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THE KOBAYASHI-MASKAWA MIXING MATRIX*

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Review of Particle Properties – Particle Data Group

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In the "standard model" with $SU(2) \times U(1)$ as the gauge group of electroweak interactions, both the quarks and leptons are assigned to be left-handed doublets and right-handed singlets. The quark mass eigenstates are not the same as the weak eigenstates, and the matrix connecting them has become known as the Kobayashi-Maskawa[1] matrix since an explicit parametrization in the six-quark case was first published by them in 1973.

By convention, the three charge 2/3 quarks (u, c, and t) are unmixed, and all the mixing is expressed in terms of a 3×3 unitary matrix V operating on the charge $-1/3$ quarks (d, s, b):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{1}$$

The values of individual K-M matrix elements can in principle all be determined from weak decays of the relevant quarks, or, in some cases, from deep inelastic neutrino scattering. Using the constraints discussed below (in the full-sized edition only), together with unitarity, and assuming only three generations, the 90%

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confidence limits on the magnitude of the elements of the complete matrix are:

$$\begin{pmatrix} 0.9742 \text{ to } 0.9756 & 0.219 \text{ to } 0.225 & 0 \text{ to } 0.008 \\ 0.219 \text{ to } 0.225 & 0.973 \text{ to } 0.975 & 0.037 \text{ to } 0.053 \\ 0.002 \text{ to } 0.018 & 0.036 \text{ to } 0.052 & 0.9986 \text{ to } 0.9993 \end{pmatrix} . \tag{2}$$

The ranges shown are for the individual matrix elements. The constraints of unitarity connect different elements, so choosing a specific value for one element restricts the range of the others.

There are several parametrizations of the K-M matrix. The form due to Maiani[2] has a number of convenient properties:

$$V = \begin{pmatrix} c_\beta c_\theta & c_\beta s_\theta & s_\beta \\ -s_\gamma c_\theta s_\beta e^{i\delta'} - s_\theta c_\gamma & c_\gamma c_\theta - s_\gamma s_\beta s_\theta e^{i\delta'} & s_\gamma c_\beta e^{i\delta'} \\ -s_\beta c_\gamma c_\theta + s_\gamma s_\theta e^{-i\delta'} & -c_\gamma s_\beta s_\theta - s_\gamma c_\theta e^{-i\delta'} & c_\gamma c_\beta \end{pmatrix} \tag{3}$$

where $c_\beta = \cos\beta$, $s_\beta = \sin\beta$, etc. With $\beta = \gamma = 0$, the first two generations of quarks decouple from the third, and θ is directly the Cabibbo angle.

In view of the need for a "standard" parametrization in the literature, we propose this form and request public comment. [Continuation of this discussion found in full-sized edition of Review of Particle Properties only.]

Kobayashi and Maskawa[1] chose a parametrization involving the four angles, θ_1, θ_2, θ_3, δ:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{4}$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$ for $i = 1,2,3$. In the limit $\theta_2 = \theta_3 = 0$, this reduces to the usual Cabibbo mixing with θ_1 identified (up to a sign) with the Cabibbo angle. The angles θ_1, θ_2, θ_3 can all be made to lie in the first quadrant (so that all s_i, c_i are positive) by an appropriate redefinition of quark field phases.

Slightly different forms of the Kobayashi-Maskawa (K-M) parametrization are found in the literature. The K-M matrix used in the 1982 Review of Particle Properties is obtained by letting $s_1 \to -s_1$ and $\delta \to \delta + \pi$ in the matrix given above. An alternative used in another review[3] is to change Eq. (4) by $s_1 \to -s_1$ but leave δ unchanged. With this change in s_1, θ_1 becomes the usual Cabibbo angle, with the "correct" sign (i.e. $d' = d\cos\theta_1 + s\sin\theta_1$) in the limit $\theta_2 = \theta_3 = 0$. The angles θ_1, θ_2, θ_3 can, as before, all be taken to lie in the first quadrant by adjusting quark field phases. Since all these parametrizations are referred to as "the" Kobayashi-Maskawa form, some care about which one is being used is needed when the quadrant in which δ lies is under discussion.

