to the following:

$$
\begin{align*}
\Gamma\left(Z^{\prime} \rightarrow Z H^{0}\right) & \approx g_{Z}^{2} \frac{M_{Z^{\prime}}}{192 \pi}\left(\frac{M_{Z^{\prime}}}{M_{Z}}\right)^{4} \frac{\cos ^{2} \theta_{M I X} \sin ^{2} \theta_{M I X}}{\left(\cos ^{2} \theta_{M I X}+\left(\frac{M_{Z^{\prime}}}{M_{Z}}\right)^{2} \sin ^{2} \theta_{M I X}\right)^{3}},  \tag{21}\\
\Gamma\left(Z^{\prime} \rightarrow f \bar{f}\right) & \approx \frac{1}{12 \pi} M_{Z^{\prime}} \frac{L^{\prime 2}+{R^{\prime}}^{2}}{2},  \tag{22}\\
\Gamma\left(Z^{\prime} \rightarrow W^{+} W^{-}\right) & \approx g_{Z}^{2} \frac{M_{Z^{\prime}}}{192 \pi}\left(\frac{M_{Z}^{\prime}}{M_{Z}}\right)^{4} \frac{\sin ^{2} \theta_{M I X}}{\left(\cos ^{2} \theta_{M I X}+\left(\frac{M_{Z^{\prime}}}{M_{Z}}\right)^{2} \sin ^{2} \theta_{M I X}\right)^{2}} \tag{23}
\end{align*}
$$

Note that in this limit the decays $Z^{\prime} \rightarrow Z H^{0}$ and $Z^{\prime} \rightarrow W^{+} W^{-}$go dominantly into longitudinal gauge bosons in the final state, as witness the factor of $\left(\frac{M_{Z^{\prime}}}{M_{z}}\right)^{4}$ in Eqs. (21) and (23). It is also this factor which will make these decay modes dominate over $Z^{\prime} \rightarrow f \bar{f}$ as $M_{Z}$, becomes large.

The $e^{+} e^{-}$cross sections at the $Z^{\prime}$ resonance are given by the general expression:

$$
\begin{equation*}
\sigma\left(e^{+} e^{-} \rightarrow \text { final }\right)=\frac{12 \pi}{M_{Z^{\prime}}^{2}} B_{e^{+} e^{-}} B_{\text {final }}, \tag{24}
\end{equation*}
$$

where $B_{e^{+} e^{-}}$and $B_{\text {final }}$ are the branching ratios for the $Z^{\prime}$ decay. In terms of
the point cross section $\sigma_{p t} \equiv 4 \pi \alpha^{2} / 3 s$ at $s=M_{Z^{\prime}}^{2}$, we can express Eq. (24) as:

$$
\begin{equation*}
\frac{\sigma\left(e^{+} e^{-} \rightarrow \text { final }\right)}{\sigma_{p t}}=\frac{9}{\alpha^{2}} B_{e^{+} e^{-}} B_{\text {final }} \tag{25}
\end{equation*}
$$

The cross sections for the $\mu^{+} \mu^{-}, W^{+} W^{-}$, and $Z H^{0}$ final states are shown in Figures 2,3 , and 4 as a function of the mixing angle for three different values of $M_{Z^{\prime}} / M_{Z}$. The standard model parameters used in the computations are $M_{Z}=93$ GeV and $\sin ^{2} \theta_{W}=0.22$ given in Ref. 8. For the Higgs mass, we choose a reasonably light value $M_{H}=0.6 M_{Z}$. For all three values of $M_{Z^{\prime}}$, we use only the $\eta$ "hypercharge" from Table 1.

For large values of the neutral boson mass ratio $M_{Z^{\prime}} / M_{Z}$, and large mixing angles, $W^{+} W^{-}$production is dominant, closely followed by $Z H^{0}$ production. This is because, as just noted, the gauge bosons in the $W^{+} W^{-}$and $Z H^{0}$ final states become primarily longitudinal for large $Z^{\prime}$ masses, and the decay rates pick up associated factors of $\left(\frac{M_{Z^{\prime}}}{M_{Z}}\right)^{4}$. There are differences between these two cross sections because of the slightly different dependence of their coupling constants on $\theta_{M I X}$ and because of the masses of the final state bosons. At low values of $M_{Z^{\prime}} / M_{Z}, W^{+} W^{-}$production is lower than that for $Z H^{0}$, mainly due to the masses of the final particles (and in particular, it is a consequence of our assuming a value for the Higgs boson mass which is smaller than $M_{W}$ ). With the assumption of a more massive Higgs boson, all the above results remain valid, except for the kinematical suppression of $Z H^{0}$ production. For example, if we doubled our assumed Higgs mass so that $M_{H}=1.2 M_{Z}$ (an "intermediate mass" Higgs), there would be no $Z H^{0}$ decay channel for $M_{Z^{\prime}} / M_{Z}=2$, but for $M_{Z \prime} / M_{Z}=4$ the final state phase space decreases $\Gamma\left(Z^{\prime} \rightarrow Z H^{0}\right)$ by only $\sim 15 \%$ and for $M_{Z^{\prime}} / M_{Z}=10$ by $\sim 2 \%$.

