

GAUGE INVARIANT ACTIONS FOR STRING MODELS*

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ABSTRACT

String models of unified interactions are elegant sets of Feynman rules for the scattering of gravitons, gauge bosons, and a host of massive excitations. The purpose of these lectures is to describe progress towards a nonperturbative formulation of the theory. Such a formulation should make the geometrical meaning of string theory manifest and explain the many “miracles” exhibited by the string Feynman rules. As yet only partial success has been achieved in realizing this goal. Most of the material presented here already appears in the published literature but there are some new results on gauge invariant observables, on the cosmological constant, and on the symmetries of interacting string field theory.

1. INTRODUCTION (WITH APOLOGIES TO E. AMBLER)^[1]

A Frenchman named Chamfort, who should have known better, once said that chance was a nickname for Providence.

It is one of those convenient, question begging aphorisms coined to discredit the unpleasant truth that chance plays an important, if not predominant, part in human affairs. Yet it was not entirely inexcusable. Chance does occasionally operate with a sort of fumbling coherence readily mistakable for the workings of a self conscious Providence.

The history of the string model is an example of this. The fact that an S matrix theorist like Veneziano^[2] should discover the key to the quantum theory of gravity by looking for solutions of hadronic finite energy sum rules is alone grotesque. That Wess and Zumino^[3] should discover spacetime supersymmetry by generalizing the two dimensional supersymmetry of the Neveu-Schwarz-Ramond model, that Schwarz and Green^[4] should spend years they could ill afford probing into the shadowy structure of the superstring, that the theory of Everything should be discovered because Harvey^[5] and Thierry-Mieg^[6] noticed an odd identity for Casimir operators ... these facts are breathtaking in their absurdity.

*Work supported by the Department of Energy, contract DE-AC03-76SF00515.

Yet, when these events are seen side by side with the other facts in the case^[7] it is difficult not to become lost in superstitious awe. Their very absurdity seems to prohibit the use of the words "chance" and "coincidence". For the sceptic there remains only one consolation: if there should be such a thing as superhuman Law, it is administered with subhuman efficiency. The choice of the dual model of hadrons as an instrument for revealing the superstring to the world could have been made only by an idiot.

More than most physical theories then, the string model cries out for an ahistorical presentation which starts from simple and deep physical principles and derives the dual amplitudes in a logical manner. My attempt to present such a description of string theories will be hampered by the fact that a true Theory of Strings does not yet exist. I will therefore begin with a brief exposition of the most coherent description of string theory now available; the covariant Feynman rules for string perturbation theory. I will then describe the framework within which all current attempts to derive these rules are based: gauge invariant functional field theory (GIFFT) I am not at all sure that the ultimate description of string theory will fall within this framework. Nonetheless, I believe that GIFFT is a correct description of string theories which is based on conceptually simple generalizations of things that we know. Even if it turns out to be a bit clumsy (as it seems at present) it appears to be the likeliest avenue of access to a more satisfying description of strings.

There has been an explosion of interest in string field theory this year. I have been unable to refer to every paper on the subject in the text. Reference 49 is a long (but probably still incomplete) list of papers on string field theory. The approach to string field theory presented here is a combination of ideas developed in collaboration with Michael Peskin and Christian Preitschopf and the elegant formalism invented by Ed Witten. I would like to thank them and Dan Friedan and Emil Martinec for numerous conversations about string theory. Finally, I would like to remind the reader that the initial development of covariant string field theory was solely due to Warren Siegel (ref. 18). His papers have been a continuing source of inspiration for myself and my collaborators.

2. STRINGY FEYNMAN RULES

In ordinary field theory, the perturbation expansion may be expressed in terms of path integrals for a single spacetime degree of freedom $x^\mu(\tau)$, which propagates with the standard action of a relativistic particle. The propagation may be visualized as taking place on a net in spacetime whose topological properties depend on the field theory Lagrangian and the order of perturbation theory. The endless series of possibilities for the topology of such a net is responsible for the complexity of higher order Feynman graphs and at least partially responsible for the wide range of possible Lagrangians.

If on the other hand we consider a string propagating in space time we see a very different picture. The string world sheet is a two dimensional manifold, and the simple requirement that it be smooth considerably restricts its topology. If the manifold is to represent the propagation of a closed oriented string for example, it is completely classified by giving the number of handles and the number of boundaries (corresponding to incoming and outgoing strings). This leads one to suspect that a perturbation expansion of a theory of interacting strings would have only one diagram in each order. This is indeed correct, though as we shall see, these unique dual diagrams are related to ordinary Feynman diagrams in a manner reminiscent of the relation between the latter and the diagrams of old fashioned time ordered perturbation theory.

At the present time, the only complete derivation of the dual Feynman rules which I am about to present is in the so called light cone gauge. A theory of interacting strings is constructed in infinite momentum frame Hamiltonian formalism. The perturbation series for this Hamiltonian is worked out and then one sees how to add up the various terms in a given order to give a single dual diagram. To be frank, a complete and rigorous derivation of the dual diagrams has only been constructed (including such details as numerical factors) through one loop order. Lorentz invariance becomes obvious only at the end of the calculation, and other fabulous properties of string models such as duality and general coordinate invariance, are not obvious at all. It is tempting to regard the infinite momentum frame string theory as a gauge fixed version of a beautiful action which manifests all of these wondrous properties at a glance. In later chapters we will review the attempts that have been made to realize this dream.

The string Feynman rules are written in terms of a conformally invariant two dimensional field theory, which is also invariant under two dimensional general coordinate transformations. For the expansion around flat ten dimensional space this field theory is given by a Lagrangian first written by Brink, DiVecchia, Howe,^[8] Deser and Zumino^[9] and explored by Polyakov.^[10]

$$\mathcal{L} = \sqrt{g} g^{\alpha\beta} \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta} \quad (1)$$

String S matrix elements are given in terms of expectation values of certain generally coordinate invariant, conformally invariant operators (called vertex operators) in this field theory. This gives us the tree level S matrix. As usual the expectation values can be written as Euclidean functional integrals. The fields in these functional integrals are defined on the plane, or equivalently, the Riemann sphere. L loop corrections to the amplitudes are given by the same functional integral on a surface with L handles. (These are the rules for closed oriented strings. Open and non-orientable strings have slightly more complicated rules.)

In order to actually compute these expectation values we have to choose a gauge for two dimensional coordinate transformations and Weyl transformations and introduce the corresponding Faddeev-Popov ghosts. We also have to make sure that there are no anomalies in the classical invariances of the action. This restricts the dimension of space time to be 26 for the bosonic string. If we are expanding around a curved space time, it also restricts the geometry to be such that the corresponding non-linear sigma model has vanishing beta function. It has been argued^[7] that this restriction is equivalent to the classical equations of motion for the string field theory. We will derive the restriction to d=26 in another way below.

The ghosts for local conformal invariance are trivial, they have no kinetic energy. Those of two dimensional reparametrizations have a Lagrangian

$$\mathcal{L} = b\partial_{\bar{z}}c + \bar{b}\partial_z\bar{c}; \quad \begin{aligned} z &= \xi_1 + i\xi_2 \\ \bar{z} &= \xi_1 - i\xi_2 \end{aligned} \quad (2)$$

The total Lagrangian of the system is

$$\mathcal{L} = \frac{\partial x^\mu}{\partial z} \frac{\partial x^\mu}{\partial \bar{z}} + b\partial_{\bar{z}}c + \bar{b}\partial_z\bar{c} \quad (3)$$

It is invariant under the Becchi-Rouet-Stora-Tyutin (BRST) transformation:

$$\begin{aligned} \delta x^\mu &= \epsilon(c\partial_{\bar{z}}x^\mu + \bar{c}\partial_zx^\mu) & \delta b &= \epsilon\left(\bar{c}\partial_zb + c\partial_{\bar{z}}b + 2\partial_{\bar{z}}cb + \frac{\partial x}{\partial z} \frac{\partial x}{\partial \bar{z}}\right) \\ \delta c &= \epsilon(c\partial_{\bar{z}}c + \bar{c}\partial_zc) & & \\ \delta \bar{c} &= \epsilon(c\partial_{\bar{z}}\bar{c} + \bar{c}\partial_z\bar{c}) & \delta \bar{b} &= \epsilon\left(\bar{c}\partial_z\bar{b} + c\partial_{\bar{z}}\bar{b} + 2\partial_z\bar{c}\bar{b} + \frac{\partial x}{\partial \bar{z}} \frac{\partial x}{\partial z}\right). \end{aligned} \quad (4)$$

At the level of classical manipulations, the BRST transformation is nilpotent. However, this breaks down at the quantum level. The commutation relations for the conformal gauge variables of the string are given by (we specialize to open strings where \bar{b} and \bar{c} are fixed in terms of b, c by the boundary conditions)

$$x^\mu(z) = \hat{x}^\mu + \frac{i}{2} \sum_{n>0} \frac{1}{n} (\alpha_n^\mu - \alpha_{-n}^\mu) (z^n + \bar{z}^n)$$

$$b(z) = \sum z^{-n-2} b_n;$$

$$c(z) = \sum z^{-n+1} c_n;$$

$$[\alpha_n^\mu, \alpha_m^\nu] = n\delta(n+m)\eta^{\mu\nu}; \quad [b_n, c_m]_+ = \delta(n+m). \quad (5)$$

The vacuum state is defined by

$$\alpha_n^\mu |\Omega\rangle = b_n |\Omega\rangle = c_n |\Omega\rangle = 0; \quad n > 0. \quad (6)$$

We will normal order all operators with respect to this state.