Another parametrization, which emphasizes the relative sizes of the matrix elements by expressing them in powers of the Cabibbo angle, was introduced by Wolfenstein.[4] Still other parametrizations[5] have come into the literature in connection with attempts to define "maximal CP violation". No physics can depend on which of the above parametrizations (or any other) is used as long as it is used consistently and care is taken to be sure that no other choice of phases is in conflict.

Our present knowledge of the matrix elements comes from the following sources:

(1) Nuclear beta decay, when compared to muon decay, gives[6]

$$|V_{ud}| = 0.9729 \pm 0.0012 \ . \tag{5}$$

Recent refinements (wherein leading log radiative corrections are summed using the renormalization group and structure dependent $0(\alpha)$ terms are analyzed and estimated) have been included, thereby lowering $|V_{ud}|$ by 0.13%.

(2) Analysis of hyperon and K_{e3} decays yields[7]

$$|V_{us}| = 0.221 \pm 0.002 \ . \tag{6}$$

The isospin violation between K_{e3}^+ and K_{e3}^0 decays has been taken into account, bringing the values of $|V_{us}|$ extracted from these two decays into agreement at the 1% level of accuracy. The hyperon data alone tend to give a higher value, but theoretically they have larger possible uncertainties because of first order symmetry breaking effects in the axial-vector couplings. A simultaneous fit to both data sets shows that the difference is not statistically significant and gives the mean value given above.

(3) From neutrino and antineutrino production of charm, the CDHS group has deduced[8]

$$|V_{cd}| = 0.24 \pm 0.03 \ . \tag{7}$$

(4) Values of $|V_{cs}|$ from such experiments are dependent on assumptions about the strange quark density in the parton-sea. Using the conservative assumption that the strange-quark sea does not exceed the value corresponding to an $SU(3)$ symmetric sea, a bound on $|V_{cs}|$ results[8] which is comparable to that given below. A different source of information on $|V_{cs}|$ arises from comparing the experimental value for $\Gamma(D \to \overline{K} e^+ \nu_e)$ with the expression that follows from the standard weak interaction amplitude:

$$\Gamma(D \to \overline{K} e^+ \nu_e) = |f_+^D(0)|^2 |V_{cs}|^2 (1.54 \times 10^{11} sec^{-1}.) \ . \tag{8}$$

Here $f_+^D((p_D - p_K)^2)$ is the form factor for $D_{\ell 3}$ decay which is the analogue of $f_+((p_K - p_\pi)^2)$ for $K_{\ell 3}$ decay. With the parametrization $f_+^D(t)/f_+^D(0) =$

$M^2/(M^2-t)$ and $M = 2.1\,GeV/c^2$ from recent measurements,[9] its variation has been taken into account in deriving Eq. (8). From combining data on $BR(D^+ \to \overline{K^0}e^+\nu_e)$ and $BR(D^0 \to K^-e^+\nu_e)$ with world average values[9] of τ_{D^+} and τ_{D^0} the resulting value of the left-hand-side of Eq. (8) is $0.79 \pm 0.11 \times 10^{11} sec^{-1}$. Therefore

$$|f_+^D(0)|^2 |V_{cs}|^2 = 0.51 \pm 0.07 \; . \tag{9}$$

With sufficient confidence in a theoretical calculation of $|f_+^D(0)|$ a value of $|V_{cs}|$ follows,[10] but even with the very conservative assumption that $|f_+(0)| < 1$ it follows that

$$|V_{cs}| > 0.66 \; . \tag{10}$$

The constraint of unitarity when there are only three generations gives a much tighter bound (see below).

(5) The ratio $|V_{ub}/V_{cb}|$ is obtained from the semileptonic decay of **B** mesons by fitting to the lepton energy spectrum as a sum of contributions involving $b \to u$ and $b \to c$. The relative overall phase space factor between the two processes is calculated from the usual four-fermion interaction with one massive fermion (c quark or u quark) in the final state. The value of this factor is between 0.4 and 0.5 depending on the quark masses used. We use 0.45. The lack of observation of the higher momentum leptons characteristic of $b \to u\ell\overline{\nu}_\ell$ as compared to $b \to c\ell\overline{\nu}_\ell$ results in a limit which depends on the lepton energy spectrum assumed for each decay. As more data has accumulated the inadequacy of previously used parametrizations