In either case such a $Z^{\prime}$ would be a "Higgs boson factory." If we imagine a future electron-positron machine operating with a luminosity of $10^{32} / \mathrm{cm}^{2} \mathrm{sec}$ at the peak of a $Z^{\prime}$ with $M_{Z^{\prime}}=4 M_{Z}$, then $\sim 200 Z H^{0}$ events are produced per day
for a small value of $\theta_{M I X}=0.01$. This is to be compared with $\sim 3$ such events per day in similar circumstances, but without a $Z^{\prime}$.

In contrast, fermion pair production is less $\theta_{M I X}$-dependent; it does not go to zero as $\theta_{M I X}$ decreases because of the finite fermion "hypercharge" $Y^{\prime}$. For this reason, fermion-antifermion pair production is always dominant at sufficiently low values of $\theta_{M I X}$. Nevertheless, the range of fermion dominance is constrained to very small mixing angles if the $M_{Z^{\prime}} / M_{Z}$ ratio is large.

The inclusion of decays to exotic fermions that fill out the 27 dimensional representation of $E_{6}$, if they are light enough, would simply decrease all of the above branching ratios; their production would be comparable to, or larger than, the production of pairs of the standard fermions. For example, inclusion of all the exotic fermion pair channels for decay of an unmixed $Z_{\eta}$ increases its total width by a factor of four.

Moving away from resonance, the situation is quite different. The behavior of the cross section as a function of the center-of-mass energy, $\sqrt{s}$, is shown in Figures 5, 6, and 7, and corresponds to the expressions below:

$$
\begin{align*}
\sigma\left(e^{+} e^{-} \rightarrow Z H^{0}\right) & =\frac{1}{96 \pi}\left[\left(1-\frac{\left(M_{Z}+M_{H}\right)^{2}}{s}\right)\left(1-\frac{\left(M_{Z}-M_{H}\right)^{2}}{s}\right)\right]^{\frac{1}{2}} \\
\times & {\left[\left(\frac{f_{Z H} M_{Z} L}{s-M_{Z}^{2}}+\frac{f_{Z H}^{\prime} M_{Z^{\prime}} L^{\prime}}{s-M_{Z^{\prime}}^{2}}\right)^{2}+\left(\frac{f_{Z H} M_{Z} R}{s-M_{Z}^{2}}+\frac{f_{Z H^{\prime}}^{\prime} M_{Z^{\prime}} R^{\prime}}{s-M_{Z^{\prime}}^{2}}\right)^{2}\right] } \\
\times & \left.\times\left(\frac{E_{Z}}{M_{Z}}\right)^{2}+2\right], \tag{26}
\end{align*}
$$

where $E_{Z}$ is the $Z$ energy in the center-of-mass frame, $L, R, L^{\prime}$, and $R^{\prime}$ are the $Z$ and $Z^{\prime}$ couplings to electrons given by Eqs. (14), $f_{Z H}^{\prime}$ is given by Eq. (18), and $f_{Z H}$ by a similar expression:

$$
f_{Z H}=\frac{g_{Z}}{2} \frac{\cos ^{2} \theta_{M I X}}{\left(\cos ^{2} \theta_{M I X}+\left(\frac{M_{Z^{\prime}}}{M_{Z}}\right)^{2} \sin ^{2} \theta_{M I X}\right)^{3 / 2}}
$$

$$
\begin{align*}
\sigma\left(e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}\right)=\frac{s}{48 \pi} & {\left[\frac{4 e^{4}}{s^{2}}+\frac{\left(L^{2}+R^{2}\right)^{2}}{\left(s-M_{Z}^{2}\right)^{2}}+\frac{\left(L^{\prime 2}+R^{\prime 2}\right)^{2}}{\left(s-M_{Z^{\prime}}^{2}\right)^{2}}+2 \frac{e^{2}(L+R)^{2}}{s\left(s-M_{Z}^{2}\right)}\right.} \\
& \left.+2 \frac{e^{2}\left(L^{\prime}+R^{\prime}\right)^{2}}{s\left(s-M_{Z^{\prime}}^{2}\right)}+2 \frac{\left(L L^{\prime}+R R^{\prime}\right)^{2}}{\left(s-M_{Z}^{2}\right)\left(s-M_{Z^{\prime}}^{2}\right)}\right] \tag{27}
\end{align*}
$$