Formally, if T_a are the generators of the residual gauge group of a gauge theory:

$$[T_a, T_b] = f_{ab}{}^c T_c \quad (7)$$

then the BRST generator is given by

$$Q = c^a (T_a + \frac{1}{2} T_a^{gh}) \quad (8)$$

where $T_a^{gh} (\equiv c^b f_{ab}{}^c \frac{\partial}{\partial c^c})$ are the generators of residual gauge transformations on the ghosts. Simple manipulations involving the Jacobi identity prove that $Q^2 = 0$.

For the string the residual gauge transformations are the conformal transformations generated by the Virasoro operators:

$$L_n^x = \frac{1}{2} : \sum \alpha_{n+k}^\mu \alpha_{-k}^\mu : \quad (9)$$

$$L_n^{gh} = : \oint \frac{dx}{(2\pi i)} z^{n+1} (c \partial_x b + 2(\partial_x c) b) : -1 \delta(n).$$

These operators have anomalous commutation laws

$$[L_n^x, L_m^x] = (n-m) L_{n+m}^x + \frac{d}{12} (n^3 - n) \delta(m+n) \quad (10)$$

$$[L_n^{gh}, L_m^{gh}] = (n-m) L_{n+m}^{gh} - \frac{13}{6} (n^3 - n) \delta(m+n).$$

The operator L_0^{gh} which appears in ((9)-(10)) is not normal ordered with respect to the vacuum. Rather, its ordering is chosen so that L_0, L_{-1}, L_1 generates an $SL(2)$ subalgebra of the Virasoro algebra. For the Virasoro generators of the x^μ variables, this requirement is equivalent to normal ordering with respect to the vacuum. In other words, we normal order the Virasoro generators with respect

to an $SL(2)$ invariant state (it is unique). The vacuum defined above is related to this $SL(2)$ invariant state by the action of c_{-1} . It therefore satisfies

$$\begin{aligned} |\Omega\rangle &= c_1 |0\rangle \\ (L_0 + 1) |\Omega\rangle &= 0 . \end{aligned} \tag{11}$$

The anomalous commutation rules of the L_n and the necessity of normal ordering the BRST charge, ruin the proof that $Q^2 = 0$. These difficulties cancel when the spacetime dimension, d , equals 26. Notice that it is at precisely this point that the full conformal generators $L_n + L_n^{gh}$ have no commutator anomaly. It can be proven in general (as long as the ghosts are free fields) that the BRST charge is nilpotent if and only if the full commutator anomaly vanishes.^[11]

Given a nilpotent BRST charge, string Feynman rules are constructed in the following way: we identify a set of local vertex operators whose integrals over the world sheet are BRST invariant. These are all dimension two conformal fields.^[11] If the spacetime background which defines the conformal field theory we are studying has some isometries, the vertex operators can be classified by their transformation laws under the isometry group. In particular, if spacetime has some flat, Euclidean, dimensions, the vertex operators can be chosen to carry fixed energy and momentum. Scattering amplitudes are then defined as the vacuum expectation values of time ordered products of integrated vertex operators in the two dimensional conformal field theory. Higher loop corrections are calculated by expressing the expectation values as Euclidean functional integrals, and then performing the same functional integral on world sheets of arbitrary topology.

In order to complete our description of the machinery necessary for constructing these rules we must show how to compute expectation values in the theory at hand. The space of states in the BRST invariant theory is a tensor product with many factors. Of particular interest is the factor describing the ghost zero modes. These are fermionic operators obeying:

$$[b_0, c_0]_+ = 1 .$$

The factor space in which they act is two dimensional. Define the state $|\downarrow\rangle$ by

$$b_0 |\downarrow\rangle = 0 . \tag{12}$$

Then the space is spanned by

$$|\downarrow\rangle ; c_0 |\downarrow\rangle . \tag{13}$$

These states each have zero scalar product with themselves but their overlap is one. The piece of the vacuum $|\Omega\rangle$ which is in the zero mode factor, is the state $|\downarrow\rangle$. Thus operators which have nonzero expectation value in this state must carry precisely one factor of the zero mode c_0 .

It should be emphasized that the general framework for string perturbation theory presented above is not tied to the flat space backgrounds in which string theory is traditionally presented. Any representation of the Virasoro algebra with appropriate central charge can be the basis for a sensible string perturbation theory (actually there are further restrictions which appear only at one loop). However since we do not yet know very much about the representations of the conformal group associated with curved manifolds, practical calculations are restricted to generalized toroidal backgrounds.

Let us now introduce a useful operator: the ghost number g , by

$$g \equiv \sum_{n>0} (b_{-n}c_n + c_{-n}b_n) + \frac{1}{2}[c_0, b_0]$$

g is antihermitian and takes the values $-1/2$ and $-3/2$ respectively on the states $|\Omega\rangle$ and $|0\rangle$. The ghost number of any other state is determined by noting that $c(z)$ carries ghost number 1 and $b(z)$, -1 . It follows that $[g, Q] = Q$.

The basic equation of the first quantized string theory that we have described is $QA = 0$. Obviously solutions are determined only up to BRST exact forms $A = Q\epsilon$. It is thus of interest to determine what the nontrivial BRST invariant states look like. This has been determined by Kato and Ogawa.^[12] A more rigorous mathematical discussion has been given by Frenkel et. al.^[13] The methods of these authors are extremely powerful and elegant and should be useful for further investigations of the subject. The results of both of these groups are the following. The only nontrivial solutions of $QA = 0$ are states of ghost number $\pm 1/2$ and (but only for spacetime momentum equal to zero) $\pm 3/2$. The fact that we get both signs of the ghost number has to do with the ambiguity in the choice of the vacuum for the ghost zero mode. We resolve the ambiguity by choosing physical states to have ghost number $-1/2$. The general solution of the BRST equation with ghost number $-1/2$ is

$$A = \Psi \otimes \Omega \tag{14}$$

where Ω is the ghost vacuum annihilated by b_0 and Ψ is a physical state:

$$L_n^z \Psi = 0; \quad n > 0; \quad (L_0^z - 1)\Psi = 0 \tag{15}$$

Note that the minus one in this equation is determined in the following way. We define L_0 for any conformal field theory as the commutator of L_{-1} and L_1

divided by two. This resolves the normal ordering ambiguity. One then finds that L_0 annihilates the vacuum in the x^μ sector while it has eigenvalue -1 on the ghost vacuum, Ω .

The nontrivial BRST cohomology with nonzero momentum thus consists of states of ghost number $-1/2$ which are in one to one correspondence with the physical states of the dual resonance model. The ghost number $-3/2$ cohomology is one dimensional and consists of the state which is invariant under the $SL(2)$ subgroup of the conformal group.

This ends our brief survey of string Feynman rules. In the next chapter we will exploit the technology that we have described here to construct a gauge invariant, free string field theory.

3. FIELD THEORY OF OPEN BOSONIC STRINGS

The basic equation of the first quantized string theory that we described in the previous section is the BRST equation

$$QA = 0 . \tag{16}$$

The program of string field theory is to derive this equation as the equation of motion of a functional action $S[A]$. Since the solutions of (16) are determined only up to a BRST exact form, $\delta A = Q\epsilon$, the action we will write down has a natural gauge invariance. One may wonder why a similar gauge invariance does not arise in the transition from the relativistic particle action to the Klein-Gordon equation. In fact it does,^[14] but the gauge transformations are trivial. The usual scalar field is a gauge invariant field strength, and since it has a local action, there is no need to keep the gauge structure.

The equation $Q=0$ does not completely determine the class of allowable string states. Physical states of the string are BRST invariant equivalence classes with ghost number $-1/2$. Generically they take the form

$$\Psi \otimes \Omega \quad L_n \Psi = (L_0 - 1)\Psi = 0; \quad n > 0 \tag{17}$$

though there are still some states of the form $Q\epsilon$ within this class. An action of the form $\langle A|Q|A \rangle$ will obviously have $QA = 0$ as its equation of motion. Furthermore, this action will be nonzero only if the ghost number of A is $-1/2$. Thus we make the natural assumption that: *The classical fields of string field theory are the states of the first quantized string with ghost number $-1/2$.* The action is then

$$\langle A|Q|A \rangle \tag{18}$$

which is nonvanishing.