has become clear.[9] Conservatively using only the lepton momentum region beyond the end-point for $b \to c\ell\overline{\nu}_\ell$ results in[9]

$$\frac{\Gamma(b \to u\ell\overline{\nu}_\ell)}{\Gamma(b \to c\ell\overline{\nu}_\ell} < 0.08 \ , \tag{11}$$

which translates to

$$|V_{ub}/V_{cb}| < 0.19 \ . \tag{12}$$

Being slightly less conservative and including the last 200 MeV/c of the $b \to c\ell\overline{\nu}_\ell$ spectrum gives a stronger limit[9]

$$\frac{\Gamma(b \to u\ell\overline{\nu}_\ell)}{\Gamma(b \to c\ell\overline{\nu}_\ell)} < 0.04 \ , \tag{13}$$

which coincides with previous limits[11] and translates to

$$|V_{ub}/V_{cb}| < 0.14 \tag{14}$$

There are some theoretical uncertainties in this analysis stemming from the fact that the physical decays involve actual hadrons and not just quarks as is assumed in the calculations of the lepton spectra for $b \to u\ell\overline{\nu}_\ell$ and $b \to c\ell\overline{\nu}_\ell$.

(6) The magnitude of V_{cb} itself can be determined if the measured semileptonic bottom hadron partial width is assumed to be that of a b quark decaying through the usual $V - A$ interaction:

$$\Gamma(b \to c\ell\overline{\nu}_\ell) = \frac{BR(b \to c\ell\overline{\nu}_\ell)}{\tau_b} = \frac{G_F^2 m_b^5}{192\pi^3} F(m_b, m_c)|V_{cb}|^2 \ , \tag{15}$$

where τ_b is the b lifetime and $F(m_b, m_c)$ is the phase space factor chosen above as 0.45.

Using an average semileptonic branching ratio measured in the continuum of[9] $12.1 \pm 0.8\%$ (which from Eq. (11) is $BR(b \to c\ell\overline{\nu}_\ell)$ to within 8%), a world average bottom hadron lifetime[9] of $1.26 \pm 0.16 \times 10^{-12}$ sec, and m_b between 4.8 and 5.2 GeV/c^2:

$$0.037 < |V_{cb}| < 0.053 \ , \tag{16}$$

where the range of m_b values used in extracting V_{cb} has been treated as a theoretical systematic error on top of the errors arising from experimental measurements.

The results for three generations of quarks, from Eqs. (5), (6), (7), (10), (12) and (16) plus unitarity, are summarized in matrix (2). The ranges given in matrix (2) are different from those given in Eqs.(5)-(16) (because of the inclusion of unitarity), but are consistent with the one standard deviation errors on the input matrix elements.

The data do not preclude there being more than three generations. Moreover, the entries deduced from unitarity might be altered when the K-M matrix is expanded to accommodate more generations. Conversely, the known entries restrict the possible values of additional elements if the matrix is expanded to account for additional generations. For example, unitarity and the known elements of the first row require that[6] any additional element in the first row have a magnitude $|V_{ub'}| < 0.088$. When there are more than three generations the allowed ranges (at 90% C.L.) of the matrix elements connecting the first three generations are

$$\begin{pmatrix} 0.9710 \text{ to } 0.9748 & 0.218 \text{ to } 0.224 & 0 \text{ to } 0.01 & \cdots \\ 0.192 \text{ to } 0.288 & 0.66 \text{ to } 0.98 & 0.037 \text{ to } 0.053 & \cdots \\ 0 \text{ to } 0.14 & 0 \text{ to } 0.72 & 0 \text{ to } 0.999 & \cdots \\ \vdots & \vdots & \vdots & \end{pmatrix}$$

where we have used unitarity (for the expanded matrix) and Eqs. (5), (6), (7), (10), (12) and (16).

Further information on the angles requires theoretical assumptions. In particular, as CP-violating amplitudes involve $\sin\delta$, assuming that observed CP violation is solely related to a nonzero value of δ allows additional constraints to be brought to bear. While hadronic matrix elements whose values are imprecisely known now enter, the constraints from CP violation in the neutral kaon system are tight enough that there may be no solution at all for certain quark masses, values of δ, etc. See the reviews in Ref. 12.

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