$$
\begin{align*}
\sigma\left(e^{+} e^{-} \rightarrow W^{+} W^{-}\right)= & \frac{1}{64 \pi s}\{
\end{align*}\left\{\frac{g^{4}}{4}\left(\frac{1}{6} x^{2}+\frac{10}{3} x-8\right)+\right\}
$$

where $x \equiv s / M_{W}^{2}$ and the effective couplings $L(s)$ and $R(s)$ are given by:

$$
\begin{aligned}
& L(s)=M_{W}^{2}\left(-\frac{e^{2}}{s}+\frac{L e \cot \theta_{W} \cos \theta_{M I X}}{s-M_{Z}^{2}}-\frac{L^{\prime} e \cot \theta_{W} \sin \theta_{M I X}}{s-M_{Z^{\prime}}^{2}}\right), \\
& R(s)=M_{W}^{2}\left(-\frac{e^{2}}{s}+\frac{R e \cot \theta_{W} \cos \theta_{M I X}}{s-M_{Z}^{2}}-\frac{R^{\prime} e \cot \theta_{W} \sin \theta_{M I X}}{s-M_{Z^{\prime}}^{2}}\right) .
\end{aligned}
$$

The cross section for $e^{+} e^{-} \rightarrow Z H^{0}$ in Figure 5 shows the most striking effects of the presence of the $Z^{\prime}$ as compared to the cross section in the standard model. ${ }^{[9]}$ Even for a comparatively small value of the mixing angle, $\theta_{M I X}=0.01$, the cross section at the peak of the $Z^{\prime}$ is increased by roughly two orders of magnitude. There is an interference between the resonance and the standard model (coming from an intermediate $Z$ ) amplitudes which is constructive below resonance and destructive above for $\theta_{M I X}$ positive as we have defined the phase conventions. The cross section as $s \rightarrow \infty$ behaves like $\sim 0.04 \sigma_{p t}$.

The production of fermion-antifermion pairs such as in the process $e^{+} e^{-} \rightarrow$ $\mu^{+} \mu^{-}$has the typical pattern shown in Figure 6, with the $Z$ and $Z^{\prime}$ resonance peaks and destructive interference of their amplitudes between resonances. Inasmuch as each unmixed resonance couples to $e^{+} e^{-}$and $\mu^{+} \mu^{-}$and we are only considering relatively small values of $\theta_{M I X}$, the cross sections in Figure 6 do not show much dependence on the mixing angle. As $s \rightarrow \infty$ the cross section behaves as a multiple of $\sigma_{p t}$ with a coefficient of order unity that doesn't depend on masses or the mixing angle, but does depend on the "hypercharge" $Y^{\prime}$ assignments of the relevant fermions.
$W^{+} W^{-}$production provides a somewhat intermediate case ${ }^{[7]}$ in that the standard model cross section ${ }^{[9]}$ is fairly big ( $\sim 20 \sigma_{p t}$ in the energy region we are considering) and grows as $\sigma_{p t} \log \left(s / M_{W}\right)$, but the $Z^{\prime}$ acquires a $W^{+} W^{-}$ decay mode only through mixing. Although comparable in absolute magnitude with $e^{+} e^{-} \rightarrow Z H^{0}$ for the same values of $\theta_{M I X}$, the peaks in the cross section for $e^{+} e^{-} \rightarrow W^{+} W^{-}$shown in Figure 7 look much less impressive because of the standard model pedestal upon which they sit. As $s \rightarrow \infty$ the cross section assumes its standard model behavior.

## 4. Conclusion

We have examined the behavior of $e^{+} e^{-}$collisions when there is a resonance due to an extra neutral gange boson. This gauge boson couples to other fields through the new "hypercharge" $Y^{\prime}$. The coupling to other bosons of the theory occurs through the presence of non-diagonal terms in the gauge boson mass matrix; these mix the $Z^{\prime}$ with the standard model $Z$ of $S U(2)_{L} \times U(1)_{Y}$, thereby coupling the physical $Z^{\prime}$ to $Z H^{0}$ and $W^{+} W^{-}$. The coupling of the $Z^{\prime}$ to fermions, however, is due not only to mixing, but also to the "hypercharge" $Y^{\prime}$ of the fermions.

The magnitude of the cross sections at resonance were determined and it was found that for low values of $M_{Z^{\prime}} / M_{Z}$ the production of fermion - antifermion
pairs dominates over the whole range of experimentally allowed mixing angles. For high values ( $\sim 5$ ) of $M_{Z^{\prime}} / M_{Z}, W^{+} W^{-}$and $Z H^{0}$ final states dominate $Z^{\prime}$ decays if the mixing angle assumes values near the maximum experimentally allowed value.