As promised, this action has the gauge invariance $\delta A = Q\epsilon$, where ϵ is a general state with ghost number $-3/2$. To understand the content of this invariance we expand A , ϵ , and the action as:

$$\begin{aligned}
A &= \int d^d x \{ \phi(x) - i\alpha_{-1}^\mu A_\mu(x) + b_{-1}c_0\xi(x) + \dots \} |\Omega, x\rangle \\
\epsilon &= \int d^d x \{ b_{-1}\Lambda(x) |\Omega, x\rangle + \dots \}; \quad \hat{x}^\mu |\Omega, x\rangle = x^\mu |\Omega, x\rangle; \quad p_\mu = \frac{1}{i} \frac{\partial}{\partial x^\mu} \\
Q &= c^0 \left(\frac{1}{2} p^2 - 1 + \sum_{n>0} [\alpha_{-n}^\mu \alpha_n^\mu + n(c_{-n}b_n + b_{-n}c_n)] \right) \\
&\quad + c_1(p_\mu \alpha_{-1}^\mu + \frac{1}{2} c_{-1}b_0 + \dots) + \dots
\end{aligned} \tag{3.4}a$$

$$\begin{aligned}
\Delta A &= Q\epsilon \\
\Delta \phi(x) &= 0 \\
\Delta A_\mu(x) &= \partial_\mu \Lambda(x) \\
\Delta \xi(x) &= \frac{1}{2} p^2 \Lambda(x)
\end{aligned} \tag{3.4}b$$

$$\begin{aligned}
S &= \frac{1}{2} \int \phi(p^2 - 2)\phi + \frac{1}{2} \int A^\mu p^2 A_\mu + \frac{1}{2} \int \xi^2 + i \int \xi p^\mu A_\mu \\
&\rightarrow \frac{1}{2} \int \phi(p^2 - 2)\phi - \frac{1}{4} \int (\partial_\mu A_\nu - \partial_\nu A_\mu)^2
\end{aligned} \tag{19}c$$

Note that the scalar field ϕ is a tachyon and is gauge invariant (remember our comment about the relativistic particle and the Klein-Gordon equation). The string gauge invariance reduces to ordinary Maxwell gauge invariance for the massless vector field. At higher levels we find massive spinning fields described by Stuecklebergian^[15] gauge invariant Lagrangians. Siegel and Zweibach^[16] have conjectured that these Lagrangians can all be obtained by dimensionally reducing massless spinning Lagrangians in one higher dimension. They have carried out the expansion of the string action through particles of spin five. These explicit computations depend of course on the detailed structure of the Lagrangian describing single string propagation in flat space time. It should be emphasized that nothing else that we will say in these lectures depends on these details.

Rather, it involves only the structure of the Virasoro algebra and its superconformal generalizations. Thus most of our discussion is applicable to the description of small string fluctuations about an arbitrary background spacetime whose associated sigma model is conformally invariant. As mentioned in the previous section, these are the classical solutions of string theory.^[7]

It is important to point out that the classical gauge invariance of the string action depended on the fact that the BRST charge is nilpotent, which is only true in 26 dimensions. Classical string theory is not a gauge invariant system in any other dimension. There are probably further violations of string gauge invariance in string loop amplitudes. Eliminating them further restricts the structure of the theory. (There is a good argument, which I will not reproduce here, that all such anomalies vanish if the integrands of the loop amplitudes are invariant under modular transformations)

We now have a gauge invariant action which obviously reproduces the correct kinematics for the low lying levels of the string. In order to see that it gives a complete description of the string spectrum we must somehow fix the gauge and show that the quantum mechanics of our system is equivalent to the light cone functional field theory of strings invented by Kaku and Kikkawa.^[17] In order to show this we will first gauge fix our action to the covariant gauge discovered by Siegel.^[18] Then we will use an argument of Parisi and Sourlas^[19] to show that Siegel's formalism is equivalent to that of Kaku and Kikkawa. A direct derivation of the descent to the light cone gauge can be found in Ref. 20.

The gauge fixing condition for Siegel's gauge is simply $b_0 A = 0$. It is easy to see that it is always possible to choose the gauge parameter ϵ so that this equation is satisfied. Just expand the BRST operator in powers of the ghost zero modes:

$$Q = c_0 K + d + \delta - 2b_0 \Downarrow . \quad (20)$$

Explicit expressions for the coefficients of c_0 and b_0 are:

$$\begin{aligned} K &= \frac{1}{2} p^2 + \sum_n : \alpha_n^\mu \alpha_{-n}^\mu : + \sum_n : n b_n c_{-n} : - 1 \\ \Downarrow &= \sum_n : n c_n c_{-n} : . \end{aligned} \quad (21)$$

Nilpotency of Q implies:

$$\left[\begin{array}{c} d \\ K, \delta \\ \Downarrow \end{array} \right] = 0 \quad \left[\begin{array}{c} d \\ \Downarrow, \delta \end{array} \right] = 0 \quad d\delta + \delta d = 2K \Downarrow \quad d^2 = \delta^2 = 0 . \quad (22)$$

We can also expand the state A and the gauge parameter ϵ as:

$$\begin{aligned} A &= \Phi + c_0 \chi \\ \epsilon &= \epsilon_0 + c_0 \epsilon_1 . \end{aligned} \tag{23}$$

The gauge transformations are then:

$$\begin{aligned} \Delta \Phi &= (d + \delta) \epsilon_0 - 2 \downarrow \epsilon_1 \\ \Delta \chi &= K \epsilon_0 - (d + \delta) \epsilon_1 . \end{aligned} \tag{24}$$

It is clear that as long as the operator K is invertible we can use ϵ_0 to eliminate χ . This is similar to the elimination of the longitudinal components of the vector potential in Landau gauge electrodynamics. There is no more off shell gauge invariance. Transformations which preserve the condition $b_0 A = 0$ do not effect the Φ component, except for states with $K = 0$.

The Faddeev-Popov determinant for this gauge fixing condition is the determinant of the operator $b_0 Q$, which is then the operator which appears in the Faddeev Popov ghost action. Since $Q^2 = 0$, this operator is singular and the ghost action has its own gauge invariance $\Delta G = Q E$, with E an arbitrary state of ghost number $-5/2$. There is no similar gauge invariance for the antighost field. The operator b_0 projects out the piece of the antighost field (which is a state of ghost number $1/2$) that does not contain the operator c^0 , but since this projection is completely nondynamical, we do not have to include a Faddeev-Popov determinant to compensate for it. The ghost system thus bears some resemblance to that encountered in the theory of p -form gauge fields^[21] (there are ghosts for ghosts) but is somewhat simpler (there are no hidden ghosts^[22] and the ghost counting is trivial). Baulieu and Ouvry^[23] have shown how to introduce extra auxiliary fields to make the analogy with p form gauge fields exact. It is not clear that this is useful for studying the interacting theory.

It is clear that we can gauge fix the ghost system with the same gauge condition that we employed for the classical fields. The ghosts of ghosts will be bosons, their action is gauge invariant under a transformation identical to that of the classical field and the ghost (except that the gauge parameter has ghost number $-7/2$) and the procedure obviously continues indefinitely. The upshot of all of this is that our gauge fixed system consists of *all* states of the Hilbert space of string fields and ghosts which are annihilated by the antighost zero mode. The only piece of the BRST charge which has non-zero matrix elements between such states is the term proportional to K . Our gauge fixed action thus has the form

$$\langle \Phi | c_0 K | \Phi \rangle ; \quad b_0 | \Phi \rangle = 0 \tag{25}$$

which is the action originally proposed by Siegel.

We next want to show that Siegel's action is equivalent to the light cone gauge action of Kaku and Kikkawa. It is important to remember that the expansion coefficients of odd numbers of two dimensional ghost fields in the functional field $\Phi(x, b, c)$ are Grassmann variables since they are odd order ghosts in the spacetime sense. This rule can be easily remembered, for it is simply the requirement that the functional field have fixed statistics. This is a remarkable connection between spacetime and world sheet statistics. We will see more of it when we deal with the superstring.

Let us assume that we can choose the conformal field theory which defines our "matter" representation of the Virasoro algebra so that it is a free field theory in two space time dimensions (the two light cone directions) plus an operator K_\perp which only refers to the transverse dimensions. If we think of L_0 as the Hamiltonian of a nonlinear sigma model, we are taking the background spacetime metric to be in light cone gauge. The string field theory action now takes the form

$$\left\langle \Phi \left| c_0 \left[\frac{1}{2} (p_+ p_-) + L_0^{gh} - 1 + : \sum_n \alpha_n^+ \alpha_{-n}^- : + K_\perp \right] \right| \Phi \right\rangle . \quad (26)$$

Since

$$[\alpha_n^+, \alpha_m^-] = n \delta(n + m) \quad (27)$$

we can write this as: ($\alpha_n \equiv \alpha_n^-$)

$$\left\langle \Phi \left| c_0 \left[\frac{1}{2} (p_+ p_-) + (K_\perp) + \sum_n n \left(\alpha_{-n} \frac{\partial}{\partial \alpha_{-n}} + c_{-n} \frac{\partial}{\partial c_{-n}} \right) \right] \right| \Phi \right\rangle \quad (28)$$

which has an obvious (Parisi Sourlas) supersymmetry connecting α_n and c_n . Now consider an arbitrary "String Green's Function", of functional fields which do not depend on the ghosts or the longitudinal oscillators

$$\int d\Phi e^{iS[\Phi]} \Phi(p_+^1, p_-^1, p_\perp^1, x_\perp^1(\sigma)) \cdots \Phi(p_+^k, p_-^k, p_\perp^k, x_\perp^k(\sigma)) . \quad (29)$$

Clearly, this will have the same value as the Kaku-Kikkawa Green's function:

$$\int d\Phi e^{iS_{KK}[\Phi]} \Phi(p_+^1, p_-^1, p_\perp^1, x_\perp^1(\sigma)) \cdots \Phi(p_+^k, p_-^k, p_\perp^k, x_\perp^k(\sigma))$$

$$S_{KK}[\Phi] = \left\langle \Phi \left| \frac{1}{2} (p_+ p_-) + (K_\perp - 1) \right| \Phi \right\rangle \quad (30)$$

if

$$\det(K_{\perp} + p^2) = s \det \left(K_{\perp} + \sum_n n \left(\alpha_n \frac{\partial}{\partial \alpha_n} + c_n \frac{\partial}{\partial c_n} \right) + p^2 \right). \quad (31)$$

Note that integration over the Siegel gauge functional fields gives a superdeterminant because the coefficients of odd numbers of ghost coordinates have Fermi statistics. The superdeterminant is given by:

$$\begin{aligned} & s \det \left(K_{\perp} + \sum_n n \left(\alpha_n \frac{\partial}{\partial \alpha_n} + c_n \frac{\partial}{\partial c_n} \right) + p^2 \right) \\ &= \int_0^{\infty} \frac{dt}{t} \operatorname{tr} e^{-t(K_{\perp} + p^2)} \prod_n \left[\operatorname{tr} e^{-tn\alpha\partial_{\alpha}} \operatorname{str} e^{-tnc\partial_c} \right] \\ &= \int_0^{\infty} \frac{dt}{t} \operatorname{tr} e^{-t(K_{\perp} + p^2)} \prod_n \left[\frac{1}{1 - e^{-nt}} (1 - e^{-nt}) \right] \\ &= \det(K_{\perp} + p^2) \end{aligned} \quad (32)$$

and the equivalence with the Kaku Kikkawa formalism is proven.

So there we have it. Free string theory is a gauge invariant functional field theory and the appearance of gauge bosons and gravitons is no longer a mystery. As in any gauge theory, it is interesting to enquire about the gauge invariant observables of the theory. This is particularly true for string theories, where at present the only sensible quantity one can compute is the S -matrix - an unacceptable situation. Indeed, most of the massive states of the string model are unstable, and their S -matrix is meaningless (they have widths of order the Planck mass). Among the massless states, only neutral ones like the graviton have an S -matrix which is free of infrared divergences. In Yang-Mills theory, the S -matrix of physical states is derived from gauge invariant Green's functions. Are there analogs of such Green's functions in string theory?

The equation of motion $QA = 0$ is a statement of the vanishing of the field strength of our string one form. Let us try to construct a general gauge invariant linear functional of A . It has the form $\langle G|A \rangle$ and is non-zero only if G has ghost number $1/2$. It is gauge invariant only if G is BRST invariant. If G is BRST exact, then $\langle G|A \rangle$ vanishes when A satisfies the equations of motion. On the other hand we have seen that the only BRST invariant quantities which are not exact are on shell states which satisfy $K|G \rangle = 0$. This would seem to lead to the disappointing conclusion that the only gauge invariant quantities in

string theory are on shell. Before despairing however, let us compute the two point functions of $\langle G|A \rangle$ for some simple G 's. We will compute in the Siegel gauge and so we might as well take G 's which are proportional to c_0 . BRST invariance then implies

$$(d + \delta)G = \downarrow G = 0 \quad (33)$$

Simple solutions of this are:

$$G = c_0 \int d^d x J(x) |\Omega, x\rangle$$

$$G = \int d^d x J_\mu(x) c_0 \alpha_{-1}^\mu |\Omega, x\rangle ; \partial^\mu J_\mu = 0 . \quad (34)$$

The two point function of $\langle G|A \rangle$ computed in the Siegel gauge is $\langle G | \frac{b_0}{K} | G \rangle$. For the first example, this just gives the two point function of the scalar tachyon field while for the second (if we write $J^\mu = \epsilon^{\mu\nu\lambda\kappa} \partial_\nu M_{\lambda\kappa}$) we get the two point function of the Maxwell field strength tensor. Thus our argument that gauge invariant off shell quantities vanish was fallacious. It is analogous to the claim that the off shell Green's function of a scalar field vanishes because any off shell source can be written as the Klein-Gordon operator acting on some other function. It will be a major challenge to generalize these gauge invariant quantities to the interacting theory that we will discuss in the last lecture.

4. OPEN SUPERSTRINGS

In the previous section we constructed an elegant gauge invariant action for bosonic open string theory. The basic objects from which the action was built were differential forms on the Virasoro group, otherwise known as Faddeev-Popov ghosts for reparametrizations. In order to repeat our performance for the superstring we will have to understand the corresponding ghosts for super-reparametrizations. Lack of time (and its conjugate variable, energy) force me to forgo a detailed discussion of the reparametrization invariant Lagrangian for the first quantized spinning string, its gauge fixing in the superconformal gauge and the derivation of the corresponding BRST formalism. I will refer you to the excellent discussions of these points in the existing literature^[24] Thus, as in the case of the bosonic string, I will begin by writing down the algebra of residual gauge transformations in the superconformal gauge:

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{d}{8} (n^3 - n) \delta(m + n)$$

$$[L_n, F_{\dot{m}}] = \left(\frac{1}{2} n - \dot{m}\right) F_{\dot{m}+n} \quad (35)$$

$$[F_{\dot{n}}, F_{\dot{m}}] = 2L_{\dot{m}+\dot{n}} + \frac{d}{2} (\dot{n}^2 - \frac{1}{4} \delta(\dot{m} + \dot{n})) .$$

The dotted indices take on integer values in the so called Ramond sector, and half integer values in the Neveu-Schwarz sector. These sectors arise because the fermion fields ψ^μ of the spinning string can satisfy two different sorts of boundary conditions on the world sheet. Both sectors are needed in order to construct a unitary string theory. The superconformal generators $F_{\dot{n}}$ (in the Neveu Schwarz sector they are usually denoted G) are built out of odd numbers of fermion fields and so carry integer or half integer values of L_0 depending on the moding of the fermions. The ground state energy (L_0) is zero in the Ramond sector and $-1/2$ in the Neveu-Schwarz sector, when we take the ghosts into account.

We introduce the usual ghosts b_n and c_n for the Virasoro generators and corresponding superghosts $\gamma_{\dot{n}}$ and $\beta_{\dot{n}} = -\frac{\partial}{\partial \gamma_{-\dot{n}}}$. The ghost fields are $\gamma(z) = \sum z^{\frac{1}{2}-\dot{n}} \gamma_{\dot{n}}$ and $\beta(z) = \sum z^{-\frac{3}{2}-\dot{n}} \beta_{\dot{n}}$. Since we are dealing with a superalgebra, the superghosts are commuting variables. The commutation relations *between* ordinary and superghosts are a matter of convention which can be changed by a Klein transformation. In ^[25] we chose them to anticommute so that they mimic the properties of superdifferential forms. However, it appears to be more convenient to follow the main body of literature on the subject and have superghosts and ghosts commute like ordinary fermi and bose fields.

There are many inequivalent representations of the superghost commutation relations. This is true for all Bose systems, but usually a positive definite Hamiltonian picks a unique representation based on its ground state. The superghost Lagrangian is however first order and no such principle applies. Indeed, the inequivalent representations are essential for understanding the covariant fermion vertex operator and supersymmetry. ^[26] We define the q th Bose sea vacuum by

$$\begin{aligned} \beta_n |q\rangle &= 0 & n > -q - 3/2 \\ \gamma_n |q\rangle &= 0 & n \geq q + 3/2 \end{aligned} \quad (36)$$

q is integer in the Neveu-Schwarz sector and half integer in the Ramond sector. If we represent the states of the superghost system as functions of the γ_n , the Bose sea vacua are given by:

$$|q\rangle = \prod_{n=q+3/2}^{\infty} \delta(\gamma_n) . \quad (37)$$

It is easy to see from this representation that we cannot transform one Bose sea vacuum into another by acting with polynomials in β and γ . In a moment we will construct operators that do this job.

We will define the scalar product in the superghost state space in such a way that $\beta_{\dot{n}} = -\beta_{-\dot{n}}^\dagger$ and $\gamma_{\dot{n}} = \gamma_{-\dot{n}}^\dagger$. Using these hermiticity properties and the commutation relations, it is easy to show that

$$\langle p|q\rangle = \delta(p+q+2) . \quad (38)$$

All other scalar products can be obtained from this one by using the commutation relations. Finally, we define the superghost number by

$$g_F = \sum_n [\beta_{\dot{n}}, \gamma_{-\dot{n}}]_+ + \frac{1}{2} [\beta_0, \gamma_0]_+ . \quad (39)$$

Note that it is an antihermitian operator.