Even for comparatively small values of the mixing angle the cross sections for $Z H^{0}$ and $W^{+} W^{-}$production receive large enhancements at the peak of such a $Z^{\prime}$ resonance. Particularly in the channel $e^{+} e^{-} \rightarrow Z H^{0}$, the presence of a $Z^{\prime}$ above the threshold for this reaction would likely provide a unique place to study Higgs boson properties.

## Table I

Values of $Y^{\prime}$ for fermions in the 27 dimensional representation of $E_{6}$, corresponding to a $Z_{\chi}, Z_{\psi}$, and $Z_{\eta}$. The $D$ is a charge $-e / 3$ quark; the $N$ an $S U(2)_{L}$ singlet, neutral lepton; and the $E_{0}, E^{-}$an $S U(2)_{L}$ doublet of leptons. The coupling of this $U(1)$ symmetry is normalized to $g_{0}=\sqrt{5 / 3}\left(e / \cos \theta_{W}\right)$.

| $S O(10)$ | $S U(5)$ | $\sqrt{10} Y_{\chi}^{\prime}$ | $\sqrt{6} Y_{\psi}^{\prime}$ | $\sqrt{15} Y_{\eta}^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: |
| 16 | $10\left(u, d, \bar{u}, e^{+}\right)$ | -1 | 1 | -2 |
|  | $\overline{5}\left(\bar{d}, \nu, e^{-}\right)$ | 3 | 1 | 1 |
|  | $1(\bar{N})$ | -5 | 1 | -5 |
| 10 | $5\left(D, \bar{E}^{0}, E^{+}\right)$ | 2 | -2 | 4 |
|  | $\overline{5}\left(\bar{D}, E^{0}, E^{-}\right)$ | -2 | -2 | 1 |
| 1 | $1\left(S^{0}\right)$ | 0 | 4 | -5 |

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## FIGURE CAPTIONS

1. Allowed region for the mass and mixing angle of a possible $Z^{\prime}$ following from $\Delta M_{Z} \leq 3 \mathrm{GeV}$ (region below the solid curve), and following from the Higgs structure of the model (region below the dashed curve).
2. The cross section for production of $Z H^{0}, \mu^{+} \mu^{-}$and $W^{+} W^{-}$at the peak of a $Z^{\prime}$ with $M_{Z^{\prime}} / M_{Z}=2$ in units of $\sigma_{p t} \equiv 4 \pi \alpha^{2} / 3 s$ as a function of mixing angle for the range allowed in Figure 1.
3. The cross section for production of $Z H^{0}, \mu^{+} \mu^{-}$and $W^{+} W^{-}$at the peak of a $Z^{\prime}$ with $M_{Z^{\prime}} / M_{Z}=4$ in units of $\sigma_{p t} \equiv 4 \pi \alpha^{2} / 3 s$ as a function of mixing angle for the range allowed in Figure 1.
4. The cross section for production of $Z H^{0}, \mu^{+} \mu^{-}$and $W^{+} W^{-}$at the peak of a $Z^{\prime}$ with $M_{Z^{\prime}} / M_{Z}=10$ in units of $\sigma_{p t} \equiv 4 \pi \alpha^{2} / 3 s$ as a function of mixing angle for the range allowed in Figure 1.
5. The cross section in units of $\sigma_{p t}$ for $e^{+} e^{-} \rightarrow Z H^{0}$ as a function of center of mass energy, $\sqrt{s}$, for $M_{H}=0.6 M_{Z}, M_{Z^{\prime}} / M_{Z}=4$, and mixing angle values of 0.04 (dashed curve), 0.01 (dash-dot curve), and 0.001 (solid curve). The resonance peak in this last case is 0.2 units high and too small to discern given the vertical scale. The dotted curve is the cross section without a $Z^{\prime}$.
6. The cross section in units of $\sigma_{p t}$ for $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$as a function of center of mass energy, $\sqrt{s}$ for mixing angles values of 0.04 (dashed curve), 0.01 (dash-dot curve), and 0.001 (solid curve) for $M_{Z^{\prime}} / M_{Z}=4$. The dotted curve is the cross section without a $Z^{\prime}$.
7. The cross section in units of $\sigma_{p t}$ for $e^{+} e^{-} \rightarrow W^{+} W^{-}$as a function of center of mass energy, $\sqrt{s}$ for mixing angles values of 0.04 (dashed curve), 0.01 (dash-dot curve), and 0.001 (solid curve) for $M_{Z^{\prime}} / M_{Z}=4$. The dotted curve is the cross section without a $Z^{\prime}$.


Fig. 1


Fig. 2


Fig. 3


Fig. 4


Fig. 5


Fig. 6


Fig. 7


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