Superghost number is the integral of a conserved world sheet current

$$g_F = \int : \beta \gamma : \equiv \int j(z) . \quad (40)$$

We would like to use this current to obtain another representation of the superghost Hilbert space analogous to the bosonization of fermions.^[27] Thus we introduce a scalar field $\phi(z) = \int^z j(z')$. The operator product expansions of the superghost energy momentum tensor with itself and the superghost number current are:

$$\begin{aligned} T(z)j(w) &\sim \frac{-2}{(z-w)^3} + \frac{j(z)}{(z-w)^3} \\ T(z)T(w) &\sim \frac{11}{2(z-w)^4} + \frac{2}{(z-w)^2} T(w) + \frac{1}{z-w} \partial_w T(w) . \end{aligned} \quad (41)$$

To reproduce the first of these we must write

$$T(z) = : \frac{1}{2} j^2(z) + \partial_z j(z) : \quad (42)$$

but then the c-number in the second OPE comes out 13 instead of 11. This indicates that ‘‘bosonization’’ does not work in a naive manner. Another indication of this comes when we try to write the β and γ fields as exponentials of ϕ . The exponentials are fermions rather than bosons.

In ref. 26, both of these problems are solved by introducing a free fermion system whose c number anomaly is -2 . The fields are called ξ and η and they have dimensions 0 and 1 respectively. The energy momentum tensor is

$$T_{\xi\eta} = -\eta\partial_z\xi \quad (43)$$

and

$$[\eta_m, \xi_n]_+ = \delta(m+n) . \quad (44)$$

Fields with the properties of β and γ are constructed as

$$\begin{aligned} \gamma &= e^\phi\eta \\ \beta &= e^{-\phi}\partial_z\xi . \end{aligned} \quad (45)$$

The conserved $\xi - \eta$ charge is

$$g_{\xi\eta} = \sum : \xi_n\eta_{-n} : + \frac{1}{2} [\xi^0, \eta^0] . \quad (46)$$

It takes the value $-1/2$ on the vacuum state defined by

$$\begin{aligned} \eta_n |\Omega\rangle &= 0 \quad n \geq 0 \\ \xi_n |\Omega\rangle &= 0 \quad n > 0 . \end{aligned} \quad (47)$$

The q th Bose sea vacuum is related to this state by

$$|q\rangle = e^{q\phi(0)} |\Omega\rangle . \quad (48)$$

Finally, by combining the superghost number g_F with the $\xi - \eta$ charge we get a quantity $B = g_F + g_{\xi\eta}$ which commutes with β and γ . B is thus the label for the inequivalent Bose sea Hilbert spaces: the q th sea has $B = q + \frac{1}{2}$.

The BRST charge for the spinning string is constructed in the same way as that of the bosonic string:

$$Q = \sum \left[c^{-n} \left(L_n + \frac{1}{2} L_n^{gh} \right) + \gamma^{-n} \left(F_n + \frac{1}{2} F_n^{gh} \right) \right] . \quad (49)$$

$Q^2 = 0$ if the spacetime dimension is 10.

We would now like to build a spinning string field theory action in terms of the BRST charge. We must first choose the quantum numbers of physical states, the ghost number and Bose sea charge. In the bosonic case there was a unique non-trivial cohomology of the BRST operator with ghost number $-1/2$. Here this is no longer true because of the occurrence of Bose seas. We will make the conventional choice of Neveu Schwarz sector as the $q = -1$ Bose sea.

The vacuum state in this sector has total ghost number (ghost + superghost number) $-1/2$ and Bose sea charge $-1/2$ as well. We will restrict all classical Neveu-Schwartz string fields to have these quantum numbers. We do not make a separate restriction on ghost and super ghost number because the BRST charge does not have a homogeneous transformation law under these symmetries.

It is now clear that an expression like $\langle A|Q|A\rangle$ must vanish. It has zero ghost number (Q has ghost number 1) but Bose sea charge -1 . Clearly we must find an operator with ghost number zero and Bose sea charge 1 to insert in the action. To see what is going on let us examine the zero mode subspace of the $\xi - \eta$ Hilbert space. It is isomorphic to the zero mode subspace of the bosonic ghosts. η_0 annihilates the vacuum, the vacuum expectation value of 1 is 0, and the vacuum expectation value of ξ_0 is 1. Clearly, then, the expectation value of Q vanished above because Q (which is built from β and γ) contains no ξ_0 . ξ_0 is just the operator with $B = 1$ and $g = 0$ that we needed above. We thus write the action for the Neveu-Schwarz model as

$$\langle A|\xi_0 Q|A\rangle \tag{50}$$

If we restrict the A field by $\eta_0 A = 0$, this is gauge invariant under $A \rightarrow A + Q\epsilon$. To prove this we have to compute the anticommutator of Q with ξ_0 . This comes only from the terms proportional to η_0 in Q and has no term proportional to ξ_0 . Thus if we restrict A to be annihilated by η_0 (which anticommutes with Q) the anticommutator has zero expectation value and the action is gauge invariant.

Let us now expand the BRST charge in ghost zero modes, just as we did for the bosonic string:

$$Q = c_0 K + d + \delta - 2b_0 \downarrow \tag{51}$$

We have used the same names for the expansion coefficients here, not because they are the same operators that we encountered for the bosonic string, but because they have the same properties. Indeed, the hermiticity properties and commutation relations of these operators are determined by the nilpotence and hermiticity of Q and by the properties of the zero modes.

Given this decomposition of Q we can now proceed to gauge fix the Neveu Schwarz string field theory by following exactly the manipulations we used for the bosonic string. Siegel gauge fixing is thus trivially carried out. When we attempt to fix the light cone gauge, two complications arise. First, we must decompose the fermionic partners of the string coordinates into transverse and longitudinal components. This is easily done. More importantly, we must discuss the statistics of the various space time fields that appear in our action. In the case of the bosonic string we recognized a remarkable correlation between space time and world sheet statistics. Coefficients of fermionic world sheet fields obeyed

fermi statistics in space time. If we insist on retaining this principle for the spinning string we come to a remarkable conclusion: half of the fields in the model must be thrown out if we wish to preserve the spin statistics connection for space-time fields. All of the fields in the Neveu Schwarz model carry integer spin. Consider however, the classical sector with $Q = B = -1/2$. Since the ψ^μ are anticommuting, half of these fields will be fermions, and should be omitted from the action. Which half are fermions depends on the overall statistics we choose for the string field A . The quadratic form $\langle A|\xi_0 Q|A\rangle$ is antisymmetric since $(\xi_0 Q)^\dagger = Q\xi_0 = -Q\xi_0 + \{Q, \xi_0\}$ and the anticommutator has zero expectation value. Thus A must be an anticommuting variable in order to get non-zero action. Schwinger^[28] invented a similar argument for Majorana spinors many years ago. If A is a fermion, then we must throw away all coefficients of even powers of ψ^μ if we do not wish to violate the spin statistics theorem. This is precisely the GSO projection!

One may object that such a classical argument should not be able to discriminate between bosons and fermions. Indeed, Schwinger's argument can be circumvented at the classical level by introducing extra degrees of freedom.^[28] A quantum mechanical calculation is necessary to show that this leads to a violation of unitarity. It is not at all clear how the present argument is related to those based on modular invariance^[29] or world sheet locality^[30] which also show the necessity of the GSO projection for the spinning string. Note that we have not shown the necessity of including the Ramond sector in the action. We only show that the NS sector must be projected (we will show the same thing for the Ramond sector below). This ends our discussion of the Neveu-Schwarz sector of the spinning string.

The Ramond sector of the spinning string has created a lot of confusion in the literature on string field theory. Many papers^[31] have been written about it, not all of which are correct. Several groups^[32] found an action which could be gauge fixed to the correct set of light cone degrees of freedom, but the connection of this action to the BRST charge remained obscure. Yamron^[33] found an action based on the BRST charge which can be partially gauge fixed to give the action of Ref. 32.^[34] Finally, an elegant form of the Ramond action which generalizes easily to the interacting case, was found by Witten.^[35] We will describe Witten's action and show how it is related to that of Ref. 32. The reader is asked to consult the literature for a discussion of how to gauge fix the action to light cone gauge. The gauge fixing is a simple generalization of what we have already done.

There are two obvious problems with any simple minded attempt to generalize what we have done to the case of Ramond superstrings. First of all the coefficient of c_0 in the BRST charge, which gives the diagonal term of the bosonic

string action, is a second order differential operator. This is appropriate for a bosonic action but not for a fermionic one. The second problem has to do with ghost number. The simplest Bose sea for the Ramond sector is $q = -1/2$. The vacuum state in this sector has vanishing ghost number and Bose sea charge. We take all classical Ramond fields to have these quantum numbers.

As in the Neveu Schwarz sector we see that the BRST charge has the wrong quantum numbers to give a nonvanishing action. Anticipating a necessary factor of ξ_0 we write

$$S = \langle \Psi | \xi_0 Y Q | \Psi \rangle \quad (52)$$

where Y must have $B = -1 = g$. We have used Witten's conventions for the space time Dirac algebra. Henceforth we drop the factor ξ_0 to simplify the notation. The action will be gauge invariant if $[Q, Y] = 0$. The authors of Refs. 26 have found a BRST invariant local operator with $g = B = -1$. They call it the "inverse" picture changing operator. Witten has shown that it can be written as

$$Y = \left[\beta_0, c \left(\frac{\pi}{2} \right) e^{-\phi(\pi/2)} \right] . \quad (53)$$

Let us now expand Q and Ψ as

$$\begin{aligned} Q &= c_0 K + J + b_0 M \\ M &= -\frac{1}{4} \gamma^{02} - 2 \downarrow \\ J &= -\frac{1}{2} \gamma_0 F + (d + \delta) - 2\beta_0 [F, \downarrow] \\ F &= F_0^{(x)} + \sum_{m>0} [m(\beta_{-m} c^m - c^{-m} \beta_m) + b_{-m} \gamma^m + \gamma^{-m} b_m] \\ \downarrow &= \sum_{m>0} [m c^{-m} c^m + \frac{1}{4} \gamma^{-m} \gamma^m] \\ \Psi &= \Psi_0 + c_0 \Psi_1 \\ Q\Psi &= c_0 (K\Psi_0 - J\Psi_1) + J\Psi_0 + M\Psi_1 . \end{aligned} \quad (54)$$

From these formula it is easy to prove that Ψ_0 can be gauge transformed to be annihilated by M and that the required gauge fixing is algebraic and does not produce a Faddeev-Popov determinant. Furthermore the Ψ_1 field equation

$$M\Psi_1 = -J\Psi_0 \quad (55)$$

is also algebraic (Ψ_1 is an auxiliary field) so we can use it to simplify the action.

The elimination of Ψ_1 and the gauge condition $M\Psi_0 = 0$ reduce the number of fields in Witten's action to the number employed in ref. 32. The gauge condition has two linearly independent functions of γ_0 in its solution. Therefore the field content is equivalent to two functions of the nonzero modes of the ghosts and superghosts. We will combine these two functions into a single state by introducing fermionic operators b and c with an algebra isomorphic to that of the reparametrization ghost zero modes. It should be emphasized that they have no conceptual connection with the ghost zero modes. The two states represent the two independent solutions of the gauge conditions.

The proof that Witten's action for these two fields reduces to the one we will present has not appeared in the literature. We use the new fermion operators b and c to define a nilpotent operator \tilde{Q}

$$\tilde{Q} = cF + d + \delta + b[F, \Downarrow]_+$$

We make the convention that b and c commute with all the other operators in the problem. d and δ are the BRST charges for the positive and negative frequency parts of the Ramond algebra. Nilpotence of \tilde{Q} then follows from the fact that F anticommutes with d and δ and that $(d + \delta)^2 = F[F, \Downarrow]_+$. Both of these are consequences of nilpotence of the full BRST charge.

From this point onwards, the algebra involved in gauge fixing the action to the light cone gauge is a straightforward generalization of that which we did in the Neveu-Schwarz sector. A notable feature of the covariant gauge fixing in this sector is that the complications usually associated with Nielsen-Kallosh ghosts for high spin fermionic gauge fields are completely bypassed. The discussion of spin and statistics parallels that in the Neveu-Schwarz sector. Since Q and Y commute, the Ψ field must still be anticommuting, but now, since the coefficient fields carry half integer spin, we must project out states with odd numbers of fermion operators. Again this coincides with the proper GSO projection. We therefore have a completely sensible picture of the free superstring in terms of gauge invariant functional field theory. Witten has extended this formalism to the interacting open superstring.

5. CLOSED STRINGS

It is easy to be fooled into thinking that the field theory of closed strings is a trivial generalization of that for open strings. The Hilbert space of the closed bosonic string is the direct product of two open string spaces (corresponding to

left and right moving modes), except for the zero mode of the x^μ field. There are two copies of the Virasoro algebra and a BRST charge $Q = Q_L + Q_R$ for the whole system. The non-trivial cohomology of the BRST charge is given by physical states of the form

$$|\Phi\rangle \otimes |\Omega\rangle$$

where $|\Omega\rangle$ is the ghost vacuum and:

$$L_n |\Phi\rangle = \bar{L}_n |\Phi\rangle = 0; \quad n > 0$$

$$(L_0 + \bar{L}_0 - 2) |\Phi\rangle = (L_0 - \bar{L}_0) |\Phi\rangle = 0$$

These states have total (left moving plus right moving) ghost number equal to -1 .

If we try to make a field theory action that reproduces these equations by using the BRST charge in the action we find that the ghost number does not add up correctly and the naive action is zero. The obvious cure is to insert an operator of ghost number one in the action, in analogy with what we did for the Neveu Schwarz superstring. This has been done for example by Lykken and Raby.^[36] However, it gives a gauge invariant action only if the string functional is subjected to the a priori constraint $(L_0 - \bar{L}_0) |A\rangle = 0$. Thus one of the Virasoro conditions must be imposed as an off shell constraint rather than an equation of motion or a gauge condition. The resulting formalism can be described in a subspace of the closed string Hilbert space in which one of the zero modes of the ghost is discarded, and the states are subjected to the above constraint. If c_0 is the remaining zero mode, the operator: (barred and unbarred quantities refer to left and right movers respectively)

$$\tilde{Q} = c_0 \left(\frac{K + \bar{K}}{2} \right) + d + \bar{d} + \delta + \bar{\delta} + b_0(\downarrow + \bar{\downarrow})$$

is nilpotent and can be used to form the gauge invariant action $\langle A | \tilde{Q} | A \rangle$. The gauge invariance of this action includes linearized general coordinate transformations. Gauge fixing proceeds in precisely the same manner as it did for the open string (the algebra is completely identical in the constrained subspace) and we obtain the Kaku-Kikkawa light cone action for closed strings. Note that in this action the field is also constrained to be annihilated by $L_0 - \bar{L}_0$.

Although one of the Virasoro conditions is being treated in a very different manner from the others, there is no apparent inconsistency in the action we have written. Indeed in the light cone gauge one can write an interacting action with fields which satisfy the constraint. This action is the basis for the only manifestly unitary calculations of string scattering amplitudes and as far as one knows^[37]

gives results in agreement with the world sheet path integral. Nonetheless I feel a bit uneasy with the present formulation of the theory and I have the feeling that we have to do a lot more to truly understand the closed string. I will present two more pieces of evidence that something is wrong.

The first has to do with superstrings. A closed spinning type II string has four sectors, corresponding to the choice of Ramond or Neveu-Schwarz boundary conditions for left movers and right movers separately. The heterotic string has two sectors, the left movers can be described by a compactified bosonic string while the right movers are either the NS or R sectors of the spinning string. When describing Ramond sectors, we will use the formalism of Ref. 25 in which all zero modes of the superghosts are discarded. By throwing away one of the bosonic ghost zero modes as well, we can in every case except the Ramond-Ramond sector, construct a modified BRST operator which is nilpotent in the subspace of states with equal left moving and right moving energy. For the NS-NS sector of the type II string or the NS sector of the heterotic string, this operator has the same form as the one we constructed for the closed bosonic string. The d 's and δ 's must be reinterpreted to stand for BRST generators of the NS algebra, but their algebra remains the same. In the NS-R sectors of type II, or the R sector of the heterotic string we write:

$$\tilde{Q} = c_0 F + (-1)^{N_F} (d + \delta + \bar{d} + \bar{\delta}) - b_0 (F \Downarrow + \Downarrow F + F \bar{\Downarrow} + \bar{\Downarrow} F) .$$

This operator is again nilpotent in the subspace of states with equal left and right moving energy.

In the Ramond-Ramond sector this analogy breaks down. It is tempting to write down a straightforward generalization of what has gone before, gluing together two Ramond open strings. A gauge invariant action can be constructed in this manner - but it has the wrong spectrum! Knowledgeable readers will be reminded of a well known difficulty with the field theory formulation of the massless level of the type IIB string. This sector contains a massless particle which can be described by a rank four antisymmetric tensor gauge field. The field equation for this gauge potential is the self duality of its (rank five) field strength. This is the ten dimensional analog of a right moving scalar field in two dimensions. No one has ever found a covariant bilinear lagrangian formulation of this system. If one simply introduces a Lagrange multiplier field, one doubles the number of degrees of freedom. This is precisely what occurs for the covariant action of the Ramond-Ramond sector described above. For both type I and type II systems it gives twice the correct number of degrees of freedom.

The only way that we have found to resolve this problem is extremely ugly. We write an action with no spacetime derivatives in its diagonal terms:

$$S = \langle B | \tilde{Q} | B \rangle \tag{56}$$

$$\tilde{Q} = c^0 + (d + \delta + \bar{d} + \bar{\delta}) - b_0(F\{F, \downarrow\} + \bar{F}\{\bar{F}, \bar{\downarrow}\}) .$$

\tilde{Q} is nilpotent in the space of states satisfying:

$$(F - \bar{F}) |\lambda\rangle = 0 \tag{57}$$

so the action is gauge invariant. In (57) F and \bar{F} are the world sheet supersymmetry generators for left and right movers. Unfortunately, this constraint is dynamical, unlike the $K = \bar{K}$ constraint we have encountered in other sectors. This is a consequence of the fact that the left moving and right moving fermions ψ^μ have independent zero modes.

In effect, a dynamical constraint is an equation of motion that does not follow from the Lagrangian. To get a sensible system which we can quantize, we must solve the constraint. This should be done without inverting any time derivatives, otherwise the resulting action will be non-local in time. The only simple way we have found to solve the constraint is to use light cone coordinates. The action can be gauge fixed to the light cone gauge by arguments exactly analogous to those already presented. The result is

$$S = \frac{1}{2} \text{tr} B^T \frac{p^2 + K_\perp}{2p^+} B \tag{58}$$

where B is an $O(8)$ bispinor. This is the correct light cone gauge Lagrangian for this sector and contains (after GSO projection) the correct spectrum of states.

This treatment of the Ramond-Ramond sector of the type II superstrings is a major disaster. It is not a manifestly covariant formulation of the theory. One gets the feeling (as one already did for the closed bosonic string) that we are leaving out some gauge invariance but the precise nature of the missing ingredients is unclear.

A further indication that something is wrong comes from the calculation of the one loop ‘‘cosmological constant’’ in string field theory. We will describe this computation below and find that our computation disagrees with that done by Polchinski in the world sheet path integral formalism^[38] One should emphasize that the ‘‘cosmological constant’’, defined as the logarithm of the string field theory vacuum amplitude is not necessarily a physical object in string theory.

At present the only fully gauge invariant objects that we know are scattering amplitudes. In light cone string field theory, one can calculate one loop scattering amplitudes which “contain” the cosmological constant (as the coefficient of the dilaton tadpole) and find an answer which agrees with Polchinski’s. However when one calculates the *vacuum amplitude* in the light cone field theory one gets the result we will present below. Thus it is not clear whether we should worry about the disagreement.

In any free field theory, the logarithm of the vacuum amplitude is the trace of the log of the kinetic operator K . For open strings this gives:

$$sTr \ln K = \int_0^\infty \frac{dt}{t} sTr \exp(-tK) = \int_0^\infty \frac{dt}{t} Tr \exp(-tK_\perp)$$

In the second equality, K_\perp is the Kaku-Kikkawa light cone kinetic energy which we obtained by the Parisi-Sourlas trick. This expression coincides with that derived from the world sheet formalism. t represents the modulus (ratio of circumference to length) of the world cylinder, and it is integrated over the correct domain.

An analogous calculation for closed strings gives

$$\int_0^\infty \int_0^{2\pi} \frac{dt}{t} d\theta Tr \exp(-t(K_\perp + \bar{K}_\perp) + i\theta(K_\perp - \bar{K}_\perp))$$

The θ integral imposes the constraint that left moving and right moving energies match. It is easy to see that this result is simply the sum of the one loop vacuum energies of all of the physical particle states contained in the closed string. Polchinski has shown that this is the same, up to an infinite factor, as the vacuum bubble given by the world sheet path integral. The (t, θ) integration region is a strip of width 2π in the upper half plane. Polchinski shows that the integrand is invariant under modular transformations of the integration region. Thus, our result is the integral over a fundamental region of the modular group (which is the world sheet result) times the infinite order of that group. I have been unable to motivate dropping this infinite factor on the basis of the rules for the closed string field theory that we have formulated. It cannot be a consequence of some additional discrete gauge invariance of the string field which is not fixed by Siegel’s condition. This would lead to an infinite factor multiplying the vacuum amplitude, while we have an infinite factor in the log of the vacuum amplitude. I do not know if this annoying discrepancy is a red herring or a profound clue to the construction of a correct closed string field theory.

6. INTERACTING STRINGS

Several proposals have appeared in the literature for generalizing the gauge invariant free string actions that we have discussed to include interactions. Peter West has described the Siegel-CERN- Kyoto^[39] approach to this problem in his lectures at this school. I will describe the elegant approach invented by E. Witten. A gauge invariant string field theory action should satisfy at least three criteria. It should generalize non-abelian gauge invariance (and for closed strings, general coordinate invariance) in an interesting way, and provide an explanation for the ubiquitous occurrence of gauge bosons and gravitons in string theory. It should provide a *simple* derivation of the elegant dual Feynman rules, and it should be simple enough to allow us to understand non-perturbative effects in string theory. Witten's action satisfies the first criterion, at least for open string theories. It does not appear to satisfy the second. The third, which is obviously the most important, may or may not be satisfied. It is too early to tell.

The starting point of Witten's construction is the idea^[40] that string fields are differential forms on the Virasoro group. More precisely, they are semi infinite differential forms. In physicists language, an ordinary differential form on the Virasoro group would be a functional of the ghost field $c(z)$. Alternatively, one could describe it as a state built on the empty Dirac sea of the ghosts. However, it is clear that in order to have a sensible action of the Virasoro algebra on the forms (in particular to have finite eigenvalues of L_0) we must fill the Dirac sea. Thus in effect, all forms with finite L_0 have infinite rank. As usual it is convenient to introduce a new concept of rank that measures the difference of the rank of a form from the rank of the "vacuum" form (the full Dirac sea). This is essentially the ghost number. In this language, the BRST charge Q is the exterior derivative on forms.

Our free action for string forms was a bilinear in the form A and its field strength or exterior derivative QA . Readers with the right kind of background will be immediately reminded of a Chern Simons form. Witten's formalism is a fleshing out of this analogy. Firstly, we clearly want to call A a one form. This means that the rank of a form is given by its ghost number plus $3/2$. The gauge parameter is then a zero form. In order to construct a topological invariant such as the Chern-Simons form, we have to find analogs of the wedge product and the integral of a differential form. Let us call the analog of the wedge product of two differential forms A and B

$$A * B .$$

It should take forms of rank n and m and multiply them together to give a form of rank $n + m$. Thus $A * B$ should have ghost number equal to $g_A + g_B - 3/2$. The exterior derivative Q should be a graded derivation of wedge multiplication:

$$Q(A * B) = QA * B + (-)^A A * QB$$

In this formula $(-1)^A$ is minus one if A is a form of odd rank and plus one if the rank of A is even.

The standard wedge product for forms turns the space of forms into a Grassman algebra. However, from Yang-Mills theory, we are familiar with forms that take values in matrix algebras, for which the wedge product is an associative but not anticommutative operation. Thus we should only require that $*$ be associative. Given a wedge product we are almost ready to define most of the usual operations of exterior calculus. What is lacking as yet is the notion of the integral of a differential form. This is a linear c -number valued function on forms which satisfies:

$$\int A * B = (-1)^{AB} \int B * A \quad \int QA = 0 .$$

Finally, we should require that $*$ and \int are such that the abelian Chern-Simons invariant $\int A * QA$ is the free string action $\langle A|Q|A \rangle$ that we constructed above.

If we can define such a product and integral then we can repeat many of the usual constructions of Yang-Mills theory. Define gauge transformations of a 1 form A by:

$$\delta A = Q\epsilon + A * \epsilon - \epsilon * A$$

Then the field strength:

$$F = QA + A * A$$

transforms covariantly:

$$\delta F = F * \epsilon - \epsilon * F$$

and we can make various gauge invariant objects by integrating products of F and A to make Chern-Simons invariants. Usually one can only form a single Chern-Simons invariant for a given dimension. The same will be true here if we insist that the integral operation carries a fixed ghost number. In particular, if we want the integral of $A * QA$ to be nonzero, the integral must vanish unless its integrand carries ghost number $3/2$. With this restriction we are led to a unique gauge invariant action:

$$S(A) = \int A * QA + 2/3 A * A * A$$

Note that it is automatically trilinear, as expected for a string theory. The field equation which follows from $S(A)$ is $F = 0$. It is thus an integrability condition, and raises the exciting possibility that string theory is a completely integrable system. Witten^[41] and Friedan and Shenker^[42] have suggested this in the context of rather different considerations.

The introduction of a product and integral in the space of string forms thus offers us the possibility of building an elegant and unique interacting string action. Witten has found explicit definitions of \int and $*$ which obey the rules that we have outlined above.^[43] He has argued that it gives the familiar three point couplings of the Veneziano model. Giddings^[44] has computed the four point amplitude from this formalism. We do not have time here to go into the details of the construction, but will content ourselves with making some general comments about invariance properties and uniqueness of Witten's definitions. (Recently Giddings, Martinec and Witten^[45] have shown that the Feynman rules of this theory generate the world sheet form of the dual amplitudes to all orders in the loop expansion.)

The dual Feynman rules are invariant under reparametrizations of the string world sheet. In actual computations one must choose a gauge, but even the gauge fixed Feynman rules are invariant under conformal transformations. One might have expected the string field theory action to be conformally invariant. This is indeed the case for the kinetic energy term that we have constructed, since the BRST charge commutes with all the conformal generators. However, the interaction term of Witten's action does not have full conformal invariance. I believe that this was inevitable and that any string field theory will have this problem. The world sheet diagram for an elementary open string interaction is shown in figure 1. Any description of this interaction in terms of an action for fields which are functionals of $X(\sigma)$, must make a cut along the world sheet to identify the surface on which the strings interact (assuming that the interaction is local). Witten's cut is shown in figure 2. This identification obviously violates conformal invariance.

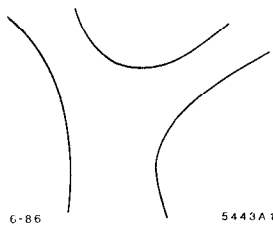


Fig. 1. The world sheet for 3-string scattering.

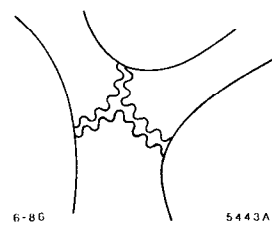


Fig. 2. The cut across the world sheet that defines Witten's interaction.

More formally, consider any action defined in terms of a $*$ product and an integral. The free action is conformally invariant by itself, so the interaction term must be as well. The only apparent way to enforce this is to insist that the conformal generators L_n be derivations of the $*$ algebra and that $\int L_n A = 0$ for all A . The integral is a linear functional on the space of string states and

thus can be represented as the scalar product with some state $|I\rangle$. I must be a state of ghost number $-3/2$, and if the action is conformally invariant it must be annihilated by all the conformal generators. In particular, it must be annihilated by L_0 . But there is only one state of ghost number $-3/2$ which is annihilated by L_0 . It is the SL_2 invariant state $|0\rangle$, described in Section I. This state is not annihilated by L_n with $n < -2$. Thus there is no state annihilated by all the conformal generators. A given definition of \int and $*$ is characterized by the subgroup of the conformal group which leaves it invariant. The integral is defined by a BRST invariant state I of ghost number $-3/2$. Thus $|I\rangle = |0\rangle + Q|B\rangle$ where $|0\rangle$ is the $SL(2)$ invariant state. I is probably completely specified by the Virasoro subalgebra which leaves it invariant. For example, if the subalgebra contains L_0 then I is the $SL(2)$ invariant state. I will now argue that with a few assumptions, one can say the same about the product.

It is natural to assume that the Hilbert space contains an identity element for the star product. We will call this I, using the same symbol as that used for the integral for reasons which will become obvious in a moment. By definition:

$$A * I = I * A = A$$

for any A. Since Q and any symmetries of the action are (possibly graded) derivations, they must annihilate the identity. It is also clear that the identity has ghost number $-3/2$. If, as we conjectured above, the subalgebra of the conformal algebra which annihilates I completely determines it, then the identity state is the state which defines the integral. This would certainly be the case if the symmetry of the action contains L_0 .

With one more assumption, we can show that the symmetry group of the action completely determines the $*$ product. Any $*$ product induces a correspondence between states and operators. The state A corresponds to the operator \hat{A} defined by

$$\hat{A}(B) = A * B.$$

We can write the state A as $\hat{A}(I)$, where I is the identity. If there were a unique operator that created A when applied to I we would be able to construct the product operation uniquely from the state I. Every state would be written as the corresponding operator acting on I, and the $*$ product would just be the ordinary operator product. Note that this definition is associative, and has the right ghost number properties. The integral is defined as above by the scalar product with I. The integral of a BRST exact form then vanishes. The two properties of Witten's formalism that are not immediately apparent in this formulation are the graded commutativity of the integral of $A*B$ and the fact that Q is a graded derivation.

Of course, the relation $A = \hat{A}(I)$ for all states does not completely define the operators \hat{A} . We can only make progress by putting further restriction on these operators. A natural restriction exists when I is the $SL(2)$ invariant state. In that case, if we require that \hat{A} be the integral of a local density, then standard results of conformal field theory ensure that there is indeed a unique connection between states and operators. It is then fairly easy to prove the remaining Witten axioms. For example:

$$\begin{aligned} Q(A * B) &= Q\hat{A}\hat{B}(I) = [Q, \hat{A}\hat{B}](I) \\ &= ([Q, \hat{A}]_{\pm}\hat{B} + (-1)^A\hat{A}[Q, \hat{B}]_{\pm})(I) = Q(A) * B + (-1)^A A * Q(B) \end{aligned}$$

In this equation we have used the fact that if \hat{A} is the local operator which creates A from the $SL(2)$ invariant vacuum then the local operator which creates $Q(A)$ is the commutator or anticommutator of Q and A , according to whether A is bosonic or fermionic. Using the operator product expansion one can also prove the graded commutativity of the integral of $A*B$.

Thus, in the case where we choose the identity state to be the $SL(2)$ invariant vacuum, there seems to be a unique definition of $*$ product and integral that satisfies Witten's axioms. The Feynman rules for this formalism involve $SL(2)$ invariant expectation values of vertex operators and seem to be rather directly related to the Koba-Nielsen formulas. (There are some troublesome points which I have not sorted out so I will not present the details here). Unfortunately, Witten's choice of identity is not the $SL(2)$ invariant state. It is a state annihilated by the set of Virasoro generators which preserve a fixed time slice and preserve the midpoint of the string in that slice. These are the generators $K_n = L_n - (-1)^n L_{-n}$. I believe that the uniqueness theorems that hold for the $SL(2)$ case are also valid here, so that Witten's formalism is completely defined by this invariance group.

We seem then to have a number of different gauge invariant actions for string field theory. What is the relation between them, and do they give equivalent results for scattering amplitudes? I can give only a partial answer to this question. Consider for definiteness, Witten's definition of the $*$ product. Let U be a conformal transformation which is not an automorphism of $*$ (e.g. e^{iaL_0}). Now define

$$A *_U B = U^{-1}(U(A) * U(B))$$

We must also define a new integral by

$$\int_U A = \int U(A)$$

It is easy to verify that this is an associative product which satisfies all of Witten's axioms. Thus, for example, we can go from Witten's product to one which is invariant under σ reparametrizations. This will define a different string field theory action. Using the fact that a conformal transformation of an on shell physical state is that same state plus a BRST exact form, it is easy to show that the S-matrices defined by these two actions are identical. Thus there is a large class of definitions of the $*$ product which give an action different from but physically equivalent to Witten's. An action based on the $SL(2)$ invariant state is not in this class. The symmetry subalgebra of Witten's formalism is not conjugate to the invariance algebra of the $SL(2)$ vacuum. Nonetheless I suspect that a manifestly SL_2 invariant formalism exists and is equivalent to Witten's.

Recently, Giddings and Martinec^[46] have described the connection between string field theory actions and triangularizations of Riemann surfaces. They also find that there should be a multitude of equivalent string field theory actions. It would be extremely interesting to relate the present discussion to theirs, for we seem to have an algebraic (according to invariance group) classification of possible actions whereas their classification is geometrical. Clearly both presentations are incomplete and much work remains to be done.

Although many of the details are cloudy, it seems clear that an elegant gauge invariant formulation of open string field theory exists. How should it be extended to the case of closed strings? We have seen already, that even free closed string field theory is not in the best of shape. A question of great importance, is whether we need to introduce separate closed string fields or whether closed strings are somehow already contained in the formalism at hand. Remember that closed string intermediate states automatically appear in the unitarization of covariant open string amplitudes.^[47] In the light cone gauge, where closed strings are needed for covariance rather than unitarity, a separate closed string field is certainly needed. However, Witten has argued that the loop diagrams of his open string field theory already contain closed string contributions. He has made a beautiful proposal which suggests why this comes about.

Let us notice that the field equations of Witten's action can be rewritten in an interesting way by using the operators \hat{A} described above. It is easy to verify that the vanishing of the field strength is equivalent to the statement that:

$$(Q + d_A)^2 = 0.$$

$$d_A(B) \equiv A * B - (-1)^{AB} B * A$$

or in other words that $Q + d_A$ is itself a BRST operator. This is a realization of the suggestion of Friedan^[48] that the equations of string field theory are the requirement that the background classical field define a two dimensional

conformal field theory with the right value of the central charge. Closed strings fit very nicely into this framework. We have emphasized repeatedly that everything in our formalism goes through for any field theory of the x^μ field which is conformally invariant. Thus different consistent background space time geometries are equivalent to different choices of Q in the above formalism. Solutions of the open string equations correspond to different BRST operators which differ only by an inner derivation of the $*$ algebra. Different consistent space time geometries (which we suspect correspond to different classical solutions of closed string field theory) correspond to BRST generators whose difference is not an inner derivation. Thus, according to Witten, we should think of a general state of the system of closed plus open strings as a general graded derivation of the $*$ algebra. The classical equation of motion of this system is just $Q^2 = 0$. In order to complete this program we must find an action from which this equation follows and a suitable parametrization of the set of all derivations. I refer the reader to Witten's paper for further discussion.

7. CONCLUSIONS

It is clear that string field theory is still in a state of flux. The beautiful results obtained by Witten have pointed the way to a full understanding of the underlying structure of strings, but much work remains to be done. I believe that a proper understanding of all possible (equivalent?) realizations of Witten's axioms is necessary in order to make a clean connection with the dual Feynman rules and Riemann surfaces. The details of Witten's outline for closed string field theory must be filled in, in particular one must find an action and a useful parametrization of the space of all derivations of the $*$ algebra. Witten has generalized his open string field theory to the superstring but the heterotic string still looks like a sport of nature. The ugly duckling has yet to be recognized as a swan. We must clearly find a better framework for describing it.

There remain many other obscure points and open problems. We must gain a much better understanding of the nature of the cosmological constant in string theory. We must find a Hamiltonian formulation of string theory analogous to the Wheeler-DeWitt formulation of quantum gravity. We must understand the "Wave Function of the Universe" and the tunneling processes between different classical vacua of string theory. We must find out whether we can attack the parts of string theory relevant to low energy physics by weak coupling and/or "effective field theory" techniques or whether there are some low energy questions which have "essentially stringy" answers. Most important of all, we must find some way to subject string theory to experimental tests, at least by calculating the parameters in an effective field theory which describes accessible physics. The developments in string field theory over the past year have made it seem that the answers to these questions may not lie asymptotically far in the future.

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